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Low & high scale MSSM inflation, gravitational waves and constraints from Planck

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Abstract. In this paper we will analyze generic predictions of an inflection-point model of inflation with Hubble-induced corrections and study them in light of the Planck data. Typically inflection-point models of inflation can be embedded within Minimal Supersymmetric Standard Model (MSSM) where inflation can occur below the Planck scale. The flexibility of the potential allows us to match the observed amplitude of the TT-power spectrum of the cosmic microwave background radiation with low and high multipoles, spectral tilt, and virtually mild running of the spectral tilt, which can put a bound on an upper limit on the tensor-to-scalar ratio, \( r \leq 0.12 \). Since the inflaton within MSSM carries the Standard Model charges, therefore it is the minimal model of inflation beyond the Standard Model which can reheat the universe with the right thermal degrees of freedom without any dark-radiation.

Keywords: particle physics - cosmology connection, supersymmetry and cosmology, inflation, CMBR theory

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1 Introduction

The observational success of primordial inflation arising from the cosmic microwave background (CMB) radiation \cite{1, 2} has lead to an outstanding question how to embed the inflationary paradigm within a particle theory \cite{3}. Since inflation dilutes all matter except for the quantum vacuum fluctuations of the inflaton, it is pertinent that end of inflation creates all the relevant Standard Model degrees of freedom for the success of Big Bang Nucleosynthesis \cite{4}, without any extra relativistic degrees of freedom, i.e. dark radiation \cite{5}.\footnote{Embedding the last 50–60 e-foldings of inflation within string theory has a major disadvantage. Due to large number of hidden sectors arising from any string compactifications, it is likely that the inflaton energy density will get dumped into the hidden sectors instead of the visible sector \cite{6}. The branching ratio for the inflaton decay into the visible sector is very tiny, therefore reheating the Standard Model degrees of freedom is one of the biggest challenges for any string motivated models of inflation. Furthermore, many of the compactifications generically lead to extra dark radiation (massless axions) which are already at the verge of being ruled out by the present data \cite{7}.}

This immediately suggests that the inflationary vacuum cannot be arbitrary and the inflaton must decay solely into the Standard Model degrees of freedom. Furthermore, the recent Planck data \cite{5} indicates that the perturbations in the baryons and the cold dark matter are adiabatic in nature, it is evident that there must be a single source of perturbations which is responsible for seeding the fluctuations in all forms of matter.\footnote{In principle more than one fields can still participate during inflation, but they must do so in such a way that here exists an attractor solution which would yield solely adiabatic perturbations and no isocurvature perturbations, such as in the case of assisted inflation \cite{9}.}

Strictly speaking this can happen only if the inflaton itself carries the Standard Model charges as in the case of Minimal Supersymmetric Standard Model (MSSM) flat-directions \cite{8},
where the lightest supersymmetric particle could be the dark matter candidate and can be created from thermal annihilation of the MSSM degrees of freedom. MSSM inflation was first discussed in refs. [10–12], and in recently, see refs. [13–18]. The inflaton candidates are made up of 

\[ \text{gauge invariant} \] combinations of squarks (supersymmetric partners of quarks) and sleptons (supersymmetric partners of leptons).

One of the key ingredients for embedding inflation within MSSM is that the inflaton VEV must be below the Planck scale, \( M_{\text{PL}} = 2.4 \times 10^{18} \text{ GeV} \). This justifies the application of an effective field theory treatment at low energies. It is well-known that the potential for the MSSM flat-direction inflaton has high degree of \textit{flexibility} — the potential can accommodate \textit{inflection-point} below the Planck VEV, which allows a rich class of flat potentials, with a vanishing effective mass, which has been studied analytically and numerically [11, 19, 20]. The application of inflection point inflation is not just limited to particle theory, but such potentials have also found their applications in string theory [21].

It has also been known that inflection-point models of inflation can occur for a wide range of Hubble values, \( H_{\text{inf}} \), ranging from \( 10^{-1} \text{ GeV} \leq H_{\text{inf}} \leq 10^{13} \text{ GeV} \). For very high scale inflation there is a possibility of obtaining signatures for the primordial gravitational waves, namely the B-modes [22] below the Planck scale.\(^3\)

The aim of this paper is to show explicitly how large scale inflection-point inflation can match the current CMB observables, namely the TT-part of the temperature anisotropy spectrum, low and high multipoles, spectral tilt, running of the spectral tilt, and running of running of the spectral tilt. We will also provide the ranges of tensor-to-scalar ratio \( r \), which is compatible with all the data sets.\(^4\)

In section 2, we will discuss the generality of inflection-point potential. In section 2.1, we will provide a brief discussion on supergravity corrections, and the two regimes of inflation will be discussed in section 3. In section 4, we will discuss the cosmological observables relevant for CMB, in section 5, we will discuss reheat temperature and the number of e-foldings. In section 6, we will discuss tensor-to-scalar ratio of high and low scale models of inflation. In section 7, we will provide the TT-power spectrum (low and high multipoles) for a particular realisation, and in section 8, we will conclude our results.

2 Flat potential around the inflection-point

The most generic inflection-point potential can be recast as [20, 22]:

\[
V(\phi) = \alpha + \beta(\phi - \phi_0) + \gamma(\phi - \phi_0)^3 + \kappa(\phi - \phi_0)^4 + \cdots ,
\]

where any generic potential, \( V(\phi) \), has been expanded around the inflection-point, \( \phi_0 \), where \( \alpha \) denotes the cosmological constant, and coefficients \( \beta, \gamma, \kappa \) determine the shape of the potential in terms of the model parameters. Typically, \( \alpha \) can be set to zero by fine tuning, but here we wish to keep this term for generality. Note that not all of the coefficients are independent once we prescribe inflaton within MSSM.

