Symmetry violations in nuclear and neutron $\beta$ decay


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The role of $\beta$ decay as a low-energy probe of physics beyond the standard model is reviewed. Traditional searches for deviations from the standard model structure of the weak interaction in $\beta$ decay are discussed in light of constraints from the Large Hadron Collider and the neutrino mass. Limits on the violation of time-reversal symmetry in $\beta$ decay are compared to the strong constraints from electric dipole moments. Novel searches for Lorentz symmetry breaking in the weak interaction in $\beta$ decay are also included, where the unique sensitivity of $\beta$ decay to test Lorentz invariance is discussed. In the conclusion a possible road map for future $\beta$-decay experiments is presented.

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I. INTRODUCTION

The study of nuclear and neutron $\beta$ decay has played a major role in uncovering the structure of the weak interaction, and therefore in the development of the electroweak sector of the standard model (SM) of particle physics. The intensity and the variety of $\beta$ emitters, combined with the high precision with which $\beta$-decay parameters can be measured, ensured that $\beta$ decay remained important in searches for new physics beyond the SM (BSM). Novel techniques of laser cooling and atom trapping (Sprouse and Orozco, 1997; Behr and Gwinner, 2011) made it possible to detect the momentum of the recoiling nucleus, allowing for further searches in unexplored observables that became available. New sources for slow neutrons enabled progress in the study of neutron $\beta$-decay observables (Abele, 2008; Nico, 2009; Dubbers and Schmidt, 2011). The motivation for these modern experiments is, on the one hand, to improve the accuracy of SM parameters, and, on the other hand, to search for physics BSM.

Searches for BSM physics in $\beta$ decay look for deviations from the left-handed vector-axial-vector (“$V-A$”) space-time structure of the weak interaction: see Severijns, Beck, and Naviliat-Cuncic (2006) and Holstein (2014) and references therein. High-precision $\beta$-decay experiments are sensitive to possible contributions of non-SM (or exotic) currents, in particular, right-handed vector, scalar, and tensor currents, that couple to hypothetical new, heavy particles. These exotic currents can also give additional violations of the discrete symmetries parity ($P$), charge conjugation ($C$), and time-reversal invariance ($T$).

Traditionally, $\beta$ decay has been viewed as complementary to the direct searches for new, heavy particles at high-energy colliders. However, with the availability of meson factories the emphasis of searching for new physics in precise measurements of semileptonic decay parameters has shifted from $\beta$ decay. New physics has also been severely constrained by the
emergence of the new field of neutrino oscillations and by the precise measurements of static observables such as the weak charges of quarks and electrons and the P- and T-odd electric dipole moments (EDMs) of particles, atoms, or molecules. Moreover, theoretical developments made it clear how various observables are interconnected, and therefore how the discovery potential of β-decay experiments compares to that of other fields.

Recently, another twist has been added to β decay as a promising precision laboratory to test the invariance of the weak interaction under Lorentz transformations, that is, boosts and rotations. The available evidence for the Lorentz invariance of the weak interaction is, in fact, surprisingly poor. The possibility to break Lorentz and the closely related CPT invariance (Greenberg, 2002) occurs in many proposals that attempt to unify the SM with general relativity, one of the central open issues in theoretical high-energy physics. During the last decade, the phenomenological consequences of such a breakdown of Lorentz symmetry have been charted (Colladay and Kostelecký, 1998), and recently such theoretical studies have been extended to β decay (Noordmans, Wilschut, and Timmermans, 2013b).

This review gives a broad overview of the searches for symmetry violations in nuclear and neutron β decay and discusses their significance compared to various other observables, both in precision measurements and in collider searches. In this way, it attempts to identify which β-decay studies are the most relevant to pursue. In Sec. II we first introduce the effective field theory (EFT) framework, which enables us to compare various experiments in a model-independent approach. We define the β-decay observables in Sec. III.

In Sec. IV we review the best bounds on exotic right-handed vector, scalar, and tensor couplings. We first address the most sensitive β-decay experiments, in which we also include limits from pion-decay experiments.

Second, we discuss how the neutrino mass and data from the Large Hadron Collider (LHC) experiments constrain BSM physics. We compare the bounds from these two sectors with the bounds from β-decay experiments. The violation of time-reversal invariance is discussed in Sec. V. In β decay, T violation manifests itself in nonzero imaginary parts of the couplings, which are probed by triple-correlation observables in β decay. We discuss how these bounds compare to those derived from the stringent upper limits on the values of EDMs.

In Sec. VI, we address the possibility that the weak interaction violates Lorentz symmetry, and, in particular, rotational invariance, in nuclear and neutron β decay. Such Lorentz violation (LV) would give rise to unique signals with no SM “background,” which, even when extremely small, could be experimentally detectable. Nuclear and neutron β decay offer a unique sensitivity to some Lorentz-violating parameters, especially in the gauge and neutrino sector, which we discuss separately.

We conclude with a roadmap for the opportunities in future β-decay studies, in light of the obtained and foreseen bounds from other frontiers.

II. FORMALISM

Nuclear and neutron β decay are semileptonic processes, mediated by the W gauge boson of the electroweak interaction. This interaction is described by a spontaneously broken SU(2)L × U(1)Y gauge symmetry. Under SU(2)L symmetry, left-handed leptons transform as a doublet, while right-handed particles are SU(2)L singlets. This is denoted by

\[ L_A = (\nu_A, l_A)_L, \quad R_A = (l_A)_R, \]  

where A is the flavor index and the left- and right-handed fields are

\[ \psi_L = \frac{1}{2} (1 - \gamma_5) \psi, \quad \psi_R = \frac{1}{2} (1 + \gamma_5) \psi. \]  

The W boson interacts only with left-handed fermions, which reflects the maximal violation of parity (P) symmetry in the weak interaction. In the minimal SM neutrinos are assumed to be massless, and right-handed neutrinos are absent. The role of the neutrino mass is discussed in Sec. IV.C.

The β− (β+ ) decay transition \( d \rightarrow u e^- \bar{\nu}_e \) (\( u \rightarrow d e^+ \nu_e \)) is, in the limit of infinite W-boson mass, described by the effective Lagrange density

\[ \mathcal{L}_{SM} = \frac{G_F}{\sqrt{2}} \bar{\nu}_e (1 - \gamma_5) \nu_e \bar{u}d (1 - \gamma_5) d + \text{H.c.}, \]  

where \( G_F \) is the Fermi coupling constant, \( V_{ud} \) is the ud entry of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, and H.c. denotes the Hermitian conjugate. We work in natural units, \( h = c = 1 \), and use \( \gamma^5 \equiv i \gamma^0 \gamma^2 \gamma^3 \) and \( \epsilon_{0123} = -\epsilon_{0123} = 1 \).

At the nucleon level, all possible quark bilinears and their associated form factors need to be inserted (Weinberg, 1958), such that

\[ \langle p | \bar{u} \gamma_{\mu} d | n \rangle = \frac{p}{M} \left[ g_{V}(q^2) \gamma_{\mu} + g_{M}(q^2) \sigma_{\mu\nu} q^\nu \right] n, \]

\[ \langle p | \bar{u} \gamma_{\mu} \gamma_5 d | n \rangle = \frac{p}{M} \left[ g_{A}(q^2) \gamma_{\mu} \gamma_5 + g_{T}(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 \right] n, \]

where \( q = p_n - p_p \) is the momentum transfer and M is the nucleon mass. The vector form factor \( g_V \) and the axial-vector form factor \( g_A \) give the leading contributions to β decay, because the nuclei can be treated nonrelativistically. In the isospin limit, the induced form factor \( g_M \) called weak magnetism, is given by \( (\mu_p - \mu_n)/2 \), i.e., the difference between the magnetic moments of the proton and the neutron. Given the current experimental precision, this form factor can be neglected, but future experiments might reach a level of precision for which weak magnetism has to be taken into account; see Sec. IV.D. In the isospin limit the induced scalar form factor \( g_S \) and tensor form factor \( g_T \) vanish (Weinberg, 1958), and we can neglect them at present. The induced pseudoscalar form factor \( g_P \) gets an additional suppression of
The leading-order SM expression for neutron decay is

$$L_{\text{SM}} = \frac{G_F V_{ud}}{\sqrt{2}} g_V (q^2) \epsilon_\mu (1 - γ_S) \nu_e \bar{p} p^\mu \left(1 - \frac{|g_A|^2}{g_V}γ_5 \right) n + \text{H.c.}$$  \hspace{1cm} (5)

In the limit of $q^2 \to 0$, the vector charge is $g_V (0) = 1$, up to small corrections. This is dictated by the hypothesis of the conserved vector current (CVC). The axial-vector charge $g_A$ is only partially conserved (PCAC). The best current value is derived from neutron $β$-decay experiments, $|g_A| = 1.2723(23)$ (Olive et al., 2014).

In nuclear $β$ decay one can exploit the properties of the parent and daughter nucleus to select particular parts of the interaction. Pure Fermi (F) transitions probe the vector currents ($γ^μ$), while pure Gamow-Teller (GT) transitions probe the axial-vector currents ($γ_5 γ^μ$). Mixed transitions always require knowledge of the Fermi and Gamow-Teller transition matrix elements, $M_F ≡ \langle f | J | i \rangle$ and $M_{GT} ≡ \langle f | [γ_5 J] | i \rangle$, respectively. The conditions for spin change ($ΔJ$) and parity change ($π_i, π_f$) for Fermi and Gamow-Teller transitions are given in Table I. This table also lists for which aspect in SM and BSM research these transitions are used. We have defined the Fermi-Gamow-Teller mixing ratio

$$\rho ≡ g_A M_{GT} / g_V M_F,$$  \hspace{1cm} (6)

and

$$\lambda ≡ |g_A| / g_V.$$  \hspace{1cm} (7)

It is desirable to reduce the uncertainties of nuclear structure and select the simplest isotopes. For Fermi transitions the superallowed $0^+ → 0^+$ transitions are of the most interest. For mixed transitions, mirror nuclei are preferred. For general mirror nuclei $ρ$ has to be measured, while neutron decay ($J^π = 1/2^+ → J^π = 1/2^+$, $|M_F|^2 = 1$, and $|M_{GT}|^2 = 3$) allows for the determination of the value of $λ$ (Abele, 2008; Nico, 2009; Dubbers and Schmidt, 2011). An elaborate compilation of neutron-decay amplitudes is given in Ivanov, Pitschmann, and Troitskaya (2013). When searching for physics BSM, nuclei serve as “micro-laboratories” that can be judiciously chosen to look for certain manifestations of new physics. In this review, we address both the traditional searches for exotic couplings and the novel searches for Lorentz violation. In the latter, the possibility of angular-momentum violation needs to be considered, where the simplest of the forbidden decays, first-forbidden unique transitions, become relevant (Noordmans, Wilschut, and Timmermans, 2013a). Both fields search for BSM physics generated by an unknown fundamental theory at a high-energy scale. To study the effect of new physics at low energies, we work in an EFT approach. Within this framework the effects of new physics at low energies are described in a model-independent way with an effective Lagrangian of the form

$$L^{(\text{eff})} = L_{\text{SM}} + L_{\text{BSM}}.$$  \hspace{1cm} (8)

The search for exotic couplings focuses on right-handed vector, scalar, and tensor couplings. These non-SM interactions can be included in the Lagrangian by adding higher-dimensional operators to $L_{\text{BSM}}$. The effects of Lorentz violation can also be described in an EFT framework (Colladay and Kostelecký, 1998; Noordmans, Wilschut, and Timmermans, 2013b). We discuss both frameworks separately.

