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On a possible node in the Sivers and Qiu–Sterman functions

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The possibility of a node in the x dependence of the Sivers and Qiu–Sterman functions is discussed in light of its importance for the experimental check of the overall sign change of the Sivers effect between semi-inclusive DIS and the Drell–Yan process. An x-dependent version of the Ehrnsperger–Schäfer–Greiner–Mankiewicz relation between the Qiu–Sterman function and a twist-3 part of g_2 is presented, which naturally suggests a node in the Qiu–Sterman function. This relation could be checked experimentally as well and could provide qualitative information on the gluonic field strength inside the proton. Satisfying the Burkardt sum rule by means of a node is briefly discussed and the importance of modelling the Sivers function including its full Wilson line is pointed out.

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The Sivers [1] and Qiu–Sterman [2] effects have been proposed as possible explanations of single transverse spin asymmetries A_N observed in the process p+p→πX [3]. In recent years it has become clear that these two effects are intimately related [4,5]. In this Letter the x dependence of these effects is discussed, in particular the possibility of a node.

The Sivers effect is described by a transverse momentum dependent parton distribution function (TMD) and generates azimuthal spin asymmetries in the Drell–Yan (DY) process and in semi-inclusive DIS (SIDIS). The Sivers effect asymmetry in SIDIS has been clearly observed in the HERMES [6] and COMPASS [7] experiments. The Sivers function, here denoted by \( f_{1T}(x, k_T^2) \), describes the difference between the probability to find a quark with lightcone momentum fraction x and transverse momentum k_T inside a hadron polarized transversely to its momentum direction and the one where the polarization points in the opposite direction. As the Sivers function describes a difference of probabilities it is not necessarily positive definite. In fact, the major interest in extracting the Sivers function from the DY process is that it is expected to have the opposite sign compared to the one extracted from SIDIS [8]. The gauge invariant definition of the Sivers function is in terms of a nonlocal operator involving a Wilson line:

\[
\begin{align*}
\int_1^{\pm1} C \left( x, k_T^2 \right) = & \, \frac{M}{2} \mathcal{F}_T (P, S_T | \bar{\psi}(0) \mathcal{C}(0, \xi) \mathbb{P}_{-Y}(\xi) | P, S_T ) \bigg|_{\xi^+ = 0}, \\
\end{align*}
\]

where \( \mathcal{C} \) denotes the Wilson line along contour \( C \); \( S_T \) denotes the transverse spin vector; and \( \mathcal{F}_T \) denotes the Fourier transform, where \( \xi^- \) and \( \xi^+ \) are the Fourier conjugate variables of \( x \) and \( k_T \), respectively. The Sivers function is not uniquely defined, as it depends on the contour of the Wilson line, which in turn depends on the process considered. The Sivers function appearing in SIDIS contains a future pointing Wilson line, whereas in DY it is the same except past pointing, leading to the following overall sign relation [8]:

\[
f_{1T}^{\pm \text{SIDIS}} (x, k_T^2) = - f_{1T}^{\pm \text{DY}} (x, k_T^2). \tag{2}
\]

This is a prediction of the TMD formalism that remains to be tested. A related sign test in W and Z production at RHIC has been put forward in Refs. [9,10]. In more complicated processes that allow TMD factorization, Sivers functions with other Wilson lines can appear, which are not simply related by an overall sign to the Sivers function of SIDIS to which we will refer as “the” Sivers function from now on.

It is important to emphasize that the above sign relation is about the overall sign and that the Sivers function itself need not be of fixed sign as a function of x. It can have one or more nodes in the x dependence, even though its present extraction from SIDIS data in a restricted x range does not display a node [11]. Nevertheless, the possibility of a node should be kept in mind when comparing the extraction from SIDIS with the future one from DY. A node position is generally expected to be Q^2 dependent, therefore, unless one compares the functions at the same x and Q^2 values, the sign change test need not be conclusive. Moreover, such a node need not be at the same position for the different flavors,
possibly complicating the comparison further. Unless one makes sure that nodes do not play a role in the comparison, one may wrongly jump to the conclusion that the TMD formalism is flawed in some way if the overall sign change between SIDIS and DY is not confirmed in experiment. Motivated by the importance to know whether the Sivers function has a node, we investigate if there are any other indications in favor or against such a node.

