Anti-analog giant dipole resonances and the neutron skin of nuclei

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ABSTRACT

We examine a method to determine the neutron-skin thickness of nuclei using data on the charge-exchange anti-analog giant dipole resonance (AGDR). Calculations performed using the relativistic proton-neutron quasiparticle random-phase approximation (pn-RQRPA) reproduce the isotopic trend of the excitation energies of the AGDR, as well as that of the spin-flip giant dipole resonances (IVGDR), in comparison to available data for the even–even isotopes 112–124 Sn. It is shown that the excitation energies of the AGDR, obtained using a set of density-dependent effective interactions which span a range of the symmetry energy at saturation density, supplemented with the experimental values, provide a stringent constraint on value of the neutron-skin thickness. For 124 Sn, in particular, we determine the value \( \Delta R_{pn} = 0.21 \pm 0.05 \) fm. The result of the present study shows that a measurement of the excitation energy of the AGDR in \((p, n)\) reactions using rare-isotope beams in inverse kinematics, provides a valuable method for the determination of neutron-skin thickness in exotic nuclei.

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1. Introduction

An interesting phenomenon in nuclear structure is the formation of a skin of neutrons on the surface of a nucleus, and its evolution with mass number in an isotopic chain [1]. A precise measurement of the thickness of neutron skin is important not only because this quantity represents a basic nuclear property, but also because it constrains the symmetry energy term of the nuclear equation of state [2–7]. A detailed knowledge of the symmetry energy is essential for describing the structure of neutron-rich nuclei, and for modeling properties of neutron-rich matter in applications relevant for nuclear astrophysics.

The difference between the neutron and proton rms radii is rather small (few percent) and a precise measurement of the neutron-skin thickness presents a considerable challenge. Several methods have been used to determine this quantity [2,8–13], but almost all of these are applicable only to stable nuclei and the results are model dependent [10]. Methods based on coherent nuclear motion include excitations of the isovector giant dipole resonance (IVGDR) [9], the isovector spin giant dipole resonance (IVSGDR) [10], the Gamow–Teller resonance (GTR) measured relative to the isobaric analog state (IAS) [14], and high-resolution study of the electric dipole polarizability [3].

The Pb radius experiment (PREX) at JLAB [5] has initiated a new line of research based on the parity-violating elastic electron scattering to measure the neutron density radius \( R_n \), which in turn allows to determine the neutron-skin thickness from \( \Delta R_{pn} = R_n - R_p \), where \( R_p \) is the radius of the proton density distribution. Although parity-violating elastic electron scattering provides a model independent measurement of \( \Delta R_{pn} \), its current precision is far from satisfactory and the method cannot be applied to unstable isotopes.

Radioactive ion beams (RIBs) have recently been employed to determine the neutron-skin thickness in unstable nuclei, specifically in measurements of reaction cross-sections and pygmy dipole resonances [1,15,16]. For an accurate determination of this quantity using RIBs, it is imperative to find a feasible method that employs reactions with low-intensity RIBs in inverse kinematics. We have recently introduced a new method [17,18] based on the excitation of the anti-analog giant dipole resonance (AGDR) observed in \((p, n)\) reaction [19]. As pointed out by Krmpotić, the excitation of the AGDR depends sensitively on the neutron-skin thickness [20] and, therefore, \( \Delta R_{pn} \) could be deduced from the measurement of AGDR excitation energy.

The main objective of this work is to test the method that determines the neutron-skin thickness in nuclei from AGDR data. By calculating excitation energies \( E(AGDR) \) and \( \Delta R_{pn} \) in a fully self-consistent theoretical approach, and comparing to available data, the feasibility of the method will be tested with the aim to provide a basis for future studies with RIBs.
The Letter is organized as follows. In Section 2 we present a short outline of the AGDR mode and discuss its properties with the aim to constrain the neutron-skin thickness. The experimental results on AGDR and IVSGDR obtained from $(p, n)$ and $(^3He, t)$ reactions for Sn isotopes are reviewed in Section 3. The theoretical framework and model calculations of excitation energies of the AGDR and IVSGDR, and neutron-skin thickness for the even Sn isotopes, are described in Section 4. The centroid of the AGDR for $^{124}$Sn is deduced from an earlier $(p, n)$ measurement in Section 5, and used to determine the corresponding neutron-skin thickness by comparing with model calculations. Section 6 includes the summary and a brief outlook for future studies.

