Time-reversal violation in threshold $\bar{n}p$ scattering

C.-P. Liu *, R.G.E. Timmermans

Theory Group, Kernfysisch Versneller Instituut, University of Groningen, Zernikelaan 25, NL-9747 AA Groningen, The Netherlands

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Abstract

We investigate parity and time-reversal violation in neutron–proton scattering in the optical regime. We calculate the neutron spin rotation and analyzing power in scattering on polarized protons. This allows us to quantify the sensitivity that such experiments should aim for in order to be competitive to present-day measurements of the neutron electric dipole moment in constraining the $P$- and $T$-odd two-nucleon interaction. While state-of-the-art techniques fall short by some three orders of magnitude for the neutron–proton case, specific neutron–nucleus experiments look promising, provided certain experimental and theoretical challenges are met.

The neutron is an excellent laboratory for the study of fundamental symmetries and interactions. Its lifetime can be used to determine $V_{ud}$, one of the Cabbibo–Kobayashi–Maskawa matrix elements [1]. The correlations between the various momenta and spins in neutron $\beta$-decay are sensitive probes of non-$(V − A)$ currents [1]. The photon asymmetry $A_\gamma$ associated with radiative capture of polarized neutrons by nuclei, and the spin rotation $\phi_{\text{spin}}$ picked up by polarized neutrons traversing through a medium, can be used to constrain the strangeness-conserving, hadronic weak interaction (see, e.g., Refs. [2,3] for reviews). The results of these measurements provide important tests of the electroweak sector of the Standard Model, and in particular its aspect of parity violation ($P$).

Neutrons can play an equally important, and in some sense even more fundamental, role in the aspect of time-reversal violation ($T$). Because of $CPT$ invariance, $T$ violation [4] is equivalent to $CP$ violation, whose origin and role in generating the matter–antimatter asymmetry of the universe are among the great mysteries of particle and astroparticle physics. The search for a permanent neutron electric dipole moment (EDM), which violates both $P$ and $T$ invariance ($P T$), has been continuously considered (see, e.g., Refs. [6,7] for reviews). The study of this report falls into the latter category.

Modern high-flux, continuous or pulsed, neutron sources are able to provide neutrons over a wide energy spectrum, ranging from very fast ($\gtrsim$ MeV) neutrons all the way down to ultra-cold ($\lesssim 10^{-7}$ eV) neutrons. For the study of the $P$ or $T$ hadronic interaction, low-energy neutrons, from the epithermal ($\sim$ eV) to the cold ($\sim$ meV) region, are particularly useful for several reasons: (i) The large flux can be maintained. (ii) Because of the long de Broglie wave-length of the neutrons, the scatterers contribute coherently. In other words, in this energy regime “neutron optics” works well. (iii) Low-energy neutrons are better suited to study the short-ranged $T$ hadronic interaction than charged particles, which are kept apart by the repulsive Coulomb force.

The Spallation Neutron Source, which is currently under construction at Oak Ridge National Laboratory, is expected to improve fundamental neutron physics to a new level. For example, a proposal to measure the $P$ neutron spin rotation in para-hydrogen (with unpaired proton spins) is aiming to reach an accuracy of $2.7 \times 10^{-7}$ rad/m [8]. Motivated by this remarkable advance, we investigate here $T$ violation in scattering of polarized neutrons ($\bar{n}$) on polarized protons ($p$), for which the $T$ signal can be calculated reliably by using modern high-quality strong $np$ potentials together with the general $P$ and $T$ interaction. The observables that we are interested in violate both $P$ and $T$, and hence they address the same physics.
as the neutron EDM, $d_n$ (or the EDM of a diamagnetic atom, such as $^{199}$Hg [9]). Our main purpose, in fact, is to quantify how such a neutron-optics experiment, now with a polarized target but assuming the same experimental accuracy, competes with modern EDM measurements in constraining the underlying $\mathcal{P}\mathcal{T}$ interaction.1 Also, a number of studies indicate that $\mathcal{P}$ observables can be greatly enhanced in certain neutron–nucleus scattering processes (see, e.g., Refs. [10–13]). We will use these results to justify some reasonable assumptions that will allow us to extrapolate our results from the $\bar{n}p$ system to $T$ violation in neutron–nucleus scattering. Our calculations can thus serve as a benchmark for gauging the sensitivity of $\mathcal{T}$ observables in neutron transmission experiments that aim to compete with EDM measurements.