In refs. [10, 11, 13, 28, 29] the authors have recognized two \textit{D-flat} directions which can be the ideal inflaton candidates. Both \( \tilde{u}d\bar{d} \), where \( \tilde{u}, \bar{d} \) correspond to the right handed squarks,\(^3\)

\(^3\)We are assuming that the gravitational modes can be quantized with quantum initial conditions. If the gravity waves behave classically, then the amplitude of the gravitational waves in a simple scalar field driven model will be absolutely zero [23], as there is no source term for the gravitational waves.

\(^4\)Similar studies were undertaken for hybrid inflation model [24, 25], after the \textit{Planck} data release, see [26]. For power law inflation, see [27].
and $\tilde{L}\tilde{e}$, where $\tilde{L}$ is the left handed slepton, and $\tilde{e}$ is the right handed (charged) leptons, flat directions are lifted by higher order superpotential terms of the following simple form:

$$W(\Phi) = \frac{\lambda}{6} \Phi^6 M_{\text{PL}}^3,$$  

(2.2)

where $\lambda \sim \mathcal{O}(1)$ coefficient. The scalar component of $\Phi$ superfield, denoted by $\phi$, is given by

$$\phi = \frac{\tilde{u} + \tilde{d} + \tilde{d}}{\sqrt{3}}, \quad \phi = \frac{\tilde{L} + \tilde{L} + \tilde{e}}{\sqrt{3}},$$  

(2.3)

for the $\tilde{u}\tilde{d}\tilde{d}$ and $\tilde{L}\tilde{L}\tilde{e}$ flat directions, respectively.

2.1 Brief discussion on Hubble induced corrections

In addition to eq. (2.2), there are many possible contributions to the vacuum energy. It is conceivable that at high energies the universe is dominated by a large cosmological constant arising from a string theory landscape [30]. Our own patch of the universe could be locked in a false vacuum within an MSSM landscape [31], or there could be hidden sector contributions [32, 33], or there could be a combination of these effects. For simplicity we may attribute such a vacuum energy to a hidden sector. A hybrid model of inflation [24] also provides a source of vacuum energy density during inflation. For the purpose of illustration, we will model earlier phases of inflation driven by the superpotential of type:

$$W(S) = M^2 S,$$  

(2.4)

where $M$ is some high scale which dictates the initial vacuum energy density, and $S$ is the hidden sector superfield. The total Kähler potential can be of the form [34–36]:

$$K = S^\dagger S + \Phi^\dagger \Phi + \delta K,$$  

(2.5)

where the non-minimal term $\delta K$ can be any of these functional forms:

$$\delta K = f(\Phi^\dagger \Phi, S^\dagger S), \quad f(S^\dagger \Phi \Phi), \quad f(S \Phi^\dagger \Phi), \quad f(S^\dagger \Phi \Phi)$$

We will always treat the fields $s, \phi \ll M_{\text{PL}}$. The higher order corrections to the Kähler potential are extremely hard to compute. It has been done within a string theory setup [37] but only in very special circumstances, and not for MSSM fields. The scalar potential in $\mathcal{N} = 1$ supergravity can be written in terms of superpotential, $W$, and Kähler potential, $K$, as

$$V = e^{K(\Phi, \Phi^\dagger)/M_{\text{PL}}^2} \left[ (D_\Phi W(\Phi)) K^{\Phi, \Phi_j} (D_\Phi W^*(\Phi^\dagger)) - \frac{3}{M_{\text{PL}}^2} |W(\Phi)|^2 \right] + (\text{D-terms}),$$  

(2.6)

where $F_\Phi \equiv D_\Phi W = W_\Phi + K_\Phi W/M_{\text{PL}}^2$, and $K^{\Phi, \Phi_j}$ is the inverse matrix of $K_{\Phi, \Phi_j}$, and the subscript denotes derivative with respect to the field. Hereafter, we neglect the contribution from the D-term, since the MSSM inflatons are D-flat directions. In the above potential $W = W(S) + W(\Phi)$.

After minimizing the potential along the angular direction, $\theta$ ($\Phi = \phi e^{i\theta}$), we take the real part of $\phi$ by rotating it to the corresponding angle $\theta$, resulting in [34–36]:

$$V(\phi, \theta) = V_0 + \frac{(m_\phi^2 + c_H H^2)}{2} |\phi|^2$$

$$+ \left( a_H H + \lambda a_\lambda \right) \frac{\lambda}{6 M_{\text{PL}}^3} \sqrt{3} \cos(6\theta + \theta_{a_H} + \theta_{a_\lambda}) + \frac{\lambda^2 |\phi|^{10}}{M_{\text{PL}}^3},$$  

(2.7)
where the cosmological constant will be determined by the overall inflationary potential, $V_0 \approx 3H^2M_{\text{Pl}}^2$. Usually this bare cosmological term can be set to zero from the beginning by tuning the graviton mass. We will consider scenarios, where we will have $V_0 \neq 0$ and $V_0 = 0$.

Note that $m_\phi$ and $a_\lambda$ are soft-breaking mass and the non-renormalizable $A$-term respectively ($A$ is a positive quantity since its phase is absorbed by a redefinition of $\theta$ during the process). The potential also obtains Hubble-induced corrections, with coefficients $c_H, a_H \sim \mathcal{O}(1)$. Their exact numerical values will depend on the nature of Kähler corrections and compactification, which are hard to compute for a generic scenario \cite{37}, but the corrections typically yield $\sim \mathcal{O}(1)$ coefficients. The non-renormalizable terms have a periodicity of $2\pi$ in $(\phi, \theta)$ 2D plane, $\theta_{a_H}, \theta_{a_\lambda}$ are the extra phase factors.