2. Charge-exchange AGDR and neutron-skin thickness

The AGDR corresponds to the $\Delta J^T = 1^-$, $\Delta L = 1$ resonant excitation, and represents the anti-isospin giant dipole resonance, i.e., the $T_0 - 1$ component of the charge-exchange GDR ($T_0$ is the ground-state isospin of the target nucleus). Fig. 1 illustrates the ground state and the giant dipole resonance (GDR) state of a target nucleus ($T_z = T_0$), and the corresponding resonant states in the daughter nucleus reached by the $(p, n)$ charge-exchange reaction: the IAS (isospin $= T_0$), and the anti-analog (isospin $= T_0 - 1$) states: GTR, IVSGDR, and AGDR [19].

The transition strength of dipole excitations is fragmented into the $T_0 - 1$, $T_0$ and $T_0 + 1$ components because of the isovector nature of the $(p, n)$ reaction. The pertinent Clebsch–Gordan coefficients [21] show that the $T_0 - 1$ component (AGDR) is favored with respect to the $T_0$ and $T_0 + 1$ components, by factors of $T_0$ and $2T_0^2$, respectively. Accordingly, Fig. 1 shows only components that are strongly favored by isospin selection rules for large $T_0$ [19].

For the charge-exchange dipole operator

$$\hat{\mathcal{O}}_\pm = \sum_i r_i Y_{10}(\hat{r}_i) r^{(i)}_\pm,$$

(1)

where $r^{(i)}_\pm$ denotes isospin raising and lowering operators, one obtains the non-energy-weighted sum rule (NEWSR) [22],

$$S^- - S^+ = \frac{1}{2\pi} \left( N(r^-_n) - Z(t^-_p) \right).$$

(2)

$S^-$ and $S^+$ denote the sums of transition strengths in the $\beta^-$ and $\beta^+$ channels calculated using Eq. (1), respectively. The AGDR mode is mediated by the $\hat{\mathcal{O}}_-$ operator. We note that the sum rule Eq. (2) is proportional to the one of the IVSGDR (up to the factor three that corresponds to different spin components in the latter case), previously used to determine the neutron-skin thickness [10,23]. In Ref. [22], Auerbach et al. derived an energy-weighted sum rule (EWSR) for the dipole strength excited in charge-exchange reactions. However, this EWSR is not sensitive to the neutron-skin thickness [22].

Although the NEWSR provides information on the neutron-skin thickness, in practice it is not straightforward to determine $\Delta R_{pn}$ from the sum rule. The problem is that usually the non-energy-weighted strengths are not simultaneously available for both $\beta^-$ and $\beta^+$ channels, i.e., experiments are mainly performed in the $\beta^-$ channel and provide $S^-$ only. To deduce $\Delta R_{pn}$ from the NEWSR additional approximations or theoretical input are needed, e.g., the estimate for $S^+$ and the normalization constant [10]. One can, however, intuitively understand the relation between the neutron-skin thickness and the energy of the AGDR if one considers that for nuclei with $N - Z \gg 1$ one can neglect $S^+$ because of Pauli blocking. In such a case, the EWSR, which is a constant, can be expressed as the product of the NEWSR and the AGDR energy and thus one can easily understand the inverse proportionality between the AGDR energy and neutron-skin thickness, i.e., $E_{AGDR}$ decreases if $\Delta R_{pn}$ increases and vice versa.

Instead of the NEWSR constraint on $\Delta R_{pn}$, in this work we aim to establish an alternative approach motivated by the study of Ref. [20], where a simple schematic model indicated strong sensitivity of the AGDR excitation energy on $\Delta R_{pn}$. In Section 4 we will explore this relation using a fully microscopic theoretical approach and available data on the AGDR energies.

3. Isovector giant resonances excited by $(p, n)$ reactions

The first identification of a giant dipole transition excited in charge-exchange $(p, n)$ reactions was reported by Bainum et al. [24] for the case $^{90}$Zr$(p, n)^{90}$Nb at 120 MeV. In addition to the pronounced excitation of the GTR, a broad peak was observed at an excitation energy of 9 MeV above the GTR, with an angular distribution characteristic of a $\Delta L = 1$ transfer. This excitation energy is about 4 MeV below the location of the $T = 3$ analog of the known GDR in $^{90}$Zr, and thus it was suggested that this state is the $T = 4$ anti-analog of the GDR.