The optics of low-energy neutron transmission through a medium (see, e.g., Refs. [14,15]) can be described by the corresponding index of refraction, $n$, which is a coherent sum of individual scatterings and which is related to the neutron-target scattering amplitude at forward angle ($\theta = 0$), $f$, by

$$n = 1 - 2\pi N f/k^2,$$

(1)

where $N$ is the target density; $k \equiv |k|$ is the neutron momentum, which is assumed to be in the $+z$ direction from now on. When $f$ contains some non-vanishing component $f_\mathcal{P}$ which depends on $\bf{\sigma} \cdot \bf{k}$ due to $\mathcal{P}$ interactions, neutrons with a $+z$ polarization have a different value of $n$ compared to the ones with a $-z$ polarization. Neutron wave functions of opposite polarizations then pick up different phases, viz. $n_{+}, k\ell$ and $n_{-}, k\ell$, after travelling a distance of $l$ in a uniform medium. This optical dichroism manifests itself in two major ways: (i) a neutron spin rotation $\phi_z$ along the $z$-axis, and (ii) a longitudinal polarization $P_z$ of an unpolarized incident beam or a longitudinal asymmetry $A_z$ between $+z$- and $-z$-polarized neutrons [16–18]. The former depends on the real part of $f$, while the latter on the imaginary part, as

$$\phi_z = -2\pi l/kN \text{Re}(f_{+z} - f_{-z}),$$

(2)

$$P_z = -2\pi l/kN \text{Im}(f_{+z} - f_{-z}).$$

(3)

These ideas for $P$ violation were generalized to study $T$ violation by Kabir [19] and Stodolsky [18]. With a polarized target (with polarization $S$), the scattering amplitude can acquire, in principle, a $\mathbf{P}\mathbf{T}$ component $f_{\mathbf{P}\mathbf{T}}$ proportional to the triple correlation $\bf{\sigma} \cdot \bf{k} \times \bf{S}$. Bunakov and Gudkov, however, argued later [20] that the combined actions of the magnetic interaction, which introduces a $\bf{\sigma} \cdot \bf{S}$-dependent component $f_M$ in $f$, and the weak interaction, generate a much larger scattering amplitude of the same $\bf{\sigma} \cdot \bf{k} \times \bf{S}$ form. This effect mimics $T$ violation—similar to how final-state interactions can mimic the $\mathbf{P}\mathbf{T}$ correlation coefficient $R$ in $\beta$-decay. Such a pseudo-$\mathbf{P}\mathbf{T}$ amplitude ultimately spoils the unambiguous identification of a true $\mathbf{P}\mathbf{T}$ signal. Several ways to circumvent this difficulty have been proposed in Refs. [21–23]. Here, we analyze two observables and show what they can reveal about the underlying $\mathbf{P}\mathbf{T}$ nucleon–nucleon ($NN$) interaction.

Without loss of generality, we assume that both neutron and proton are polarized in the $+x$ direction. Because $k \times S$ defines a specific direction ($+y$ in our case), similar to $k$ for the above $\mathcal{P}$ case, the quantities $\phi_y$, $\bar{P}_y$, and $\bar{A}_y$ can be obtained via the scattering amplitudes $f_{y+}$ and $f_{y-}$:

$$\bar{\phi}_y = -2\pi l/k\bar{N} \text{Re}(f_{y+} - f_{y-}),$$

(4)

$$\bar{P}_y = -2\pi l/k\bar{N} \text{Im}(f_{y+} - f_{y-}).$$

(5)