3 Two regimes of inflation: low & high

There are two regimes where one can describe the dynamics of the potential:

3.1 $m_\phi \gg H$: low scale inflation

Since $c_H, a_H \sim \mathcal{O}(1)$ the Hubble-induced terms do not play any crucial role in this case, and the scale of inflation remains very low. As a result the tensor-to scalar ratio, $r$, becomes too small to be ever detectable. This was the scenario studied in refs. \cite{10, 11}. In this case we can set $V_0 = 0$ from the beginning by tuning the gravitino mass \cite{38}. The potential can be minimized along the $\theta$ direction, which reduces to \cite{10, 11}:

$$V(\phi, \theta) = \frac{m_\phi^2}{2} |\phi|^2 - a_\lambda m_\phi \frac{\lambda \phi^6}{6 M_{\text{Pl}}^3} + \frac{\lambda^2 |\phi|^10}{M_{\text{Pl}}^6}$$ \hspace{1cm} (3.1)

For,

$$\alpha = V(\phi_0) = 4 \frac{m_\phi^2}{15} \phi_0^2 + \mathcal{O}(\delta^2),$$ \hspace{1cm} (3.5)

$$\beta = V'(\phi_0) = 4 \alpha^2 m_\phi^2 \phi_0 + \mathcal{O}(\delta^4),$$ \hspace{1cm} (3.6)

$$\gamma = V''(\phi_0) = 32 \frac{m_\phi^2}{\phi_0} + \mathcal{O}(\delta^2).$$ \hspace{1cm} (3.7)

\footnote{The masses are given by:

$$m_\phi^2 = m_{\tilde{L}}^2 + m_{\tilde{L}}^2 + m_{\tilde{D}}^2, \quad m_\phi^2 = m_{\tilde{u}}^2 + m_{\tilde{d}}^2 + m_{\tilde{d}}^2$$

for $\tilde{L}\tilde{e}$ and $\tilde{u}\tilde{d}$ directions respectively. Typically these masses are set by the scale of supersymmetry, in the low scale case the masses will be typically of order $\mathcal{O}(1)\text{TeV}$.}

For, $\frac{a_\lambda^2}{40} = 1 - 4\delta^2,$ \hspace{1cm} (3.2)
and $\delta^2 \ll 1$, there exists a point of inflection $(\phi_0)$ in $V(\phi)$, where

$$\phi_0 = \left( \frac{m_\phi M_{\text{Pl}}^3}{\lambda \sqrt{10}} \right)^{1/4} + \mathcal{O}(\delta^2),$$ \hspace{1cm} (3.3)

$$V''(\phi_0) = 0,$$ \hspace{1cm} (3.4)

at which
The potential is specified completely by \( m_\phi \) and \( \lambda \). However \( m_\phi \) is determined by the soft-SUSY breaking mass parameter, which is well constrained by the current ATLAS \( [39] \) and CMS \( [40] \) data, and we shall take \( m_\phi = 1 \text{ TeV.} \) For \( m_\phi \sim 1 \text{ TeV,} \ H \sim 0.1 \text{ GeV,} \) and our assumption of neglecting \( H \) in such a case is well justified. We will always consider \( \lambda = 1 \) in our analysis.

### 3.2 \( H \gg m_\phi \): high scale inflation

The supergravity corrections become important, the Hubble-induced terms dominate the potential. This can happen quite naturally if there exists a previous source of effective cosmological constant term described in refs. \( [14, 22] \). In this case one can safely ignore the soft SUSY breaking mass term, and since \( a_\lambda \sim \mathcal{O}(1) \), one can safely consider only the Hubble-induced non-renormalizable term. One advantage of considering such a potential is to obtain large tensor-to-scalar ratio, \( r \), which can be within the range of Planck and other future CMB B-mode polarization experiments. We will keep \( V_0 \) in this case, and the potential simplifies to \( [14] \):

\[
V(\phi) = V_0 + \frac{c_H H^2}{2} |\phi|^2 - \frac{a_H H \phi^6}{6 M_{\text{PL}}^3} + |\phi|^{10} M_{\text{PL}}^6.
\]

where we have taken \( \lambda = 1 \). The potential admits inflection point for

\[
a_H^2 \approx 40 c_H^2.
\]

When \( |\delta| \) is small, a point of inflection \( \phi_0 \) exists such that \( V''(\phi_0) = 0 \), with

\[
\phi_0 = \left( \frac{c_H}{10} H M_{\text{PL}}^3 \right)^{1/4}.
\]

For \( \delta < 1 \), we can Taylor-expand the inflaton potential around the inflection point \( \phi = \phi_0 \) similar to eq. (2.1), where the coefficients are now given by:

\[
\alpha = V(\phi_0) = V_0 + \left( \frac{4}{15} + \frac{4}{3} \delta^2 \right) c_H H^2 \phi_0^2 + \mathcal{O}(\delta^4),
\]

\[
\beta = V'(\phi_0) = 4 \delta^2 c_H H^2 \phi_0 + \mathcal{O}(\delta^4),
\]

\[
\gamma = \frac{V''(\phi_0)}{3!} = \frac{c_H H^2}{\phi_0} \left( 32 - 80 \delta^2 \right) + \mathcal{O}(\delta^4),
\]

\[
\kappa = \frac{V'''(\phi_0)}{4!} = \frac{c_H H^2}{\phi_0^3} \left( 384 - 1260 \delta^2 \right) + \mathcal{O}(\delta^4).
\]

The shape and behavior of inflationary potential including Hubble induced corrections have been clearly shown in figure 1 and figure 3. Note that once we specify \( c_H \) and \( H \), all the terms in the potential are determined. In this regard the potential indeed simplifies a lot to study the cosmological observables. In section 7, we will be scanning over \( c_H, \ H \) to obtain the best fits with the current observations.

One must also ensure that the vacuum energy density which generated the large cosmological constant in the first place vanishes by the end of slow-roll inflation. This typically happens in the case of hybrid inflation \( [24, 25] \), and as discussed in \( [20, 22, 49] \). In the string
landscape [30], or in the case of MSSM [31], this can happen through bubble nucleation, provided the rate of nucleation is such that \( \Gamma_{\text{nucl}} \gg H \). In the latter case all the bubbles will belong to the MSSM vacuum — similar to the first order phase transition in the electroweak symmetry breaking scenario. However, one has to make sure that the cosmological constant disappears in the MSSM vacuum right at the end of inflation [58].