Dipole resonances have also been studied systematically in $(p, n)$ reactions at $E_p = 45$ MeV by Sterrenburg et al. [19], for 17 different targets from $^{92}$Zr to $^{208}$Pb. Nishihara et al. [25] measured also the dipole strength distributions at $E_p = 41$ MeV. It was shown [26,27] that the observed $\Delta L = 1$ resonance in general corresponds to a superposition of all possible spin-flip dipole (IVSGDR) and non-spin-flip dipole (AGDR) modes. According to Osterfeld [21], the non-spin-flip to spin-flip ratio is favored at low bombarding energy (below 50 MeV), and also at very high bombarding energy (above 600 MeV). Properties of the IVSGDR were further investigated by Gaarde et al. [28] using $(p, n)$ reactions on targets with mass of $40 \leq A \leq 208$, and by Pham et al. [29] in $(^3He, t)$ reactions. In every experimental spectrum a peak was observed at an energy several MeV above the GTR, with an angular distribution characteristic of a $\Delta L = 1$ transfer. The observed excitation energies of the AGDR [19] and IVSGDR [23,29], relative to the IAS, are shown in Fig. 2 as functions of the mass number for the even–even Sn isotopes. For both modes one observes a systematic decrease of the excitation energy along the Sn isotopic chain.
detected by a low-energy neutron-array (LENA) ToF spectrometer [31,32], developed in Debrecen. The spectrometer was placed at 1 m from the target, covering a laboratory scattering-angle region of $65^\circ \leq \Theta_{LAB} \leq 75^\circ$. In this way, the excitation energy of the AGDR and IAS can be determined, and new data enable studies of neutron-skin thickness in nuclei. One expects that future progress with RIBs and novel experimental techniques should provide data on the AGDR in exotic nuclei with even more pronounced neutron skin. The experimental feasibility of the suggested method is also supported by a recent publication in which the strength distribution of the Gamow–Teller giant resonance was studied by the $(p,n)$ reaction with RIBs [33] using a similar neutron spectrometer [34].

4. Theoretical analysis

To describe the evolution of excitation energies of the AGDR relative to the IAS, and their relation to $\Delta R_{pn}$, we perform a microscopic theoretical analysis based on relativistic nuclear energy density functionals. The theoretical framework is realized in terms of the fully self-consistent relativistic proton–neutron quasiparticle random-phase approximation (pn-RQRPA) based on the relativistic Hartree–Bogoliubov model (RHB) [35]. The pn-RQRPA is formulated in the canonical single-nucleon basis of the RHB model in Ref. [36], and extended to the description of charge-exchange excitations (pn-RQRPA) in Ref. [37]. The RHB+pn-RQRPA model is fully self-consistent: in the particle-hole channel effective Lagrangians with density-dependent meson-nucleon couplings are employed, and pairing correlations are described by the pairing part of the finite-range Gogny interaction [38].

For the purpose of the present study we employ a family of density-dependent meson-exchange (DD-ME) effective interactions for which the constraint on the symmetry energy at saturation density was systematically varied, and the remaining model parameters were adjusted to reproduce empirical nuclear-matter properties (binding energy, saturation density, compression modulus), and the binding energies and charge radii of a standard set of spherical nuclei [39]. These effective interactions were used to provide a microscopic estimate of the nuclear-matter incompressibility and symmetry energy in relativistic mean-field models [39], and in Ref. [15] to study a possible correlation between the observed pygmy dipole strength in $^{130,132}$Sn and the corresponding values for the neutron-skin thickness. In addition to a set of effective interactions with systematically varied values of the symmetry energy at saturation density, the relativistic functional DD-ME2 [30] is also used here to calculate the excitation energies of the AGDR with respect to the IAS, as a function of the neutron skin. Pertinent to the present analysis is the fact that the relativistic RPA with the DD-ME2 effective interaction predicts for the dipole polarizability [3]

$$\alpha_D = \frac{8\pi}{9} \pi^2 m^{-1}$$

(directly proportional to the inverse energy-weighted moment $m^{-1}$) of $^{208}$Pb the value $\alpha_D = 20.8 \text{ fm}^3$, in very good agreement with the recently measured value: $\alpha_D = (20.1 \pm 0.6) \text{ fm}^3$ [3].

In addition to the experimental excitation energies, Fig. 2 also includes the theoretical results obtained with the RHB+pn-RQRPA model using the DD-ME2 effective interaction. The difference in the excitation energy of the AGDR and the IAS, as well as between the IVSGDR and the IAS, for the even–even Sn isotopes are shown as functions of the mass number. For the excitation energies of the AGDR and the IVSGDR we take the centroids of the theoretical strength distributions: $m_1/m_0$, whereas a single peak is calculated for the IAS. Within the experimental uncertainty, we find a reasonable agreement between the data and the theoretical values for the AGDR. The largest deviations, $\approx 0.4$ MeV, correspond to $^{122,124}$Sn. The agreement is less satisfactory for the IVSGDR, with the discrepancy being especially large for $^{114,116}$Sn. Part of the discrepancy is probably caused by the overlapping of the two resonances. Experimentally it is possible to enhance the excitation of the AGDR with respect to the IVSGDR by choosing a low bombarding energy ($\leq 45$ MeV), but it is expected that the suppressed IVSGDR can still cause some lowering of the energy centroid of the AGDR observed in the $(p,n)$ reaction since the former has a lower excitation energy. On the other hand, the excitation of the IVSGDR can be enhanced at higher bombarding energy (around 150–200 MeV) but a small fraction of the AGDR still remains, raising the centroid energy as the energy of the AGDR is higher than that of the IVSGDR. Since the IVSGDR has three components with $\Delta J^\pi = 0^+$, $1^-$ and $2^-$, its strength generally spreads over a larger energy interval compared to the AGDR. The model calculation cannot reproduce these structures as precisely as the centroid energy of the AGDR, and this is the reason why in this work we make use of the AGDR to determine the neutron-skin thickness.