Since we are interested in the $x$ dependence here, we will not address the transverse momentum dependence for a discussion of possible nodes in the $k_T$ dependence cf. [12] and restrict to the first transverse moment of the Sivers function, i.e. the Sivers function weighted with the transverse momentum squared:

$$f_{1T}^{(1)}(x) = \int d^2k_T \frac{\rho_T^2}{2M^2} f_{1T}(x, \rho_T^2).$$  

This quantity is of interest because of its direct relation to the twist-3 Qiu–Sterman (QS) function $T(x, S_T)$ [2]:

$$T(x, S_T) = i \frac{M}{p_T} \int \frac{d\lambda}{2\pi} e^{i\lambda(x(P, S) - \bar{\psi}(0)\Gamma_{\alpha}} \times \int d\eta F^{++}(\eta n_\perp) \psi(\lambda n_\perp|P, S),$$  

where $\Gamma_{\alpha} \equiv \epsilon_{\alpha\beta\mu\nu} F_{\mu\nu}(P, S)$ (like in [2] we take $\bar{S}_T^2 = 1$) and $\epsilon_{\beta\mu\nu}^{\perp} = \epsilon_{\alpha\beta\mu\nu} n_\alpha n_\beta$. The relation between the first moment of the Sivers function (of SIDIS) and the QS function is a direct proportionality [4]:

$$f_{1T}^{(1)}(x) = -\frac{g}{2M} T(x, S_T).$$  

The $x$ dependence of the two functions is therefore the same, apart from an overall proportionality constant. From now on we will absorb the coupling constant $g$ into the definition of $T$, but it is displayed here explicitly since it determines the relative sign in front of it (for a discussion of this issue cf. [12]).

The above definition of the QS function is given in the $A^+ = 0$ lightcone gauge for simplicity and contains the operator $\int d\eta F^{++}(\eta n_\perp)$, which renders it intrinsically nonlocal along the lightcone, even in the $A^+ = 0$ gauge. Below we are going to discuss two different assumptions about this lightcone integral over the gluonic field strength, one by Qiu and Sterman [2] and one by Ehrnsperger, Schäfer, Greiner and Mankiewicz (ESGM) [16].

Qiu and Sterman considered the following parametrization [2]:

$$T^q(x, S_T) = k_q \lambda f_1^q(x),$$  

where $f_1(x)$ is the ordinary unpolarized parton distribution function and $q$ denotes the quark flavor. This parametrization follows when the QS matrix element is viewed as yielding the average value of $\int d\eta F^{++}(\eta n_\perp)$ inside the unpolarized proton and hence is expected to be simply a number times the unpolarized distribution function:

$$f_1(x) = \frac{1}{2p_T} \int \frac{d\lambda}{2\pi} e^{i\lambda x(P)|\bar{\psi}(0)\eta_- \psi(\lambda n_-)|P).$$  

The function $f_1(x)$ is a probability distribution and hence of definite sign. Using this parametrization, $T(x, S_T)$ can have different signs for different flavors, but cannot exhibit any node. In order to roughly describe the single transverse spin asymmetries (SSA) experimentally measured in the process $p^+ p \rightarrow \pi X$ at $\sqrt{s} \approx 20$ GeV [3], the following choices were made for the parameters: $k_q = +1 = -k_{q' D}$, $x_S = 0$, yielding $\lambda \sim 100$ MeV [2]. This parametrization was subsequently used to predict SSA in $\pi$ production at $\sqrt{s} = 200$ GeV [13] and in Drell–Yan [14,15]. No conclusive evidence in favor of the above parameterization has been obtained yet though.