4 CMB observables and inflection point

In this section our primary focus is to study the cosmological observables to match the CMB data for an inflection-point inflation whose potential is given by eq. (3.8). First we use the following parameterizations for the amplitude of the scalar perturbations, \( P_S \), tensor pertur-
butions, $P_T$, and the tensor-to-scalar ratio $r$, in terms of the slow-roll parameters [41, 42]:

$$
P_S(k) = P_S(k_\star) \left( \frac{k}{k_\star} \right)^{n_S - 1 + \frac{\alpha_S}{2} \ln \left( \frac{k}{k_\star} \right) + \frac{\kappa_S}{2} \ln^2 \left( \frac{k}{k_\star} \right) + \ldots} , \quad (4.1)$$

$$
P_T(k) = P_T(k_\star) \left( \frac{k}{k_\star} \right)^{n_T - 1 + \frac{\alpha_T}{2} \ln \left( \frac{k}{k_\star} \right) + \frac{\kappa_T}{2} \ln^2 \left( \frac{k}{k_\star} \right) + \ldots} , \quad (4.2)$$

$$
r(k) = \frac{P_T(k)}{P_S(k)} = r(k_\star) \left( \frac{k}{k_\star} \right)^{n_T - n_S + 1 + \frac{\alpha_T - \alpha_S}{2} \ln \left( \frac{k}{k_\star} \right) + \frac{\kappa_T - \kappa_S}{2} \ln^2 \left( \frac{k}{k_\star} \right) + \ldots} . \quad (4.3)$$

where the observables are now given in terms of the inflationary potential, running of the spectral tilt $\alpha_S$, $\alpha_T$, and running of the running, given by $\kappa_S$, $\kappa_T$, where the subscripts $S$ and $T$ denote scalar and tensor modes. Here the consistency relations are modified at the second order due to the presence of running. The above parametrization are realized by expanding the scale dependent slow-roll parameters around the pivot scale $k = k_\star$ and they have been listed in an appendix (see eqs. (A.1), (A.2)).

Cosmological parameter estimation can be done more precisely once we allow the higher order radiative corrections to the slow-roll parameters [43]. We have listed all the relevant cosmological observables in an appendix (see eqs. (A.3)–(A.10)). In our case we have obtained the predicted power spectrum from the higher order radiative corrections to the slow-roll parameters, and also numerically we have evolved the perturbations which we will discuss below.

In order to illustrate this, let us consider the case when $H \gg m_\phi$. In this case there is a possibility of detecting large tensor-scalar ratio, $r$. We start our analysis numerically by using cosmological code CAMB [44] with the well known Bunch-Davies in-in vacuum state [45], and also perform the quantum fluctuations via cosmological linear perturbation theory, see ref. [41, 46]. In figure 2, we have shown the evolution of the power spectrum for the entire range of momentum scale across the 17 e-folds of inflation which has been observed by the Planck satellite [1, 47]. In this plot the orange dotted curve is obtained from the radiative corrections to the higher order slow-roll corrections. On the other hand the red curve is the outcome of our numerical analysis. As we can see that the higher order radiative corrections to the slow-roll parameters is quite a good approximation when compared to the results obtained from the numerical analysis for the observed range of $k_{\text{max}} = l_{\text{max}}/\eta_0 \pi \sim O(0.056 \ Mpc^{-1})$ for $l_{\text{max}} = 2500$ down to $k_{\text{min}} \sim O(10^{-5} \ Mpc^{-1})$ for $l_{\text{min}} = 2$, where $\eta_0 \sim 14000 \ Mpc$ is the conformal time at present epoch [1, 47]. The slow roll approximations differ very minutely from the numerical estimation, but for very large values of $k$, or very small wavelength regimes beyond $l \gg 2500$.

5 Estimation of reheat temperature and the scale of inflation

We need to compute the pivot scale, $k_\star$, when the relevant perturbations had left the Hubble patch during inflation. We can compute by expressing the number of e-foldings during inflation, which is given by [1, 48, 49]:

$$
N_\star \approx 71.21 - \ln \left( \frac{k_\star}{k_0} \right) + \frac{1}{4} \ln \left( \frac{V_\star}{M_P^4} \right) + \frac{1}{4} \ln \left( \frac{V_\star}{\rho_{\text{end}}} \right) + \frac{1 - 3w_{\text{int}}}{12(1 + w_{\text{int}})} \ln \left( \frac{\rho_{\text{rh}}}{\rho_{\text{end}}} \right) , \quad (5.1)
$$

where $\rho_{\text{end}}$ is the energy density at the end of inflation, $\rho_{\text{rh}}$ is an energy scale during reheating, $k_0 = a_0 H_0$ is the present Hubble scale, $V_\star$ corresponds to the potential energy when
Figure 2. We show the evolution of the power spectrum, \( P(k) \), for high scale model of inflation when \( H \gg m_\phi \). The specific values of the model parameters are: \( \delta \sim 10^{-4} \) (see eq. (3.9), \( \lambda = 1 \), \( c_H = 2 \), \( a_H = 2.108 \), \( \phi_0 = 1.129 \times 10^{16} \) GeV for the entire range of momentum, \( k \), that crosses the Hubble patch during the 17 e-folds of inflation. Here the orange dotted curve is obtained from the higher order radiative corrections to the slow-roll parameters, see appendix eq. (A.3), and the red curve from numerically integrating the cosmological perturbations. The blue vertical line corresponds to \( k_{\text{max}} = 0.056 \, \text{Mpc}^{-1} \) for \( l_{\text{max}} = 2500 \), which is the highest probing limit of recent Planck data. On the other hand the black vertical line corresponds to \( k_{\text{min}} = 4.488 \times 10^{-5} \, \text{Mpc}^{-1} \) for \( l_{\text{min}} = 2 \).