The calculated values of the neutron-skin thickness for the Sn isotopes as a function of mass number are compared to available data in Fig. 3. The RHB neutron-skin thicknesses obtained using the DD-ME2 effective interaction are in very good agreement with the experimental values obtained using various methods [2,10,11], together with the consistent RHB+pn-RQRPA model calculation of $\Delta R_{pn}$ and the AGDR energy, is used to constrain the value of the neutron-skin thickness. We consider the available data for the AGDR for $^{124}$Sn from Sterrenburg et al. [19] ($E(AGDR) - E(IAS) = 10.60 \pm 0.20$ MeV), but slightly increased to $E(AGDR) - E(IAS) = 10.93 \pm 0.20$ MeV in order to approximately compensate the effect.
of the energy shift caused by the mixing with the IVSGDR. Below we explain how this energy shift is determined.

Austin et al. [27] developed a phenomenological model to describe the variation with bombarding energy of the peak positions of the AGDR and IVSGDR observed in \((p, n)\) reactions. They assumed that the position \(C\) of the centroid of the \(\Delta I = 1\) excitations (including both the AGDR and IVSGDR) at a bombarding energy \(E_p\) is given by the weighted average of the energies:

\[
C = \frac{\sigma_0 E_0 + \sigma_1 E_1}{\sigma_0 + \sigma_1} = E_0 - \frac{\sigma_1/\sigma_0}{1 + \sigma_1/\sigma_0} \Delta,
\]

where \(E_0\) (\(E_1\)) is the energy of the AGDR (IVSGDR), \(\Delta = E_0 - E_1\) and \(\sigma_0\) (\(\sigma_1\)) is the cross-section for \(S = 0\) (\(S = 1\)) transfer. They estimated the \(\sigma_1/\sigma_0\) ratio by \(\sigma_1/\sigma_0 \approx (E_p/(\text{MeV})/55)^2\) [27] and obtained the energy of the AGDR in \(^{124}\text{Sn}\) to be \(14.4 \pm 2.2\) MeV, which is completely different from any theoretical prediction [27]. In reality, the centroid of the dipole strength distribution is usually determined by fitting the distribution by a Gaussian or a Lorentzian curve. This makes a significant difference in case of \(^{124}\text{Sn}\), where the AGDR and the IVSGDR display very different widths: 3.6 MeV [19] and 9 MeV [29], respectively.

To determine the energy shift of the AGDR peak at \(E_p = 45\) MeV from the empirical peak energy, we simulate the mixing of the AGDR and IVSGDR by using their real widths of 3.6 MeV and 9 MeV, the ratio of their intensities as approximated by Austin et al. [27], and their energy difference \(\Delta = 2.3\) MeV obtained from Fig. 2. The composite spectrum is then fitted by a Gaussian curve in a reasonably wide energy range \((\pm 5\) MeV) around the position of the peak, and this yields an energy shift of 0.33 MeV for the AGDR.

The sensitivity of the centroid energy of the AGDR to the neutron-skin thickness of \(^{124}\text{Sn}\) is explored by performing RHB + pn-RQRPA calculations using a set of the effective interactions with different values of the symmetry energy at saturation: \(a_4 = 30, 32, 34, 36\) and 38 MeV (and correspondingly different slopes of the symmetry energy [6]) and, in addition, the DD-ME2 effective interaction \((a_4 = 32.3\) MeV). In Fig. 4, the resulting energy differences \(E(\text{AGDR}) - E(\text{IAS})\) are plotted as a function of the corresponding neutron-skin thickness \(\Delta R_{pn}\) predicted by these effective interactions.