Another view on the lightcone integral over the gluonic field strength is taken by ESGM. In Ref. [16] it is assumed that the field strength is only significantly contributing inside the proton and that the following approximation holds:

$$\int d\eta F^{++}(\eta n_\perp) \approx F^{++}(0) \propto 2s M R_0,$$  

where $R_0$ is the proton “lightcone” radius in the rest frame of the proton (taking into account that it is being probed by a highly relativistic probe, which sees the proton as Lorentz contracted) and $\sqrt{s} \equiv \int d\eta = 2s M R_0$, where $c$ is argued to be a constant between 1/3 and 1 [16,17]. ESGM considered the field strength at $\eta = 0$, because that corresponds to the position of the quark fields upon integration of $T(x, S_T)$ over $x$, such that a local operator is obtained. A priori it is not known whether this approximation is legitimate, except that it may simply be true numerically for some $A^+$ configurations, but of course there is no way to select such configurations. Nevertheless, it leads to an interesting result, which is the ESGM relation between the lowest (zeroth) Mellin moment of the QS function and the second moment of the twist-3 part of the distribution function $g_2$, which according to [16] is:

$$\int x^2 g_2(x)|_{\text{twist-3}} dx = -12c M^2 R_0 \int x^2 g_2(x)|_{\text{twist-3}} dx.$$  

It should be emphasized that this is not an exact relation. But if ESGM’s approximation is fine, then one can relate the magnitude of the above mentioned SSA to $g_2$; this would be very useful, since it is known that $\frac{x^2 g_2(x)|_{\text{twist-3}}}{dx}$ is very small.

The structure function $g_2$, which in the parton model is directly related to the distribution function $g_2^q$ via $g_2(x) = \frac{1}{2} \sum_n q_n g_2^q$, with $q_n$ the quark charge squared in units of the electron charge, has been measured by the E155 experiment at SLAC. Also the twist-3 part of $g_2$ was extracted, yielding a value for its second moment $d_2$ which is defined as

$$d_2 = 3 \int x^2 g_2(x)|_{\text{twist-3}} dx.$$  

The E155 experiment obtained $d_2 = 0.0032 \pm 0.0017$ for the proton [18], when taking into account all available SLAC data. Assuming there are no large cancellations among the quark flavors, one concludes that also $d_2^u$ and $d_2^d$ are both very small. This conclusion is supported by lattice QCD evaluations [19].

Combining the ESGM relation and the above parametrization (6) of the QS function would suggest a very small SSA in $p^+ p \rightarrow \pi X$ contrary to observations. Given the very small size of $d_2$, one would conclude that either $\lambda$ is much smaller than expected from the SSA data or $R_0$ has to be unnaturally large (\gg 1 fm). Failure of the ESGM relation is one possibility and may indicate some qualitative features of the lightcone integral of the gluonic field strength inside the proton, such as that it is not slowly varying inside the proton or is significantly contributing outside the proton too. But one could also question the validity of parametrization (6), because one way to allow for large SSA and simultaneously accommodate small $d_2$ through the ESGM relation is to consider the option that $T(x, S_T)$ changes sign as a function of $x$, having large absolute value in certain $x$ regions, but having a small integral. This is the view we will explore here, leaving aside the idea that $T(x, S_T)$ is proportional to $f_1(x)$. 
The approximation (8) can be extended in various ways to obtain an \(x\)-dependent version of the ESGM relation. For instance, assuming that the gluonic field strength is a slowly varying function (or constant even), within the proton, leads to the approximation
\[
\int d\eta \, F^{+\alpha}(\eta n_\perp) \approx F^{+\alpha}(\eta_0 n_\perp) \times 2cM R_0, \quad (11)
\]
for some (or any) \(\eta_0\) within the proton lightcone radius \(R_0\). To reduce to the ESGM relation upon integration over \(x\), one has to consider \(\eta_0\) at the position of either one of the quark fields. This is automatically satisfied if the field strength is taken to be constant within the proton. It should be mentioned that if one views the above approximation as giving the average field strength times the integration region, that in that case the estimate of \(c\) of Refs. [16,17] may be far off if the gluonic field is in fact heavily fluctuating. This can be experimentally investigated.

