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Haddadi, Z.; Kalantar-Nayestanaki, Nasser; Kavatsyuk, Myroslav; Löhner, Herbert; Messchendorp, Johannes; Tiemens, M.; BESIII Collaboration

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Evidence of Two Resonant Structures in $e^+e^- \rightarrow \pi^+\pi^- h_c$
K. J. Zhu,1†, S. Zhu,1 S. H. Zhu,4 X. L. Zhu,39 Y. C. Zhu,46†, Y. S. Zhu,1 Z. A. Zhu,1 J. Zhuang,1†
L. Zotti,49a,49c B. S. Zou,1 and J. H. Zou1 (BESIII Collaboration)

1Institute of High Energy Physics, Beijing 100049, People’s Republic of China
2Beihang University, Beijing 100191, People’s Republic of China
3Beijing Institute of Petrochemical Technology, Beijing 102617, People’s Republic of China
4Bochum Ruhr-University, D-44780 Bochum, Germany
5Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
6Central China Normal University, Wuhan 430079, People’s Republic of China
7China Center of Advanced Science and Technology, Beijing 100190, People’s Republic of China
8COMSATS Institute of Information Technology, Lahore, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan
9GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany
10Guangxi Normal University, Guilin 541004, People’s Republic of China
11Guangxi University, Nanning 530004, People’s Republic of China
12Hangzhou Normal University, Hangzhou 310036, People’s Republic of China
13Helmholtz Institute Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
14Henan Normal University, Xinxiang 453007, People’s Republic of China
15Henan University of Science and Technology, Luoyang 471003, People’s Republic of China
16Huangshan College, Huangshan 245000, People’s Republic of China
17Indiana University, Bloomington, Indiana 47405, USA
18INFN Laboratori Nazionali di Frascati, I-00044 Frascati, Italy
19INFN and University of Perugia, I-06100 Perugia, Italy
20INFN Sezione di Ferrara, I-44122 Ferrara, Italy
21University of Ferrara, I-44122 Ferrara, Italy
22Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
23Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
24Justus–Liebig-Universitaet Giessen, H. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
25KVI-CART, University of Groningen, NL-9747 AA Groningen, The Netherlands
26Lanzhou University, Lanzhou 730000, People’s Republic of China
27Liaoning University, Shenyang 110036, People’s Republic of China
28Nanjing Normal University, Nanjing 210023, People’s Republic of China
29Nanjing University, Nanjing 210093, People’s Republic of China
30Nankai University, Tianjin 300071, People’s Republic of China
31Peking University, Beijing 100871, People’s Republic of China
32Seoul National University, Seoul 151-747, Korea
33Shandong University, Jinan 250100, People’s Republic of China
34University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
35University of Hawaii, Honolulu, Hawaii 96822, USA
36University of Minnesota, Minneapolis, Minnesota 55455, USA
37University of Rochester, Rochester, New York 14627, USA
38Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
39Tsinghua University, Beijing 100084, People’s Republic of China
40Ankara University, 06100 Tandogan, Ankara, Turkey
41Istanbul Bilgi University, 34060 Eyup, Istanbul, Turkey
42Uludag University, 16059 Bursa, Turkey
43Near East University, Nicosia, North Cyprus, Mersin 10, Turkey
44University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
45University of Science and Technology Liaoning, Anshan 114051, People’s Republic of China
46University of Science and Technology of China, Hefei 230026, People’s Republic of China
47University of South China, Hengyang 421001, People’s Republic of China
48University of the Punjab, Lahore-54590, Pakistan
49University of Turin, I-10125 Turin, Italy