A. Exotic couplings

In EFT, deviations from the $V − A$ structure due to exotic couplings are generated by higher-dimensional operators, which are suppressed by the high-energy scale $Λ$. The effective Lagrangian is parametrized as

$$L^{(\text{eff})} = L_{\text{SM}} + \frac{1}{Λ^k} L^{(4+k)},$$  \hspace{1cm} (9)

where

$$L^{(4+k)} = \sum_i c_i O_i^{(4+k)},$$  \hspace{1cm} (10)

and where $c_i$ are dimensionless constants and $O_i^{(4+k)}$ are dimension-(4 + k) operators. The SM only contains operators with mass dimension 3 or 4. For Lorentz-symmetric BSM physics, the lowest term we could add is $L^{(5)}$. There is, however, only one dimension-5 operator, namely, the operator that generates Majorana neutrino masses (Weinberg, 1979). In

| TABLE I. Classification of nuclear $β$ decays and their characteristic use in the SM and in the search for BSM physics, |
|---|---|---|---|---|---|---|
| | F | GT | Mixed | First unique forbidden |
| | ΔJ = 0 | ΔJ = 0, ±1 | $π_iπ_f = +1$ | $π_iπ_f = −1$ | ΔJ = ±2 | Section |
| SM parameter | $V_{ud}$ | $ρ$ | $ρ, V_{ud}, λ$ | $α_{L,R}$ | III |
| BSM | $A_{L,R}$ | $α_{L,R}$ | $α_{L,R}$ | $α_{L,R}$ | $α_{L,R}$ | IV.A |
| T even | | | | | | |
| BSM | | | | | | |
| T odd | $X^{µν}$ | $X^{µν}$ | $X^{µν}$ | $X^{µν}$ | $X^{µν}$ | VI |
| LV | | | | | | |

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searches for exotic couplings we assume the neutrino mass to be small, and therefore we neglect this operator. We focus only on $\mathcal{L}^{(6)}$, as even higher-dimensional terms are suppressed by additional powers of the large scale $\Lambda$.

The $\mathcal{O}_{i}^{(6)}$ that contribute to semileptonic charged decays are listed in Cirigliano, Jenkins, and González-Alonso (2010) and Cirigliano, González-Alonso, and Graesser (2013). At low energies these dimension-6 operators generate the original vector ($C_{V}$), axial-vector ($C_{A}$), scalar ($C_{S}$), pseudoscalar ($C_{P}$), and tensor ($C_{T}$) couplings of Lee and Yang (1956). At the quark level, the effective Lagrangian for $\beta$ decay, with nonderivative four-fermion couplings, is

$$\mathcal{L}^{(eff)} = \frac{4G_{F}V_{ud}}{\sqrt{2}} \sum_{\delta=L,R} \left\{ \alpha_{\delta5} \bar{\nu}_{\delta} \gamma^{\mu} \nu_{\delta} \cdot \bar{u}_{\delta} d_{\delta} + \alpha_{\tau} \bar{\nu}_{\tau} \gamma^{\mu} \nu_{\tau} \cdot \bar{u}_{\tau} \gamma_{\mu} d_{\tau} \right\},$$

where we sum over the chirality ($L, R$) of the final states.

The coefficients represent
- $\alpha_{\delta5}$: all possible $V$ and $A$ couplings.
- $A_{\delta5}$: exotic scalar and pseudoscalar couplings (where $\delta$ denotes the chirality of the neutrino and $\delta$ the chirality of the $d$ quark).
- $\alpha_{\tau}$: exotic tensor couplings (where $\epsilon$ denotes the chirality of both the neutrino and the $d$ quark).

These coefficients are related to the coefficients $C_{i}$ and $C_{i}^{c}$ ($i = S, V, A, T, P$) of Lee and Yang (1956) by Eqs. (A3) and (A4) of Appendix A. In the SM all couplings except $a_{L L} = 1$ are zero. For tensor couplings, only $\alpha_{L}$ and $\alpha_{R}$ occur, since $\sigma_{\mu\nu}T_{5} = (i/2)\epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma}$. The constants $\alpha_{\delta5}, A_{\delta5}$, and $\alpha_{\tau}$ can be related to $c_{i}$, by matching their values at the low-energy scale with standard EFT techniques. The chiral structure of the coefficients is expressed by the first and second indices, which denote the chirality of the neutrino and the $d$ quark, respectively. All couplings with first index $R$ involve a right-handed neutrino. In the SM, right-handed neutrinos are absent, but they are present in many new-physics models. The role of the right-handed neutrino is discussed in Sec. IV.C. The new exotic couplings can be complex, representing the possibility of time-reversal ($T$) violation (Sec. V). The introduction of left-handed and right-handed couplings leads to parity violation when the coefficients differ. In the absence of right-handed couplings, parity violation is maximal.

To describe $\beta$ decay of the nucleon we define the hadronic matrix elements (Herczeg, 2001)

$$\langle p | m_{\mu} n_{\bar{m}} | n \rangle = g_{V}(q^{2}) \bar{m}_{\mu} m_{n},$$

modifying the effective Lagrangian in Eq. (11) accordingly. As before, the vector charge is $q_{V} = q_{V}(0) = 1$. The other couplings $g_{A}, g_{S}, g_{P}$, and $g_{T}$ can be calculated theoretically by using lattice QCD. Estimates for $g_{A}$ on the lattice are currently not competitive with the experimental value $|g_{A}(0)| = 1.2723(23)$ determined from neutron $\beta$ decay (Olive et al., 2014). The scalar, pseudoscalar, and tensor constants, $g_{S}, g_{P}$, and $g_{T}$, are determined theoretically. They are further discussed in Sec. IV.

Searches for exotic coupling also include searches for right-handed $V + A$ currents. Such currents are predicted for instance by left-right (LR) models, which add an $SU(2)_{R}$ gauge symmetry to the SM. This extends the SM with an additional gauge boson $W_{R}$, which mixes with the original SM $W$ boson $W'_{L}$. The weak eigenstates can be expressed in the mass eigenstates $W_{1}$ and $W_{2}$ as

$$W_{L} = W_{1} \cos \xi + W_{2} \sin \xi,$$

where $\xi$ is the mixing angle and $\omega$ is a $CP$-violating phase. The coupling of $W_{R}$ to quarks and leptons introduces the right-handed coupling $g_{R}$ and the right-handed CKM element $V_{uR}$, the equivalents of the SM parameters. The expressions for $a_{L R}, a_{R L}$, and $a_{R R}$ in terms of these parameters are given in Herczeg (2001). A specific class of LR models are the symmetric LR models, in which $P$ or $C$ symmetry of the Lagrangian is imposed, which implies $g_{L} = g_{R}$. We focus on bounds for such models in Sec. IV.B.

### B. Lorentz violation

The study of Lorentz violation is motivated by the possibility of spontaneous breaking of Lorentz invariance predicted by theories of quantum gravity (Kostelecký and Samuel, 1989; Liberati and Maccione, 2009; Liberati, 2013). The natural energy scale for these theories of quantum gravity is the Planck scale, which lies 17 orders of magnitude higher than the electroweak scale. This precludes the direct detection of Planck-scale physics, but the effects of Lorentz violation at the Planck scale can become manifest at much lower energies, providing a “window on quantum gravity.” At low energy, Lorentz violation can be systematically described by the standard model extension (SME) (Colladay and Kostelecký, 1998), by using an EFT approach. The SME contains all possible Lorentz-violating terms that obey the SM gauge symmetries, which include $CPT$-violating terms, since Lorentz violation allows for the breaking of $CPT$ invariance. In fact, $CPT$ violation can occur only if Lorentz symmetry is also broken (Greenberg, 2002).

Spontaneous Lorentz violation arises as Lorentz-tensor fields acquire a vacuum-expectation value (VEV), resulting in Lorentz-violating tensor coefficients in the SME Lagrangian. These coefficients can be understood as constant background tensor fields. Because of these tensor fields, the Lagrangian is no longer invariant under particle or active...
Lorentz transformations, i.e., boosts or rotations of the particles, because the background fields do not change under the Lorentz group (Colladay and Kostelecký, 1998). However, the low-energy theory remains invariant under observer Lorentz transformations, i.e., boosts or rotations of the observer’s inertial frame. Because Lorentz symmetry is spontaneously broken, the underlying fundamental theory at the Planck scale remains Lorentz invariant, implying that important features such as energy-momentum conservation and microcausality are still valid. A possible experimental signature of Lorentz violation is a sidereal variation of observables, which arise as the laboratory moves through the Lorentz-violating background field when Earth rotates [other examples are given in, e.g., Mattingly (2005)].

Schematically, terms in $L_{\text{BSM}}$ in Eq. (8) can be written as (Colladay and Kostelecký, 1997)

$$L_{\text{ND}} = \lambda^{(3)} \langle T \rangle \cdot \bar{\psi} \Gamma \psi + \frac{\lambda^{(4)}}{\Delta} \langle T \rangle \cdot \bar{\psi} \Gamma (i \partial) \psi + \frac{\lambda^{(4+k)}}{\Lambda^k} \langle T \rangle \cdot \mathcal{O}^{(4+k)},$$

(14)

where we summed over repeated indices and where $\lambda^{(i)}$ are dimensionless constants, $\langle T \rangle$ is the expectation value of tensor $T$, $\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}$ represents the gamma-matrix structure, and $\mathcal{O}^{(4+k)}$ are higher-dimensional operators. Furthermore, $\Lambda$ represents the scale of the fundamental theory, which is naturally the Planck scale. The higher-dimensional operators are suppressed by powers of this high scale. The first two terms in Eq. (14) have mass dimensions 3 and 4, respectively. These terms are described in the original SME papers by Colladay and Kostelecký (1998) and are now referred to as the minimal standard model extension (mSME). For our present discussion we limit ourselves to the mSME, although higher-dimensional coefficients have also been described (Bolokhov and Pospelov, 2008; Kostelecký and Mewes, 2009, 2012, 2013).

From an EFT point of view, the introduced Lorentz-violating dimension-3 and dimension-4 operators are unnatural. Naively, one would expect the dimension-3 operators to scale linearly with the large scale $\Lambda$, while the coefficients of the dimension-4 operators should be of order unity. The experimental bounds on these dimension-3 and dimension-4 operators are much smaller, of course. This problem does not occur for higher-dimensional operators, which are naturally suppressed by the scale $\Lambda$. To evade these naturalness problems, the current limits on dimension-3 and dimension-4 coefficients require either large fine-tuning, or a symmetry that forbids these coefficients. However, even if dimension-3 and dimension-4 operators are forbidden at tree level, they will be induced by quantum corrections generated by higher-dimensional nonrenormalizable operators. These corrections scale quadratically with the cutoff scale, which might be as large as $\Lambda$. This can be circumvented by introducing new physics between the weak scale and the Planck scale. In that case, radiative corrections scale with a significantly lower cutoff scale (Mattingly, 2008). Such a scenario occurs in supersymmetry (SUSY) (Bolokhov, Groot Nibbelink, and Pospelov, 2005; Groot Nibbelink and Pospelov, 2005). SUSY restricts Lorentz-violating operators to dimension 5 and higher, and forbids those of dimensions 3 and 4. Dimension-3 and dimension-4 operators are generated by loop corrections if SUSY is broken. This would naturally lead to a suppression of $m^2/\Lambda$ and $m/\Lambda$ for dimension-3 and dimension-4 operators, respectively, where $m$ is the SUSY-breaking scale (Bolokhov, Groot Nibbelink, and Pospelov, 2005; Groot Nibbelink and Pospelov, 2005). In the mSME, it is assumed that dimension-3 and dimension-4 operators are suppressed by some unspecified higher-scale mechanism, and the experimental constraints are studied without any assumptions on the nature of this suppression mechanism (Colladay and Kostelecký, 1998; Kostelecký and Russell, 2011).