the relevant modes left the Hubble patch during inflation and \( w_{\text{int}} \) characterizes the effective equation of state parameter between the end of inflation and the energy scale during reheating. For our model we have \( w_{\text{int}} = 1/3 \) exactly for which the contribution from the last term in eq. (5.1) vanishes. For inflation driven by the MSSM flat directions the time scale for the transfer of energy to the MSSM relativistic species can be computed exactly as in [50]. This happens roughly within one Hubble time. This is due to the gauge couplings of the inflaton to gauge/gaugino fields. Within 10 – 20 inflaton oscillations radiation-dominated universe prevails, as shown in ref. [50]. The resultant upper-bound on the reheat temperature at which all the MSSM degrees of freedom are in thermal equilibrium (kinetic and chemical equilibrium) is given by [50]

\[
T_{\text{rh}} = \left( \frac{30}{\pi^2 g_*} \right)^{\frac{1}{4}} \sqrt{V_\star} \leq 6.654 \times 10^{15} \sqrt{\frac{T_\star}{0.12}} \, \text{GeV},
\]

(5.2)

where we have used \( g_* = 228.75 \) (all the degrees of freedom in MSSM). Since the temperature of the universe is so high, the lightest supersymmetric particle (LSP) relic density is then given by the standard (thermal) freeze-out mechanism [51]. In particular, if the neutralino is the LSP, then its relic density is determined by its annihilation and coannihilation rates [28, 29, 52]. The advantage of realizing inflation in the visible sector is that it is possible to nail down the thermal history of the universe precisely. At temperatures below 100 GeV there will be no extra degrees of freedom in the thermal bath except that of the SM, therefore BBN can proceed without any trouble.
For low scale models of inflation, i.e. $m \phi \gg H$, the tensor modes are utterly negligible. For $m \phi \sim 1 \text{TeV}$, and $\phi_0 \sim 3 \times 10^{14} \text{GeV}$, the value of $H_* \sim 10^{-1} \text{GeV}$, see eq. (3.3). The estimation of the reheat temperature is given by the equality of the above eq. (5.2). The reheat temperature is typically $3 \times 10^8 \text{GeV}$ for the above parameters. Note that for $m \phi \gg H$, the tensor to scalar ratio $r$ does not scale with the reheat temperature.

Note that saturating the upper-bound on $r \sim 0.12$ would yield a large reheating temperature of the universe. It is sufficiently large to create gravitino from a thermal bath \cite{53, 54}. The gravitino production from the direct decay of the inflaton will be suppressed \cite{55}. In this case, the gravitino abundance is compatible with the BBN bounds, provided the gravitino mass, $m_{3/2} \geq \mathcal{O}(10) \text{TeV}$, see \cite{56}. The bound holds only for a decaying gravitino, for which the graviton will decay before the BBN. If gravitino happens to be the LSP, then such a high scale model of inflation with large Hubble-induced corrections will be ruled out, unless there is some late entropy injection or there are some kinematical reasons for which the gravitino production is highly suppressed \cite{57}.

### 6 Tensor to scalar ratio for high & low scale models of inflation

The Planck constraint implies that the tensor-to-scalar ratio, $r$, at the pivot scale $k = k_*$, corresponds to an upper bound on the energy scale of the Hubble induced inflection point inflation:

$$V_* \leq \frac{3}{2} \pi^2 P_S(k_*) r_* M_{\text{Pl}}^4 = (1.96 \times 10^{16} \text{GeV})^4 \frac{r_*}{0.12}$$

(6.1)

For an example, at the pivot scale $k_* = 0.002 \ Mpc^{-1}$, the corresponding upper bound on the energy density becomes $V_* = 1.89 \times 10^{16} \text{GeV}$. The equivalent statement can be made in terms of the upper bound on the numerical value of the Hubble parameter during inflation as

$$H_* \leq 9.241 \times 10^{13} \sqrt{\frac{r_*}{0.12}} \text{GeV}.$$  

(6.2)

Here in eq. (5.2), eq. (6.1) and eq. (6.2) the equalities hold good in high scale inflationary regime ($H \gg m_\phi$). The inequalities are more significant once we enter low scale inflationary ($m_\phi \gg H$) region.

We scan the model parameters for obtaining large tensor to scalar ratio, $r$, for the following values:

$$c_H \sim \mathcal{O}(1 - 10),$$

$$a_H \sim \mathcal{O}(10 - 100),$$

$$\lambda \sim \mathcal{O}(1),$$

$$\phi_0 \sim \mathcal{O}((1 - 3) \times 10^{16} \text{GeV})$$

(6.3)

Now including the higher order corrections to the slow-roll parameters, the inflationary observables are estimated from our model as following:

$$2.092 < 10^3 P_S < 2.297,$$

$$0.958 < n_S < 0.963,$$

$$r < 0.12,$$

$$-0.0098 < \alpha_S < 0.0003,$$

$$-0.0007 < \kappa_S < 0.006$$

(6.4)
Figure 3. For large scale inflation, $H \gg m_\phi$, we have shown the variation of $P_S$ vs $n_S$. The red curve shows the model parameters, $\delta \sim 10^{-4}$, $\lambda = 1$, $c_H = 2$, $a_H = 2.108$, $\phi_0 = 1.129 \times 10^{16}$ GeV, for the pivot scale $k_* = 0.002$ Mpc$^{-1}$. The green shaded region shows the $2\sigma$ CL. range in $n_s$ allowed by the Planck data [1]. Instead of getting a single solid red curve we get a black shaded region if we consider the full parameter space for high scale ($H \gg m_\phi$) inflation given by eq. (6.3).

which confronts the Planck+WMAP-9+BAO data set, [1, 5, 47, 59], well within $2\sigma$ CL. Furthermore, we consider the following values of the model parameters which match the TT-spectrum of the CMB data for high scale model of inflation, i.e. $H \gg m_\phi$,

$$
\begin{align*}
\delta &\sim 10^{-4}, & \lambda &= 1, & c_H &= 2, & a_H &= 12.650, \\
\phi_0 & = 1.129 \times 10^{16} \text{ GeV} = 4.704 \times 10^{-9} M_{PL}.
\end{align*}
$$

(6.5)

Using eq. (6.5), in figure 3 we have shown the behavior of the amplitude of the the power spectrum, $P_S$ with respect to spectral tilt, $n_S$ at the pivot scale $k_* = 0.002$ Mpc$^{-1}$ by a red curve. If we take care of the full parameter space, see eq. (6.3), there are solutions which have been shown in a black shaded region.