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The cross sections of $e^+e^- \rightarrow \pi^+\pi^- h_c$ at center-of-mass energies from 3.896 to 4.600 GeV are measured using data samples collected with the BESIII detector operating at the Beijing Electron Positron Collider. The cross sections are found to be of the same order of magnitude as those of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ and $e^+e^- \rightarrow \pi^+\pi^- \psi(2S)$, but the line shape is inconsistent with the $\psi$ states observed in the latter two modes. Two structures are observed in the $e^+e^- \rightarrow \pi^+\pi^- h_c$ cross sections around 4.22 and 4.39 GeV/c², which we call $Y(4220)$ and $Y(4390)$, respectively. A fit with a coherent sum of two Breit-Wigner functions results in a mass of $(4218.4^{+5.5}_{-4.5} \pm 0.9)$ MeV/c² and a width of $(66.0^{+12.3}_{-8.3} \pm 0.4)$ MeV for the $Y(4220)$, and a mass of $(4391.5^{+6.3}_{-6.8} \pm 1.0)$ MeV/c² and a width of $(139.5^{+16.2}_{-20.6} \pm 0.6)$ MeV for the $Y(4390)$, where the first uncertainties are statistical and the second ones systematic. The statistical significance of $Y(4220)$ and $Y(4390)$ is 10σ over one structure assumption.

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In the last decade, a series of charmoniumlike states have been observed at $e^+e^-$ colliders. These states challenge the understanding of charmonium spectroscopy as well as QCD calculations [1,2]. According to potential models, there are five vector charmonium states between the 1D state $\psi(3770)$ and 4.7 GeV/c², namely, the 3S, 2D, 4S, 3D, and 5S states [1]. From experimental studies, besides the three well-established structures observed in the inclusive hadronic cross section [3], i.e., $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$, five $Y$ states, i.e., $Y(4008)$, $Y(4220)$, $Y(4260)$, $Y(4360)$, and $Y(4660)$ have been reported in initial state radiation (ISR) processes $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^- J/\psi$ or $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^- \psi(2S)$ at the B factories [4–11] or in the direct production processes at the CLEO and BESIII experiments [12,13]. The overpopulation of structures in this region and the mismatch of the properties of the potential model prediction and experimental measurements make them good candidates for exotic states. Various scenarios have been proposed, which interpret one or some of them as hybrid states, tetraquark states, or molecular states [14].

The study of charmoniumlike states in different production processes supplies useful information on their properties. The process $e^+e^- \rightarrow \pi^+\pi^- h_c$ was first studied by CLEO [15] at center-of-mass (c.m.) energies $\sqrt{s}$ from 4.000 to 4.260 GeV. A 10σ signal at 4.170 GeV and a hint of a rising cross section at 4.260 GeV were observed. Using data samples taken at 13 c.m. energies from 3.900 to 4.420 GeV [16], BESIII reported the measurement of the cross section of $e^+e^- \rightarrow \pi^+\pi^- h_c$ [17]. Unlike the line shape of the process $e^+e^- \rightarrow \pi^+\pi^- J/\psi$, there is a broad structure in the high energy region with a possible local maximum at around 4.23 GeV in $e^+e^- \rightarrow \pi^+\pi^- h_c$. Based on the CLEO measurement at $\sqrt{s} = 4.170$ GeV and the BESIII measurement, two assumptions were made to describe the cross section in Ref. [18]. In both assumptions, a narrow structure exists at around 4.23 GeV, while the situation in the high energy region is unclear due to the lack of experimental data.

In this Letter, we present a follow-up study of $e^+e^- \rightarrow \pi^+\pi^- h_c$ at c.m. energies from 3.896 to 4.600 GeV using data samples taken at 79 energy points [19] with the BESIII detector [20]. Two resonant structures are observed at $\sqrt{s} = 4.22$ and 4.39 GeV [hereafter referred to as $Y(4220)$ and $Y(4390)$]. The integrated luminosity at each energy point is measured with an uncertainty of 1.0% using large-angle Bhabha events [21,22]. There are 17 energy points where the integrated luminosities are larger than 40 pb⁻¹ (referred to as “XYZ data sample” hereafter), while the integrated luminosities for the other energy points are smaller than 20 pb⁻¹ (referred to as “R-scan data sample” hereafter). The c.m. energies for the XYZ data sample are measured with $e^+e^- \rightarrow \gamma_{\text{ISR/FSR}}\mu^+\mu^-$ events with an uncertainty of ±0.8 MeV [23], which is dominated by the systematic uncertainty. A similar method is used for the R-scan data sample with multihadron final states [24].

