The shape of dark matter haloes – V. Analysis of observations of edge-on galaxies

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ABSTRACT

In previous papers in this series, we measured the stellar and H I content in a sample of edge-on galaxies. In the present paper, we perform a simultaneous rotation curve and vertical force field gradient decomposition for five of these edge-on galaxies. The rotation curve decomposition provides a measure of the radial dark matter potential, while the vertical force field gradient provides a measure of the vertical dark matter potential. We fit dark matter halo models to these potentials. Using our H I self-absorption results, we find that a typical dark matter halo has a less dense core (0.094 ± 0.230 M⊙ pc−3) than that for an optically thin H I model (0.150 ± 0.124 M⊙ pc−3). The H I self-absorption dark matter halo has a longer scale-length Rc of 1.42 ± 3.48 kpc, versus 1.10 ± 1.81 kpc for the optically thin H I model. The median halo shape is spherical at q = 1.0 ± 0.6 for self-absorbing H I, while it is prolate at q = 1.5 ± 0.6 for the optically thin case. Our best results were obtained for ESO 274-G001 and UGC 7321, for which we were able to measure the velocity dispersion in Paper III. These two galaxies have very different halo shapes, with one oblate and one strongly prolate. Overall, we find that the many assumptions required make this type of analysis susceptible to errors.

Key words: galaxies: haloes – galaxies: kinematics and dynamics – galaxies: photometry – galaxies: spiral – galaxies: structure.

1 INTRODUCTION

Modern-day cosmological dark matter simulations predict that dark matter has a significant effect on the formation and evolution of galaxies. The dark matter clumps into haloes, which serve as gravitational sinks for baryonic matter. These baryons then form the galaxies and other visible structures of the Universe. The size and the shape of these haloes are influenced by the type of dark matter particle and its merger history. As such, ascertaining the shape of a halo offers a potential constraint on the dark matter model (Davis et al. 1985).

The shape of dark matter haloes can be classified by the shape parameter q, using the ratio between the vertical axis c and the radial axis a, such that q = c/a. This strategy enables the potential halo shapes to be divided into three classes: prolate (q > 1), oblate (q < 1) and spherical (q ∼ 1). This is, of course, only a simplified version of reality, because we can also expect triaxial shapes and changes of shape with radius and history (Vera-Ciro et al. 2014). For haloes with masses $\gtrsim 10^{12.3} h^{-1} M_\odot$, however, the haloes can be adequately described by a single vertical-to-radial axis ratio (Schneider, Frenk & Cole 2012).

Direct observation of the shape of haloes is tricky, as there are only a few tracers that offer a clear view of the vertical gravitational potential of the halo. The flat rotation curves of galaxies, while an excellent tracer of the radial potential of the halo, provide no information on the vertical direction. Fortunately, some tracers do exist. The stellar streams of stripped, in-falling galaxies can be used to measure the potential that the stream is traversing. Helmi (2004) analysed the stream of Sagittarius, as found in the 2MASS survey, around the Galaxy and found that the data best fits a model with a prolate shape, with an axis ratio of 5/3. In a similar fashion, the satellites of the Galaxy offer such a tracer further out. Globular cluster NGC 5466 was modelled by Lux et al. (2012), with results favouring an oblate or triaxial halo while excluding spherical and prolate haloes with a high confidence. The stellar stream was re-analysed by Vera-Ciro & Helmi (2013), who reported an oblate halo with q = 0.9 for r ≤ 10 kpc.

Gravitational lensing offers another measure of the vertical gravitational potential. Strong lensing uses the Einstein lens of background sources around a single galaxy or cluster. By modelling the lens, it is possible to create a detailed mass map of the system. By combining this map with gas and stellar kinematics, it is possible to

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calculate the dark matter mass distribution (Treu 2010). For example, Barnabé et al. (2012) applied this method to the lensed galaxy SDSS J2141, finding a slightly oblate halo ($q = 0.91^{+0.15}_{-0.13}$).

Weak lensing lacks the clear gravitational lens seen in strong lensing. Therefore, this approach cannot measure the halo of a single galaxy. Instead, it provides an average halo shape from a statistically large sample of sources, by modelling the alignment of background galaxies to a large series of foreground galaxies. This means that sources can be used to probe the outer edges of haloes. A recent analysis by van Uitert et al. (2012) found, for a sample of $2.2 \times 10^7$ galaxies, that the halo ellipticity distribution favours oblate configurations, with $q = 0.62^{+0.25}_{-0.26}$.

Polar ring galaxies are also of interest in studies of the dark matter halo shape, as the orbits of the stars in the polar rings are very sensitive to the gravitational potential. The method was pioneered by Schweizer, Whitmore & Rubin (1983), who noted that the polar ring in galaxy A0136-0801 indicated a massive halo that was more spherical than flat. Whitmore, McElroy & Schweizer (1987) studied this galaxy in more detail and found $q = 0.98 \pm 0.20$. These authors also studied galaxies NGC 4650A and ESO 415-G26, for which they reported $q = 0.86 \pm 0.21$ and $q = 1.05 \pm 0.17$, respectively. Galaxy NGC 4650A has also been studied by Sackett et al. (1994), who reported a flattened halo with $q$ between 0.3 and 0.4.

Another way to place constraints on the halo shape is through the careful modelling of local edge-on galaxies. The thickness of the H I layer in spiral galaxies corresponds to directly on the local hydrostatic equilibrium. The H I layer flares out at large radii, as there is less matter gravitationally binding it to the central plane. Because of this, flaring provides a sensitive tracer of the vertical potential as a function of radius in the disc. By combining this information with the horizontal potential as obtained from the rotation curve, and estimates for the gas and stellar mass distributions, it is possible to fit the potential well created by the dark matter halo. This method was first applied to the Galaxy by Celnik, Rohlf & Braunfurst (1979) and by van der Kruit (1981) to the edge-on galaxy NGC 891. The latter study found that the halo was spherical rather than flattened like the stellar disc. NGC 4244 was analysed by Olling (1995), who found a highly flattened halo with $q = 0.2^{+0.5}_{-0.1}$. O'Brien, Freeman & van der Kruit (2010) aimed to measure the velocity dispersion as a function of radius. Using that approach to measure the halo shape of UGC 7321, they found a spherical halo ($q = 1.0 \pm 0.1$).

This paper is the fifth in a series. We will refer to earlier papers of the series simply as Papers I to IV. In this Paper V, we perform a similar analysis to the previous papers, using measured parameters for the H I disc (Paper III) and stellar disc (Paper IV). The series is based on a large part of the PhD thesis of the first author (Peters 2014). The paper is organized as follows. Section 2, provides a detailed description of the hydrostatic models we use and of the fitting strategy. Section 3 presents and discusses the results. The results are summarized in Section 4.

2 MODELLING STRATEGY

2.1 Overall strategy and sample

Celnik et al. (1979) proposed a new strategy in which the flaring of H I layers is used as a tracer of the vertical gravitational potential in the Galaxy. This method was extended to edge-on galaxy NGC 891 by van der Kruit (1981). When this method is combined with a traditional rotation curve decomposition, it is possible to obtain information on both the radial and the vertical direction of the gravitational potential. The dark matter halo model can be fitted to this information. Previous work on this subject, using the flaring of gas layers, includes studies by Olling (1995, 1996), Olling & Merrifield (2000), Combes & Becquaert (1997), Narayan, Saha & Jog (2005), Kalberla (2003), Kalberla et al. (2007), Baneree & Jog (2008), and others.

We will use this same strategy to model the dark matter halo shape for the sample of eight galaxies detailed in Paper I and extensively studied in subsequent papers in this series. In Paper III, we measured the structure and kinematics of H I in these galaxies, using modelling procedures that allow for the correction of self-absorption in the H I for assumed spin temperatures. The stellar discs were modelled in Paper IV. Based on the quality of the results from Papers III and IV, we decided to model five out of the original eight galaxies in our sample, namely IC 5249, ESO 115-G021, ESO 138-G014, ESO 274-G001 and UGC 7321. All five galaxies are late-type Sd, with self-absorption-corrected H I masses between $(4.1 \pm 0.1) \times 10^8$ and $(7.8 \pm 0.8) \times 10^8$ $M_\odot$ (see table 1 in Paper III). The distances to these galaxies vary greatly. ESO 274-G001 is at a distance of only 3.0 Mpc, while the distance to IC 5249 is estimated to be 32.1 Mpc.

