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Wang, Ruoqi; Wei, Yanji; van Rooij, Marijn; Jayawardhana, Bayu; Vakis, Antonis I.

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INFLUENCE OF A TAUT CABLE ON THE PERFORMANCE OF A POINT-ABSORBER WAVE ENERGY CONVERTER

R. Wang Y. Wei M. van Rooij B. Jayawardhana A. I. Vakis
Faculty of Science & Engineering, University of Groningen,
Nijenborgh 4, Groningen 9747AG, The Netherlands

ABSTRACT
In recent years, wave energy converters (WECs) have received considerable attention as an efficient way to harvest alternative energy sources. Within this class of systems, point-absorbers are popular and have become one of the most widely used renewable energy harvest designs all over the world, at least in the preliminary R&D stage, with many relevant research works having been published as well. However, unlike the single buoy and PTO systems which already have a comprehensive research basis, the connection cable has received little attention. The traditional taut cable analysis approach, initiated from the needs of the oil&gas industry, has been applied for WEC investigations. However, this approach utilizes an essential assumption that the oscillating term (PTO force) is much smaller than the static term of the cable force (pre-tension) and could be neglected, which may not be proper for WEC applications. In this work, a conventional frequency domain model is utilized to test and verify the validity of the previously mentioned assumption by presenting the ratio between two force terms. Then the ratio could be applied in combination with sea state contours to reveal the critical state of the cable. Then, a fully nonlinear time domain method of a numerical solution of the point-absorber wave energy converter is presented. According to the critical states obtained from the frequency domain analysis, an improved model of a slack cable is proposed. Its influence on the energy extraction performance is investigated using the open source code–WEC-Sim. This work provides insight into simulating a proper model of the cable and how the design of the cable influences the WEC performance.

Introduction
Due to the increase in oil prices, the limitation of fossil fuels, the steady growth of population, and the commitment to reduce CO₂ emissions, the world is searching for alternative renewable energy options to fulfill the gap between energy demand and consumption. Among all alternatives, wave energy shows extraordinary promise both in scientific and economic aspects due to its abundance, cleanness and predictability. Correspondingly, numerous researchers have worked in this field. Modern ocean wave energy research began during the oil crisis of the 1970s, when Salter [1] and Evans [2] contributed much of the early research basis. After years of research and study, various designs of wave energy converters (WECs) have been proposed and modeled to different degrees of success. However, many of them are still at the R&D stage, while only a small range of devices has been tested at a large scale in the ocean [3]; e.g. Elwood et al. [4] present an overview of the Sea Beaver project which began in 2006 and achieved the ocean test in 2007.

Among the various technologies developed in the past years, the point-absorber design has been recognized as both efficient and effective, and has been widely utilized and researched worldwide, featuring extensive modeling efforts. Their typical configuration comprises a single buoy, a power take-off (PTO) system, and a mooring system. The PTO system is usually a seabottom-fixed structure while alternatively could be located inside the buoy as well. The mooring system in these devices consists of a cable, which connects the buoy to the PTO system and is pretensioned by a spring or equivalent device.

The response of a similar system was partially researched previously. Harleman et al. [5] researched the behavior of a moored submerged buoyant sphere in an oscillatory wave by
vibration theory. The mooring line tension and motion of the sphere were predicted relative to the incident wave characteristics. Mavarakos et al. [6] investigated the power absorption performance of a tightly moored symmetric WEC. The emphasis was on the influence of the floaters hydrodynamics by applying several geometries of floaters. Ye et al. [7] provided a systematic review of hydrodynamic modeling methods for point-absorbers, providing a good understanding of device performance and constituting a bridge between the pre-commercial and full-scale commercial stages. A submerged point-absorber TWED has been studied under regular wave conditions for optimization by Guanche et al. [8], and was then tested under irregular waves. Hann et al. [9] performed experimental measurements of the interaction of a taut moored floating body representing a WEC with extreme waves. The dependence of mooring load and device motion on both wave steepness and wave breaking point was explored.