The SME contains a large number of coefficients that parametrize possible Lorentz violation. We list the relevant coefficients for $\beta$ decay, which are the lepton, Higgs, and gauge terms. The Lorentz-violating terms for leptons are (Colladay and Kostelecký, 1998)

$$L_{\text{lepton}} = L_A \left[ i (e_L^{a\nu}) \mu_{\alpha \beta} \phi^\alpha D^\nu - (a_L^{a\nu}) \mu_{\alpha \beta} \phi^\alpha \right] L_B + R_A \left[ i (e_R^{a\nu}) \mu_{\alpha \beta} \phi^\alpha D^\nu - (a_R^{a\nu}) \mu_{\alpha \beta} \phi^\alpha \right] R_B,$$

(15)

where $L$ denotes the $SU(2)_L$ doublet and $R$ denotes the singlet, defined in Eq. (1). The subscripts $A, B$ are flavor indices, and $D^\mu$ is the covariant derivative. This introduces the Lorentz-violating coefficients $a_L^{a\nu}$ and $a_R^{a\nu}$, which are $CPT$ odd and $CPT$ even, respectively. We introduced the superscript $LV$ for these coefficients, in order not to confuse them with the coefficients in Eq. (11).

Before electroweak symmetry breaking, the Higgs and gauge sector are described by (Colladay and Kostelecký, 1998)

$$L_{\text{Higgs+gauge}} = \left[ 2 k_{\phi \phi}^\mu (D^\mu \phi) (D_\mu \phi + \text{H.c.}) + [i k_{\phi}^\mu \phi^a D_\mu \phi^a + \text{H.c.}] - \frac{1}{2} k_{BB}^\mu \phi B^\mu B + \frac{1}{2} k_{GB}^\mu \phi G_{\mu \nu} B^\mu B^\nu \right] \text{Tr}(G^{\nu \mu} G^\nu) - \frac{1}{2} k_{W}^\mu \phi W^\mu W^\nu - \frac{1}{2} k_{B}^\mu \phi B^\mu B^\nu \right],$$

(16)

where $G^{\mu \nu}, W^\mu$, and $B^\mu$ are the $SU(3)_c, SU(2)_L$, and $U(1)_Y$ field-strength tensors, respectively, and $\phi$ is the Higgs doublet. The coefficient $k_{\phi}$ is $CPT$ odd, and the only coefficient with dimension of mass. The other coefficients are $CPT$ even and dimensionless. The coefficient $k_{\phi \phi}$ has symmetric real and antisymmetric imaginary components. The $k_{\phi W}$ and $k_{\phi B}$ coefficients are real and antisymmetric. The gauge couplings $k_G, k_W$, and $k_B$ are real and have the symmetry properties of the Riemann tensor (Colladay and Kostelecký, 1998).

The SME parameters have been studied in a wide range of experiments (Kostelecký and Russell, 2011). The electromagnetic and gravity sectors have been studied extensively, whereas the number of searches in the weak interaction is rather low. This changed recently (Müller et al., 2013; Noordmans, Wilschut, and Timmermans, 2013a, 2013b), and the search for Lorentz violation has been extended to weak decays, in particular, $\beta$ decay. $\beta$ decay places strong constraints on Lorentz-violating coefficients in the Higgs and gauge sector. In addition, $\beta$ decay has a unique sensitivity to some coefficients in the neutrino sector (Díaz, Kostelecký, and Lehnert, 2013). We discuss these constraints in Sec. VI.
III. OBSERVABLES IN $\beta$ DECAY

A. Correlation coefficients in $\beta$ decay

In $\beta$ decay, the correlations differ between different observables, such as the $\beta$ momentum and the nuclear spin, can be measured. The amount of correlation is expressed in terms of correlation coefficients. These correlation coefficients depend on SM couplings and possible new of correlation coefficients. These correlation coefficients depend on Eq. (11), we can write the decay-rate distribution for polarized nuclei as (Jackson, Treiman, and Wyld, 1957b)

$$\omega_0(\vec{J})|E_e, \Omega_e, \Omega_\nu_0|dE_e d\Omega_e d\Omega_\nu = \frac{F(\pm Z, E_e)}{(2\pi)^4} p_e E_e (E_0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu$$

$$\times \tilde{\xi} \left\{ 1 + a \frac{\tilde{p}_e \cdot \tilde{p}_\nu}{E_e} + b \frac{m_e}{E_e} \right\}$$

$$+ c \left( \frac{1}{E_e} \right) \left[ \tilde{p}_e \cdot \tilde{p}_\nu + \frac{(\tilde{p}_e \cdot \tilde{p}_\nu)(\tilde{p}_e \cdot \tilde{p}_\nu)}{E_e} \right]$$

$$+ \frac{(\tilde{J}) J}{E_e} \left[ A \frac{\tilde{p}_e}{E_e} + B \frac{\tilde{p}_e}{E_e} + D \frac{\tilde{p}_e}{E_e} \cdot \frac{\tilde{p}_e}{E_e} \right],$$

where $E_0(\nu)$, $\Omega_\nu(\nu)$, and $p_\nu(\nu)$ denote the total $\beta(\nu)$ energy, direction, and momentum, respectively, $E_0$ is the energy available to the electron and the neutrino, $\tilde{J}$ is the expectation value of the spin of the initial nuclear state, and $\tilde{J}$ is the unit vector in this direction; $F(\pm Z, E_e)$ is the Fermi function which modifies the phase space of the electron due to the Coulomb field of the nucleus. Also affecting the phase space is the Fierz-interference term, factorized with the coefficient $b$. This term is zero in the SM. We defined $\tilde{\xi} = G_F^2 V_{ud}^2 / 2\xi$, where $\xi$ gives the strength of the interaction. The remaining terms describe the $\beta$-correlation coefficients: the $\beta$-neutrino asymmetry $c$, the $P$-odd “Wu parameter,” the $\beta$ asymmetry $A$, the neutrino asymmetry $B$, and the triple-correlation coefficient $D$.

The $c$ coefficient vanishes for nonoriented nuclei and for nuclei with $J = 1/2$, such as the neutron. The $c$ coefficient has not been taken into account in any experiment to date. However, in future experiments, which use laser beams to trap and cool samples, the expectation value $\langle \tilde{J}_e \cdot \tilde{J}_\nu \rangle$ may be affected, such that the $c$ coefficient can play a role.

The decay rate integrated over the neutrino direction, but taking into account electron polarization, is (Jackson, Treiman, and Wyld, 1957b)

$$\omega_0(\tilde{J})|E_e, \Omega_e, \Omega_\nu|dE_e d\Omega_e = \frac{F(\pm Z, E_e)}{(2\pi)^4} p_e E_e (E_0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu$$

$$\times \tilde{\xi} \left\{ 1 + b \frac{m_e}{E_e} + \frac{\tilde{p}_e \cdot \tilde{p}_\nu}{E_e} \right\}$$

$$+ \tilde{\sigma}_e \cdot \left[ N \frac{(\tilde{J})}{J} + Q \frac{\tilde{p}_e}{E_e} + \frac{(\tilde{J})}{J} \cdot \frac{\tilde{p}_e}{E_e} + R \frac{(\tilde{J})}{J} \cdot \frac{\tilde{p}_e}{E_e} \right],$$

where $\tilde{\sigma}_e$ is the spin vector of the $\beta$ particle. This introduces the longitudinal $\beta$ polarization $G$, the spin-correlation coefficients $N$ and $Q$, and the triple-correlation coefficient $R$.

TABLE II. Overview of symmetry properties under parity ($P$) transformations and time reversal ($T$) of the most relevant correlation coefficients in allowed $\beta$ decay.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Correlation</th>
<th>$P$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0 (\vec{p} \nu$ angular correlation)</td>
<td>$\tilde{p}<em>e \cdot \tilde{p}</em>\nu / E_e$</td>
<td>Even</td>
<td>Even</td>
</tr>
<tr>
<td>$b$ (Fierz-interference term)</td>
<td>$\frac{m_e}{E_e}$</td>
<td>Even</td>
<td>Even</td>
</tr>
<tr>
<td>$A (\beta$ asymmetry)</td>
<td>$\tilde{J} \cdot \tilde{p}_e / E_e$</td>
<td>Odd</td>
<td>Even</td>
</tr>
<tr>
<td>$B (\nu$ asymmetry)</td>
<td>$\tilde{J} \cdot \tilde{p}_e / E_e$</td>
<td>Odd</td>
<td>Even</td>
</tr>
<tr>
<td>$G$ (longitudinal polarization)</td>
<td>$\tilde{\sigma}_e \cdot \tilde{p}_e / E_e$</td>
<td>Odd</td>
<td>Even</td>
</tr>
<tr>
<td>$N$ (longitudinal polarization)</td>
<td>$\tilde{J} \cdot \tilde{\sigma}_e / E_e$</td>
<td>Even</td>
<td>Even</td>
</tr>
<tr>
<td>$Q$ (longitudinal polarization)</td>
<td>$\tilde{\sigma}_e \cdot (\tilde{p}<em>e \times \tilde{p}</em>\nu) / E_e$</td>
<td>Even</td>
<td>Odd</td>
</tr>
<tr>
<td>$R$ (triplet correlation)</td>
<td>$\tilde{\sigma}_e \cdot (\tilde{J} \times \tilde{p}_e) / E_e$</td>
<td>Odd</td>
<td>Odd</td>
</tr>
</tbody>
</table>

Integrating the decay rate over all kinematical variables gives the inverse lifetime,

$$\frac{1}{\tau} = \frac{m_e^5}{2\pi^3} f_\xi \left( 1 + b \left( \frac{m_e}{E_e} \right) \right),$$

where $f$ contains the integration over the modified phase space and $\langle m_e / E_e \rangle$ is the average inverse energy in units of the electron mass.

In Appendix A we list the relevant correlation coefficients in terms of the couplings defined in Eq. (11) and the Fermi-Gamow-Teller matrix elements. The different correlation coefficients contain combinations of the complex $V, A, S, T, P$, and $T$ couplings. Given the current experimental precision, we have neglected Coulomb corrections. These corrections mainly introduce additional imaginary couplings (except for the $D$ and $R$ coefficients) (Jackson, Treiman, and Wyld, 1957b).

We proceed by discussing how $\beta$-decay correlation experiments, combined with lifetime measurements, are used to obtain precise values for the SM $V$ and $A$ coupling strengths. In Sec. IV we discuss constraints on exotic couplings.

B. Standard model parameters in $\beta$ decay

The correlation coefficients in Appendix A reduce to the SM expressions when putting the scalar and tensor couplings to zero, $A_{LL,RR,RL,LR} = 0$ and $a_1 = 0$, and by using only $V - A$ couplings, $a_{L,R,RR,RL} = 0$. The Fierz-interference coefficient $b$ is zero in the SM. The lifetime in Eq. (19) can be derived from the $ft$ value, using the measured half-life $t$ instead of $\tau$. In the SM,
\[ \frac{1}{f_t} = \frac{m_e^5}{2\pi^3 \ln(2)} G_F^2 V_{ud}^\dagger G_{\text{GT}} \beta |M_F|^2 (1 + |\beta|^2). \] (20)

The SM value for \( G_F \) is obtained from muon decay (Webber et al., 2011). It is important to note that if one considers non-SM contributions these may influence muon decay as well. In principle, \( g_4 \) is calculable using lattice QCD, but as mentioned before, current lattice calculations are not as accurate as values derived from experiments and henceforth \( \lambda = |g_A|/g_\beta \) is considered a free parameter. In general, \( M_F \) and \( M_{\text{GT}} \) need to be derived from nuclear model calculations. For superallowed Fermi transitions \( \rho = 0 \) and \( M_F = \sqrt{2} \) in the isospin limit. Hardy and Towner (2009) analyzed all available superallowed Fermi transitions and derived a value for the \( ud \) CKM matrix element. Since the \( f_t \) values of superallowed transitions should be equal, a large number of measurements could be combined, leading to the most precise value of \( V_{ud} = 0.97425(22) \) (Hardy and Towner, 2009). In the analysis, details of the isotope-dependent nuclear-structure corrections on the matrix element \( M_F \) (e.g., isospin breaking) and the phase-space modifications are also considered. The superallowed transitions also give the best bound on the Fierz coefficient \( b \) in Eq. (19) by considering the energy dependence of the lifetime (Sec. IV.A.1).