In Paper III, we successfully measured the velocity dispersion of the H I as function of radius in ESO 274-G001 and UGC 7321. In ESO 115-G021, the velocity dispersion appeared to increase with radius from about 7 km s$^{-1}$ in the inner parts to 12–14 km s$^{-1}$ at the outer measured point. Because we are skeptical of this result, we will adopt a constant velocity dispersion of 10 km s$^{-1}$ for this galaxy. We also adopt this constant velocity dispersion for IC 5249 and ESO 138-G014. For UGC7321, the velocity dispersion drops from 10 km s$^{-1}$ in the central parts to 8 km s$^{-1}$ at 8 kpc, then remains constant at this level out to 10 kpc, after which it increases to 12 km s$^{-1}$ at 14 kpc. We have sufficient confidence in these results to adopt them in our analysis.

Galaxy UGC 7321 has a total $B$-band magnitude of 13.75 and a $V$-band magnitude of 13.19 (Taylor et al. 2005), giving a $B-V$ band difference of 0.56. In order to estimate the stellar mass-to-light ($M_*/L_R$) ratio, we use the model interpolation engine$^1$, which is based on the Worthey (1994) and Bertelli et al. (1994) stellar population models. Using a single-burst, Salpeter (slope $-2.35$) initial mass function (IMF) we found a good fit using a single population of 950 Myr and [Fe/H]$=0.0$, with $B-V = 0.554$. This would place $M_*/L_R$ in the $R_C$ band at 0.55. We repeated this exercise for our other galaxies, each time finding $M_*/L_R \approx 0.5$. We therefore adopt a mass-to-light ratio $M_*/L_R = 0.5$ as a lower boundary to the stellar disc mass, and use the reported $R$-band total luminosities reported in table 3 of Paper IV. As an upper boundary, we adopt $M_*/L_R < 3$. At solar metallicity, this would imply a single stellar population of $\sim$8 Myr. At metallicities lower than solar, this value would increase to an even greater age.

2.2 Decomposition strategy

We start with the assumption that there are three components: stars, gas+dust and dark matter. Each of these components adds to the gravitational potential, so ideally one would write down and solve the combined Poisson–Boltzmann equation. This approach is internally consistent, but requires simplifying assumptions, for example regarding the properties of the stellar velocity tensor (e.g. that it is Gaussian and isothermal or a superposition of such components).

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$^1$ Available at astro.wsu.edu/worthy/dial/dial_a_pad.html.
and its variation in space. This approach, which indeed is self-consistent, has been used by the authors referred to above, starting with Olling (1996).

Our strategy is as follows. We infer the radial force near the plane of the galaxy from the H I rotation curve, assuming that, owing to the relatively low velocity dispersion of order 10 km s⁻¹, the asymmetric drift term in the Jeans equation can be ignored. Next we model the light distribution and, assuming a constant mass-to-light ratio, estimate the contribution of the disc from the free parameter M/L. We do the same from the inferred mass distribution of gas, adding the usual contribution (25 per cent of the total) for helium. We assume also that the molecular gas content in these late-type dwarfs is small. This gives us the gradient in the radial force resulting from the halo. We next estimate the vertical force from the thickness and velocity dispersion of the H I, using the vertical Jeans equation. Because we evaluate the forces at z = 100 pc, this should be an excellent approximation, even though the equations used are not internally self-consistent as in the approach in the previous paragraph. This strategy has previously been used by O’Brien et al. (2010).

2.2.1 Radial tracer

The radial force gradient is calculated using a classic rotation curve decomposition performed at the mid-plane of the galaxy (van Albada et al. 1985):

\[ v_{\text{total}}^2(R) = v_{\text{gas}}^2(R) + v_{\text{stellar}}^2(R) + v_{\text{halo}}^2(R). \]  

The total rotation \( v_{\text{total}}(R) \) is the observed rotation curve of the H I gas, as measured in Paper III. The theoretical rotation curves of the gas, \( v_{\text{gas}}(R) \), and stars, \( v_{\text{stellar}}(R) \), represent the contributions from the stellar and gaseous mass components. We calculate the theoretical rotation curve of the stellar and gaseous components using equation (A.17) of Casertano (1983):

\[ v^2(R) = -8GR \int_0^\infty r \int_0^\infty \frac{\partial \rho(r,z)}{\partial r} K(p) - E(p) \ (Rr)^{1/2} \ dz \ dr, \]

with

\[ p = x - (x^2 - 1)^{1/2}, \]

\[ x = \frac{R^2 + r^2 + z^2}{2Rr}. \]

The equation of the density distribution \( \rho(r,z) \) is the complete elliptical integrals of the first and second kind. The equations are evaluated numerically.

The equation of the density distribution \( \rho(r,z) \) of the H I disc was previously given in equation (15) in Paper II. We presented the measured densities in Paper III. The equations for the stellar disc and bulge luminosity distributions were given in equations (1) and (3) in Paper IV. We convert to a combined stellar mass distribution from these equations using

\[ \rho(r,z) = (M_*/L_R) \left( j_{\text{disc}}(R,z) + j_{\text{bulge}}(R,z) \right). \]

The measurements for the stellar disc were presented in Paper IV. Note that we adopt a single mass-to-light ratio \( M_*/L_R \). While this may be slightly unrealistic, we noted in Paper IV that the fits for some of the bulges indicate that they may be part of the disc, rather than model a central component. Treating the two as distinct would therefore be invalid. A fixed \( M_*/L_R \) ratio also has the advantage of limiting the complexity of the parameter space in which we will be fitting.

2.2.2 Vertical tracer

We follow the method of O’Brien et al. (2010) to calculate the gradient of the vertical force. The gradients for each mass component add up as follows:

\[ \frac{dF_{z,\text{total}}(R,z)}{dz} = \frac{dF_{z,\text{gas}}(R,z)}{dz} + \frac{dF_{z,\text{stellar}}(R,z)}{dz} + \frac{dF_{z,\text{halo}}(R,z)}{dz}. \]  

Following O’Brien et al. (2010), the disc is assumed to be in vertical hydrostatic equilibrium, such that the vertical gas pressure gradient and total vertical gravitational force of the galaxy potential \( \Phi_{\text{total}} \) resulting from all mass components balance perfectly:

\[ \nabla \left( \frac{\sigma_{\text{gas}}^2 \rho_{\text{gas}}}{\Phi_{\text{total}}} \right) = \rho_{\text{gas}} \nabla \Phi_{\text{total}}. \]

We next assume that the gas velocity dispersion is isothermal in \( z \). In that case, equation (7) reduces to

\[ \frac{\partial}{\partial z} \left[ \frac{\log \rho_{\text{gas}}(R,z)}{z^2} \right] = -\frac{\partial F_{z,\text{total}}(R,z)}{\partial z}. \]

Because our H I disc is modelled as a Gaussian distribution (see equation 15 of Paper II), this equation becomes

\[ \frac{\partial F_{z,\text{total}}(R,z)}{\partial z} = \frac{\sigma_{\text{gas}}^2(R)}{z^2 v_t^2(R)}, \]

such that the vertical gradient of \( F_{z,\text{total}} \) is constant with height \( z \).

The gradients of the stellar and gas force components were calculated using the Poisson equation, assuming that the disc is axisymmetric and that the circular rotation is constant with height (O’Brien et al. 2010):

\[ \frac{\partial F_{z}(R,z)}{\partial z} = -4\pi G p(R,z) + \frac{1}{R} \frac{\partial v^2(R)}{\partial R}, \]

where we use the density \( p \) and rotation \( v \) of each of the two components. The squared velocity gradient is calculated numerically. Our modelling of the vertical tracer will use a plane at a height \( z \) of 100 pc.

2.3 Halo potential

We make use of the flattened, but axisymmetric, pseudo-isothermal halo model proposed by Sackett et al. (1994), in which we assume that the equatorial plane of the halo matches that of the galaxy. In this model the density is stratified in concentric ellipsoids, as given by

\[ \rho_{\text{halo}}(R,z) = \frac{\rho_{\text{halo}} R_z^2}{R_z^2 + R^2 + z^2/q^2}. \]

The ellipsoids formed by this density distribution have an axis ratio \( q \equiv c/a \), with core radius \( R_c \).