We use here tildes as a reminder that we consider the case which involves polarized targets and that it is the observables which violate not only $P$ but also $T$ that are of interest. Analogously to what has been concluded in Ref. [23], one finds that (i) $\phi_y$ and (ii) $\bar{P}_y + \bar{A}_y$ are unambiguous measures of $T$ violation. This can be easily illustrated in Figs. 1 and 2: Although pseudo-effects can mimic true $\mathcal{T}$ effects in the scattering amplitude and some observables, their invariances under $T$ and $R$ ($\pi$), a $180^\circ$ rotation around the $y$-axis, will render that

$$\bar{\phi}_y^\text{pseudo} = R_y(\pi)T \bar{\phi}_y^\text{pseudo} T^{-1}R_y^{-1}(\pi) = -\bar{\phi}_y^\text{pseudo},$$

$$\bar{A}_y^\text{pseudo} = R_y(\pi)T \bar{A}_y^\text{pseudo} T^{-1}R_y^{-1}(\pi) = -\bar{P}_y^\text{pseudo}.$$

Therefore, neither (i) nor (ii) can be faked by a pseudo-effect. It is also worth to point out that only one experiment is needed for measuring $\phi_y$, but two are needed for the $\bar{P}_y + \bar{A}_y$ comparison. In other words, the spin rotation represents a true null experiment to test $\mathcal{T}$, and therefore has some advantage [24].

The calculations of $f_\mathcal{P}$ and $f_{\mathcal{P}\mathcal{T}}$ are briefly outlined in the following. Since both $\mathcal{P}$ and $\mathcal{P}\mathcal{T}$ interactions, $H_\mathcal{P}$ and $H_{\mathcal{P}\mathcal{T}}$, are much smaller than the strong interaction, the first-order Born approximation is sufficient to calculate the scattering amplitudes. Resolving the spin states for both neutron and proton explicitly in terms of spinors quantized in the $z$-direction, one

![Fig. 1](image-url)
obtains
\[
\begin{align*}
 f_{+z} - f_{-z} &= 1/2 \{ H_P(\uparrow\uparrow, \uparrow\uparrow) + H_P(\downarrow\downarrow, \downarrow\downarrow) \\
 - H_P(\downarrow\uparrow, \uparrow\downarrow) - H_P(\uparrow\downarrow, \downarrow\uparrow) \}, \\
 f_{+y} - f_{-y} &= -i/\sqrt{2} \{ H_{TF}(\uparrow\uparrow, \uparrow\uparrow) + H_{TF}(\uparrow\downarrow, \downarrow\downarrow) \\
 - H_{TF}(\downarrow\uparrow, \uparrow\downarrow) - H_{TF}(\downarrow\downarrow, \uparrow\uparrow) \}
\end{align*}
\]
(6)

with
\[
H(m_{s1}m_{s2}, m_{s1}m_{s2}) \equiv (-)^{m_s}m_{s2} |H| m_{s1}m_{s2}\rangle \langle (+),
\]
(8)

where \(H\) is \(H_P\) or \(H_{TF}\). The distorted (by the strong interaction) wave functions are obtained by solving the Lippmann-Schwinger equation
\[
|m_{s1}m_{s2}\rangle = |m_{s1}m_{s2}\rangle^{(0)} + \frac{1}{E - H_0 - H_S \pm i\epsilon} |m_{s1}m_{s2}\rangle^{(\pm)},
\]
(9)

where \(|m_{s1}m_{s2}\rangle^{(0)}\) is simply a plane wave. We have used several high-quality local \(n\bar{p}\) potentials, viz. AV18 [25], Reid93 and Nijm-II [26], as input for \(H_S\). The \(H_P\) and \(H_{TF}\) used in this work are both built upon the one-meson-exchange model and parametrized by the corresponding \(\vec{P}\) and \(\vec{PP}\) meson-nucleon coupling constants \(h_M^I's\) and \(g_M^3's\) ("\(M\)" for the type of meson and "\(I\)" for isospin), respectively. The former is the well-known, so-called DDH potential [27], which contains 6 \(\vec{P}\) couplings (with \(h_P^I\) usually being ignored) due to one \(\pi\)-, \(\rho\)-, and \(\omega\)-exchanges, and the most complete form of the latter, which contains 10 \(\vec{PP}\) couplings due to one \(\pi\)-, \(\eta\)-, \(\rho\)- and \(\omega\)-exchanges, can be found in Ref. [28]. In the low-energy region, only the lowest partial waves are important, and the results depend on three \(S-P\) amplitudes: \(3 S_1-3 P_1 (\Delta I = 1), 3 S_{1-1}^{-1} P_1 (\Delta I = 0)\), and \(1 S_{0-3}^3 P_0 (\Delta I = 0, 2)\). The small admixture of \(3 D_1\) to \(3 S_1\) by the tensor force can be ignored safely.