The parameters \( \lambda \) and \( V_{ud} \) can also be determined from \( \beta \)-decay correlations in neutron decay and from the neutron lifetime (Abele, 2008; Nico, 2009; Dubbers and Schmidt, 2011; Wietfeldt and Greene, 2011). The best current values are \( \lambda = 1.2723(23) \) (Olive et al., 2014) and \( V_{ud} = 0.9742(12) \) (Dubbers and Schmidt, 2011). The latter is more than 5 times less precise; see also Fig. 22 in Dubbers and Schmidt (2011). The strong Gamow-Teller dependence of neutron decay and the precision of the neutron-decay parameters is such that neutron decay also plays an important role in searches for tensor currents, as discussed in Sec. IV.A.3.

Another class of nuclei for which the nuclear structure is relatively well known are the mirror nuclei (Severijns et al., 2008). Like neutron decay, mirror decays are mixed Fermi-Gamow-Teller transitions. Extraction of \( V_{ud} \) from lifetime measurements requires knowledge of the mixing parameter \( \rho \), such that an additional measurement of at least one of the correlation coefficients is necessary. Naviliat-Cuncic and Severijns (2009) found \( V_{ud} = 0.9719(17) \), using five available transitions. The important structure corrections to Eq. (20) for mirror nuclei have been evaluated (Severijns et al., 2008), in analogy to the work of Hardy and Towner (2009) for superallowed Fermi decays. This new class of nuclei will broaden the spectrum of data and remove any possible bias in selecting only superallowed Fermi transitions in the determination of \( V_{ud} \). Measurements with this motivation were undertaken. For example, Triambak et al. (2012), Broussard et al. (2014), and Shidling et al. (2014) measured the lifetime of two relevant mirror nuclei \(^{19}\text{Ne} \) and \(^{37}\text{K} \). We will not review the status of this field here, but comment on their relevance in limiting left-handed tensor couplings via the Fierz-interference term in the next section. It demonstrates that the contribution of nuclear physics to high-precision SM data goes hand in hand with the searches for new physics in \( \beta \) decay.

IV. CONSTRAINTS ON EXOTIC COUPLINGS

\( \beta \) decay played an important role in establishing the \( V-A \) structure of the SM, initially eliminating to a large extent the possible contributions of scalar and tensor interactions. Modern searches in nuclear \( \beta \) decay consider again scalar and tensor currents as possible very small deviations from the SM due to new physics (Severijns, Beck, and Naviliat-Cuncic, 2006; Severijns and Naviliat-Cuncic, 2011).

The searches in \( \beta \) decay are part of a much wider search in subatomic physics for new physics. Comparison between different searches has become possible in an EFT framework by using the effective Lagrangian in Eq. (11). At the quark level the relations between different observables are clean, but at the nucleon level they involve the nuclear form factors \( g_A, g_S, g_P \), and \( g_T \). Accurate values for these parameters are necessary in order to compare different limits. Recently, significant progress on the accuracy of both \( g_S \) and \( g_T \) has been reported. First results for \( g_T \) are also available. The most precise value for \( g_T \) is calculated with lattice QCD. Two recent results are from Green et al. (2012), \( g_T = 1.038(16) \), and Bhattacharya et al. (2014), \( g_T = 1.047(61) \).

The calculation method used in these works gives a much larger uncertainty for \( g_S \). Estimates range from \( g_S = 0.72(32) \) (Bhattacharya et al., 2014) to \( g_S = 1.08(32) \) (Green et al., 2012). A value for \( g_S \) can also be derived using the CVC relation and lattice calculations (González-Alonso and Camalich, 2014),

\[ g_S(0) = \frac{\delta M_{\text{QCD}}}{\delta m_q} = 1.02(11), \] (21)

where both \( \delta M_{\text{QCD}} = (M_n - M_p)_{\text{QCD}} \) (González-Alonso and Camalich, 2014) and \( \delta m_q = m_q - m_n \) (Colangelo et al., 2011) are obtained separately from lattice calculations. However, the determination of \( g_S \) with Eq. (21) might underestimate the error, because correlations between the numerator and denominator are neglected. Such errors could be avoided by calculating the ratio in Eq. (21) directly on the lattice. Further efforts to reduce the error for \( g_S \) directly on the lattice are being pursued (Bhattacharya et al., 2012, 2014).

The pseudoscalar constant \( g_P \) can be calculated by using the PCAC relation. Combined with lattice QCD results (González-Alonso and Camalich, 2014) one finds

\[ g_P(0) = \frac{\bar{M}_N}{m_q} g_A = 349(9), \] (22)

where \( \bar{M} = (M_p + M_n)/2 \) is the average nucleon mass and \( m_q = (m_n + m_p)/2 = 3.42(9) \) MeV is the average light-quark mass determined on the lattice (Colangelo et al., 2011). According to Beringer et al. (2012), \( m_q = 3.5(0.2) \) MeV, which gives a much larger error \( g_P = 340^{+68}_{-59} \). Nevertheless, this shows that the pseudoscalar form factor is of the order of \( O(10^2) \). In \( \beta \) decay, pseudoscalar interactions are generally neglected, because they occur only as higher-order recoil corrections. This suppresses pseudoscalar interactions compared to scalar and tensor interactions. The large value of \( g_P \) cancels this suppression to a large extent, and
β-decay experiments may be sensitive to pseudoscalar couplings after all. There are, however, already strong constraints on pseudoscalar couplings from pion decay, as discussed in Sec. IV.A.5.

In the remainder of this section we comment on searches for exotic couplings in β decay (Sec. IV.A), but consider only real couplings. We compare these results with constraints from the LHC experiments (Sec. IV.B) and due to the nonzero mass of the neutrino (Sec. IV.C). Bounds on imaginary couplings are discussed separately in Sec. V.

### A. Constraints from β decay

In nuclear β decays, exotic couplings are mainly searched for in either pure Fermi or pure Gamow-Teller decays. Pure Fermi transitions depend on vector and possibly scalar couplings, while pure Gamow-Teller transitions depend on axial-vector and possibly tensor couplings. The use of mixed transitions is necessary when searching for interference terms. Preferred are isotopes with a relatively simple nuclear structure, e.g., mirror nuclei, or the neutron. We discuss the constraints from Fermi, Gamow-Teller, and mixed decays separately, focusing on the best current experimental data. We discuss the constraints on scalar and tensor couplings, while assuming no additional vector or axial-vector interactions. For a fit of the data including these interactions we refer to Severijns, Beck, and Naviliat-Cuncic (2006), where also a review of the experimental techniques is given. We discuss V + A couplings in Sec. IV.B.

Most β-correlation coefficients are measured by constructing asymmetry ratios. For example, the β asymmetry is measured from the quantity

\[
A_{\text{measured}} = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)},
\]

where \(N(\uparrow)\) and \(N(\downarrow)\) are the decay rates derived from measuring β particles in a particular detector while the polarization \(P\) of the nucleus changes sign. The arrows indicate the direction of polarization. The rates \(N(\uparrow), N(\downarrow)\) correspond to the integration of Eq. (17) over all unobserved degrees of freedom, which removes the dependence on the neutrino direction. In the numerator only the \(P\)-odd term remains, while in the denominator the odd term drops out. However, the Fierz-interference term remains in the sum \(N(\uparrow) + N(\downarrow)\), so that

\[
\tilde{A} = \frac{A}{1 + b(m_e/E_e)},
\]  

The inverse average energy is approximated by

\[
\left\langle \frac{m_e}{E_e} \right\rangle = \frac{\int_{E_{\text{min}}}^{E_0} F(\pm Z, E_e) p_e(E_0 - E_e)^2 dE_e}{\int_{E_{\text{min}}}^{E_0} F(\pm Z, E_e)p_e(E_0 - E_e) dE_e},
\]

which depends on the specific isotope and the experimental setup. In principle, the average energy could also depend on the angular distribution \((\theta_e)\). This makes it preferable that the analysis of \(\langle m_e/E_e \rangle\) is done and published together with the observed correlation coefficients. At present, many of the values for \(\langle m_e/E_e \rangle\) are derived by using the β-energy threshold \(E_{\text{min}}\) (Severijns, Beck, and Naviliat-Cuncic, 2006; Pattie, Hickerson, and Young, 2013; Wauters, Garcia, and Hong, 2014).

For the measured quantity \(\tilde{X}, X = a, A, B, G, \ldots\), Eq. (25) applies. Except for \(B\) and \(N\), the numerator of Eq. (25) depends only on the square of the coupling constants, while \(b\) has a linear dependence on left-handed couplings. In such cases one is most sensitive to \(b\), and the measurement of \(\tilde{X}\) provides in the first place a measurement of the Fierz coefficient \(b\). Therefore, the exact value of the \(\langle m_e/E_e \rangle\) will become increasingly important with increasing experimental precision.

1. **Nuclear scalar searches**

Throughout the discussion of limits on scalar and tensor couplings, we assume conventional left-handed vector couplings for the \(V - A\) part, such that \(a_{LL} = 1\), and \(a_{LR, RL, RR} = 0\). These and the other couplings are defined in Eq. (11). The notation is chosen such that the difference between the left-handed and right-handed couplings of the neutrino is emphasized, i.e., for the scalar couplings \(A_L = A_{LL} + A_{LR}\) (left-handed neutrino coupling) and \(A_R = A_{RR} + A_{RL}\) (right-handed neutrino coupling). Further details on the notation and some relevant expressions can be found in Appendix A.

For pure Fermi transitions

\[
\xi = 2|M_F|^2 \gamma_0 \left(1 + \frac{\gamma_0}{g_0} \right) [A_L^2 + A_R^2],
\]

from Eqs. (A9) and (A11), where \(b_F\) is the Fermi part of the Fierz coefficient \(b\), the upper (lower) sign is for \(\beta^- (\beta^+)\) decays and \(\gamma = \sqrt{1 - Z^2 m_e^2}\), with \(Z\) the atomic number of the daughter nucleus and \(\alpha\) the fine-structure constant. For the positron-emitting superallowed \(0^+ \rightarrow 0^+\) Fermi decays
\[
\frac{1}{fT_F} = \frac{m_e^5}{2\pi^3\ln(2)} G_F^2 V_{ud}^2 gV^2 |M_F|^2 \\
\times \left\{ 1 + \left( \frac{gS}{gV} \right)^2 [A_L^2 + A_R^2] - 2\gamma \frac{m_e}{E_e} \frac{gS}{gV} A_L \right\}. \quad (29)
\]

Hardy and Towner (2009) obtained an average of all \( fT_F \) values, \( \overline{fT_F} \), after the appropriate corrections for radiative and nuclear-structure effects. The current best value of \( V_{ud} \) is derived from \( \overline{fT_F} \), assuming no exotic couplings. Allowing for scalar terms one can exploit (Hardy and Towner, 2005) the different values of \( (m_e/E_e) \) to put a stringent limit on the Fermi Fierz-interference coefficient (Hardy and Towner, 2009),

\[
b_F = -0.0022(26) \\
= -2 \frac{(gS/gV)A_L}{1 + (gS/gV)(A_L^2 + A_R^2)} \approx -2 \frac{gS}{gV} A_L. \quad (30)
\]

Although \( b_F \) is not sensitive to right-handed scalar currents, the value of \( \overline{fT_F} \) is sensitive to these. In fact, the bound on right-handed couplings is more than an order of magnitude larger than that of left-handed couplings, such that both contributions to the \( \overline{fT_F} \) values are of the same order, as can be seen in Eq. (29). Therefore, in searches for BSM physics one may not assume \( V_{ud} \) as given by the Particle Data Group (PDG) when such a search concerns also right-handed scalar terms. In the correlation coefficients, the value of \( V_{ud} \) mostly drops out, but in limits derived from measured lifetimes the actual value of \( V_{ud} \) is required.