In this study, the $h_c$ is reconstructed via its electric-dipole transition $h_c \rightarrow \gamma h_c$ with $h_c \rightarrow X_i$, where $X_i$ is one of 16 exclusive hadronic final states: $p\bar{p}$, $2(\pi^+\pi^-)$, $2(K^+K^-)$, $\pi^+\pi^- K^+ K^-$, $\pi^+\pi^- p\bar{p}$, $3(\pi^+\pi^-)$, $2(\pi^+\pi^-)K^+ K^-$, $K_S^0 K^+\pi^+$, $K_L^0 K^+\pi^+$, $K^+ K^-$, $p\bar{p}n\bar{n}$, $K^+ K^-\eta$, $\pi^+\pi^-\eta$, $2(\pi^+\pi^-)\eta$, $\pi^+\pi^-\pi^0\eta$, and $2(\pi^+\pi^-\pi^0)$. Here, the $K^0_S$ is reconstructed using its decay to $\pi^+\pi^-$, and the $\pi^0$ and $\eta$ from the $\gamma\gamma$ final state.

Monte Carlo (MC) simulated events are used to optimize the selection criteria, determine the detection efficiency, and estimate the possible backgrounds. The simulation of the BESIII detector is based on GEANT4 [25] and includes
the geometric description of the BESIII detector and the detector response. For the signal process, we use an MC sample for $e^+e^- \rightarrow \pi^+\pi^- h_c$ process generated according to phase space. ISR is simulated with KKMC [26] with a maximum energy for the ISR photon corresponding to the mass window around the nominal mass [27]. For the data samples at the other mass points, we fit the mass spectrum summed over all decay modes simultaneously with the number of signal events.

We select signal candidates with the same method as that described in Ref. [17]. Figure 1 shows the scatter plot of the invariant mass of the $\eta_c$ candidate vs the one of the $h_c$ candidate and the invariant mass distribution of $\gamma \eta_c$ in the $\eta_c$ signal region for the data sample at $\sqrt{s} = 4.416$ GeV. A clear $h_c \rightarrow \gamma \eta_c$ signal is observed. The $\eta_c$ signal region is defined by a mass window around the nominal $\eta_c$ mass [3], which is within $\pm 50$ MeV/$c^2$ with efficiency about 84% ($\pm 45$ MeV/$c^2$ with efficiency about 80%) from MC simulation for final states with only charged or $K^0$ particles (for those including $\eta^0$ or $\eta$).

We determine the number of $\pi^+\pi^- h_c$ signal events ($n_{h_c}^{\text{sig}}$) from the $\gamma \eta_c$ invariant mass distribution. For the XYZ data sample, the $\gamma \eta_c$ mass spectrum is fitted with the MC simulated signal shape convolved with a Gaussian function to reflect the mass resolution difference between the data and MC simulation, together with a linear background. The fit to the data sample at $\sqrt{s} = 4.416$ GeV is shown in Fig. 1. The tail on the high mass side is due to events with ISR (ISR photon undetected); this is simulated with KKMC in MC simulation, and its fraction is fixed in the fit. For the data samples with large statistics ($\sqrt{s} = 4.226, 4.258, 4.358, 4.416$ GeV), the fit is applied to the 16 $\eta_c$ decay modes simultaneously with the number of signal events in each decay mode constrained by the corresponding branching fraction [27]. For the data samples at the other energy points, we fit the mass spectrum summed over all $\eta_c$ decay modes. For the $R$-scan data sample, the number of signal events is calculated by counting the entries in the $h_c$ signal region [3.515, 3.535] GeV/$c^2$ ($n_{h_c}^{\text{sig}}$) and the entries in the $h_c$ sideband regions [3.475, 3.495] GeV/$c^2$ and [3.555, 3.575] GeV/$c^2$ ($n_{h_c}^{\text{side}}$) using the formula $n_{h_c}^{\text{obs}} = n_{h_c}^{\text{sig}} - f n_{h_c}^{\text{side}}$. Here, the scale factor $f = 0.5$ is the ratio of the size of the signal region and the background region, and the background is assumed to be distributed linearly in the region of interest.