With the approximation (11) for \(\eta_0 = \lambda\), the following unintegrated relation can be obtained:
\[
T^q(x, S_T) = -2cM^2 R_0 x^2 \tilde{g}_T^q(x). \quad (12)
\]
This is an unintegrated version of the ESGM relation, which holds for each flavor separately. The twist-3 distribution function \(\tilde{g}_T^q(x)\) is the quark–gluon–quark correlation part of \(\tilde{g}_T^q(x) = g_1^q(x) + g_2^q(x)\) split off by means of the equations of motion (for \(m = 0\)): \(\tilde{g}_T^q(x) = g_T^q(x) - \tilde{g}_1^q(x)/x\) (cf. e.g. Refs. [21–23]). It can for instance be measured in the Drell–Yan process in double spin asymmetries \(A_{LT}\) of longitudinally polarized hadrons colliding with transversely polarized hadrons [20,21] or using SIDIS [23].

Upon taking the lowest Mellin moment of Eq. (12), one arrives at:
\[
\int_0^1 T^q(x, S_T) \, dx = -cM^2 R_0 [d_1^q + d_2^q], \quad (13)
\]
where
\[
d_2^q = \int_0^1 x^2 \tilde{g}_2^q(x)|_{\text{twist-3}} \, dx = 2 \int_0^1 x^2 \tilde{g}_2^q(x) \, dx. \quad (14)
\]
Summing over quark flavors, one obtains a relation in terms of the twist-3 part of the structure function \(g_T^q\):
\[
\sum_q c_q^2 \int_0^1 T^q(x, S_T) \, dx = -6cM^2 R_0 \int_0^1 x^2 g_2(x)|_{\text{twist-3}} \, dx. \quad (15)
\]

which apart from the sum over flavors and the quark charge squared factor is a factor of 2 different from Eq. (9). Given the uncertainty in the proportionality constant \(c\) this factor is not of importance, but we do note that the flavor dependence of the ESGM relation was not addressed properly in Ref. [16].

The question here is whether \(\tilde{g}_T^q(x)\) has a sign change as function of \(x\), since the unintegrated “ESGM” relation implies similar behavior for \(T(x, S_T)\). The bag model [24] suggests that \(g_2^q(x)|_{\text{twist-3}}\) is a sign changing function of \(x\). However, \(g_2(x)|_{\text{twist-3}}\) and \(\tilde{g}_T^q(x)\) correspond to different operator matrix elements, even though the second moments are directly related through Eq. (14).

Instead, one can look at the first moment of \(\tilde{g}_T^q(x)\), which in the \(A^+=0\) gauge is given by:
\[
\int_0^1 x \tilde{g}_T^q(x) \, dx \approx \Re \langle P, S | \bar{\psi}(0) \gamma^\mu g A^\mu(0) \psi(0) | P, S \rangle n_{\lambda\mu} \hat{S}_{TV} = 0, \quad (16)
\]
where the vanishing of this particular matrix element is shown in Ref. [25] on the basis of Lorentz invariance, analogous to the derivation of the Burkhardt–Cottingham sum rule \(f_{-1}^1 \tilde{g}_T^q(x) \, dx = 0\). Since the first moment of \(\tilde{g}_T^q(x)\) deals with a local operator that appears in the Operator Product Expansion (OPE) for charged currents, one can more specifically conclude that also \(\int_0^1 x^2 \tilde{g}_T^q(x) \, dx\) corresponds to a local operator matrix element (obtained through Taylor expansion) that vanishes due to Lorentz invariance, in contrast to \(\int_0^1 x \tilde{g}_T^q(x) \, dx\) which does not appear in the local OPE [26,27]. In other words, the vanishing integral in Eq. (16) is not due to a cancellation among the quark \((x > 0)\) and antiquark \((x < 0)\) contributions.

The vanishing of \(\int_0^1 x^2 \tilde{g}_T^q(x) \, dx\) implies that \(\tilde{g}_T^q(x)\) has a node and through the unintegrated “ESGM” relation also the QS function would have a node. In that case, \(\int T(x, S_T) \, dx\) can be much smaller than the maximum value of \(T(x, S_T)\), such that a sign change of \(T(x, S_T)\) can accommodate both small \(d_2\) and large SSA in a limited \(x\) range in a natural way. The unintegrated relation is useful in this respect, since asymmetries can be measured as function of \(x\), therefore, the relation can be checked in experiment. The option that the QS function has a node should then be kept in mind and also that the node can change position as a function of \(Q^2\), which is relevant when comparing different experiments.