The threshold behavior is examined across a wide range of neutron energy \(E_n\) from epithermal \(\sim\) eV to very cold \(\sim 10^{-4}\) eV. Our numerical results agree very well with the qualitative predictions by Stodolsky [18] that \(\tilde{\phi}_n\) is constant and \(\tilde{P}_n\) decreases as \(\sqrt{E_n}\). Stodolsky also pointed out that the existence of exothermic processes, i.e., inelastic channels, could possibly lead to a non-zero contribution to \(\tilde{P}_n\) at zero energy for neutron–nucleus scattering. However, this is not the case for \(np\) scattering: As it is known that the neutron-helicity–dependent differential cross section for radiative capture, i.e., \(\tilde{n} + p \rightarrow d + \gamma\), takes the form \(d\sigma / d\Omega \propto (1 \pm A_y \cos \theta)\) (see, e.g., Ref. [29]), the total cross sections for neutrons of opposite helicities are the same; hence, no total asymmetry arises from this particular exothermic process.²

The target density, to which all optical observables are proportional, is certainly an important factor affecting the feasibility of a neutron transmission experiment. For the \(\tilde{n}p\) case, high-purity liquid para-hydrogen, with \(N \sim 0.4 \times 10^{23}/\text{cm}^3\), provides a good choice for the \(\vec{P}\) study [8]. For the \(\tilde{n}p\) case, a target containing polarized protons with a reasonably high density is required. A novel technique to produce a polarized solid HD target [30], called SPHICE (Strongly Polarized Hydrogen ICE), with a 95% proton polarization in a molecular volume 20 cm³/mol, suggests that \(N \sim 0.3 \times 10^{23}/\text{cm}^3\) is possible. Therefore, for the following numerical results, we adopt \(N = \tilde{N} \sim 0.4 \times 10^{23}/\text{cm}^3\).

Assuming that the target density is uniform, the differential observables \(d\tilde{\phi}_P / dz\) and \(d\tilde{P}_P / dz\) for neutrons at thermal energy, \(E_n = 0.025\) eV, are given in Tables 1 and 2. The dominance of pion exchange, due to its comparatively long range, is obvious. Also its model dependence is very small. Of the three contributing \(S-P\) amplitudes, the \(1 S_0-3 P_0\) transition plays the most important role. It gives a \(g_\rho^0 - 4 g_\pi^2\) dependence on the \(\vec{P}\) pion–nucleon couplings. Since the heavy-meson contributions are more sensitive to the short-range wave functions, the difference between various strong potentials becomes more apparent. At this energy, \(d\tilde{\phi}_P / dz\) is about three orders of magnitude bigger than \(d\tilde{P}_P / dz\), therefore, spin-rotation experiments look more promising, besides the advantage already mentioned above that they are true null tests. We also calculate \(\tilde{\phi}_n\) using the same wave functions. For the AV18 model, the result is
\[
\begin{align*}
 d\phi_n / dz &= 1.130 h_2^0 - 0.238 h_0^0 + 0.008 h_1^0 + 0.250 h_0^1 \\
 &\quad - 0.269 h_0^1 - 0.024 h_1^1 \text{ rad/m.}
\end{align*}
\]
(10)

Using the DDH “best values” [27] for the \(h_M^I's\), one gets \(d\tilde{\phi}_P / dz \simeq 6.5 \times 10^{-7}\) rad/m.³

We now assume that a neutron spin rotation experiment with polarized protons as target can reach a similar sensitivity of \(2.7 \times 10^{-7}\) rad/m as what is expected for the one using para-hydrogen for the \(\vec{P}\) experiment. Our calculation then demonstrates that this null test for \(T\) violation constrains the \(\vec{PP} NN\)

² In other words, the existence of exothermic channels is only a necessary but not a sufficient condition for a non-zero total asymmetry at zero energy.