The potential arising from this density distribution is given by Sackett & Sparke (1990) as

\[ \Phi_{\text{halo}}(R,z) = 2\pi G q \rho_{\text{halo}} R_z^2 \int_0^{1/q} \left[ \frac{1}{x^2(1 - q^2) + 1} \right] \left[ 1 + \frac{x^2}{R_z^2} \left( \frac{R^2}{x^2(1 - q^2) + 1 + z^2} \right) \right] dx. \]

Note that this assumption is discussed in section 7 of Paper IV.
Sackett et al. (1994) has given the solution to the forces associated with this halo in spherical coordinates:

\[ F_R(R, z) = \frac{-v_R^2 R \gamma}{h \arctan \gamma} \left( \frac{\mu^2}{\mu^2 - 1} \left( \frac{\arctan \gamma \mu - \arctan \gamma}{\gamma} \right) - \frac{v^2}{v^2 - 1} \left( \frac{\arctan \gamma v - \arctan \gamma}{\gamma v} \right) \right), \]

where

\[ \gamma = \sqrt{1 - q^{-2}}, \quad v = \frac{2c}{b - h}, \quad \mu = \sqrt{\frac{2c}{b + h}}, \]

and

\[ h = \sqrt{b^2 - 4ac}, \]

\[ a = (1 - q^2)R_c^2, \]

\[ b = z^2 + R^2 + (1 - q^2)R_c^2, \]

\[ c = z^2. \]

Note that calculations that have \( q > 1 \) require the use of complex numbers, although these have reverted to real numbers by the end of the calculation. If only real numbers are used, the fit will be constrained to \( q < 1 \). The calculation has a singularity at \( q = 1 \), which we automatically substitute with \( q = 0.999 \) where required.

The asymptotic halo velocity \( v_H \) is defined as

\[ v_H^2 = \frac{4\pi G \rho_{\text{halo}} R_c^2 q \arccos q}{\sqrt{1 - q^2}}, \]

### 2.4 Fitting strategy

Using equation (1), we calculate the observed circular rotation curve resulting from the dark matter halo \( v_{\text{halo}} \), at the mid-plane of the galaxy over the full range of \( R \). In a similar way, we calculate the observed vertical force gradient \( \frac{dF_z}{dz}(R, z) \) resulting from the dark matter halo at a height \( z \) of 100 pc, using equation (6). We inspect the results to determine in which range of radii \( R \) they are sufficiently reliable. In contrast to O’Brien et al. (2010), we fit both the vertical and radial tracers simultaneously. Because the rotation curve decomposition is less sensitive to noise than the vertical force gradient decomposition, we can often fit a larger radial range for the rotation curve decomposition than for the vertical force gradient decomposition. We will fit the dark matter halo using equations (13) and (14), where we calculate the gradients numerically, and use the mid-plane approximation \( v^2 = -RF \) (Kuijken & Gilmore 1989).

The two tracers operate in very different numerical regimes. The total observed rotation velocity is often near 100 km s\(^{-1}\), while the observed vertical force gradient has a value of \(-0.004\) km s\(^{-1}\) pc\(^{-2}\). Because these numbers are so different, the combined \( \chi^2 \) would be dominated by the rotation curve decomposition. We therefore normalize the data of each force by its maximum value in that range, such that the total \( \chi^2 \) error is calculated as

\[ \chi^2 = \chi^2_R + \chi^2_z, \]

where the \( R \) values are all significant compared to the respective fitting ranges used for the vertical and radial directions. We tested many variations of equations (22) and (23), including converting and integrating both the tracers back into forces so that they could be compared more directly. The overall problem, however, remained, as the two forces were too different in strength and the errors in the radial component would dominate the fit. We decided to retain the units adopted by O’Brien et al. (2010), as this strategy provided the best way to compare results.

In Paper III, we performed a Monte Carlo Markov chain (MCMC) fit to the neutral hydrogen, such that a set of samples from the so-called chain together cover the likelihood distribution of the parameters. Our fitting strategy here makes use of this likelihood distribution. We took the last 1000 samples from the chain and performed a fit of the dark matter halo to each individual sample. In total, we thus obtained 1000 solutions for the halo. We made use of the PSWARM particle swarm optimization algorithm (Vaz & Vicente 2009), as implemented through the OpenOpt library (Kroshko 2007). The PSWARM algorithm is an example of a global optimization routine, which prevents the solution from becoming stuck in local optima in the solution space. The fit was performed directly to \( \chi^2 \) as defined in equation (21). In some cases, the models converge to unrealistic solutions. We therefore based our results on the 25 per cent of the samples with the lowest \( \chi^2 \) errors. The halo model uses three free parameters: the halo central density \( \rho_{\text{halo}} \), the scale-length \( R_c \), and the halo shape \( q \). Together with the mass-to-light conversion \( M_* / L_R \) for the stellar disc, we thus have four free parameters. We considered using an additional mass-to-light conversion for the bulge, but found that with these four parameters the solutions were already becoming degenerate. The addition of an additional free parameter would have exacerbated this problem.

We constrain \( \rho_{\text{halo}} \) between 0 and 3 M\(_\odot\) pc\(^{-3}\), \( R_c \) between 100 pc and 10 kpc, \( q \) between 0.1 and 2.0, and \( M_* / L_R \) between 0.5 and 3.0.

### 3 RESULTS

#### 3.1 IC 5249

We were moderately successful at modelling the dark matter halo of galaxy IC 5249. The decompositions of both the optically thin and the self-absorbing H\(^i\) results are shown in Fig. 1. As is clear from the figure, the uncertainties in the stellar halo contribution, and subsequently the dark matter halo contribution, are quite severe. This is largely because the measurements of both the vertical force gradient and the rotation curve start relatively far out (near 5.0–5.5 kpc). The data for the inward parts of the galaxy were too uncertain for a reliable measurement of the tracers. Because the dark matter halo shape \( q \) can most accurately be constrained from the vertical force gradient in the inner parts of the galaxy (see O’Brien et al. 2010), this lack of data means that we cannot place significant
constraints on \( q \). The optically thin H\(_1\) model yields \( q = 1.5^{+0.5}_{-0.5} \), while the self-absorbing H\(_1\) model yields \( q = 1.0^{+0.4}_{-0.3} \).

The lack of a significant constraint on \( q \) leads to strong correlations between the other parameters. This is reflected in the cross-correlation diagrams for the parameters, shown in Fig. 2 (left) for the optically thin case, and in Fig. 2 (right) for the self-absorbing H\(_1\) case. An oblate dark matter halo shape (\( q < 1 \)) produces a less massive stellar disc, with a shorter dark matter halo scale-length \( R_c \) and a higher dark matter halo core density \( \rho_0 \). This behaviour holds in both models. Over the whole data set, we find that for an optically thin H\(_1\) model the halo is found to have \( \rho_0 = 0.007^{+0.005}_{-0.001} \) M\(_\odot\) pc\(^{-3}\), \( R_c = 5.45^{+0.21}_{-0.13} \) kpc. The stellar disc is found to have \( M_*/L_R = 2.6^{+0.38}_{-0.19} \). The self-absorbing H\(_1\) models return \( \rho_0 = 0.004^{+0.002}_{-0.001} \) M\(_\odot\) pc\(^{-3}\), a scale-length \( R_c = 7.9^{+0.2}_{-0.0} \) kpc, and a stellar disc with \( M_*/L_R = 2.98^{+0.02}_{-0.17} \).