The Born cross section is calculated from

$$\sigma^B = \frac{n_{h_c}^{\text{obs}}}{\mathcal{L}(1 + \delta)|1 + \Pi|^2B_1 \sum_{i=1}^{16} e_i B_2(i)},$$

where $n_{h_c}^{\text{obs}}$ is the number of observed signal events, $\mathcal{L}$ is the integrated luminosity, $(1 + \delta)$ is the ISR correction factor, $|1 + \Pi|^2$ is the correction factor for vacuum polarization [28], $B_1$ is the branching fraction of $h_c \rightarrow \gamma \eta_c$ [3], $e_i$ and $B_2(i)$ are the detection efficiency and branching fraction for the $i$th $\eta_c$ decay mode [27], respectively. The ISR correction factor is obtained using the QED calculation as described in Ref. [29] and taking the formula used to fit the cross section measured in this analysis after two iterations as input. The Born cross sections are summarized in the Supplemental Material [19] together with all numbers used in the calculation of the Born cross sections. The dressed cross sections (including vacuum polarization effects) are shown in Fig. 2 with dots and squares for the $R$-scan and XYZ data sample, respectively. The cross sections are of the same order of magnitude as those of the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ and $e^+e^- \rightarrow \pi^+\pi^- \psi(2S)$ [4–12], but follow a different line shape. The cross section drops in the high energy region, but more slowly than for the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ process.

Systematic uncertainties in the cross section measurement mainly come from the luminosity measurement, the

![FIG. 1.](image1.png)  
**FIG. 1.** The $M_{\gamma\eta_c}$ distribution in the $\eta_c$ signal region of 4.416 GeV data. Points with error bars are the data and the curves are the best fit described in the text. The inset is the scatter plot of the mass of the $\eta_c$ candidate $M_{\eta_c}$ vs the mass of the $h_c$ candidate $M_{h_c}$ for the same data sample.

![FIG. 2.](image2.png)  
**FIG. 2.** Fit to the dressed cross section of $e^+e^- \rightarrow \pi^+\pi^- h_c$ with the coherent sum of two Breit-Wigner functions (solid curve). The dash (dash-dot) curve shows the contribution from the two structures $Y(4220)$ [$Y(4390)$]. The dots with error bars are the cross sections for the $R$-scan data sample, the squares with error bars are the cross sections for the XYZ data sample. Here the error bars are statistical uncertainty only.
branching fraction of \( h_c \to \gamma_{\ell c} \) and \( \eta_c \to X_i \), the detection efficiency, the ISR correction factor, and the fit. The uncertainty due to the vacuum polarization is negligible. The uncertainty in the integrated luminosity is 1% at each energy point. The uncertainty sources for the detection efficiency include systematic uncertainties in tracking efficiency (1% per track), photon reconstruction (1% per photon), and \( K^0 \) reconstruction (1.2% per \( K^0 \)). Further uncertainties arise from the \( \pi^0/\eta \) mass window requirement (1% per \( \pi^0/\eta \)), the \( \chi^2 \) requirement, \( \eta_c \) parameters, and line shape, possible intermediate states in the \( \pi^+h_c \) and \( \pi^+\pi^- \) mass spectra, intermediate states in \( \eta_c \) decays (included in the uncertainty from the branching fraction of \( \eta_c \to X_i \)), and the limited statistics of the MC simulation.