Constraints on right-handed scalar couplings can be extracted from the \( \beta-\nu \)-correlation coefficient \( a \) defined in Eq. (A10). We define \( \delta_- = |a_{SM} - a_{exp}| \) as the lower bound and \( \delta_+ = |a_{exp} - a_{SM}| \) as the upper bound, where the experimental value, at 90% confidence level (C.L.), lies between \( a_{exp} \) and \( a_{SM} \). Limits from \( a \) then give

\[
2 \left( \frac{gS}{gV} \right)^2 [A_L^2 + A_R^2] < \delta_-, \quad (31)
\]

which gives a circular bound in the \( A_L, A_R \) plane. Thus, the bound on \( A_L \) and \( A_R \) would be the same,

\[
\left| \frac{gS}{gV} A_{L(R)} \right| < \sqrt{\frac{\delta_-}{2}}. \quad (32)
\]

In practice experiments normalize the correlation to the total number of counts, and the absolute normalization is not measured. This means that in fact \( \bar{a} \) is measured, as discussed below Eq. (23). In this way the Fierz-interference term \( b \) enters. The bounds remain circular, but the bound on \( A_L \) changes to

\[
\frac{-\delta_-}{2\gamma (m_e/E_e)} < \frac{gS}{gV} A_L < \frac{\delta_+}{2\gamma (m_e/E_e)}, \quad (33)
\]

for \( \beta^+ \) and with opposite signs for \( \beta^- \).

Figure 1 shows the bounds from the best current experiments. The superallowed Fermi decays constrain only left-handed couplings and give a narrow vertical band (Hardy and Towner, 2009). The right-handed coupling \( A_R \) is constrained only by the \( \beta-\nu \) correlations and depends on the square root of the experimental error \( \delta_\beta \). The most sensitive \( \beta-\nu \) correlation measurements are from \( ^{38m}\text{K} \) (Gorelov et al., 2005) and \( ^{32}\text{Ar} \) (Adelberger et al., 1999); cf. Eq. (33). Also the bound from the mirror nucleus \( ^{21}\text{Na} \) (Vetter et al., 2008) is given, neglecting tensor contributions.
from Eq. (A13). Thus in the absence of Coulomb corrections one finds that $\bar{A}$ becomes independent of $\alpha_R$ and therefore only limits on $\alpha_L$ can be obtained from $\bar{A}$. Defining the experimental bounds of $\bar{A} - A_{\text{SM}}$ as before gives

$$\frac{-\delta_+}{4\gamma\langle m_e/E_e \rangle} < \frac{g_T}{|g_A|} \alpha_L < \frac{\delta_+}{4\gamma\langle m_e/E_e \rangle}.$$  

(36)

To obtain a bound on $\alpha_R$ one can exploit the $\beta$-$\nu$ correlation $a$. The result is similar to the result for $a$ in Fermi decay. For $\beta^-$ Gamow-Teller decay $a_{\text{SM}} = -1/3$ and the bounds are

$$\left| \frac{g_T}{g_A} \right| < \sqrt{\frac{3\delta_-}{8}},$$

$$-\frac{3\delta_-}{4\gamma\langle m_e/E_e \rangle} < \frac{g_T}{|g_A|} \alpha_L < \frac{3\delta_+}{4\gamma\langle m_e/E_e \rangle}.$$  

(37)

The limits on tensor interactions can be improved by combining scalar and tensor searches. In particular, the left-handed tensor couplings can be further constrained by using the measurements of the Fermi and Gamow-Teller-transition ratio of the longitudinal $\beta$ polarization. These measurements were performed in the first place to study the manifest left-right symmetric model (Wichers et al., 1987; Carnoy et al., 1991); see also Sec. IV.B. The ratio of longitudinal polarizations ($P$, see Appendix A) of the emitted positrons was measured in the systems $^{26}$Al$^{m=30}$P (Wichers et al., 1987) and $^{14}$O$^{10}$C (Carnoy et al., 1991), where the first nucleus decays via a Fermi and the second a Gamow-Teller transition. The two transitions have nearly identical end-point energies, which eliminates systematic errors. The measured ratio is

$$\frac{P_F}{P_{\text{GT}}} \approx \frac{\bar{G}_F}{\bar{G}_{\text{GT}}} \approx 1 - 2\left( \frac{m_e}{E_e} \right) \left( \frac{g_S}{g_T} A_L + \frac{g_T}{|g_T|} \alpha_L \right).$$  

(38)

Combining these measurements with the bounds on $b_T$ in Eq. (30) gives a more precise left-handed tensor bound, but it does not constrain right-handed couplings.

Figure 2 shows the best constraints on tensor couplings. We use the $P_F/P_{\text{GT}}$ values (Wichers et al., 1987; Carnoy et al., 1991), the $\beta$-$\nu$ correlation in $^6$He (Johnson, Pleasonton, and Carlson, 1963; Glück, 1998), and the $\beta$ asymmetry in $^{60}$Co (Wauters et al., 2010) (see Table III) to find the best bounds for nuclear searches, using $\chi^2$ minimization. For the $P_F/P_{\text{GT}}$ values we have included the limits on scalar couplings in...
FIG. 2 (color online). Bounds on left- and right-handed tensor couplings (90% C.L.). The measurement of the $\beta$--$\nu$ correlation in $^6\text{He}$ (Johnson, Pleasonton, and Carlson, 1963; Glück, 1998) gives a ring-shaped boundary. The boundary of measurements of the $\beta$ asymmetry in the pure Gamow-Teller decay of $^{60}\text{Co}$ (Wauters et al., 2010) is given by dashed lines, the measurement only constrains left-handed couplings [Eq. (35)]. The strongest bounds on left-handed couplings are from measurements of the $\beta$-longitudinal polarization $P_F/P_{\text{GT}}$ in Eq. (38) (Wichers et al., 1987; Carnoy et al., 1991), combined with the constraint on $b_F$.

Eq. (34). The combined fit for real tensor couplings gives, at 90% C.L.,

$$-0.3 \times 10^{-2} < \frac{g_T}{|g_A|} \alpha_L < 0.6 \times 10^{-2}, \quad (39a)$$

$$-6 \times 10^{-2} < \frac{g_T}{|g_A|} \alpha_R < 6 \times 10^{-2}. \quad (39b)$$

Reducing the limits will require increased statistics and experimental improvements (Sec. IV.D). Further constraints from $\beta$ decay come from mixed decays which we discuss next.

3. Tensor constraints from neutron and mirror nuclei

Mirror transitions are mixed transitions and therefore sensitive to both scalar and tensor interactions. Mirror decays might be used to improve the bounds of pure Fermi and Gamow-Teller transitions discussed previously. At this point only the neutron can be considered. The prospects of using mirror nuclei are discussed at the end of this section. The neutron can serve as a laboratory for studying a range of fundamental interactions (Abele, 2008; Nico, 2009; Dubbers and Schmidt, 2011). In neutron $\beta$ decay, the main focus lies on determining the SM parameters $V_{ud}$ and $\lambda = g_A/g_V$. Non-SM values are included by allowing $\lambda$ to be complex and/or by allowing for scalar ($A_L, A_R$) and/or tensor ($\alpha_L, \alpha_R$) interactions. We still consider only real couplings and defer to Secs. V.A.1 and V.A.2 for complex $\lambda$ and scalar and tensor couplings, respectively. To clarify the role of possible left- and right-handed scalar and tensor contributions, we keep the simplifying assumptions that the $V$ and $A$ couplings are those of the SM. For neutron decay, with $M_{\text{GT}} = \sqrt{3}$ and $M_F = 1$, the $ft$ value is given by

$$1/ft_n = \frac{m_e^2}{2\pi^2 \ln(2)} G_F^2 V_{ud}^2 g_F^2$$

$$\times \left\{ 1 + \left[ \frac{g_S}{g_V} \right]^2 [A_L^2 + A_R^2] + 2\gamma \left[ \frac{m_e}{E_e} \right] \frac{g_S}{g_V} A_L$$

$$+ 3\lambda^2 \left( 1 + \left[ \frac{g_T}{|g_A|} \right]^2 [\alpha_L^2 + \alpha_R^2] - 4\gamma \left[ \frac{m_e}{E_e} \right] \frac{g_T}{|g_A|} \alpha_L \right) \right\}.$$  \hspace{1cm} (40)

The current value recommended for the lifetime is $\tau_n = 880.3(1.1) \text{ s}$ (Olive et al., 2014), which is nearly 6 s lower, but with the same error, as the recommended value of 2008. Of course, this affects the SM values for $V_{ud}$ and $\lambda$, but cross-checks with other correlation coefficients are possible, allowing for consistency of the SM parameters (Wietfeldt and Greene, 2011). Including scalar and tensor contributions increases the number of degrees of freedom and such cross-checks are no longer possible. The observable $ft$ is most sensitive to $\alpha_L$, because of the partial Gamow-Teller nature of neutron decay. One can combine various correlation coefficients from neutron decay to extract $\lambda$, while allowing for non-SM contributions. In combination with the experimental results from the superallowed Fermi transitions ($F_F$ and $F_T$), improved bounds on tensor contributions can be obtained. For example, with the recent limits on $A$ from UCNA and PERKEOII Collaborations (Mendenhall et al., 2013; Mund et al., 2013) and neglecting right-handed neutrinos ($\lambda_R = 0, \alpha_R = 0$), it is possible to obtain an analytical bound on $\alpha_L$ (Pattie, Hickerson, and Young, 2013). Allowing for right-handed neutrinos requires a fitting procedure.

A complete set of neutron correlation data has been compiled by Dubbers and Schmidt (2011). More recent results are obtained with the PERKEOII setup (Mund et al., 2013) and from the UCNA Collaboration (Mendenhall et al., 2013). Combined with the bounds from pure Fermi and Gamow-Teller transitions a fit can be made to obtain all relevant parameters ($\lambda, A_L, A_R, \alpha_L, \alpha_R$) in a consistent way. This was recently done by Wauters, García, and Hong (2014), to extract both left-handed and right-handed tensor-coupling limits. Their fitting method entails a grid search. For all $\alpha_L$ and $\alpha_R$ values, a value of $\chi^2$ was obtained by minimizing $\chi^2$ for the other three parameters. With this $2D \chi^2$ surface a contour plot can be made, by plotting the equal $\Delta \chi^2 = \chi^2 - \chi^2_0$ lines, where $\chi^2_0$ is the minimal $\chi^2$.

Figure 3 shows the contour plot for the 1, 2, and $3\sigma$ ($\Delta \chi^2 = 1, 4, \text{ and } 9$) bounds obtained with this method and by using the most relevant experiments listed in Table III. It is important to note that the neutron lifetime requires the value of $V_{ud}$. The most precise value for $V_{ud}$ is obtained from the $F_T$ of superallowed decays (Hardy and Towner, 2014), under the assumption of no scalar interactions. We have corrected for this by using Eq. (A20) for the neutron lifetime. For the neutron lifetime we use the average value of the PDG (Beringer et al., 2012). For the correlation coefficients the averages of the PDG cannot be used, because these are obtained by assuming only SM interaction. The possible different dependence on the Fierz-interference term is therefore not included. We consider the different values of $A$
was derived from the measurement of the asymmetry $\beta$ decay of $^{19}$Ne to $^{19}$F was recently studied to determine the lifetime of $^{19}$Ne (Broussard et al., 2014). In this work, the effectiveness of the method described above is shown. For mixed decays an independent measurement of $\rho$ is necessary. For $^{19}$Ne, $\rho = 1.5995(45)$ (Calaprice et al., 1975) was derived from the measurement of the $\beta$ asymmetry $A$. Neglecting quadratic couplings in Eq. (42) and using the extracted value $\mathcal{F}_I = 1719.8(13)$ s with $\langle m_e/E_e \rangle = 0.387022(18)$ from Broussard et al. (2014) a limit on $b_{GT}$ is derived. For left-handed tensor couplings this gives at 90% C.L. (Broussard et al., 2014)

$$ -1.5 \times 10^{-2} < \frac{g_T}{g_A} \alpha_L < 0.12 \times 10^{-2}. $$

The bounds are only an order of magnitude less precise than the combined limits in Eq. (41) and show the potential for this kind of measurements for improving the existing bounds.