The two parallel solid lines in Fig. 4 delineate the region of theoretical uncertainty for the used set of effective interactions. When adjusting the parameters of these interactions [30,39], an uncertainty of 10\% was assumed for the difference between neutron and proton radii for the nuclei \(^{118}\text{Sn}, ^{124}\text{Sn},\) and \(^{208}\text{Pb}\). This set of interactions was also used to calculate the electric dipole polarizability and neutron-skin thickness of \(^{208}\text{Pb}, ^{132}\text{Sn},\) and \(^{48}\text{Ca}\), in comparison with the predictions of more than 40 non-relativistic and relativistic mean-field effective interactions [7].

By comparing the experimental result for \(E(\text{AGDR}) - E(\text{IAS})\) to the theoretical energy differences (see Fig. 4), we deduce the value of the neutron-skin thickness in \(^{124}\text{Sn}: \Delta R_{pn} = 0.205 \pm 0.050\) fm (including theoretical uncertainties). In Table 1 the value for \(\Delta R_{pn}\) determined in the present analysis is compared to previous results obtained with a variety of experimental methods. The very good agreement with previously determined values reinforces the expected reliability of the proposed method.

### Table 1

<table>
<thead>
<tr>
<th>Method</th>
<th>Ref.</th>
<th>Date</th>
<th>(\Delta R_{pn}) (fm)</th>
</tr>
</thead>
<tbody>
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<td>0.8 GeV</td>
<td>[8]</td>
<td>1979</td>
<td>0.25 \pm 0.05</td>
</tr>
<tr>
<td>((\alpha,\alpha')\ IGDR 120 MeV</td>
<td>[40]</td>
<td>1994</td>
<td>0.21 \pm 0.11</td>
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<tr>
<td>Antiproton absorption</td>
<td>[11,12]</td>
<td>2001</td>
<td>0.19 \pm 0.09</td>
</tr>
<tr>
<td>((\alpha,\alpha')\ IVGDR + AGDR</td>
<td>[10]</td>
<td>2004</td>
<td>0.27 \pm 0.07</td>
</tr>
<tr>
<td>Pygmy dipole resonance</td>
<td>[15,13]</td>
<td>2007</td>
<td>0.19 \pm 0.05</td>
</tr>
<tr>
<td>((p, p)) 295 MeV</td>
<td>[2,13]</td>
<td>2008</td>
<td>0.185 \pm 0.05</td>
</tr>
<tr>
<td>AGDR present result</td>
<td></td>
<td>2013</td>
<td>0.21 \pm 0.05</td>
</tr>
</tbody>
</table>

The very good agreement with previously determined values reinforces the expected reliability of the proposed method.

### 6. Conclusion and outlook

A method to determine the size of the neutron-skin thickness in nuclei using data on the anti-analog giant dipole resonance has been discussed. Charge-exchange \((p, n)\) reactions provide an excellent probe for the neutron-skin thickness, as already demonstrated by measurement of the IVSGDR and GTR, and the AGDR provides a complementary approach. In contrast to the IVSGDR, which displays a complex underlying structure with three overlap-
ping components and its strength spreads over a large energy interval, the AGDR represents a rather simple charge-exchange mode ($\Delta J^T = 1^-$, $\Delta L = 1$). While previous analyses were based on sum rules, this work introduces an alternative self-consistent approach that could systematically be used not only for the AGDR, but also for other modes sensitive to the neutron skin.

As a first test, we have used the self-consistent RHB plus proton–neutron RQRPA to calculate the $T_0 = 1$ dipole excitations in a sequence of Sn isotopes. By using effective interactions with density-dependent meson-nucleon couplings in the particle-hole channel, and the pairing part of the Gogny interaction D1S for the $T = 1$ pairing channel, it has been possible to reproduce the experimental results on the excitation energy of the AGDR relative to the isobaric analog state. We have also shown that the isotopic dependence of the energy difference between the AGDR and IAS provides direct information on the evolution of neutron-skin thickness along the Sn isotopic chain. Very good results have been obtained in comparison with available data on the neutron-skin thickness. The present analysis demonstrates that this quantity can be determined by measuring the excitation energies of the AGDR relative to IAS.

The accuracy of the method has been tested in the example of $^{124}$Sn. By employing a set of effective interactions that span a broad range of values for the neutron-skin thickness (as a result of variation of the symmetry energy at saturation density), the size of the neutron skin has been determined from the AGDR energies relative to IAS. The result is in very good agreement with previously published experimental values. More extensive studies, in line with recent work on the electric dipole polarizability and neutron-skin thickness [7] that has employed families of non-relativistic and relativistic energy density functionals, would allow a further reduction of theoretical uncertainties. The successful test of the method based on the AGDR holds promise for determining the size of the neutron-skin of unstable neutron-rich exotic nuclei, and this is reinforced by recent advances in the development of RIBs.

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