The uncertainty due to the \( \chi^2_{\text{MC}} \) requirement is estimated by correcting the helix parameters of the simulated charged tracks to match the resolution found in data, and repeating the analysis [30]. Uncertainties due to the \( \eta_c \) parameters and line shape are estimated by varying them in the MC simulation. When producing MC events for the \( e^+e^- \to \pi^+\pi^- h_c \) process through the intermediate states \( Z_c(3900) \) or \( Z_c(4020) \), the parameters of the \( Z_c(3900) \) and \( Z_c(4020) \) are fixed to the average values from the published measurements [11,17,31–33]. The quantum numbers of both \( Z_c(3900) \) and \( Z_c(4020) \) are assumed to be \( JP=1^+ \). The differences in the efficiency obtained from phase space MC samples and those with intermediate \( Z_c \) states are taken as the uncertainties from possible intermediate states in the \( \pi^+h_c \) system. The uncertainty from intermediate states in the \( \pi^+\pi^- \) system is estimated by reweighting the \( \pi^+\pi^- \) mass distribution in the phase space MC sample according to the data, and the resulting difference in the efficiency is considered as uncertainty. The uncertainties due to data and MC differences in the detection efficiency are determined to be between 5.5% and 10.8%, depending on the \( \eta_c \) decay modes and the c.m. energy. Combining the uncertainties for the branching fractions of \( \eta_c \) decays [27], the uncertainties for the average efficiency \( \sum_{i=1}^{16} e_i B_i(\ell) \) are between 6.4% and 9.1% depending on the c.m. energy.

The uncertainty in the ISR correction is estimated as described in Ref. [31]. Uncertainties due to the choice of the signal shape, the background shape, the mass resolution, and fit range are estimated by changing the \( h_c \) and \( \eta_c \) resonant parameters and line shapes in the MC simulation, changing the background function from a linear to a second-order polynomial, changing the mass resolution difference between the data and the MC simulation by 1 standard deviation, and by extending or shrinking the fit range.

Assuming all of the sources are independent, the total systematic uncertainty in the \( \pi^+\pi^- h_c \) cross section measurement is determined to be 9.4%-13.6% depending on the c.m. energy. The uncertainty in \( B_1 \) is 11.8% [3], common to all energy points, and quoted separately in the cross section measurement. Altogether, the quadratic sum of the common systematic errors at each energy point accounts for about 95% of the total systematic error.

A maximum likelihood method is used to fit the dressed cross sections to determine the parameters of the resonant structures. The likelihood is constructed taking the fluctuations of the number of signal and background events into account (the definition is described in the Supplemental Material [19]). Assuming that the \( \pi^+\pi^- h_c \) signal comes from two resonances, the cross section is parametrized as the coherent sum of two constant width relativistic Breit-Wigner functions, i.e.,

\[
\sigma(m) = B_1(m) \sqrt{\frac{P(m)}{P(M_1)}} + e^{i\phi} B_2(m) \sqrt{\frac{P(m)}{P(M_2)}}^2,
\]

where \( B_j(m) = \sqrt{12\pi(\Gamma_{\ell c}\Gamma_j)/(m^2 - M_j^2 + iM_j\Gamma_j)} \) with \( j = 1 \) or 2 is the Breit-Wigner function, and \( P(m) \) is the three-body phase space factor. The masses \( M_j \), the total widths \( \Gamma_j \), the products of the electronic partial width and the branching fraction to \( \pi^+\pi^- h_c \) (\( \Gamma_{\ell c}\Gamma_j \)), and the relative phase \( \phi \) between the two Breit-Wigner functions are free parameters in the fit. Only the statistical uncertainty is considered in the fit. There are two solutions from the fit, one of them is shown in Fig. 2. The second solution is very close to the one shown here. This can be proved analytically using Eq. (9) in Ref. [34], which relates the two solutions from the fit when a sum of two coherent Breit-Wigner functions is used. The parameters determined from the fit are \( M_1 = (4218.4^{+5.5}_{-4.2}) \text{ MeV} / c^2 \), \( \Gamma_1 = (66.0^{+12.3}_{-8.8}) \text{ MeV} \), and \( (\Gamma_{\ell c}\Gamma_1) = (4.6^{+2.9}_{-1.4}) \text{ eV} \) for \( Y(4220) \), \( M_2 = (4391.5^{+6.3}_{-6.8}) \text{ MeV} / c^2 \), \( \Gamma_2 = (139.5^{+16.2}_{-20.6}) \text{ MeV} \), and \( (\Gamma_{\ell c}\Gamma_2) = (11.6^{+5.4}_{-4.4}) \text{ eV} \) for \( Y(4390) \). The relative phase \( \phi \) is \( (3.1^{+0.7}_{-0.9}) \text{ rad} \). The correlation matrix of the fit parameters shows large correlation between the \( (\Gamma_{\ell c}\Gamma_j) \) and \( \phi \) (see Supplemental Material [19]).

