Chapter 1

Introduction

This thesis investigates the problem of constructing tensegrity frameworks and the related applications in formation control in the context of multi-agent systems. We will concentrate on growing tensegrity frameworks in several scenarios, including merging rigid/infinitesimally rigid/super stable tensegrity frameworks, and Henneberg constructions on super stable tensegrity frameworks. In order to make good use of the desirable features of tensegrity frameworks, such as flexible scalability and robustness, a control scheme based on virtual tensegrity frameworks is proposed to manipulate formations for carrying out different tasks. Before proceeding to the specific problems, I will briefly introduce the background, motivations, and structure of this thesis.

1.1 Background

In this chapter, the basic knowledge of tensegrity frameworks and formation control is provided. The detailed literature review will be provided at the beginning of each main chapter for specific problems.

1.1.1 Tensegrity frameworks

The English word “Tensegrity” was coined by Buckminster Fuller in the late 1950s by combining the words “tension” and “integrity” [46, 106]. This conjunction also literally implies that this class of structures is integrated by the inner tension in various geometric forms. In fact, one decade before the word tensegrity structures were named, a contemporary sculptor, Kenneth Snelson, had constructed the “X-Piece”, shown in Fig. 1.1, which has been regarded as the first widely known piece of the tensegrity structure. It is made of two plywood X’s placed in an interlaced manner and one stands over the other linked by nylon lines in tension. Apart from X-Piece, Snelson has also made a series of sculptures in the following decades, like “Needle Tower”, created in 1968, now exhibiting in Hirshhorn Museum and Sculpture Garden, Washington and “Sleeping Dragon” created in 2002-03, now “sleeping” in Kirkpatrick Oil Company Building, Oklahoma City.
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Even though a variety of tensegrity structures has been built by researchers, engineers, artists, and sculptors, there exists no general form of the definition of tensegrity due to practitioners’ different perspectives (see, for example, [38, 46, 97, 114]). A detailed introduction to the historical development and fundamental concepts of tensegrity frameworks can be found in [85]. In spite of the diverse definitions, one commonly-accepted statement is that a tensegrity structure consists of compression elements (i.e., struts) and tension elements (i.e., cables), with which the resulted pushing and pulling forces are balanced such that the whole structure is stable [64]. Because of the elements in compression or tension together with their carefully designed connections, tensegrity structures enjoy several remarkable features [113]: Efficiency in supporting loads; deployability to a large volume; easy adjustment; reliable modeling and control; and clear connections to many biological structures.

With these features, tensegrity structures have received extensive attention from different disciplines. Starting from explaining the molecular structure of the spider fiber [111, 121], researchers have applied tensegrity structures to model biologic organisms from cell cytoskeletons [60], cats’ hind legs [37] to the spines of humans [74], and in fact, one can recognize a new research community “biotensegrity”. The beauty of the tensegrity structures is admired not only by the biologists but also by the artists. As mentioned above, Snelson has created many sculptures, which are exhibited in museums and art galleries worldwide. At the same time, systematic analyses on the equilibrium conditions were initially reported in [67, 97], all of which laid theoretical foundations for the achievements afterwards. In architecture, the tensegrity concepts have also been adopted to construct shelters, bridges, roofs.

Figure 1.1: The “X-Piece” made by Kenneth Snelson in 1948.
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and even whole buildings. These constructions embrace the advantages that, on one hand, they can deform their shapes to survive drastic movements caused by an earthquake or whatever disturbances without breakdown and on the other hand, they are much lighter yet bear higher stiffness in comparison with traditional structures [112]. Besides the extensive applications of tensegrity structures in civil engineering, people also seek to explore a new paradigm in the design and control of locomotor robots using tensegrity concepts [69, 94]. Recently, a project aiming for exploring the deeper space was launched by NASA (National Aeronautics and Space Administration), in which the core platform “Super Ball Bot” based on tensegrity structure can flexibly adjust its configuration to suit the complicated environment.