As discussed in Section 2.1, the most likely \( M_*/L_R \) values lie close to 0.5. If we thus limit ourselves to the data points at \( M_*/L_R < 0.55 \), we find for the optically thin H\(_1\) model an oblate halo with \( q = 0.76^{+0.04}_{-0.03} \). The core density of the dark matter halo is \( \rho_0 = 0.017^{+0.001}_{-0.000} \) M\(_\odot\) pc\(^{-3}\) and its scale-length is \( R_c = 4.49^{+0.05}_{-0.05} \) kpc. The self-absorbing H\(_1\) model returns an even more oblate halo, with a shape given by \( q = 0.65^{+0.14}_{-0.03} \), a core density of \( \rho_0 = 0.014^{+0.003}_{-0.003} \) M\(_\odot\) pc\(^{-3}\) and a scale-length of \( h_0 = 5.19^{+1.10}_{-1.30} \) kpc. By comparing the two models, it can be seen that the dark matter halo of the self-absorbing H\(_1\) model requires a more oblate halo, with a longer scale-length \( R_c \) and a less massive central density \( \rho_0 \).}

**Figure 1.** Rotation curve and vertical force gradient decomposition of IC 5249. The left-hand panels show the results for the optically thin H\(_1\) models, while the right-hand panels show the results when we assume self-absorption at \( T_{\text{spin}} = 100 \) K.

### 3.2 ESO 115-G021

We have not been very successful in modelling ESO 115-G021. As can be seen from the results in Fig. 3, the observed vertical force gradient has a nearly flat slope near the inner parts of the galaxy (\( R < 4 \) kpc). It is problematic to fit to this, as we expect the vertical force gradient to become increasingly steep near the inner parts. We attempted fitting only beyond \( R = 3.5 \) kpc, but this left only 2 kpc in which we could fit the data, which did not result in a stable fit. Smoothing was applied on the input parameters, but this did not improve the quality of the observed vertical force gradient. As such, we present here our best fits, but caution the reader to treat them with a degree of skepticism.

The self-absorbing H\(_1\) model of the galaxy results in a dark matter core density \( \rho_0 \) of \( 0.015^{+0.001}_{-0.001} \) M\(_\odot\) pc\(^{-3}\), a scale-length \( R_c \) of \( 3.02^{+0.03}_{-0.08} \) kpc, and an oblate shape of \( q = 0.5^{+0.1}_{-0.1} \). The stellar disc is found to have a high \( M_*/L_R = 2.89^{+0.11}_{-0.04} \). The optically thin H\(_1\) model produces a more massive central core density of \( \rho_0 = 0.022^{+0.004}_{-0.001} \) M\(_\odot\) pc\(^{-3}\), a shorter scale-length of \( R_c = 2.30^{+0.10}_{-0.15} \), and a less oblate halo shape, namely \( q = 0.7^{+0.1}_{-0.1} \). We again find a high \( M_*/L_R = 2.84^{+0.16}_{-0.15} \).

The cross-correlation diagrams of the models are shown in Fig. 4.

### 3.3 ESO 138-G014

Galaxy ESO 138-G014 was initially hard to model, as the total observed vertical force gradient was already weaker than the
Figure 2. Correlation diagram for the optically thin model of IC 5249 (top) and the self-absorption model (bottom), both with a constraint of $M/L \geq 0.5$. 
Figure 3. Rotation curve and vertical force gradient decomposition for ESO 115-G021. The left-hand panels show the results for the optically thin H\textsubscript{i} models, while the right-hand panels show the results when we assume self-absorption at $T_{\text{spin}} = 100$ K.

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The galaxy seems to have quite a thick H\textsubscript{i} layer (see fig. 13 of Paper III). The most likely explanation for this is that the galaxy is not seen completely edge-on. This is consistent with the observations of the associated stellar disc reported in Paper IV, for which we measured $i = 86.8$ (table 2 of Paper IV). We attempted to correct for this by lowering the observed thickness by 30\% per cent, but will treat the results as uncertain in view of the inclination. The noise in this galaxy was too high for the velocity dispersion to be measured, so we keep this fixed at $\sigma = 10$ km s$^{-1}$.

The results for the optically thin and the self-absorbing H\textsubscript{i} models are shown in Fig. 5. The cross-correlation diagram of the optically thin model is shown in Fig. 6 (left), while that for the self-absorption model is shown in Fig. 6 (right). In both cases, the rotation curve and the vertical force gradient have been successfully fitted.

Compared with this, the self-absorbing H\textsubscript{i} model gives a halo with a higher core density of $\rho_{0,\text{halo}} = 0.620^{+0.009}_{-0.003}$ M\textsubscript{\odot} pc$^{-3}$ and a scale-length of $R_c = 0.41^{+0.03}_{-0.01}$ kpc. Again, the optimal solution favours a prolate halo with $q = 1.9^{+0.1}_{-0.3}$. In this case, the mass-to-light ratio is $M_*/L_R = 0.69^{+0.04}_{-0.06}$.

3.4 ESO 274-G001

Galaxy ESO 274-G001 is one of the two galaxies from Paper III for which we could accurately measure the velocity dispersion. We model the galaxy in the standard way, setting the $M_*/L_R$ lower boundary to 0.5. Both the rotation curve and the vertical force decomposition are shown in Fig. 7, where we give the results for both the optically thin and the self-absorbing H\textsubscript{i} model. Both models reproduce the rotation curve and the vertical force gradient reasonably well, although the self-absorbing model is more successful at the vertical force gradient.

We show the cross-correlation diagram for the optically thin H\textsubscript{i} model in Fig. 8 (left). There is a clear correlation between the various parameters, which is mostly caused by the uncertainty in $M_*/L_R = 0.96^{+0.46}_{-0.42}$. The halo is oblate with $q = 0.7^{+0.1}_{-0.1}$, and
Figure 4. Correlation diagram for the optically thin model of ESO 115-G021 (top) and the self-absorption model (bottom), both with a constraint of $M/L \geq 0.5$. 
The shape of dark matter haloes – V.

Figure 5. Rotation curve and vertical force gradient decomposition for ESO 138-G014. The left-hand panels show the results for the optically thin H\textsubscript{I} models, while the right side panels show the results when we assume self-absorption at $T_{\text{spin}} = 100$ K results.

The cross-correlation diagram of the H\textsubscript{I} self-absorption model is shown in Fig. 8 (right). Once again there is scatter in $M^*/L_R$, although its value has dropped compared with the optically thin model: it is now at $0.76^{+0.07}_{-0.26}$. The shape of the halo is identical to that in the optically thin model, with an oblate shape of $q = 0.7^{+0.1}_{-0.1}$. The other parameters are $\rho_0,\text{halo} = 0.094^{+0.009}_{-0.010} \, \text{M}_\odot \, \text{pc}^{-3}$ and $R_c = 1.49^{+0.09}_{-0.03} \, \text{kpc}$. Compared with the optically thin model, the self-absorbing H\textsubscript{I} model produces a dark matter halo with a longer scale-length $R_c$ and lower central density $\rho_0$.

If we limit the analysis to $M^*/L_R < 0.55$, we find that the halo becomes even more oblate: $q = 0.64^{+0.01}_{-0.01}$ for the optically thin model and $q = 0.67^{+0.01}_{-0.01}$ for the self-absorption model. The central density of the halo also increases to $\rho_0 = 0.171^{+0.005}_{-0.001} \, \text{M}_\odot \, \text{pc}^{-3}$ for the optically thin model, and to $\rho_0 = 0.103^{+0.001}_{-0.001} \, \text{M}_\odot \, \text{pc}^{-3}$ for the self-absorbing H\textsubscript{I} model. The scale-lengths decrease to $h_0 = 1.07^{+0.01}_{-0.001} \, \text{kpc}$ and $h_0 = 1.39^{+0.01}_{-0.01} \, \text{kpc}$, respectively. A lower mass in the stellar disc thus results in haloes that are more oblate and that have higher central core densities and slightly shorter scale-lengths.

3.5 UGC 7321

Galaxy UGC 7321 has the highest signal-to-noise ratio for the H\textsubscript{I} data from our sample. The galaxy was previously modelled by O’Brien et al. (2010), who found that the halo flattening $q$ was round ($q = 1.0 \pm 0.1$). The modelling strategy of these authors consisted of a two-pass scheme, in which they first performed a rotation curve decomposition, and only then performed a separate fit to the vertical force gradient. This second fit failed, however, and the authors had to deviate substantially from the results from the rotation curve decomposition and use a stellar disc with a very low mass in order to reproduce the observed vertical force gradient.