The likelihood contours in the mass and width planes for \( Y(4220) \) and \( Y(4390) \) are shown in Fig. 3, together with the positions of \( Y(4230), Y(4260), Y(4360), \) and \( \psi(4415) \) with the parameters taken from the latest PDG average [3]. The low-lying resonance from the study of \( e^+e^- \to \pi^+\pi^- J/\psi \) at BESIII [35], marked as \( Y(4260)^{\text{BESIII}} \) in the plot, is also compared. \( Y(4260), Y(4360), \) and \( \psi(4415) \) are located outside the \( 3\sigma \) contours, while \( Y(4230) \) and \( Y(4260)^{\text{BESIII}} \) are overlapped with the \( 3\sigma \) contour of \( Y(4220) \).

Fitting the dressed cross section with only one resonance yields a worse result, the change of the likelihood value from two resonances to one resonance is \( \Delta(-2 \ln L) = 113.5 \). Taking the change in the number of degrees of freedom (4) into account, the significance for the assumption of two resonant structures over the assumption of one resonant structure is 10\( \sigma \). The fit with the coherent
The uncertainty from c.m. energy spread is estimated by the changes in the parameters taken as uncertainty. The systematic uncertainty of the c.m. energy measurement includes the uncertainty of the c.m. energy and the assumption made in the c.m. energy measurement for each energy point in the two data samples. The differences on the parameters as uncertainties. The uncertainty in the fit to the cross section, and taking the differences on the parameters as uncertainties. The second part includes all the other sources, is common for all data points (14.8%), and only affects the $\Gamma e\bar{e}B$ measurement. Table I summarizes the systematic uncertainty in the resonance parameters.

In summary, we measure the $e^+e^-\rightarrow \pi^+\pi^-h_c$. Born cross section using data at 79 c.m. energy points from 3.896 to 4.600 GeV. Assuming the $\pi^+\pi^-h_c$ events come from two resonances, we obtain $M = (4218.4^{+1.5}_{-1.4} \pm 0.9)$ MeV/$c^2$, $\Gamma = (66.0^{+12.3}_{-8.3} \pm 0.4)$ MeV, and $(\Gamma e\bar{e}B) = (4.6^{+2.9}_{-1.4} \pm 0.8)$ eV for $Y(4220)$, and $M = (4391.5^{+6.8}_{-6.8} \pm 1.0)$ MeV/$c^2$, $\Gamma = (139.5^{+16.2}_{-20.6} \pm 0.6)$ MeV, and $(\Gamma e\bar{e}B) = (11.6^{+5.8}_{-4.4} \pm 1.9)$ eV for $Y(4390)$, with a relative phase of $\phi = (3.1^{+0.7}_{-0.9} \pm 0.2)$ rad. The first errors are statistical and the second are systematic. The parameters of these structures are different from those of $Y(4260)$, $Y(4360)$, and $\psi(4415)$ [3]. The resonance parameters of $Y(4220)$ are consistent with those of the resonance observed in $e^+e^-\rightarrow \omega h_c$ [13].