In principle, we can vary $H_*$ from high scales to low scales. Since in our case the advantage is that the thermal history is well established, we can trace the relevant number of e-foldings, i.e. $N_*$ for various ranges of $H_*$. By varying $10^{-1}$ GeV $< H_* < 9.241 \times 10^{13}$ GeV, we can probe tensor-to-scalar ratio for a wide range: $10^{-29} < r_* \leq 0.12$.

Furthermore, by using Planck+WMAP-9 and Planck+WMAP-9+BAO datasets with $\Lambda$CDM background along with different combined constraints, we have shown the status of inflection point inflationary model in the marginalized $1\sigma$ and $2\sigma$ CL. contours in figure 4. The allowed region from the model is explicitly shown by the green shaded region bounded by orange vertical lines parallel to $r$-axis. Along the vertical lines the number of e-foldings varies within $50 < N_* < 70$ (from left to right) for various ranges of $H_*$ as mentioned earlier. We have also shown a thick black line parallel to $n_S$ axis in figure 4 which discriminates between the low scale ($m_\phi \geq H$) and high scale ($H \gg m_\phi$) inflationary scenarios. Additionally, we have depicted various straight lines for the intermediate values of $H_*$ within the allowed region. The model also provides very mild running, $\alpha_s$, and running of running, $\kappa_s$, which is also shown in the marginalized $1\sigma$ and $2\sigma$ CL. contours in figure 5, where we have used Planck+WMAP-9 and Planck+WMAP-9+BAO datasets with $\Lambda$CDM background along with different combined observational constraints.
By varying $H_\star$ we can probe a wide range of tensor-to-scalar ratio: $10^{-29} < r_\star \leq 0.12$. The vertical line on the left corresponds to $N = 50$, while the right line corresponds to $N = 70$.

We show the joint $1\sigma$ and $2\sigma$ CL. contours using (a) Planck+WMAP-9 data with $\Lambda$CDM+$r$(Blue region), and $\Lambda$CDM+$r + \alpha_S$(Red region), (b) Planck+WMAP-9+BAO data with $\Lambda$CDM+$r$(Blue region) and $\Lambda$CDM+$r + \alpha_S$(Red region). The straight lines parallel to $n_S$ axis are drawn by varying the Hubble parameter $H_\star$ within the range $10^{-1}$ GeV $< H_\star \leq 9.241 \times 10^{13}$ GeV. The deep green line and the yellow line correspond to the upper and lower bound of $H_\star$ respectively. The green shaded region bounded by orange lines represent the allowed region obtained from the model. Additionally, the black thick line divides the low scale ($m_\phi \gg H$) and the high scale ($H \gg m_\phi$) regions of inflation.

7 Multipole scanning of TT-spectra and CMB anisotropy

In this section we study the TT-angular power spectrum for the CMB anisotropy. For our present setup at low $\ell$ region ($2 < \ell < 49$) the contributions from the running ($\alpha_S, \alpha_T$), and running of running ($\kappa_S, \kappa_T$) are very small. Consequently their additional contribution to the power spectrum for scalar and tensor modes becomes unity ($\sim O(1)$) (this is consistent with the initial condition at the pivot scale $k = k_\star$) and the original power spectrum becomes unchanged. As a result the proposed model will be well fitted with the Planck low-$\ell$ data.
Figure 5. We show the joint 1σ and 2σ CL contours using Planck+WMAP-9+BAO with (a) $\Lambda$CDM+$\alpha_S$(Blue region) and $\Lambda$CDM+$\alpha_S+\tau$(Red region), (b) $\Lambda$CDM+$\alpha_S+\kappa_S$(Blue region) background. The straight lines parallel to $n_S$ axis are drawn by varying the Hubble parameter $H_*$ within the range $10^{-1}$ GeV $< H_* \leq 9.241 \times 10^{13}$ GeV. The deep green line and the yellow line correspond to the upper and lower bound of $H_*$ respectively. The green shaded region bounded by orange lines represent the allowed region obtained from the model.

within high cosmic variance except for a few outliers. On the other hand, when we move towards high $\ell$ regime ($50 < \ell < 2500$) the contribution of running and running of running become stronger and this will enhance the power spectrum to a permissible value such that it will accurately fit Planck high-$\ell$ data within very small cosmic variance. In this way one can easily survey over all the multipoles starting from low-$\ell$ to high-$\ell$ using the same parameterizations as mentioned in eqs. (6.5).

From figure 6, we see that the Sachs-Wolfe plateau obtained from our model is non flat, confirming the appearance of running, and running of the running in the spectrum observed for low $\ell$ region ($l < 50$). For larger value of the multipole ($50 < \ell < 2500$), CMB anisotropy spectrum is dominated by the Baryon Acoustic Oscillations (BAO), giving rise to several ups and downs in the spectrum, see [60]. Note that high $l$ regions of our model are well fitted within the small cosmic variance observed by Planck. In the low $l$ region due to the presence of very large cosmic variance there may be other pre-inflationary scenarios which might be able to fit the TT-power spectrum better [47]. In our study we have considered only the possibility for which the model is well fitted for both low and high $l$ regions.
Figure 6. TT-power spectrum for $\ell$ ($2 < \ell < 2500$). The vertical line is drawn at $l = 50$ which separates the low-$l$ ($2 < \ell < 50$) and high-$l$ ($50 < \ell < 2500$) region. Here the TT-power spectrum is drawn for the parameter values mentioned in eq. (6.5) in the context of high scale ($H \gg m_\phi$) Hubble induced inflationary framework.