Though much research have been accomplished based on a single-point absorber model, the emphasis was put more on the buoy or PTO system, while the mooring cable dynamics receive little attention. In Harleman’s research, the mooring line was modeled as a vertical line and assumed to remain straight under tension at all times [5]. Spanos et al. [10] proposed a statistical linearization technique to describe the stochastic nonlinear dynamics of single-point harvester. Their previous work presented that the cable was always regarded as a simple connection without any dynamic characteristics. However, based on the research of Vicente et al. [11] on the dynamic response of point-absorbers deployed in a triangular array, the buoy behavior and power performance were found to be significantly affected by the presence of the slack mooring system. Four years later, the same group proposed a thorough investigation of the dynamics of the single-point absorber by a traditional taut cable analysis approach which was initiated by the oil& gas industry. They derived the equation of motion of a single-point absorber in surge and heave and pursued an approximate solution both in the frequency and time domains by assuming that the oscillating force term (PTO force) is much smaller than the static force term (pre-tension), and is therefore negligible [12]. Their research revealed that, even for that simplified model, the nonlinear dynamics introduced by the mooring system imposed significant computational difficulties.

Note that the small PTO force assumption may not be applicable in some cases, for example, when the device operates under large amplitude waves. In order to fulfill the theoretical blank, this work focuses on the fully nonlinear dynamics introduced by the mooring cable based on a more comprehensive model. The ratio between the PTO force and the pre-tension is presented in a conventional frequency domain analysis to verify the validity of the assumption used in the literature. Additionally, the influence of the cable configuration on the energy harvest performance of the device is investigated. Finally, an improved fully nonlinear cable model in the time domain is simulated.

**Numerical model**

Consider a point-absorber WEC comprising a spherical buoy with radius \(R\) connected to a PTO system on the sea floor by a taut mooring line with length \(L\) (Fig. 1). The spherical buoy allows pitch to be neglected. Thus, the two degrees-of-freedom of this system in consideration are heave and surge. The mooring line is kept tight by a spring with a pre-tension \(F_{\text{pre}}\) and connected to the center of the buoy. The linear PTO system consists of a linear damper which converts the energy absorbed from the waves to electricity. A one-directional wave is assumed.

![Figure 1: The cable model](image)

Let \(x\) and \(z\) represent, respectively, the horizontal and vertical time-varying displacements of the buoys center from its original static equilibrium position, with \(z\) increasing upwards and \(x\) aligned with the wave direction. At each moment, the cable length after elongation can be derived, formed by the combined effect of \(x\) and \(z\), and is the distance between the attachment point and the PTO system fixed on the sea bed. Then the formula of the cable elongation \(\Delta L\) is obtained:

\[
\Delta L = \sqrt{x^2 + (z + L)^2} - L. \tag{1}
\]

The angle between the cable and the vertical direction is denoted by \(\alpha\) with \(\sin \alpha = x / (L + \Delta L)\) and \(\cos \alpha = (z + L) / (L + \Delta L)\). The viscous force of the cable is neglected and small mass is assumed.

As illustrated before, the linear PTO model consists of the linear damper \(C\) and spring \(K\) mounted in parallel. Then, the PTO force could be expressed as

\[
f_{\text{PTO}} = K\Delta L + C \frac{d\Delta L}{dt}. \tag{2}
\]
The cable force comprises the static term pre-tension $F_{pre}$ and the oscillating term PTO force $f_{PTO}$, as

$$\phi = F_{pre} + f_{PTO}. \quad (3)$$

Then, the horizontal restoring force and the vertical pulling force are derived as Eq 4.

$$\phi_x = \phi \cdot \sin \alpha$$
$$\phi_z = \phi \cdot \cos \alpha \quad (4)$$

In calm seas, half of the sphere is assumed to be submerged; therefore, the center of gravity of the buoy lies at the free-surface ($z = 0$) in a state of equilibrium. The buoy mass $m$ must be:

$$m = \frac{2}{3} \pi R^3 \rho - \frac{F_{pre}}{g}. \quad (5)$$

The buoy mass is related to the value of pre-tension. Thus, the balance between these two parameters is important for the WEC performance.

**Frequency domain (FD) analysis**

First, the regular wave with angular frequency $\omega$ is considered.

$$(m + A_x) \ddot{x} + B_x x = f_{ds} - \phi_x$$
$$(m + A_z) \ddot{z} + B_z z + \rho g S = f_{dz} - \phi_z \quad (6)$$

Here, $A_x$, $A_z$, and $B_x$, $B_z$ are the hydrodynamic coefficients of added mass and radiation damping for the surge and heave motions. The forces $f_{ds}$ and $f_{dz}$ are the horizontal and vertical components of the wave-induced excitation force on the buoy. Finally, the cross-sectional area of the buoy is $S = \pi R^2$.