We performed an inspection of the codes used in the analysis of O’Brien et al. (2010). It appears that there was a restriction that allowed only models with $q \leq 1$ and it would have been impossible for them to fit a prolate halo.

Banerjee, Matthews & Jog (2010) also analysed UGC 7321 and found a spherical halo. These authors assumed a constant velocity dispersion, or at most a decreasing gradient, in their work, and used a different potential from us.

In Fig. 9, we present our own rotation curve decomposition of this galaxy, and in Fig. 10 we show the vertical force gradient...
Figure 6. Correlation diagram for the optically thin model of ESO 138-G014 (top) and the self-absorption model (bottom), both with a constraint of $M/L \geq 0.5$. 
decomposition. Rather than presenting decompositions for only the optically thin and self-absorbing H\textsc{i} models that we measured, as we did for the previous galaxies, we show here the results for six fits.

Because O'Brien et al. (2010) found a good fit at a negligible stellar mass, the first panel in both figures demonstrates a fit in which $M_\star/L_R$ can range between zero and 3, for an optically thin H\textsc{i} disc. This fit should therefore be the closest to the results obtained by O'Brien et al. (2010). The 1000 samples produce a range of solutions. Both the rotation curve and the vertical force gradient are reproduced well. The mass-to-light ratio has a median value of $M_\star/L_R = 1.58^{+0.45}_{-0.65}$, which is significantly higher than that measured by O'Brien et al. (2010). The halo is very prolate, $q = 1.90^{+0.10}_{-0.12}$, and has a high central density of $\rho_0,\text{halo} = 3.24^{+0.67}_{-0.63} M_\odot$ pc$^{-3}$ and a short scale-length of $R_c = 0.64^{+0.05}_{-0.05}$ kpc. Previously, O'Brien et al. (2010) reported $R_c = 0.52 \pm 0.02$ kpc and $\rho_0,\text{halo} = 0.73 \pm 0.05$ M$\odot$ pc$^{-3}$.

Because we estimate a minimum of $M_\star/L_R = 0.5$, the second panel raises the boundary condition for the minimal stellar mass-to-light ratio to 0.5. The mass-to-light ratio is found to be $M_\star/L_R = 1.63^{+0.48}_{-0.66}$, which is still very similar to in the previous model. The observed rotation curve and vertical velocity gradients are reproduced well, as shown in Figs 9 and 10. We find $\rho_0,\text{halo} = 0.318^{+0.042}_{-0.045} M_\odot$ pc$^{-3}$ and $R_c = 0.64^{+0.03}_{-0.02}$ kpc, values that are roughly similar to those in the previous fit. The halo shape becomes quickly towards $q = 2$, which is also the boundary condition. We tested the effect of lifting this boundary condition. When we do this, the model tends to run towards even greater values of $q$. However, because the current research question focuses primarily on prolate versus oblate, we decided to stick to an upper boundary of $q = 2$. We present a cross-correlation diagram of this fit in Fig. 11 (left).

Our next fit uses the self-absorbing H\textsc{i} model rather than the optically thin model. We again let $M_\star/L_R$ run from zero to 3. As shown in Figs 9 and 10, the stellar disc in this fit has a negligible mass assigned to it ($M_\star/L_R = 0.00^{+0.04}_{-0.06}$). The other parameters are $\rho_0,\text{halo} = 0.307^{+0.005}_{-0.006} M_\odot$ pc$^{-3}$, $R_c = 0.71^{+0.01}_{-0.01}$ kpc and $q = 1.84^{+0.12}_{-0.09}$. Compared with the optically thin model, the dark matter halo is again strongly prolate, but has a longer scale-length and a lower central density.

Similar to in the optically thin H\textsc{i} case, we again increase the lower $M_\star/L_R$ boundary to 0.5. The results are shown in...
Figure 8. Correlation diagram for the optically thin model of ESO 274-G001 (top) and the self-absorption model (bottom), both with a constraint of $M/L \geq 0.5$. 
Figure 9. Rotation curve decomposition of UGC 7321 for various models. From top-left to bottom-right: optically thin H$_I$ ($M/L \geq 0$), optically thin H$_I$ ($M/L \geq 0.5$), self-absorbing H$_I$ ($M/L \geq 0$), self-absorbing H$_I$ ($M/L \geq 0.5$), spherical halo with self-absorbing H$_I$ ($M/L \geq 0$), spherical halo with self-absorbing H$_I$ ($M/L \geq 0.2$).
Figure 10. Vertical force decomposition of UGC 7321 for various models. From top-left to bottom-right: optically thin HI ($M/L \geq 0$), optically thin HI ($M/L \geq 0.5$), self-absorbing HI ($M/L \geq 0$), self-absorbing HI ($M/L \geq 0.5$), spherical halo with self-absorbing HI ($M/L \geq 0$), spherical halo with self-absorbing HI ($M/L \geq 0.5$).
Figure 11. Correlation diagram for the optically thin model of UGC 7321 (top) and for the self-absorption model (bottom), both with a constraint of $M/L \geq 0.5$. 

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The shape of dark matter haloes – V.
Figs 9 and 10. The observed rotation curve has been modelled well, but the model fails to account for the vertical force gradient and produces too strong a vertical force gradient. This illustrates why \( M_\ast/L_c \) was zero in the previous fit, as this was the only way for the vertical force gradient to be fitted. The parameters found are 
\[
\rho_{\text{halo}} = 0.28^{+0.01}_{-0.01} \, \text{M}_\odot \, \text{pc}^{-3}, \quad R_c = 0.79^{+0.01}_{-0.01} \, \text{kpc}, \quad q = 2.00^{+0.01}_{-0.01} \, \text{and} \quad M_\ast/L_c = 0.5.
\]
The cross-correlation diagram for this fit is shown in Fig. 11 (right).

In their previous study, O'Brien et al. (2010) were unable to fit the rotation curve and the vertical force gradient simultaneously. They successfully started with a rotation curve decomposition, in which the stellar disc mass was a free parameter. In order subsequently to perform their vertical force gradient decomposition, however, they were forced to drastically lower the stellar mass. They eventually found a spherical halo, but only when they allowed very small \( M_\ast/L_c \) (the best fit was actually for \( M_\ast/L_c = 0 \)), smaller than we allowed here.

As a final test, we ran fits to the optically thin and self-absorbing H\(_\text{i}\) results in which we constrained \( q \) to be equal to 1. The results are shown in Figs 9 and 10. The rotation curve decomposition does not depend strongly on \( q \) (O'Brien et al. 2010). As such, it is again reproduced well. Clearly, however, the vertical force gradient is fitted poorly. A spherical halo simply does not work for this galaxy.

In the previous section, we presented the results for the individual galaxies. So we now explore how the results compare with each other. In Table 1, we present an overview of all the derived parameters. For ESO 138-G014, we present only the results for which the thickness of the H\(_\text{i}\) layer has been reduced by 30 per cent. For galaxy UGC 7321, we present the results for the default model, in which the mass-to-light ratio \( M_\ast/L_c \) was allowed to vary between 0.5 and 3.0, and the halo shape \( q \) to vary between 0.1 and 2.0.

We present an overview of the average of these parameters in Table 2. There is an interesting difference between the optically thin and self-absorbing H\(_\text{i}\) models. Overall, it can be seen that the halo of an optically thin H\(_\text{i}\) model has a core density that is overestimated by 150 per cent. The scale-length of the dark matter halo is 28 per cent longer in the self-absorption model than in the optically thin model. In addition, whereas the optically thin models have a median shape that is prolate, the median shape is spherical in the self-absorbing H\(_\text{i}\) models. The mass-to-light ratio of the stellar disc drops by more than half when self-absorbing H\(_\text{i}\) is accounted for.