Due to the increasing applications of tensegrity structures in different fields, researchers conducted more rigorous mathematical analysis much beyond the geometrical interpretations in the early stage. One of the most important problems is to identify an equilibrium configuration, known as form-finding [64, 80, 122], which falls out of the scope of this thesis, and thus we skip the discussion on this problem. Instead, we will focus on the theoretical study of tensegrity structures in the context of rigidity graph theory. To keep consistent with the conventions in graph theory, we will refer to tensegrity structures as “tensegrity frameworks” in the rest of this thesis. The concepts related to rigidity in terms of bar frameworks were extended to tensegrity frameworks in [104]. Then Connelly investigated the local rigidity conditions in terms of the stress-based energy function [19]. Later, the concepts of second-order rigidity and pre-stress stability for tensegrity frameworks were established in [26], where the physical parameters and the stress were linked via the Hessian matrix. In addition to these, global rigidity [23], super stability [21], and iterative universal rigidity [24] of tensegrity frameworks were also studied based on the stress matrix.

1.1.2 Distributed formation control

The last two decades witnessed sustained considerable efforts on distributed formation control, which is one of the central topics in the context of cooperative control of multi-agent systems. Initially inspired by the collective behavior of groups of animals, such as birds and fish, people gain new insight into the control of complex systems. Then distributed control schemes were proposed based on only local interactions with neighbors in contrast to the all-to-all or all-to-one communications in centralized control. Consequently, the distributed control systems can obtain more benefits because of the facts that they have low operational cost, high robustness to disturbances and system failure, and flexible scalability [17]. Due to these advantages, formations of robots have been employed to carry out various tasks, such as satellites flying in a certain shape to explore the deep space, drones
flying in formation to transport goods, and wheeled robots moving in an organized pattern to map an area [10].

The objective of formation control is to achieve some prescribed formations normally specified by relative positions or pairwise distances among agents. According to the collective behavior of the whole group, formation control can be roughly categorized into two scenarios [102]: formation producing (stabilization) refers to the convergence of team agents to some pre-defined feasible geometric shapes by running control laws; formation tracking refers to formation stabilization, in the meantime, following a given leader or a given reference signal. This implies that formation tracking control can be regarded as the integration of formation stabilization and trajectory tracking control. It is worth noting that the realization of formation tracking is more than just simple addition between formation stabilization and trajectory tracking due to the coupling effect caused by the sub-controllers to the other.

As summarized in the recent survey paper [91], the approaches to solving formation control problems can be generally classified into position-based, displacement-based, and distance-based control according to the sensing and communication variables. By invoking the position-based control, agents can move towards their desired positions individually, which means that a global coordinate system is compulsory. To remove such a restrictive requirement, the displacement-based control measures the relative positions, and thus relies on local coordinate systems but with consistent orientations. Among these three strategies, the distance-based control is the most efficient approach in practice, since the inter-agent distances can be obtained in a fully local manner in the sense that neither a global coordinate system or the same orientation is required. However, in general, only local convergence can be expected from the gradient system using distance-based control. Many results have been reported on this issue, such as [13, 29, 30, 70].

Apart from these basic tasks discussed above, formations are also required to vary in size or even in shape to adapt to the changing environment in some situations. For example, a team of flying drones needs to shrink its formation size to pass through some restricted areas. We call this type of transformation, i.e., altering only the formation size without changing the geometric shape, formation scaling. This issue will be addressed in this thesis by investigating how to control a small number of agents to recast the size of the formation. In addition, it is also a common phenomenon that as a formation of robots moves, it might split into sub-formations and then merge the small portions to form a bigger whole after a certain period of time, for the purpose of obstacle avoidance, predator avoidance, or target enclosing [5]. The merging problem will also be discussed in the context of tensegrity frameworks in this thesis.
1.2 Outline and main contributions of this thesis

Chapter 2 first provides some notations employed in this thesis, followed by the basic concepts of graphs and bar frameworks together with rigidity theory. Parallel to the bar frameworks, Section 2.3 presents the definitions and preliminaries of rigidity theory associated with tensegrity frameworks.