³ This number differs somewhat from a recent calculation by Schiavilla et al. [31]. This is because we use different strong parameters and because the Yukawa function in their work is modified by a monopole form factor. When we use the same model as they did, we get a perfect agreement with their result.
interaction at the level of
\[ \pm 2.7 \times 10^{-7} > 7.7 \bar{g}^0_\pi + 1.1 \bar{g}^1_\pi - 28 \bar{g}^2_\pi + \cdots. \tag{11} \]

On the other hand, the neutron EDM \( d_n \) can also be expressed in terms of these \( \vec{P} \vec{T} \) meson–nucleon couplings [32]. By using the recent estimate in Ref. [28], the current most stringent upper limit on \( d_n \): \( d_n < 6.3 \times 10^{-26} \) e cm [5], provides the constraint
\[ \pm 6.3 \times 10^{-11} > 14 (\bar{g}^0_\pi - \bar{g}^2_\pi) + \cdots. \tag{12} \]

Comparing Eqs. (11) and (12), a neutron EDM measurement at the \( 10^{-25} \) e cm level is 3–4 orders of magnitude more sensitive than a spin rotation measurement in polarized hydrogen at the \( 10^{-7} \) rad/m level. Given that the accuracy of \( 10^{-7} \) rad/m is already state-of-the-art and that there are many difficulties involved in keeping de-polarization effects under control, it seems very unlikely that a neutron spin rotation experiment can compete with the neutron EDM experiments in the near future.

However, the situation could be quite different when certain heavy nuclei are chosen as targets. By exploiting the low-lying \( p \)-wave resonances in neutron–nucleus scattering, the combined dynamical and resonance enhancements for \( \vec{P} \vec{T} \) signals can be as large as \( 10^6 \) [10,33]. A recent \( \vec{P} \) neutron transmission measurement that exploits the \( 7.734 \) eV \( p \)-wave resonance of \(^{139}\)La resulted in \( d_{\phi_p} / d_z = (7.4 \pm 1.1) \times 10^{-1} \) rad/m [13]. Compared to the theoretical prediction for thermal neutrons in hydrogen: \( d_{\phi_p} / d_z = 5.1 \approx 7.2 \times 10^{-7} \) rad/m [31], one does find a \( 10^6 \) enhancement factor. Therefore, if a similar \( \vec{P} \vec{T} \) measurement could be performed with a polarized \(^{139}\)La target and with a \( 10^{-7} \) rad/m sensitivity, it will be competitive to the currently planned \( d_n \) measurements that target the \( 10^{-27} - 10^{-28} \) e cm level.

While this is an optimistic conclusion, there exist several major challenges. On the experimental side, noticeably, the sensitivity reported for \( \vec{P} \) in the \(^{139}\)La case is only at the \( 10^{-1} \) rad/m level. This six-orders-of-magnitude loss of sensitivity thus neutralizes the \( 10^6 \) enhancement factor, which results in a measurement not better than the one with a hydrogen target and a \( 10^{-7} \) rad/m sensitivity. A rough theoretical estimate that a 3–4 orders of improvement is necessary to keep these measurements competitive to the current \( d_n \) limit was given in Refs. [34,35], and a possibility of such an experimental improvement

was reported in Ref. [36]. On the theoretical side, it will require a major effort to interpret the observables in neutron–nucleus scattering, in terms of \( \vec{P} \vec{T} \) meson–nucleon couplings, at a similar level of accuracy as what we have done here for the \( np \) system (1% or even, say, at the 10% level). There have been efforts to apply the theory of statistical spectroscopy to interpret \( \vec{P} \) phenomena (see, e.g., Ref. [37]), apparently, how they can constrain the underlying \( N \) interactions is then subject to statistics. Similar work for \( T \) violation will be necessary.

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