By assuming a small rotation angle, which also implies that the motion is small compared to the cable length, the expressions of the angle can be approximated as $\sin \alpha \approx x/L$ and $\cos \alpha \approx 1$. Then the force components in surge and heave are simplified as $\phi_x = (F_{pre} + f_{PTO}) \frac{x}{L}$ and $\phi_z = F_{pre} + f_{PTO}$.

Having linearized the system, the displacement in heave $z$ is a simple-harmonic function of time, which can written as:

$$z = 2e^{i\omega t}. \quad (7)$$

Therefore, the vertical pulling PTO force is

$$f_{PTO} = Re \{K_z + C \dot{z}\} = kz + i\omega C \ddot{z}, \quad (8)$$

and the time-averaged power absorbed by the buoy is

$$p = \frac{1}{2} \rho g C \ddot{z}^2. \quad (9)$$

Numerical results are presented next for regular waves with the following parameters: $A_w = 1m$, $K = 1.8 \times 10^5 N/m$, $C = 2.5 \times 10^5 N/(m/s)$, $L = 60m$, $R = 7.5m$. Different values of the pre-tension are utilized to show its influence on the force ratio and energy absorption performance.

**FIGURE 2**: The force ratio between the PTO force and the pre-tension for different pre-tension setting

While PTO force alone does not have a direct correspondence to power absorption, a greater PTO force generally leads to greater PTO power, which is desirable for WEC applications. Fig. 2 represents how the ratio between the PTO force and the pre-tension varies with the adjustment in pre-tension values. It is observed that a lower pre-tension leads to a higher force ratio. Even for the case $F_{pre} = 1.5MN$ which has the lowest ratio, the PTO force is around 10% of the pre-tension, and reaches 18% at peak frequency. This is obviously not consistent with the classical assumption used in the literature that the PTO force is much smaller than the pre-tension and can be assumed to negligible. For WEC applications, a slack cable may in fact be harmful to the PTO system, while a large force ratio implies that the system is more prone to suffer such slack; this eventuality should be avoided in WEC design.
The capture factor is generally applied as an indicator to assess the energy absorption performance. It is calculated as

\[ CF = \frac{P_{\text{absorb}}}{D_{CW} \cdot P_{\text{incoming}}} \]  (10)

where \( D_{CW} \) is the characteristic length of the device (here, the diameter of the buoy is applied); the incoming wave power is calculated as

\[ P_{\text{incoming}} = \frac{1}{8} \rho g H^2 C_g \]  (11)

where \( C_g \) is the group velocity of incoming waves.

A higher capture factor implies a better transmission from the incoming power to absorbed power. Fig. 3 shows that lower pre-tension leads to slightly better absorption performance, with little shifting of the peak frequency. Since the displacement of the linear system is simple-harmonic, the angular frequency is inversely related to the buoy mass as \( \omega = \sqrt{\frac{k}{m}} \). When a lower pre-tension also implies a heavier buoy mass according to Eq. 5, a relatively low peak frequency is expected.

The ratio between the PTO force and the pre-tension could give us insight about their comparative values, and verify the invalidity of the previously mentioned assumption. When we consider it the other way around, the ratio between the pre-tension and the PTO force is the amplitude of the critical sea state where the PTO force counteracts the pre-tension; therefore the cable force is at its critical state before getting slack. This characteristic could be applied in combination with the sea state energy contour at a given location for design considerations. If most sea states are beyond the critical limit, this implies a high probability for a slack cable and the pre-tension is not properly designed.

Here, the Irish M5 buoy is utilized as an example since it has similar water depth with what is used in our simulation (Table 1). The wave data between 2010 and 2012 was manually collected based on live information from the American National Oceanic and Atmospheric Administrations (NOAA) National Data Buoy Center (NDBC) [13]. An energy contour is generated by combining the significant wave height and peak period according to their energy output. The same color represents the same amount of energy availability, and the color changing from purple to yellow (towards the contour center) represents an increasing amount in energy availability. It is interesting to observe that some small sea states have the same energy output with extreme sea states. That is due to the fact that the energy output is related to the rate of occurrence of specific sea states as well: though extreme sea states could provide more energy (\( P_{\text{incoming}} \propto H^2 \)), they appear much more rarely than other sea states. Therefore, the energy contour plot could also be regarded as a sea state occurrence probability plot, where the extreme sea states have a lower occurrence probability and small waves have a high occurrence probability.