4. Tensor constraints from radiative pion $\beta$ decay

Bychkov et al. (2009) derived limits on tensor couplings from radiative pion decay $\pi^+ \to e^+ + \nu_e + \gamma$. These bounds can be translated into bounds on $\alpha_L$ (Bhattacharya et al., 2012) by using estimates for the pion form factor (Mateu and Portolés, 2007). Assuming no right-handed couplings and using $g_T = 1.047(61)$, a limit at 90% C.L. is found,

$$ -1.9 \times 10^{-3} < \frac{g_T}{g_A} \alpha_L < 2.3 \times 10^{-3}. $$

The extracted value of $\lambda$ has a much larger error compared to $\lambda = 1.2723(23)$ from PDG. The scalar bounds are the same as the bounds in Eq. (34), but the tensor bounds are improved because of the inclusion of the neutron data. Especially the negative bound for $\alpha_R$ is reduced as compared to Eq. (39). This is caused by the large spread in experimental values for $A$. Using only the two most recent values of the PERKEOII setup (Mund et al., 2013) and from the UCNA Collaboration (Mendenhall et al., 2013) gives $-0.3 \times 10^{-2} < \frac{g_T}{g_A} \alpha_L < 0.2 \times 10^{-2}$. For the tensor bounds, the neutron lifetime has a large influence (Wauters, García, and Hong, 2014). We therefore anticipate that the error in the neutron lifetime and the spread in $A$ will soon give the dominant error on the limit on tensor couplings.

Recently, also mirror decays have been used to constrain tensor couplings. The strong constraint on $b_F$ from superallowed Fermi decays can be combined with measurements on mirror nuclei, to derive a value for $b_{GT}$. In Severijns et al. (2008) a complete survey of $\mathcal{F}_I$ values of the available mirror transitions is given. For $I = 1/2$ transitions the relation between the $\mathcal{F}_I$ values of the mirror and superallowed $0^+ \to 0^+$ is given by (Severijns et al., 2008)

$$\mathcal{F}_I = \frac{2\mathcal{F}_I^{0^+ \to 0^+} (1 + \frac{\rho^2}{\gamma_0} [L_{\lambda} + A_{\lambda})^2] + \rho^2 \gamma_0 [A_{\lambda}^2 + A_{\lambda}^2] + 4\alpha_L^2 + 4\alpha_R^2) + 2\gamma_0 [\frac{\rho}{\gamma_0} A_{\lambda} - 2 \frac{\rho}{\gamma_0} \alpha_L \rho^2] \rho^2}{\gamma_0 [L_{\lambda} + A_{\lambda})^2] + \rho^2 \gamma_0 [A_{\lambda}^2 + A_{\lambda}^2] + 4\alpha_L^2 + 4\alpha_R^2 + 2\gamma_0 [\frac{\rho}{\gamma_0} A_{\lambda} - 2 \frac{\rho}{\gamma_0} \alpha_L \rho^2] \rho^2},$$

where $f_A/f_V = 1.0143(29)$ is the ratio of the axial-vector and vector statistical rate functions (Severijns et al., 2008). The inverse energy dependence of the superallowed Fermi decays is denoted by $\langle m_e/E_e \rangle^{0^+ \to 0^+}$ and calculated in Pattie, Hickerson, and Young (2013). If $\rho$ is known, a value for $\alpha_L$ can be extracted from $\mathcal{F}_I$.

The mirror $\beta^+$ decay of $^{19}$Ne to $^{19}$F was recently studied to determine the lifetime of $^{19}$Ne (Broussard et al., 2014). In this work, the effectiveness of the method described above is shown. For mixed decays an independent measurement of $\rho$ is necessary. For $^{19}$Ne, $\rho = 1.5995(45)$ (Calaprice et al., 1975) was derived from the measurement of the $\beta$ asymmetry $A$. Neglecting quadratic couplings in Eq. (42) and using the extracted value $\mathcal{F}_I = 1719.8(13)$ s with $\langle m_e/E_e \rangle = 0.387022(18)$ from Broussard et al. (2014) a limit on $b_{GT}$ is derived. For left-handed tensor couplings this gives at 90% C.L. (Broussard et al., 2014)

$$ -1.5 \times 10^{-2} < \frac{g_T}{g_A} \alpha_L < 0.12 \times 10^{-2}. $$

The bounds are only an order of magnitude less precise than the combined limits in Eq. (41) and show the potential for this kind of measurements for improving the existing bounds.
These bounds are the strongest bounds on tensor couplings from a single decay experiment and show that future $\beta$-decay experiments should probe $\alpha_\perp < 10^{-3}$ and beyond, in order to improve these existing limits.

5. Pseudoscalar constraints

Pseudoscalar interactions have so far been neglected in $\beta$-decay searches, since they are strongly suppressed because the nuclei are nonrelativistic. The suppression of these terms is $O(1/M)$, where $M$ is the nucleon mass. However, in $\beta$ decay, the pseudoscalar interactions are always multiplied by $g_P$, the pseudoscalar form factor discussed in Eq. (22). The large value $g_P = 349(9)$ (González-Alonso and Camalich, 2014) largely cancels this suppression, and $\beta$-decay experiments might be used to probe these interactions. There are, however, already strong constraints on pseudoscalar couplings from pion decay (Herczeg, 1994, 2001; Bhattacharya et al., 2012).

The ratio $R_\pi = \Gamma(\pi \rightarrow e\nu) / \Gamma(\pi \rightarrow \mu\nu)$ is sensitive to pseudoscalar couplings defined by

$$L = \frac{G_F V_{ud}}{\sqrt{2}} \{ A_P^L \bar{e}(1 - \gamma_5)\nu_e + A_P^R \bar{e}(1 + \gamma_5)\nu_e \} \bar{u} \gamma_5 d,$$

where we have neglected flavor-changing couplings, which can be found in Bhattacharya et al. (2012). The ratio $R_\pi/R_{\pi}^{SM}$, where $R_\pi$ is the measured value, is sensitive to electron and muon pseudoscalar couplings $A_P^{(e)}$ and $A_P^{(\mu)}$, respectively. If these couplings are such that $A_P^{(e)}/m_e = A_P^{(\mu)}/m_\mu$, their contributions to the ratio cancel and no bounds on pseudoscalar interactions can be obtained. Since there is no reason to assume such a cancellation, we can place bounds on pseudoscalar interactions, because these would show up as $R_\pi/R_{\pi}^{SM} \neq 1$. The current best value for this ratio is

$$R_\pi/R_{\pi}^{SM} = 0.996(3) \quad \text{(Beringer et al., 2012; Cirigliano and Rosell, 2007),}$$

which leads to (90% C.L.) (Bhattacharya et al., 2012; Cirigliano, González-Alonso, and Graesser, 2013)

$$-1.4 \times 10^{-7} < A_P^L < 5.5 \times 10^{-4}, \quad (46a)$$

$$-2.8 \times 10^{-4} < A_P^R < 2.8 \times 10^{-4}. \quad (46b)$$

In $\beta$ decay the pseudoscalar term shows up in Gamow-Teller and mixed decays. The most relevant to experiments are its contributions to the Fierz-interference term,

$$b_{GT} = \pm 4 \gamma \frac{g_T}{g_A} \alpha_L \pm \frac{1}{3} \gamma \frac{g_P}{g_A} A_P^L \frac{E_0 - E_\pi}{M}, \quad (47)$$

which enters with the usual $(m_e/E_\pi)$ suppression. The $(E_0 - E_\pi)/M$ term is responsible for the suppression of pseudoscalar contributions, however, because $g_P (E_0 - E_\pi)/M \approx 0.4$ pseudoscalar interactions are still suppressed compared to tensor interactions. Given the current limit on $\alpha_L$, improving the bounds in Eq. (46a) seems unlikely in the near future.

The pseudoscalar couplings in Eq. (46) can also be translated into bounds on scalar and tensor couplings. If scalar and tensor interactions are present at the new-physics scale $\Lambda$, they will mix via radiative loop corrections, and pseudoscalar couplings will radiatively be generated (Herczeg, 1994; Campbell and Maybury, 2005). Current limits are at the level of (Bhattacharya et al., 2012; Cirigliano, González-Alonso, and Graesser, 2013; Cirigliano, Gardner, and Holstein, 2013)

$$|A_L| < 8 \times 10^{-2} \quad \text{and} \quad |A_R| < 5 \times 10^{-2}, \quad (48a)$$

$$|\alpha_L| < 2 \times 10^{-3} \quad \text{and} \quad |\alpha_R| < 1.2 \times 10^{-3}, \quad (48b)$$

and depend logarithmically on the scale of new physics $\Lambda$, for which $\Lambda = 10$ TeV is used. These bounds are of the same order of magnitude as global-fit limits from $\beta$ decay in Eq. (41), except for the bound on $\alpha_R$, which is an order of magnitude better. However, because the constraints for right-handed currents rely on the flavor structure of new physics (Cirigliano, González-Alonso, and Graesser, 2013), we do not further consider these bounds.

6. Left-handed scalar versus tensor

In Sec. IV.C we discuss exotic couplings involving right-handed neutrinos. If right-handed neutrinos are absent, or too heavy to be energetically allowed in $\beta$ decay, right-handed neutrino couplings, i.e., $A_R$ and $\alpha_R$, can be neglected. The resulting reduction of parameter space allows us to use mixed decays to fit the correlations between left-handed tensor and scalar couplings. Figure 4 shows these correlations. For the complete set of data listed in Table III we find at 90% C.L.

$$-0.1 \times 10^{-2} < \frac{g_S}{g_V} A_L < 0.3 \times 10^{-2}, \quad (49a)$$

$$-0.2 \times 10^{-2} < \frac{g_T}{g_A} \alpha_L < 0.06 \times 10^{-2}, \quad (49b)$$

$$1.2715 < |\delta| < 1.2744. \quad (49c)$$

FIG. 4 (color online). Contour plot of the 1, 2, and 3$\sigma$ contours, derived from the selection of available data listed in Table III without right-handed couplings, i.e., $A_R = \alpha_R = 0$. 
These bounds are not significantly different from the bounds from the complete fit in Eq. (41). For comparison: limits on right-handed couplings from neutron decay alone are found in Konrad et al. (2010) and Dubbers and Schmidt (2011).

B. Constraints from the LHC experiments

Low-energy experiments are mostly viewed as complementary to high-energy collider searches for BSM physics. Experiments at the LHC can place bounds on new physics by looking for the on-shell production of new particles, as done in searches for a $W_R$ boson [Eq. (13)] or supersymmetric particles. We focus here on the effect of a $W_R$ boson, because this has been studied complementary by precision decay experiments and by the LHC, e.g., Dekens and Boer (2014). At the LHC, $W_R$ is searched for by considering its possible decay channels. In the $W_R \rightarrow \bar{t}b$ channel, such direct searches at the CMS experiment constrain $M_R > 2$ TeV (Chatrchyan et al., 2014). Constraints from the $W_R \rightarrow e\nu$ channel are similar, but depend on assumptions for the right-handed neutrino. Constraints from neutral-kaon mixing give $M_R > 3$ TeV (Bertolini, Maiezza, and Nesti, 2014).