8 Conclusion

In this paper we have considered a simple model of inflection-point inflation motivated by the MSSM flat directions, where we have taken potentials with both supergravity corrections are important and negligible. In the former case, we yield significantly large tensor-to-scalar ratio, $r \leq 0.12$, for $H_\star \sim 9.24 \times 10^{13}$ GeV and the VEV $\phi_0 = 1.12 \times 10^{16}$ GeV. The model fits the amplitude of the power spectrum and the spectral tilt. The model predicts mild running and running of the running of the spectral tilt well within the $2\sigma$ uncertainties. In particularly, the high scale inflection-point inflation model fits the high $l$ multipoles of the Planck data quite well with the $\Lambda$CDM parameters. The low $l$ multipoles have high uncertainties and they are within the cosmic variance. The forthcoming polarization data from Planck will hopefully further constrain the inflection-point model of inflation. The model tends to predict a perfect match for the spectral tilt even for a small tensor-to-scalar ratio, $r$, as seen in figure 4.

The perturbations created from the slow roll evolution of the inflaton are Gaussian and adiabatic. The amplitude and the spectral tilt match very well with the Planck data. One of the advantages of the proposed model is that it is embedded fully within MSSM, and therefore it predicts the right thermal history of the universe with no extra relativistic degrees of freedom other than that of the Standard Model.

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A Inflationary observables in presence of higher order slow-roll corrections

The power spectrum as well as other relevant parameters in terms of the potential can be written as [41, 42]:

\[ \epsilon_V(k) = \epsilon_V - \frac{\alpha_T}{2} \ln \left( \frac{k}{k_*} \right) + \frac{\kappa_T}{4} \ln^2 \left( \frac{k}{k_*} \right) + \ldots, \]  
\[ \eta_V(k) = \eta_V - \left( \frac{\alpha_S - 3\alpha_T}{2} \right) \ln \left( \frac{k}{k_*} \right) + \frac{1}{2} \left( \kappa_S - 3\kappa_T + \{n_T^2 + \alpha_T \} \right) \left[ \frac{n_S - 3n_T - 1}{2} \right] \]  
\[ + \frac{\xi_T^2}{2} \{ 1 - n_S - 3n_T \} \ln^2 \left( \frac{k}{k_*} \right) + \ldots, \]  
\[ \xi_V^2(k) = \xi_V^2 - \frac{1}{2} \left( \kappa_S - 4\kappa_T + 4n_T^2 \{ n_S - n_T - 1 \} \right) \ln \left( \frac{k}{k_*} \right) \]  
\[ + \frac{1}{4} \left( \xi_V^2 \left( 16n_T^2 + 9n_T + n_S - 3n_T - 1 \right) + \{ n_S - 3n_T - 1 \}^2 + 2\xi_V^2 \right) \right] \ln^2 \left( \frac{k}{k_*} \right) + \ldots, \]  
\[ \sigma_V^3(k) = \sigma_V^3 + \sigma_V^3 \left( 1 - n_S \right) \ln \left( \frac{k}{k_*} \right) \]  
\[ + \frac{\sigma_V^3}{4} \left( 30n_T^2 + 20n_T \{ n_S - 3n_T - 1 \} + \xi_V^2 + 2\{ n_S - 3n_T - 1 \}^2 \right) \ln^2 \left( \frac{k}{k_*} \right) + \ldots \]

where at the pivot scale \( k = k_* \) the slow roll parameters are defined as follows:

\[ \epsilon_V = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta_V = \frac{M_P^2}{2} \left( \frac{V''}{V} \right), \quad \xi_V^2 = \frac{M_P^2}{2} \left( \frac{V'V''}{V^2} \right), \quad \sigma_V^3 = \frac{M_P^2}{2} \left( \frac{V'^3}{V^3} \right). \]  

Additionally, the appearance of very mild running \( (\alpha_S, \alpha_T) \) and running of the running \( (\kappa_S, \kappa_T) \) of the spectrum might have implications for the Primordial black hole formation [61].

Once the higher order radiative corrections are incorporated due to the presence of running in the parameterization of the power spectrum, then the inflationary observables can be expressed as:

\[ P_S(k_*) = \left[ 1 - (2C_E + 1)\epsilon_V + C_E\eta_V \right]^2 \frac{V}{24\pi^2 M_{PL}^4 \epsilon_V}, \quad P_T(k_*) = \left[ 1 - (C_E + 1)\epsilon_V \right]^2 \frac{2V}{3\pi^2 M_{PL}^4}, \]  
\[ n_S - 1 \approx (2\eta_V - 6\epsilon_V) - 2C_E\xi_V^2 + 3\eta_V^2 + 2(8C_E + 3)\epsilon_V + 2\epsilon_V \eta_V \left( 6C_E + \frac{7}{3} \right), \]  
\[ - 4C_E(C_E + 1)\xi_V^2 \epsilon_V + 2C_E^2 \eta_V \xi_V^2, \]  
\[ n_T \approx -2\epsilon_V + 2 \left( 2C_E + \frac{5}{3} \right) \epsilon_V \eta_V - 2 \left( 4C_E + \frac{13}{3} \right) \epsilon_V^2, \]  
\[ r(k_*) = \frac{16\epsilon_V \left[ 1 - (C_E + 1)\epsilon_V \right]^2}{\left[ 1 - (2C_E + 1)\epsilon_V + C_E\eta_V \right]^2} \approx 16\epsilon_V \left[ 1 + 2C_E(\epsilon_V - \eta_V) \right], \]
\[
\alpha_S \approx (16\nu_e\epsilon V - 24\epsilon V^2 - 2\xi V^2) - 2C_E(4\nu_e\epsilon V^2 - \eta V\epsilon V - \sigma V^2) + \frac{4}{3}\eta V(2\eta V\epsilon V - \xi V^2)
\]
\[
+ 4(8C_E + 3)\epsilon V(4\epsilon V^2 - 2\eta V\epsilon V)
\]
\[
- 4C_E(C_E + 1)\left[\epsilon V(4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2) + \xi V^2(4e V\epsilon V - 2\eta V\epsilon V)\right] + 2C_E^2(2\eta V\epsilon V - \xi V^2) + 2C_E^2\eta V(4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2),
\]
\[
(A.7)
\]
\[
\alpha_T \approx (4\eta V\epsilon V - 8\epsilon V^2) + 2\left(2C_E + \frac{5}{3}\right)\left[\epsilon V(2\eta V\epsilon V - \xi V^2) + \eta V(4\epsilon V^2 - 2\eta V\epsilon V)\right]
\]
\[
- 4\left(4C_E + \frac{13}{3}\right)\epsilon V(4\epsilon V^2 - 2\eta V\epsilon V),
\]
\[
(A.8)
\]
\[
\kappa_S \approx 192e V^2 \eta V - 192e V^3 + 2\sigma V^2 - 24e V\xi V^2 + 2\nu V\xi V^2 - 32\eta V^2 e V
\]
\[
- 8C_E\left[\epsilon V(4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2) + \xi V^2(4e V^2 - 2\eta V\epsilon V)\right]
\]
\[
+ 2C_E\left[\eta V(4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2) + \xi V^2(2\eta V\epsilon V - \xi V^2)\right] + 4C_E\sigma V^3(3e V - \eta V) + \frac{4}{3}(2\eta V\epsilon V - \xi V^2)^2
\]
\[
+ \frac{4}{3}\eta V\left[2\eta V(4e V^2 - 2\eta V\epsilon V) + 2e V(2\eta V\epsilon V - \xi V^2) - (4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2)\right]
\]
\[
+ 4(8C_E + 3)(4\epsilon V^2 - 2\eta V\epsilon V)^2 + 16(8C_E + 3)\epsilon V\left(2\epsilon V - \eta V\right)(4\epsilon V^2 - 2\eta V\epsilon V) - e V(2\eta V\epsilon V - \xi V^2)
\]
\[
+ 4\left(6C_E + \frac{7}{3}\right)(2\eta V\epsilon V - \xi V^2)(4\epsilon V^2 - 2\eta V\epsilon V)
\]
\[
+ 2\left(6C_E + \frac{7}{3}\right)\epsilon V\left[2(2\eta V\epsilon V - \xi V^2)\epsilon V + 2\nu V(4e V^2 - 2\eta V\epsilon V) - (4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2)\right]
\]
\[
- 4C_E(C_E + 1)(4e V^2 - 2\eta V\epsilon V)(4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2)
\]
\[
- 4C_E(C_E + 1)\epsilon V\left[(4e V - \eta V)(4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2)
\]
\[
+ (16e V^2 + \xi V^3 - 10\eta V\epsilon V)\xi V^2 - 2\sigma V^3(3e V - \eta V)\right]
\]
\[
- 4C_E(C_E + 1)\left[(4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2)(4e V^2 - 2\eta V\epsilon V)\right]
\]
\[
+ 2\xi V^2\left[(4e V - \eta V)(4e V^2 - 2\eta V\epsilon V) - e V[2\eta V\epsilon V - \xi V^2]\right]
\]
\[
+ 2C_E\left[4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2\right](2\eta V\epsilon V - \xi V^2) + \xi V^2(2\eta V(4e V^2 - 2\eta V\epsilon V)
\]
\[
+ 2e V(2\eta V\epsilon V - \xi V^2 - \eta V\xi V^2 - \sigma V^2)
\]
\[
+ 2C_E^2\left[2(2\eta V\epsilon V - \xi V^2)(4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2)\right] + \eta V\left[4e V - \eta V\right][4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2]
\]
\[
+ \xi V^2(16e V^2 + \xi V^2 - 10\eta V\epsilon V - 2\sigma V^3(3e V - \eta V))]
\]
\[
(A.9)
\]
\[
\kappa_T \approx 56\eta V\epsilon V^2 - 64\epsilon V^3 - 8\eta V^2 e V - 4e V\xi V^2 + 2\left(2C_E + \frac{5}{3}\right)\left[2\eta V\epsilon V - \xi V^2\right](4e V^2 - 2\eta V\epsilon V)
\]
\[
+ e V(2\eta V[4e V^2 - 2\eta V\epsilon V] + 2e V[2\eta V\epsilon V - \xi V^2] - [4e V\xi V^2 - \eta V\xi V^2 - \sigma V^2])
\]
\[
+ \eta V\left[8(\epsilon V^2 - 2\eta V\epsilon V) + 2(2\eta V\epsilon V - \xi V^2) - 2\eta V[4e V^2 - 2\eta V\epsilon V] - 2e V[2\eta V\epsilon V - \xi V^2]\right]
\]
\[
- 4\left(4C_E + \frac{13}{3}\right)[(4e V^2 - 2\eta V\epsilon V)^2 + e V((8e V - 2\eta V)[4\epsilon V^2 - \epsilon V] - 2e V[2\eta V\epsilon V - \xi V^2])].
\]
\[
(A.10)
\]
\[
where \ C_E = 4(ln 2 + \gamma_E) - 5 with \ \gamma_E = 0.5772 \ is \ the \ Euler-Mascheroni \ constant \ originating \ in \ the \ expansion \ of \ the \ gamma \ function. \ For \ details \ discussion \ on \ these \ aspects \ see \ refs. \ [43].
\]
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