<table>
<thead>
<tr>
<th>TABLE 1: Information of Irish buoy M5.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designation</td>
</tr>
<tr>
<td>ID</td>
</tr>
<tr>
<td>Operator</td>
</tr>
<tr>
<td>Longitude</td>
</tr>
<tr>
<td>Latitude</td>
</tr>
<tr>
<td>Water depth</td>
</tr>
</tbody>
</table>
As shown in Fig. 4, for \( F_{\text{pre}}=0.5 \text{MN} \) half of the contour is beyond the cable critical state, which implies a higher for the cable becoming slack during operation. Therefore, in conclusion, a cable with lower pre-tension could lead to slightly better energy absorption performance but would also introduce additional challenges. Achieving a balance in this respect is essential and preferred for an optimal design with both efficiency and safety considered. A more accurate time domain analysis is proposed for this case.

**Time domain (TD) analysis**

In the previous section, linearizing approximations were introduced to enable the FD analysis. Nevertheless, the horizontal and vertical components of the forces applied on the buoy by the cable are in fact not linear functions of the displacements \( x \) and \( z \) and their derivatives. Our Simulink TD model consists of a rigid body at sea level representing the buoy, another slender rigid body with light weight representing the connection cable, a constraint between them transferring the force and energy, and a PTO system with a translational and angular joint; hence, translational and rotational motions are both taken into account.

The fully nonlinear equations governing the surge and heave motions are, respectively:

\[
(m + A_{\infty})(\ddot{x} + \int_{-\infty}^{t} K_{x}(t-\tau)\dot{x}(\tau) d\tau) = f_{dx} - \phi_{x} \]

\[
(m + A_{\infty})(\ddot{z} + \int_{-\infty}^{t} K_{z}(t-\tau)\dot{z}(\tau) d\tau) = f_{dz} - \phi_{z} \tag{12}
\]

Here, \( A_{\infty}(j=x, z) \) is the limiting value of the added mass coefficient \( A_{j}(\omega) \) for \( \omega = \infty \); the term \( \int_{-\infty}^{t} K_{j}(t-\tau)\dot{j}(\tau) d\tau \) is the convolution integral which represents the radiation damping; and \( \phi_{j} \) is derived as in Eq. 4 without the small angle assumption for \( \alpha \).

Firstly, the TD model is verified by comparing it with the FD model. Two sea states of regular waves are selected. A sea state with \( H=1 \text{m} \) represents small waves with high occurrence probability: it has smaller wave height than the critical one. On the contrary, a sea state with \( H=5 \text{m} \), larger than the critical wave height, represents large waves leading to cable slack. Both sea states are simulated and their energy absorption performance is compared. As Fig. 5 shows, the TD simulation results are consistent with the FD results, and no significant difference could be observed. This implies that the simplification of the rotation angle \( \alpha \) doesn’t result in significant differences in the system response and proves the validity of the small angle assumption used in the FD model. For \( H=1 \text{m} \), no slack cable or nonlinearity is observed. For \( H=5 \text{m} \), the same trend of \( CF \) variation is observed with a slight difference in peak amplitude. A possible reason for this discrepancy is that the system configuration and incident waves used in the simulation did not result in large horizontal displacements; therefore, the small angle assumption is still valid. The WEC located in shallower waters and encountering larger incident waves may yield a greater difference, but this is something that will be investigated in the future.

As previously explained in Fig. 4, it is possible to have a slack cable under large wave conditions, especially for low pretensions, but this is an undesired phenomenon for an energy harvester that should be avoided to avoid damaging the PTO system (e.g. due to fatigue) and dissipating energy through the cable’s repeated deformation. When the cable is slack, the cable force would become zero; however, the original model shown in Fig. 6(a) predicts a negative cable force when the cable becomes slack.

**FIGURE 6:** Models for TD analysis: (a) the cable is regarded as single element; (b) numerous small elements consist the cable

**FIGURE 5:** The comparison between FD and TD in capture factor:
slack during simulation. This is not physical since an elastic cable will easily buckle under compression.