![Table 1. Measured parameters for the various haloes. OT denotes the optically thin H\(_\text{i}\) models, while SA denotes the self-absorbing H\(_\text{i}\) models.](https://example.com/table1.png)

<table>
<thead>
<tr>
<th>Name</th>
<th>H(_\text{i}) model</th>
<th>( \rho_0 ) [M(_\odot) pc(^{-3})]</th>
<th>( R_c ) [kpc]</th>
<th>( q )</th>
<th>( M_\ast/L_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC 5249</td>
<td>SA</td>
<td>0.004(^{+0.003}_{-0.001})</td>
<td>9.79(^{+0.21}_{-2.06})</td>
<td>1.0(^{+0.4}_{-0.3})</td>
<td>2.98(^{+0.02}_{-1.17})</td>
</tr>
<tr>
<td>IC 5249</td>
<td>OT</td>
<td>0.007(^{+0.005}_{-0.004})</td>
<td>3.02(^{+0.03}_{-0.08})</td>
<td>0.5(^{+0.1}_{-0.4})</td>
<td>2.89(^{+0.11}_{-0.19})</td>
</tr>
<tr>
<td>ESO 115-G021</td>
<td>SA</td>
<td>0.015(^{+0.001}_{-0.001})</td>
<td>5.45(^{+0.21}_{-0.52})</td>
<td>1.5(^{+0.5}_{-0.5})</td>
<td>2.62(^{+0.38}_{-1.19})</td>
</tr>
<tr>
<td>ESO 115-G021</td>
<td>OT</td>
<td>0.025(^{+0.04}_{-0.09})</td>
<td>9.90(^{+0.6}_{-0.44})</td>
<td>0.5(^{+0.1}_{-0.4})</td>
<td>2.89(^{+0.11}_{-0.19})</td>
</tr>
<tr>
<td>ESO 138-G014</td>
<td>SA</td>
<td>0.620(^{+0.02}_{-0.07})</td>
<td>7.36(^{+0.10}_{-0.15})</td>
<td>0.7(^{+0.1}_{-0.1})</td>
<td>2.89(^{+0.16}_{-0.11})</td>
</tr>
<tr>
<td>ESO 138-G014</td>
<td>OT</td>
<td>0.261(^{+0.02}_{-0.03})</td>
<td>1.42(^{+0.09}_{-0.03})</td>
<td>0.7(^{+0.1}_{-0.1})</td>
<td>0.76(^{+0.57}_{-0.26})</td>
</tr>
<tr>
<td>ESO 274-G001</td>
<td>SA</td>
<td>0.094(^{+0.009}_{-0.009})</td>
<td>1.16(^{+0.04}_{-0.03})</td>
<td>0.7(^{+0.1}_{-0.1})</td>
<td>0.90(^{+0.46}_{-0.42})</td>
</tr>
<tr>
<td>UGC 7321</td>
<td>SA</td>
<td>0.150(^{+0.001}_{-0.001})</td>
<td>0.72(^{+0.01}_{-0.01})</td>
<td>2.0(^{+0.1}_{-0.1})</td>
<td>0.50(^{+0.02}_{-0.01})</td>
</tr>
<tr>
<td>UGC 7321</td>
<td>OT</td>
<td>0.318(^{+0.06}_{-0.04})</td>
<td>0.64(^{+0.03}_{-0.02})</td>
<td>1.9(^{+0.1}_{-0.3})</td>
<td>1.62(^{+0.48}_{-0.66})</td>
</tr>
</tbody>
</table>

![Table 2. Overview of the global parameters of our halo sample. The median and standard deviations of the parameters are given. The units of \( \rho_0 \) are in M\(_\odot\) pc\(^{-3}\), and those of the radius \( R_c \) are in kpc.](https://example.com/table2.png)

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Optically thin</th>
<th>Self-absorbing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_0 )</td>
<td>0.150 \pm 0.124</td>
<td>0.094 \pm 0.230</td>
</tr>
<tr>
<td>( R_c )</td>
<td>1.10 \pm 1.81</td>
<td>1.42 \pm 3.48</td>
</tr>
<tr>
<td>( q )</td>
<td>1.5 \pm 0.6</td>
<td>1.0 \pm 0.6</td>
</tr>
<tr>
<td>( M_\ast/L_c )</td>
<td>1.63 \pm 0.91</td>
<td>0.76 \pm 1.12</td>
</tr>
</tbody>
</table>

All of our five discs are submaximal. This was demonstrated previously by O'Brien et al. (2010) and Banerjee et al. (2010) for UGC 7321, who reported a stellar disc \( M/L_R \) at a maximum of 2.5, although their final decomposition found a maximum of \( M/L_R \approx 0.2 \). The result is consistent with work by Bershady et al. (2011), who argued that all galaxies are submaximal based on an analysis of the central vertical velocity dispersion of the disc stars and the maximum rotation of 30 face-on galaxies. Similar conclusions were reached by Bottema (1997) and Kregel, van der Kruit & Freeman (2005). Martinsson et al. (2013a) confirmed these results after performing dynamically determined rotation curve mass decompositions for all their 30 galaxies.

Fig. 12, gives the correlation between the four free parameters from our fit. Various data-points from other authors are also plotted in this figure (see Table 3 for an overview: note that multiple halo models are used and that as such the core radius can be expected to vary). From the figure, it can be seen that the most notable correlation is the one between the core radius \( R_c \) and halo core density \( \rho_0 \). With the exception of one point, ESO 138-G14 by Hashim et al. (2014) using a Navarro–Frenk–White (NFW) halo, all of the points seem to follow the relationship \( R_c \approx 1/\rho_0 \). Our results for ESO 138-G14 are uncertain owing to the possibility of a residual inclination compared to edge-on, and any results for this galaxy, including ours at \( q \approx 2 \), should be treated with caution. This relationship is similar to the degeneracy between the two parameters in an individual galaxy, as for example in Fig. 11, and it is interesting to observe a similar trend visible across multiple galaxies and halo models. If this is a true relationship, then the implication is that there are two families of haloes: one compact halo family with a high core density \( \rho_0 \) and scale-length \( R_c \), and a second non-compact halo family with a low core density \( \rho_0 \) and scale-length \( R_c \).

Our best results are for ESO 274-G001 and UGC 7321, for which we were able to include a measurement of the velocity dispersion of...
that becomes progressively more prolate with radius. Vera-Ciro &
Helmi (2013) reported an oblate halo with \( q = 0.9 \) for the inner
10 kpc, based on stellar streams. Using lensing, Barnabè et al.
(2012) also found a slightly oblate halo at \( q = 0.91^{+0.15}_{-0.13} \) for
the galaxy SDSS J2141. The large weak-lensing galaxy sample of van
Uitert et al. (2012), containing \( 2.2 \times 10^7 \) galaxies, produced a
halo ellipticity distribution that also favours oblate haloes. The
responding halo shape parameter \( q \) was \( 0.62^{+0.25}_{-0.22} \). The three
polar ring galaxies studied by Whitmore et al. (1987), A0136-0801,
NGC 4650A and ESO 415-G026, had slightly oblate to spherical
halo shapes \( q \), namely \( 0.98 \pm 0.20, 0.86 \pm 0.21 \) and \( 1.05 \pm 0.17 \).
From this selected sample of papers, it is apparent that our result
for ESO 274-G001, with \( q = 0.7 \pm 0.1 \), is consistent with results
found by other authors.

While ESO 274-G001 is clearly consistent with measurements of
other galaxies, the other case of our two best fits, UGC 7321, is more
‘problematic’ (Figs 9, 10 and 11). With a halo shape of \( q = 1.9^{+0.1}_{-0.3} \)
for the self-absorption model, the dark matter halo shape is very
strongly prolate. As noted in Section 3.5, our upper boundary con-
dition for the halo shape is \( q \leq 2 \). If we removed this boundary,
some of the fits returned results as high as \( q \sim 5000 \), which are
clearly not physical. The galaxy was previously analysed by Baner-
jee et al. (2010), who successfully modelled the dark matter halo
shape for a spherical halo. O’Brien et al. (2010) had problems fitting
the dark matter halo shape. They had to lower their initially mea-
sured asymptotic halo rotation (see equation 20) in order to obtain
a successful fit to their data at \( q = 1.0 \pm 0.1 \), although they were
limited to \( q \leq 1 \) in their analysis. Had their boundary condition been
higher, it would have been likely that they too would have found
higher values for \( q \).

3.6 Concerns regarding reliability and degeneracy

Given that the two galaxies with the best results produce such
dramatically different results, how reliable is our methodology? To
answer this question, let us recap the underlying assumptions from
this paper and Paper IV.