Chapter 3 deals with the problem of how to preserve infinitesimal rigidity and rigidity of the tensegrity frameworks after the merging operation in $\mathbb{R}^2$. We show that in the case of merging separate infinitesimal rigid tensegrity frameworks, there exists a set of proper self-stresses for the post-merged tensegrity framework, where the type of a pre-existing member can be maintained by checking the sign of the new stress. Then based on this self-stress, it can be shown mathematically that the combined tensegrity framework is infinitesimally rigid. Furthermore, we also show that rigidity can be expected when merging two rigid tensegrity frameworks via analyzing the distance perturbation on the linking members. For appropriate assignment of the new members, a novel method is proposed by morphing the rigidity matrix. To the best of our knowledge, no results on analyzing infinitesimal rigidity or rigidity of the merged tensegrity framework have been reported in the existing literature. The results of this chapter can serve as the theoretical foundation for formation control strategies design in practical applications.

In Chapter 4, we focus on the problem of growing super stable tensegrity frameworks. We first investigate the vertex addition and edge splitting operations on a super stable tensegrity framework along the line of classic Henneberg construction for bar frameworks. It is shown that these operations can preserve the super stability with appropriate selection of struts or cables inserted during the growing process in $\mathbb{R}^2$ ($\mathbb{R}^3$). In addition, Chapter 4 studies the merging problem for two super stable tensegrity frameworks in $\mathbb{R}^d$ under the condition that they share at least $d + 1$ vertices. We present one mild sufficient condition to ensure super stability by looking into the stress matrix. When the dimension of the working space is constrained to two or three, we give detailed procedures on how they may be merged to generate a super stable framework. In the last section of Chapter 4, comparisons are made with bar frameworks in terms of the quantity of the members to accomplish the growing procedure. The results of this chapter are important not only in enriching the rigidity graph theory with respect to super stable tensegrity frameworks, but also in constructing large-scale stable tensegrity frameworks in civil engineering.

Chapter 5 explores how to construct a universally rigid tensegrity framework given any configuration in general positions. We present a numerical algorithm to derive a stress matrix, based on which a universally rigid tensegrity framework can be built accordingly. We then consider the formation control problem with constraints on the inter-agent distances, in which the strict upper or lower bounds
for pairs of agents are imposed. By projecting the multi-agent system into a virtual tensegrity framework, we propose distributed control laws to stabilize prescribed formations. One can easily use our control strategies to tackle the distance constraints within the subject of formation control in applications. For example, the vehicles cannot diverge from each other too much due to the existence of cables in tethered robots or pipes in aerial refueling.

We study the formation scaling problem in Chapter 6, aiming at changing the formation size by controlling only a small portion of the agents. The virtual tensegrity framework is again employed to model the connecting relationships among the agents as well as the weight on each edge. We first show that the size of the formation in $\mathbb{R}^d$ can be altered by $d$ pairs of agents whose position vectors span the whole space $\mathbb{R}^d$. To further reduce the number of informed agents, we design another class of stress-based formation scaling control laws involving orthogonal projections used to drive the agents to correct directions. In this circumstance, it is shown that one pair of informed agents is sufficient to determine the size of the whole formation. Moreover, we discuss the equilibria when the stress agrees with a generic universally rigid tensegrity framework. As an extreme case, when only one agent is informed of the size of the formation, we design a new type of distributed estimator-based control algorithms, with which the formation scaling problem can be solved. This greatly improves the feasibility of the control laws in the terms of communication and sensing requirements. Last but not least, by introducing the negative weights, more interaction models can be involved to comprehensively study the mechanism of coordination.

In Chapter 7, we address the formation tracking problem for multi-agent systems, where the centroid of the formation needs to be controlled to follow a given reference signal. In light of the fact that the centroid of the formation is a global variable that cannot be computed easily using only local information, we design a finite-time centroid estimator for each agent. In comparison with existing results, our proposed estimator can get rid of the explicit knowledge of the bound of the agents’ speed. Using the centroid estimation, distance-based formation tracking control laws are designed and stability is proved by invoking rigidity graph theory. What also deserves to be highlighted in this chapter is that the proposed control scheme can be accomplished in local coordinate frames, which can definitely broaden its applications in practice.

Chapter 8 presents the conclusions of this thesis, and provides some possible directions of interest, from my point of view, for future research.
1.3 List of publications

Journal papers:


Conference papers