In order to have a more accurate model to describe the slack phenomenon, the cable is divided into \( n \) elements hinged with each other as shown in Fig. 6(b). The interaction between each element will not transfer the negative force, making the model closer to the real case.

A five-element model is utilized (designated \( M_5 \), compared to the original single-element model designated \( M_1 \)). Both models are tested under select sea states and their energy harvest performance is plotted in Fig. 7. For small waves, there is no difference between \( M_1 \) and \( M_5 \) in capture factor and the cable force is always positive. For large waves, \( M_1 \) still shows similar behavior as in the previous two plots for small waves, i.e. there is no slack; on the contrary, \( M_5 \) shows nonlinear behavior around \( \omega = 1.1 \text{ rad/s} \), and a deviation from the other plots is clearly observed. It is reasonable to infer that the discrete model has direct influence on the system response. Therefore the heave motion of the buoy and the cable force are inspected at \( \omega = 1.1 \text{ rad/s} \). Dimensionless parameters are defined as follows: \( Z^* = \{Z\} / (H / 2) \), \( \phi^* = \phi / F_{pre} \), \( T^* = t / T \).

Referring to the displacement plots of Fig. 8 only the case with multiple elements and tested under large waves (\( M_5, H=5m \)) differentiates from the other three cases, similarly with what was observed in Fig. 7. This case exhibits slightly larger heave motion than others. When examining the cable force in Fig. 9 several important observations can be made. Firstly, large waves always lead to a higher cable force as the cable will be tensioned or elongated to a larger degree. The cases where \( H=5m \) have a much higher force amplitude than the cases where \( H=1m \). Secondly, both models exhibit the same response under small wave excitation, as their force plots for \( H=1m \) are indistinguishable. In these cases, the cable will not become slack and the cable force is always positive. Thirdly, large waves lead to higher probability for cable slack. For case ”\( M_1, H=5m \)”, the cable force periodically becomes negative, corresponding to cable slack. As an improved model, case ”\( M_5, H=5m \)” eliminates the negative force and models the cable slack phenomenon accurately. The cable force realistically drops to zero when the cable becomes slack. Additionally, a slack cable may lead to undesired energy loss. Contrary to case “\( M_1, H=5m \)”, where the cable force varies smoothly, the hinged model “\( M_5, H=5m \)” exhibits more oscillations when the cable is still loaded. Each small element may interact with its neighboring elements and encounter sudden and high loads during the process of becoming slack and then being tensioned again. That partly explains the reason why case ”\( M_5, H=5m \)” has a lower capture factor than other cases.
Conclusions

The present paper proposed a model for a single-point absorber, both in a conventional linearized form and as a fully nonlinear model. Several conclusions can be drawn from the simulations:

- The conventional assumption applied for the cable dynamics is not proper for WEC applications since the PTO force has a comparable value to the pre-tension and should not be neglected.
- A lower pre-tension force in a cable (also implying a heavier buoy on top) could lead to better energy absorption performance, but with a higher probability of having a slack cable at a given installation location (and sea state). Hence, it is essential to find an optimum pre-tension force when designing such systems.
- The small rotation angle assumption is valid when the displacements of the buoy are too small compared to the cable length to bring any impact to the system. Then, the results of the FD and TD simulations are consistent with each other.
- Due to the limitation of the original single-element cable model, a multi-element hinged cable model is introduced. The new model will not transfer a negative force so that the slack cable phenomenon may be modeled in a more accurate and realistic way. Cable slack occurs more often in large wave conditions.
- A slack cable will lead to undesired energy loss during the transition between slacking and tensioning. Therefore the hinged cable model accordingly predicts a lower capture factor under large waves.

The present work focused in understanding the dynamic response of the cable in a single-point absorber and how it affects the energy absorption. The current work only deals with regular waves as input; it could be extended to irregular waves for a more realistic environmental modeling. Since the cable is a slender element, the viscous force could affect the cable dynamics in an important way, which is already an investigation in progress. As mentioned before, cable slack will lead to significant energy loss; however, this loss has not been quantified in this work and will be investigated in the future. Last but not least, one of the advantages of a single-point absorber is that it is flexible and more effective when deployed within an array. In this case, buoys can also be moored to each other, and it will be interesting to investigate how additional moorings may affect the energy extraction.

REFERENCES