3.6.1 Concerning the neutral hydrogen

We start with the neutral hydrogen. In Paper I, we argued that the \( \text{H}_1 \)
in edge-on galaxies could suffer from significant self-absorption. In
order to model the \( \text{H}_1 \) more accurately, we developed a new tool that
allowed the neutral hydrogen in galaxies to be fitted automatically,
while incorporating a treatment for the self-absorption of the gas.
Indeed, we saw in section 7 of Paper II that the visible mass of

Table 3. Overview of haloes measured in other papers whose results are used in Fig. 12 as data points.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>( \rho_0 ) ( [M_{\odot} \text{pc}^{-3}] )</th>
<th>( R_0 ) [kpc]</th>
<th>( q )</th>
<th>( M_\bullet / L_R )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>UGC 7321</td>
<td>0.73 \pm 0.05</td>
<td>0.52 \pm 0.02</td>
<td>1.0</td>
<td>0.2</td>
<td>O’Brien et al. (2010)</td>
</tr>
<tr>
<td>UGC 7321</td>
<td>0.048 \pm 0.009</td>
<td>2.7 \pm 0.2</td>
<td>1</td>
<td></td>
<td>Banerjee et al. (2010)</td>
</tr>
<tr>
<td>NGC 4244</td>
<td>0.6595 \pm 0.6495</td>
<td>6.15 \pm 5.85</td>
<td>0.2</td>
<td></td>
<td>Olling (1996)</td>
</tr>
<tr>
<td>M 31</td>
<td>0.011</td>
<td>0.4</td>
<td></td>
<td></td>
<td>Banerjee &amp; Jog (2008)</td>
</tr>
<tr>
<td>Milky Way (NFW)</td>
<td>( 0.0413^{+0.013}_{-0.016} )</td>
<td>( 9.26^{+5.6}_{-0.046} )</td>
<td></td>
<td></td>
<td>Nesti &amp; Salucci (2013)</td>
</tr>
<tr>
<td>Milky Way</td>
<td>( 0.0147^{+0.0095}_{-0.0099} )</td>
<td>( 16.1^{+7.8}_{-5} )</td>
<td></td>
<td></td>
<td>Nesti &amp; Salucci (2013)</td>
</tr>
<tr>
<td>Milky Way</td>
<td>12</td>
<td>1/3</td>
<td></td>
<td></td>
<td>Helmi (2004)</td>
</tr>
<tr>
<td>Milky Way</td>
<td>7.1</td>
<td>0.7</td>
<td></td>
<td></td>
<td>Olling &amp; Merrifield (2000)</td>
</tr>
<tr>
<td>Lensing</td>
<td>( 23.9^{+0.2}_{-0.05} )</td>
<td>( 0.62^{+0.25}_{-0.25} )</td>
<td></td>
<td></td>
<td>van Uitert et al. (2012)</td>
</tr>
<tr>
<td>ESO 138-G014</td>
<td>( 0.013 \pm 0.002 )</td>
<td>7.5 \pm 0.5</td>
<td></td>
<td></td>
<td>Hashim et al. (2014)</td>
</tr>
<tr>
<td>ESO 138-G014</td>
<td>( 0.077 \pm 0.042 )</td>
<td>10.7 \pm 2.7</td>
<td></td>
<td></td>
<td>Hashim et al. (2014)</td>
</tr>
</tbody>
</table>
a galaxy decreases as it is rotated from face-on to fully edge-on. In Paper II, we developed a method to model the \textsc{hi} content of a galaxy that was edge-on. In section 3 of Paper III, we tested this method on a series of simulated galaxies, showing that we could reproduce the input parameters reasonably well using our method. We also demonstrated that the assumption of an optically thin \textsc{hi} disc, which in reality was self-absorbing, could lead to an incorrect measure of the face-on surface density, the thickness of the \textsc{hi} layer and the velocity dispersion. We have continued to use the optically thin \textsc{hi} results in this paper in order to demonstrate how the dark matter halo measurement is affected by this. As discussed in the previous section, the effect of taking self-absorption into account changes the outcome drastically. So, how certain are we of our conclusions now?

One of the key assumptions made in Paper II was an effective spin temperature of the neutral hydrogen of 100 K. While this has proved to be a very successful value on which to base our results, it is an assumption based purely on what seemed to work best. In reality, the neutral hydrogen probably consists of multiple phases, such as the cold neutral medium (CNM) and warm neutral medium (WNM). The effective spin temperature is a result of the mix of the phases of the CNM, which has a median spin temperature of 80 K, and the WNM, with temperatures between 6000 and 10 000 K. In section 4 of Paper II, we demonstrated how the interplay of \textsc{hi} gas phases could lead to an effective median spin temperature. So what would be the consequence of an incorrect estimate of the spin temperature? Suppose that the spin temperature were $T_{\text{spin}} = 90$ K rather than 100 K. In that case, the face-on surface density of the neutral hydrogen would be higher, which in this paper would lead to a larger contribution to the rotation curve from the gas components in the galaxy and stronger vertical force gradients than are currently found. Simultaneously, the thickness of the disc would be smaller, and thus the total observed vertical force gradient would be higher (equation 9). Although it is hard to estimate the exact effect, the phases of \textsc{hi} all have different distributions, together producing the observed thickness of the disc (Lockman & Gehman 1991). The effective spin temperature could thus be height above the plane.

Another assumption is the uniform density of the \textsc{hi} as a function of radius. In reality of course, galaxies have spiral arms, supernovae, shocks, gravitational collapse, and other features, all of which create a non-uniform \textsc{hi} disc. The question thus is how strongly the parameters are affected by this non-uniformity. Kamphuis et al. (2013) made a valiant attempt to model the density waves in galaxies NGC 5023 and UGC 2082, demonstrating that these could be detected in edge-on galaxies. Indeed, as we discussed in section 5 of Paper I, the position–velocity (XY)-diagrams are not symmetric on both side of the galaxies. This problem most strongly affects the velocity dispersion, which is dependent on small-scale features. In most cases, this leads to an overestimation of the \textsc{hi} velocity dispersion, as the fitting algorithm tries to ’smooth over’ the small-scale fluctuations. The result is an overestimation of the observed vertical force gradient through equation (9). This overestimation could be the reason why UGC 7321 has such a distinctive halo shape.

We have also their results, assumed that the \textsc{hi} has an isotropic velocity dispersion tensor; that is, that it has the same value in the $R, z$ and $\theta$ directions (see section 2.2 of Paper II). While currently there is no observational proof that this is an invalid assumption, it remains untested. If the velocity dispersion tensor were in reality anisotropic, this would imply that our observed vertical force gradient is incorrect (equation 9). Simultaneously, it would affect the amount of observed \textsc{hi} in the self-absorption mode, as gas with a low velocity dispersion would give a larger effect than gas with a high velocity dispersion (equation 19 of Paper II). It would affect the rotation curve measurements to a minor degree.

We also have assumed that the velocity dispersion is isothermal in $z$; that is, that it does not vary with height. In section 4.7 of Paper III, we attempted to measure the velocity dispersion in ESO 274-G001. We found a very small increase of 1 km s$^{-1}$ in the slice above the central 290 pc of the disc. If this were confirmed in other galaxies, it would mean that equation (8) is false, and thus our observed total force gradient would be incorrect. Previously, Lockman & Gehman (1991) attempted to model the vertical structure of the Galaxy using multiple \textsc{hi} phases, which had different scale-heights and velocity dispersions. The phases with the highest scale-heights also have the highest velocity dispersions. It is quite possible that the effective spin temperature of the different phases would increase for those with larger scale-heights. Assuming that our currently observed velocity dispersion is due to the combination of high- and low-$z$ gas, the total vertical force gradient that we are currently reporting would be weaker in the mid-plane, and stronger at high values of $z$.

The thickness of the \textsc{hi} disc was assumed to follow a Gaussian form (see equation 15 of Paper II) for mathematical convenience in equation (9). Other possible candidates for the \textsc{hi} model would have been the sech and sech$^2$ functions. Olling (1995) previously discussed the various types of discs and concluded that the changes arising from this would be minor. A sech function has more extended wings and steeper inner slopes compared with the Gaussian function. If a galaxy has gas at high latitudes, as a result, for example, of warps or \textsc{hi} haloes, then it is possible that a fit with a Gaussian function would find the full width at half-maximum (FWHM) of the disc to be unrealistically large. A fit with a sech or even a sech$^2$ function could then be a better approximation to the shape of the \textsc{hi} disc.