The two resonances observed in $e^+e^-\rightarrow \pi^+\pi^-h_c$ process are located in the mass region between 4.2 and 4.4 GeV/$c^2$, where the vector charmonium hybrid states are predicted from various QCD calculations [37–39]. The mass of $Y(4220)$ is lower than that of $Y(4260)$ observed in the $e^+e^-\rightarrow \pi^+\pi^-J/\psi$ process. The smaller mass is consistent with some of the theoretical calculations for the mass of $Y(4260)$ when explaining it as a $D_1\bar{D}$ molecule [40,41].

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![Graphical representation of resonance parameters](image-url)

**FIG. 3.** The likelihood contours in the mass and width planes for $Y(4220)$ (left panel) and $Y(4390)$ (right panel). The filled areas are up to $3\sigma$ likelihood contours and the dots with error bars are the locations of $Y$ or $\psi$ states. The parameters of $Y(4260)^{\text{PDG}}$ are taken from the PDG average [3] and $Y(4260)^{\text{BESIII}}$ from the measurement of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ at BESIII [35].

The systematic uncertainties in the resonance parameters mainly come from the absolute c.m. energy measurement, the c.m. energy spread, and the systematic uncertainty on the cross section measurement. The uncertainty from the c.m. energy measurement includes the uncertainty of the c.m. energy and the assumption made in the measurement for the $R$-scan data sample. Because of the low statistics at each energy point in the $R$-scan data sample, we approximate the difference between the requested and the actual c.m. energy by a common constant. To assess the systematic uncertainty connected with this assumption, we replace the constant by a c.m. energy-dependent second-order polynomial. The systematic uncertainty of the c.m. energy is common for all the energy points in the two data samples and will propagate to the mass measurement (0.8 MeV). The changes on the parameters are taken as uncertainty. The uncertainty from c.m. energy spread is estimated by convoluting the fit formula with a Gaussian function with a width of 1.6 MeV, which is beam spread, measured by the Beam Energy Measurement System [36]. The uncertainty from the cross section measurement is divided into two parts. The first one is uncorrelated among the different c.m. energy points and comes mainly from the fit to the $\gamma h_c$ invariant mass spectrum to determine the signal yields. The corresponding uncertainty is estimated by including the uncertainty in the fit to the cross section, and taking the differences on the parameters as uncertainties. The second part includes all the other sources, is common for all data points (14.8%), and only affects the $\Gamma e\bar{e}B$ measurement. Table I summarizes the systematic uncertainty in the resonance parameters.

Table I. The systematic uncertainty in the measurement of the resonance parameters. c.m. energy $y_{1,2}$ represent the uncertainty from the systematic uncertainty of c.m. energy measurement and the assumption made in the c.m. energy measurement for $R$-scan data sample, respectively. Cross section $s_{a,b}$ represents the uncertainty from the systematic uncertainties of the cross section measurement which are un-correlated (common) in each energy point.

<table>
<thead>
<tr>
<th>Sources</th>
<th>$M$ (MeV/$c^2$)</th>
<th>$Y(4220)$</th>
<th>$(\Gamma e\bar{e}B)$ (eV)</th>
<th>$M$ (MeV/$c^2$)</th>
<th>$Y(4390)$</th>
<th>$(\Gamma e\bar{e}B)$ (eV)</th>
<th>$\phi$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c.m. energy $y_{1,2}$</td>
<td>0.8(0.1)</td>
<td>-(0.1)</td>
<td>-(0.2)</td>
<td>0.8(0.1)</td>
<td>-(0.2)</td>
<td>-(0.3)</td>
<td>-(0.1)</td>
</tr>
<tr>
<td>c.m. energy spread</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>Cross section $s_{a,b}$</td>
<td>0.1(-)</td>
<td>0.2(0.7)</td>
<td>0.6(-)</td>
<td>0.5(-)</td>
<td>0.4(1.7)</td>
<td>0.1(-)</td>
<td>0.2</td>
</tr>
<tr>
<td>Total</td>
<td>0.9</td>
<td>0.4</td>
<td>0.8</td>
<td>1.0</td>
<td>0.6</td>
<td>1.9</td>
<td>0.2</td>
</tr>
</tbody>
</table>