What speaks against a node is that most model calculations of the Sivers function [28–34] do not show a node (ignoring some small bag model artifacts), except for those of Refs. [35,36] which obtain down quark Sivers functions with a node, albeit very different ones, and for Ref. [37] which obtains an up quark Sivers function with a node. These model calculations of the Sivers functions all consider the gauge link to lowest nontrivial order in the coupling constant, in other words, the first order expansion of the Wilson line. It is unclear what the size of the higher order corrections is and whether these could change the sign in a particular \(x\) region.

A common feature of the model results is that the up and down quark Sivers functions are opposite in sign. This is also expected from the large \(N_c\) limit, which leads to \(f_{+1}^T(x, k_T^2) = -f_{-1}^T(x, k_T^2) + O(N_c^{-1})\) [38,39], and on the basis of the signs of the quark anomalous magnetic moments through an integral relation of the Sivers function and GPDs [40,41]. This suggests that if a node occurs, it is present in both up and down quark Sivers functions.

SIDIS experiments off proton and deuteron targets in the valence region seem to indicate that up and down Sivers functions are indeed opposite in sign and moreover similar in magnitude. In some models the magnitude of the up quark Sivers function was found to be much larger than that of the down quark [30,35], but in more recent model calculations they are found to be of comparable magnitude [31,33,34,36,37]. If indeed of opposite sign, but similar in magnitude, the so-called Burkardt sum rule [42]
\[
\sum_{a=q,g} \lambda f_{+1}^{T}^{(10)}(x) \, dx = 0, \quad (17)
\]
can be satisfied by a cancellation among contributions of the valence quarks, not requiring large contributions (and accompanying
The large cancellations) from non-valence contributions. This would be one possibility. See Ref. [43] for a check of the Burkardt sum rule in a model calculation that includes the gluon Sivers function. A second possibility to satisfy the Burkardt sum rule is that the $x$ integral of the Sivers moment is zero for each flavor separately. The latter is not in contradiction with the measured Sivers asymmetries in SIDIS, since those do not provide full information on the integral over $x$ of the Sivers moment. A future observation of a node in the Sivers function could point to this second possibility, as does a small gluon Sivers function. The latter can be seen by rewriting the Burkardt sum rule in the form:

$$\sum_q \int_{-1}^{1} T^q(x, S_T) \, dx = - \int_{-1}^{1} T^g(x, S_T) \, dx,$$

and by comparing it to the integrated ESGM relation of Eq. (15), where the l.h.s. of the latter includes an additional quark charge squared factor. If the gluon QS (or Sivers) function turns out to be small, then combined with the fact that $d_2$ is small, it implies that both $\sum_q \int_{-1}^{1} T^q(x, S_T) \, dx$ and $\sum_q \int_{-1}^{1} T^g(x, S_T) \, dx$ are small, disfavoring large cancellations among the flavors, but rather suggesting the $x$ integral of the QS function to be small for each flavor separately. In this way a small gluon QS/Sivers function and a node in the quark Sivers functions would go hand in hand.

In conclusion, the $x$-dependent version of the ESGM relation presented here implies that the QS function changes sign as a function of $x$, but it is based on some assumptions that may not hold. It would be very interesting to test this relation in experiment, even outside the region of a possible node, since the proportionality relation should hold for all $x$ values. Confirmation of or deviations from the relation could teach us about the qualitative features of the gluonic field strength inside the proton. Despite this uncertainty a node should be considered as a serious possibility, since twist-3 functions are not probability densities, therefore need not be positive definite. It would be just as natural for the QS function to have a node as it is for the twist-3 part of $g_2$. A sign change as function of $x$ also offers another way to satisfy the Burkardt sum rule, without requiring large cancellations among the different flavors. This sum rule together with the integrated ESGM relation obtained here suggest that a small gluon QS/Sivers function disfavors such large cancellations. A node is found in a few model calculations for the Sivers function, but not in most, and not for up and down quarks simultaneously, contrary to expectation from the large $N_c$ limit. Extending these model calculations of the Sivers function to include its full Wilson line, or equivalently performing model calculations of the Qiu–Sterman function, would therefore be very interesting.

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References