Another assumption in our model is the perfect edge-on nature of the \textsc{hi} disc. In section 3.5 of Paper III, we tested how our \textsc{hi} fitting strategy worked on a galaxy at $i = 88.8$. We found that the parameters were well recovered. However, suppose that some galaxies are even further from edge-on than that, as formally indicated by our stellar decompositions (table 4 of Paper IV). In that case, the thickness of the \textsc{hi} is probably overestimated, and the circular rotation underestimated. This has a profound effect on the rotation curve decomposition, which would require a larger rotational contribution owing to the dark matter. In a similar vein, the total observed vertical force gradient would be underestimated, and would thus require a more massive dark matter halo in the decomposition. On the other hand, the density of the \textsc{hi} would be lower at a height of 100 pc, such that in equation (10) the vertical force gradient resulting from the \textsc{hi} would be lower.

3.6.2 Concerning the stellar disc

In Paper IV, we set out to use the \texttt{fitpsf} tool to model the stellar disc in our sample of galaxies. We modelled the galaxies using a stellar disc and a bulge component. While the results were acceptable in most cases, these galaxies were selected to be relatively bulge-less (section 3 of Paper I). Because of this, the fitting routine was found to ‘misuse’ the bulge component as a tool to better model the stellar disc. Thus, the bulges in most of the galaxies act like an extension of the stellar disc, rather than like a separate central component (see table 4 of Paper IV for the parameters). In some cases, the amount of light from the bulge component is similar to that from the stellar disc.
The shape of dark matter haloes – V.

3.6.3 Concerning the cross-correlation between parameters

An advantage of our MCMC method is that we have not one but a whole range of parameter sets for each galaxy. This allows us to explore the interplay between the cross-correlations, as is demonstrated in Fig. 12 for the self-absorption model of ESO 274-G001. As is clear from this figure, a whole range of solutions can be valid. For example, one parameter set returned a core density of 0.1 M⊙ pc⁻³, with a scale-length of 1.4 kpc, a halo shape q of 0.67, and a value Mₚ/Lₚ of 0.4. A different parameter set returned a core density of 0.06 M⊙ pc⁻³, with a scale-length of nearly 1.6 kpc, a shape q = 0.84 and a value of Mₚ/Lₚ of 1.7. These results are drastically different, yet both are accepted parameter sets.

The largest source of uncertainty is the stellar disc parameter Mₚ/Lₚ. It was beyond the scope of this project to measure this parameter in each of our galaxies, which is why we treated it as a free parameter. A different approach would have been to adopt the maximum value of Mₚ/Lₚ permitted in the rotation curve decomposition, the so-called maximum disc parameter (e.g. Carignan & Freeman 1985; van Albada et al. 1985). However, these galaxies were selected to be dark matter-dominated at all radii, and as such this approach would have been invalid (section 3 of Paper I). In addition, the applicability of the maximum disc criteria has already been questioned by Kregel et al. (2005) and Martinsson et al. (2013b), who both report submaximal stellar discs. Although beyond the scope of this project, the best approach would be to perform a full stellar population synthesis analysis of each galaxy. For examples see Bruzual & Charlot (2003) and Maraston (2005).

By measuring Mₚ pc⁻³ rather than using it as a free parameter, the solution space would become far less degenerate and the parameters could thus be fixed far more accurately.

Another cause for concern is the boundary conditions imposed on our data. We have done our best to impose realistic boundary conditions. For the dark matter halo shape, we adopted 0.1 < q < 2.0, as we believe that more oblate or prolate haloes would be unrealistic. In Section 2.1, we adopted 0.5 < Mₚ/Lₚ < 3.0 as a likely boundary, based on the stellar population models by Worthey (1994) and Bertelli et al. (1994). As can clearly be seen from the various cross-correlation diagrams, the models are often constrained by our boundary conditions. While it is possible to raise or remove the boundary conditions, we do not believe that this would lead to realistic results and we have therefore refrained from doing so.

3.6.4 Halo model

In this work, we adopted the dark matter halo model by Sackett et al. (1994). With this model, we can create flattened, axisymmetric, pseudo-isothermal haloes. In this model, the density is stratified in concentric ellipsoids. We chose this model in order to enable comparison with O’Brien et al. (2010), who also used this model.

There are many other halo models. Carignan & Freeman (1985, 1988) used isothermal, rather than pseudo-isothermal, haloes to model their galaxies. Kormendy & Freeman (2004) compared the merits of isothermal and pseudo-isothermal haloes. As is shown in that paper (and reproduced in O’Brien et al. 2010), the rotation curve of an isothermal halo initially rises above the asymptotic velocity vₜₐₜ, before dropping towards it again. In contrast, the pseudo-isothermal rotation curve approaches the asymptotic velocity gradually from lower values. This behaviour would affect the results for the rotation curve decomposition.

There are more models, such as the NFW and the Burkert halo model, each of which has some mathematical or theoretical advantage (Burkert 1995; Navarro, Frenk & White 1996). Even more exotic models exist in which the dark matter halo shape can vary with radius (Vera-Ciro & Helmi 2013). This can, for example, lead to haloes that become progressively more prolate with radius (Banerjee & Jog 2011). While all these haloes are very interesting, we believe that the quality of the data, as discussed in this section, does not warrant such a detailed exploration of the properties of the various halo models.

An altogether different solution would have been the use of Modified Newtonian Dynamics (MOND), which would have removed the need for a dark matter halo altogether (Milgrom 1983). We find that in many of our vertical force decompositions, a slight increase in the mass of the H i and stellar disc would be sufficient to account for the total observed vertical force gradient. As we argued previously, additional mass in both the stellar and the H i discs is allowed for by the data. While it is beyond the scope of this project to test MOND on our data, it is an interesting avenue for further research.

4 CONCLUSIONS

We have attempted to measure the shape of the dark matter halo in five galaxies, using a simultaneous decomposition of the rotation curve and of the vertical force gradient at the mid-plane. For the dark matter halo model, we adopted the Sackett et al. (1994) dark matter halo. Both optically thin and self-absorbing H i models were used. We found that these choices lead to very different results. As we argued in Papers I, II and III, the H i self-absorption models are the more accurate representation of galaxies. Using H i self-absorption, we found that a typical dark matter halo has a less dense core (0.094 ± 0.230 M⊙ pc⁻³)⁴ compared with an optically thin H i model (0.150 ± 0.124 M⊙ pc⁻³). The H i self-absorption dark matter halo had a longer scale-length R_c of 1.42 ± 3.48 kpc, versus 1.10 ± 1.81 kpc for the optically thin H i model. The median halo shape was spherical, at q = 1.0 ± 0.6, for the self-absorbing model, while it was prolate, at q = 1.5 ± 0.6, for the optically thin model.

Our best results were obtained for ESO 274-G001 and UGC 7321, for which we were able to measure the velocity dispersion in Paper III. These two galaxies have very different halo shapes. ESO 274-G001 was found to be oblate, at q = 0.7 ± 0.1 (models with and without self-absorption), while UGC 7321 returns a distinctly prolate halo, at q = 1.9 ± 0.1 (optically thin) and q = 2.0 ± 0.1 (self-absorbing). The halo of ESO 274-G001 is similar to those found in other studies, but UGC 7321 is more problematic. In UGC 7321, the most likely cause for concern is the presence of spiral arms and an H i halo.

⁴ The central value is the median; the error is the standard deviation.
With these very different results, we concluded that the question whether haloes are oblate or prolate is not settled. The results for both of our best galaxies appear to be accurate. A larger set of galaxies needs to be analysed, before it becomes clear if one of these galaxies is an outlier, or if prolate and oblate haloes are equally likely in nature.

We have discussed extensively the various assumptions and sources of uncertainty in our models, of which there are many. While we have done our best to minimize the effects of these assumptions, for example using MCMC fits to the H i cube, we found that fitting the hydrostatics of the dark matter halo using the vertical force gradient near the mid-plane of the galaxy will always be tricky.

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REFERENCES


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