In $\beta$ decay, strong limits come from CKM unitarity tests, for which the best bound is (Hardy and Towner, 2014)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.00085 (56),$$

which uses the value of $V_{us}$ from Moulson (2013). The error has equal contributions from $V_{ud}$ and $V_{us}$. Following Hardy and Towner (2009), this leads to a constraint on $a_{LR}$, i.e., left-handed lepton couplings and right-handed quark couplings, of

$$-4 \times 10^{-3} < a_{LR} < 5 \times 10^{-3},$$

at 90% C.L. The precision of both $V_{ud}$ and $V_{us}$ should improve simultaneously for such a test to remain significant.

In $\beta$ decay, some correlation coefficients are sensitive to $a_{LR}, a_{RL}$, and $a_{RR}$, where the latter two are present only if light right-handed neutrinos are assumed. For example, the measurements of $P_F/P_{GT}$ (Wichers et al., 1987; Carnoy et al., 1991) and $A_{GT}$ in $^{56}$Co (Wauters et al., 2010) are used to constrain parameters of manifest $LR$-symmetric models. Such models have a $P$ symmetry, such that for the CKM matrices $V^T_{ud} = \pm V^T_{ad}$. There is no additional spontaneous $CP$ violation, so $\omega = 0$. In this simplified model, $a_{RL} = \pm a_{LR} \sim -\bar{\xi}$ and $a_{RR} = \delta = (M_1/M_2)^2$. Measurements of $P_F/P_{GT}$ limit the combination $\delta - \bar{\xi}$ and do not give additional bounds, because of the strong bound on $\bar{\xi}$ from unitarity tests given in Eq. (51). Because $\bar{\xi}$ is strongly constrained, $\beta$-decay experiments can constrain only $a_{RR}$ and thus the mass of the $W_R$. Derived limits are of the order of 200 GeV (Gorelov et al., 2005; Wauters et al., 2010), an order of magnitude below the bound from the LHC experiments presented above. In fact, when assuming manifest $LR$ symmetry, the strongest bound on $W_R$ comes from the $K^-\Lambda_s$ mass difference, from which $W_R > 20$ TeV was derived (Maiezza and Nemevšek, 2014).

Besides constraining new physics by searching for direct on-shell production, it is also possible for colliders to constrain exotic couplings. When the mass of the non-SM particle exceeds the energy accessible at the LHC, the new particles cannot be produced on shell, but their effects can still be found in deviations from the SM predictions. In that way, the exotic interactions in Eq. (11) will also manifest themselves in proton-proton collisions. This makes it possible for the LHC data to constrain the same tensor and scalar couplings relevant in $\beta$ decay (Bhattacharya et al., 2012; Cirigliano, González-Alonso, and Graesser, 2013).

In particular, the $pp \rightarrow e + \text{MET} + X$ channel is considered, where MET signifies missing transverse energy. This channel is closely related to $\beta$ decay, since it involves the $\bar{u}d \rightarrow e\nu$ process at quark level. At the LHC, both the ATLAS and CMS detectors are used to search for new physics in this channel (Aad et al., 2012; Chatrchyan et al., 2012), by searching for an excess of events predicted at a large lepton transverse mass cut $m_T$. At large $m_T$, the SM cross section approaches zero more rapidly than the cross sections for new physics, making the sensitivity to non-SM physics larger at high momenta. The total cross section is

$$\sigma(m_T > \bar{m}_T) = \sigma_{SM}(1 + |a_{LR}|^2 + |a_{RL}|^2) + \sigma_{R}(|a_{RR}|^2 + \frac{1}{2} \bar{m}_T(|a_{LR}|^2 + |a_{RL}|^2).$$

(52)

where $\sigma_{SM}$ is the SM cross section and $\sigma_{R,S,T}$ are the cross sections for new physics. The explicit form of $\sigma_{SM}$ and $\sigma_{R,S,T}$ is given, to lowest order in QCD corrections, in Cirigliano, González-Alonso, and Graesser (2013). The coefficients $a_{LR}$ and $a_{RL}$ cannot be constrained, because their contribution is proportional to $\sigma_{SM}$, and therefore small at large $m_T$.

With the expected number of background events and the number of actual observed events, one can place an upper limit on the number of new-physics events, $n_{SM}^\text{up}$ (Bhattacharya et al., 2012). This translates into an upper limit for $\sigma$, and finally into bounds on exotic couplings. First bounds were derived by Bhattacharya et al. (2012), and updated bounds are given in Naviliat-Cunic and González-Alonso (2013).

The bounds are derived by using the experimental data of Khachatryan et al. (2014) at an integrated luminosity of 20 fb$^{-1}$ and at a center-of-mass energy of $\sqrt{s} = 8$ TeV. Naviliat-Cunic and González-Alonso (2013) also gave the combined limits for scalar and tensor couplings, assuming only left-handed couplings. In Table IV we give the 90% C.L. bounds, obtained by allowing one exotic interaction and putting all other couplings to zero. To compare these results with $\beta$-decay constraints, we use the values from the global fit in Eq. (41) and the form factors $g_S = 1.02(11)$ (González-Alonso and Camalich, 2014) and $g_T = 1.047(61)$.

| \hline
<table>
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<tr>
<th>$\beta$ decay</th>
<th>$A_L$</th>
<th>$A_R$</th>
<th>$a_L$</th>
<th>$a_R$</th>
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<td>$3 \times 10^{-3}$</td>
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<tr>
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<td>$6 \times 10^{-3}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$2 \times 10^{-3}$</td>
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<tr>
<td>$A_L$</td>
<td>$1 \times 10^{-3}$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$1 \times 10^{-3}$</td>
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</table>

TABLE IV. Comparison between $\beta$-decay limits on left- and right-handed scalar $A_L$ and $A_R$ and tensor couplings $a_L$ and $a_R$, constraints from the LHC data (Naviliat-Cunic and González-Alonso, 2013), and from the neutrino mass (Ilo and Prezeau, 2005). Constraints are at 90% C.L., and all couplings are assumed to be real.
(Bhattacharya et al., 2014). Because the errors on the form factors are not Gaussian, we use the R-fit method described in Bhattacharya et al. (2012), which treats all the values in a 1σ interval with equal probability. Therefore, only the lower bounds are important. We stress again that the reduction of the error in $g_S$ and $g_T$ is important to make meaningful comparisons between the different experiments.

Table IV shows that the LHC constraints on left-handed couplings are comparable to $\beta$-decay constraints, while for right-handed couplings the LHC constraints are an order of magnitude better than the $\beta$-decay limits. The current status is discussed in Sec. IV.D. Naviliat-Cuncic and González-Alonso (2013) also made a projection for the 14 TeV run at 50 fb$^{-1}$ and found that the expected bounds are a factor of 3 better.

### C. Neutrino-mass implications

Besides strong bounds from the LHC experiments on right-handed interactions, there are also bounds from the neutrino mass. In the SM, neutrinos are assumed to be massless, but neutrino oscillations indicate the existence of at least two massive neutrinos. A direct upper limit on the neutrino mass comes from the shift of the end point of the $\beta$ spectrum. Recent measurements of the $\beta$ spectrum of $^3$H give $m_\nu < 2$ eV (95% C.L.) (Kraus et al., 2005; Aseev et al., 2011). The experiment of the KATRIN Collaboration aims to improve these limits by an order of magnitude (Otten and Weinheimer, 2012), which treats all the values in a 1σ interval with equal probability. Therefore, only the lower bounds are important. We stress again that the reduction of the error in $g_S$ and $g_T$ is important to make meaningful comparisons between the different experiments.

Table IV shows that the LHC constraints on left-handed couplings are comparable to $\beta$-decay constraints, while for right-handed couplings the LHC constraints are an order of magnitude better than the $\beta$-decay limits. The current status is discussed in Sec. IV.D. Naviliat-Cuncic and González-Alonso (2013) also made a projection for the 14 TeV run at 50 fb$^{-1}$ and found that the expected bounds are a factor of 3 better.

![FIG. 5. The two-loop contribution to the neutrino mass, where the boxes indicate the exotic couplings. The crosses indicate mass insertions, with (a) $m_q = 4$ MeV and (b) $m_e = 0.511$ MeV (Ito and Prezeau, 2005). For Majorana neutrinos one can substitute $\nu_R \to \nu_L$.](image)

Neutrino masses can be either Dirac ($\nu \to \gamma l \nu_D$) or Majorana ($\nu \to l \nu_L$), where $\nu_L = i\gamma_5 \nu_D$, or a combination of the two. However, the following results are general and apply to both types. Couplings to right-handed neutrinos contribute to the neutrino mass via loop interactions. Figure 5 shows the leading two-loop contribution to the neutrino mass, where the boxes indicate the non-SM couplings to right-handed particles. The crosses indicate the mass insertions needed to couple two fermions with different chiralities. Here the chirality-changing interactions are either proportional to (a) the quark or (b) the electron mass. In a power-counting scheme, one-loop contributions are in general less suppressed than two-loop contributions. However, the two-loop diagrams in Fig. 5 are enhanced by the $W$-boson mass, while the one-loop diagrams are suppressed only by the light-fermion mass. This makes the two-loop contribution dominant, as the additional loop suppression of $1/(4\pi)^2$ is diminished by the heavy $W$-boson mass.

One can estimate the two-loop contribution to the neutrino mass by considering only the logarithmic part of Fig. 5. The analytic parts are renormalization-scheme dependent and are therefore neglected (Prezeau and Kurylov, 2005). By using dimensional regularization the contribution to $\delta m_\nu$ is estimated as (Ito and Prezeau, 2005)

$$\delta m_\nu \approx 3g^2 G_F \tilde{a} \frac{m_f M_W^2}{(4\pi)^4} \left( \frac{\ln \frac{\mu^2}{M_W^2}}{2} \right)^2,$$

where $\tilde{a} = \{A_{RL}, A_{RR}, \alpha_R, \alpha_{RL}\}$ are the exotic couplings from Eq. (11), $g = 0.64$ is the gauge coupling, $m_f$ is the inserted fermion mass, and $\mu$ is the renormalization scale, which should exceed the heaviest mass in the interaction $\mu > m_\nu$, where $m_i$ is the top-quark mass. Assuming that the loop corrections do not exceed the mass of the neutrino, i.e., $\delta m_\nu < m_\nu$, setting $m_q = 4$ MeV, $\mu = 1$ TeV, and $m_\nu < 0.15$ eV in Eq. (53) gives

$$|a_{RL}| \lesssim 10^{-2},$$

$$|A_{RR}|, |A_{RL}|, |\alpha_R| \lesssim 10^{-3}.$$

---

There might be scenarios in which this is not obeyed, but these scenarios would have to be fine-tuned.
In Table IV we compare these limits with current right-handed $\beta$-decay bounds and bounds from the LHC. The estimates from the neutrino are currently the strongest bounds on right-handed currents. They are more than an order of magnitude stronger than the $\beta$-decay bounds and comparable to the LHC bounds. For the bounds in Eq. (54) we have used the updated neutrino mass from the Planck space observatory, which might further improve in the future. The given bounds are conservative estimates, but nevertheless they show the large impact of the neutrino mass on $\beta$-decay measurements. Even stronger constraints of $O(10^{-5})$ from the neutrino mass have been derived in Wang (2007).

D. Conclusions and outlook

We summarized the current status of the bounds on real right-handed vector, scalar, pseudoscalar, and tensor interactions in $\beta$ decay. We compared these bounds with those obtained from proton-proton collisions at the LHC experiments and the upper limit on the neutrino mass, mainly focusing on scalar and tensor interactions. The best current bounds are given in Table IV. We distinguished between bounds on left- and right-handed scalar and tensor interactions, where left or right denotes the chirality of the neutrino. The constraints on left-handed interactions are equally constrained by the LHC and $\beta$-decay experiments. On the other hand, $\beta$-decay experiments measuring right-handed interactions would have to improve orders of magnitude to compete with the bounds from the LHC experiments and the neutrino mass. This is illustrated in Fig. 6 for scalar interactions and in Fig. 7 for tensor interactions. Table V projects the competitive accuracy required for different $\beta$-decay parameters. For left-handed currents we give the necessary precision to compete with projected future LHC bounds (Naviliat-Cunic and González-Alonso, 2013). For right-handed bounds, we give two accuracies. The first corresponds to the required sensitivity to compete with current LHC bounds; the number in brackets corresponds to the required precision to compete with the bounds from the neutrino mass (see Table IV).

The bounds on left-handed couplings are best pursued via measurements of the Fierz-interference coefficient $b$. For left-handed scalar couplings $A_L$, the bound is most stringent because of the vast effort in the study of superallowed Fermi transitions. These studies also provide the best current value for $V_{ud}$. The left-handed tensor coupling $\alpha_T$ requires a larger effort, for which several measurements need to be combined. The best current bounds are from the global fit in which neutron and nuclear data are combined. In this fit, especially the uncertainties in the neutron lifetime and the $A$ coefficient of the neutron have a significant impact. We pointed out that the large spread in the available $A$ measurements influences the obtained bound significantly. The Gamow-Teller part $b_{GT}$ of the Fierz-interference term and $V_{ud}$ can also be constrained in mirror nuclei, in analogy to the superallowed Fermi transitions. However, this also requires the measurement of at least one correlation coefficient. Measurements with this aim are undertaken (Ban et al., 2013).

In Gamow-Teller transitions, measurements of the Fierz-interference term $b_{GT}$ allow for bounds on the left-handed tensor terms. In Seattle, a $^6$He factory has been set up to study this term. The lifetime of $^6$He was already measured with high precision (Knecht et al., 2012), but the shell-model calculations are not sufficiently accurate as yet to search for tensor interactions. One straightforward, but not so simple, approach is to measure the decay spectrum precisely. This would give access to $b_{GT}$. These measurements would also have to consider contributions from the SM weak magnetism [cf. Eq. (4)]. Measurements of $b_{GT}$ from electron-antineutrino correlation $\alpha_{GT}$ and the spectrum are both ongoing and being set up (Fléchard et al., 2008, 2011; Knecht et al., 2011; Aviv et al., 2012; Naviliat-Cunic, 2014; Severijns, 2014). If these measurements reach $b < 10^{-3}$, they would allow for a strong limit on $\alpha_T$. Such a precision is necessary to compete with the projected bounds from the 14 TeV run of the LHC. In neutron decay, many efforts are undertaken to improve the measurements of $\alpha_{GT}$ and $A$ (Baessler et al., 2008, 2014; Märtisch et al., 2009; Počanić et al., 2009; Wietfeldt et al., 2009; Konrad et al., 2012). For comparison, limits on the Fierz terms from

![Fig. 6](image1.png) **FIG. 6** (color online). Scalar bounds from nuclear $\beta$ decay as in Fig. 1 combined with limits derived from the neutrino mass (horizontal lines) and constraints from the LHC experiments (circular bounds).

![Fig. 7](image2.png) **FIG. 7** (color online). Tensor bounds from nuclear $\beta$ decay as in Fig. 2 combined with limits derived from the neutrino mass (horizontal lines) and constraints from the LHC experiments (circular bounds).
neutron decay alone are found in Konrad et al. (2010) and Dubbers and Schmidt (2011), including limits derived from the electron energy dependence of the $\beta$ asymmetry $A_{\text{exp}}(E)$ alone.

Right-handed interactions, which imply the existence of a light right-handed neutrino, do not interfere with the SM interactions and can therefore only be measured directly, i.e., via quadratic terms. This makes it difficult to reach the sensitivity obtained for left-handed couplings. In $\beta$ decay, the right-handed tensor coupling $a_R$ can be constrained by measuring the $\beta$-$\nu$ correlation, $\tilde{a}_{\mu\nu}$. The best measurement in pure Gamow-Teller decays of $\mu_B$ stems from the measurement in $^6$He (Johnson, Pleasonton, and Carlson, 1963). Many efforts are undertaken to improve this limit in $^6$He (Knecht et al., 2011; Aviv et al., 2012; Couratin et al., 2012). A dedicated effort to limit right-handed tensor couplings is ongoing in $^8$Li, for which the daughter nucleus $^8$Be breaks up into two $\alpha$ particles, $^8\text{Li} \to e^- + \nu + 2\alpha$. The $a_{\text{GT}}$ coefficient can be measured by measuring the $\beta$-$\alpha$ correlation, and by taking advantage of the increased sensitivity due to the population of a $2^+$ state in $^8$Be. After putting the Fierz term $b = 0$, such that only right-handed interactions are constrained (Li et al., 2013), one finds

$$\frac{g_T}{g_A} |a_R| < 8 \times 10^{-2}. \quad (55)$$

The bound reaches the precision of the combined fits, but when considering the LHC or neutrino bounds the experiment would have to improve by more than 3 orders of magnitude to compete (see Table V).

When comparing tensor and scalar bounds from different fields, the form factors $g_A$ and $g_T$ are important. Lattice QCD calculations have made great progress and will continue to do so in the next period. The lattice prediction of $g_A$ will hopefully reach the experimental precision soon, which would allow for a cross-check between the experimental value and the theoretical lattice value.

Besides scalar and tensor searches, we also discussed searches for $V + A$ and pseudoscalar interactions.

### Table V. Required experimental precision on $\beta$-decay parameters to remain competitive with the LHC bounds; cf. Naviliat-Cuncic and González-Alonso (2013). Only the Fermi ($F$) and Gamow-Teller (GT) parts of the Fierz-interference term $b$ and the $\beta$-$\nu$ correlation $a$ are listed. The third column gives the corresponding limit on scalar couplings $A_1$ and $A_3$ and tensor couplings $a_R$ and $a_T$. The Fierz term is the leading term in most $\beta$-correlation experiments (Sec. IV.A). The indicated bounds for $b$ assumes that future LHC data lead to bounds indicated in the last column. The $a$ parameter is the most direct way to obtain a bound on right-handed couplings, which should be the motivation to measure $a$. Here the current bounds of the LHC are assumed, while the values in parentheses are the required accuracies when the bounds derived from the limit of the neutrino mass are considered (Table IV).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bound</th>
<th>Constraint at 90% C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{\text{GT}}$</td>
<td>$10^{-3}$</td>
<td>$a_L &lt; 3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$b_F$</td>
<td>$10^{-3}$</td>
<td>$A_L &lt; 5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$a_{\text{GT}}$</td>
<td>$10^{-4}$ ($5 \times 10^{-6}$)</td>
<td>$a_T &lt; 6 \times 10^{-3}$ ($a_R &lt; 10^{-3}$)</td>
</tr>
<tr>
<td>$a_F$</td>
<td>$8 \times 10^{-6}$ ($2 \times 10^{-6}$)</td>
<td>$A_R &lt; 2 \times 10^{-3}$ ($A_R &lt; 5 \times 10^{-3}$)</td>
</tr>
</tbody>
</table>

Pseudoscalar interactions are less suppressed than previously thought, due to the large value of $g_P$. However, strong bounds exist from radiative pion decay, and pseudoscalar interactions can still be neglected in the upcoming $\beta$-decay experiments. Strong constraints on $V + A$ currents are extracted from CKM unitarity tests, to which $\beta$-decay experiments contribute by providing the most accurate value of $V_{ud}$. Besides this, measurements of correlation coefficients can be used to constrain parameters of (manifest) left-right symmetric models. For these specific models, strong limits from the LHC experiments and the neutral-kaon mass difference exist. Therefore, the significance of $\beta$ experiments in these experiments is limited to specific models.

### V. Limits on Time-Reversal Violation

So far we have considered only the real parts of the exotic couplings. In this section we focus on their imaginary parts. A nonzero measurement of an imaginary coupling would imply that time-reversal ($T$) symmetry and, by the CPT theorem, $CP$ symmetry is violated.\(^4\) Because of the matter-antimatter asymmetry of the Universe, new sources of $CP$ violation are expected (Sakharov, 1967). Many models of BSM physics predict such additional sources of $CP$ violation; see, e.g., Dekens and de Vries (2013), Ibrahim and Nath (2008), and Branco, González Felipe, and Joaquim (2012). This makes $T$ or $CP$ violation one of the main portals to search for new physics. These searches range from experiments at the LHC to atomic-physics experiments. As such the observables can be quite diverse. With advances in theory, in particular, via EFT methods, relations between the different observables have become more clear (cf. Secs. IV.B and IV.C).

In this section we focus on the connection between $T$-violating observables in $\beta$ decay and the bounds on EDMs. The $P$- and $T$-odd EDM measurements are a powerful probe of $CP$ violation beyond the SM (Pospelov and Ritz, 2005). High-precision EDM searches have been made for the neutron, paramagnetic and diamagnetic atoms, and molecules. The EDM is a static observable, and, therefore, allows for very precise atomic-physics experiments. It is also a background-free observable, because the electroweak SM contributions to the EDM are strongly suppressed. Therefore, EDM experiments give strong limits on new $T$-violating physics. BSM physics contributions to the EDM can be parametrized by dimension-6 operators (de Vries, Higa et al., 2011; de Vries, Mereghetti et al., 2011; de Vries, Timmermans et al., 2011; de Vries et al., 2013; Bsaisou et al., 2015). At low energy this leads to a relation between the $T$-violating correlations in $\beta$ decay and EDMs.

Many correlation coefficients in $\beta$ decay depend on the square of the underlying coupling constants. As such they depend only on the imaginary couplings squared, which are therefore difficult to access. A more direct way to probe imaginary couplings is to consider the $T$-odd triple correlations $\tilde{J} \cdot (\tilde{p}_e \times \tilde{p}_\nu)$ and $\tilde{J} \cdot (\tilde{J} \times \tilde{p}_e)$ multiplied by the $D$ [Eq. (17)]

\(^4\)In any Lorentz-symmetric local field theory, $CP$ violation is equivalent to $T$ violation, according to the $CPT$ theorem. For $CPT$ violation, see Sec. VI.
and $R$ [Eq. (18)] coefficients, respectively. The first is $P$ even and $T$ odd, while the latter is $P$ and $T$ odd. They probe left-handed imaginary couplings, which are absent in the SM.

Since the interactions contributing to $D$, $R$, and EDMs are generated by the same operators, a limit on the EDM also limits the $D$ and $R$ coefficients. We consider these relations and discuss the relative precision of the two types of experiments.

A. Limits on triple-correlation coefficients in $\beta$ decay

A finite $D$ coefficient arises from the interference between the imaginary parts of the left-handed vector couplings and is proportional to $\text{Im} \alpha_{LR}$. The $R$ coefficient arises from the interference between the imaginary parts of scalar or tensor couplings and SM couplings, making this coefficient sensitive to both $\text{Im} \alpha_L$ and $\text{Im} \alpha_{LL}$.

The SM contributes to both the $R$ and $D$ coefficients through electromagnetic final-state interactions (FSI) and through SM $CP$ violation. The FSI are only motion-reversal odd, i.e., the initial and final states are no longer interchangeable, due to radiative corrections. In this way, FSI mimic time-reversal violation, but in fact are $T$ even. We denote their contributions by $R_f$ and $D_f$ and write $D = D_i + D_f$ and $R = R_i + R_f$ (Herczeg, 2005), where $D_i$ and $R_i$ are the true $T$-violating contributions. The contributions from FSI are comparable to the current experimental precision and depend on the momentum of the $\beta$ particle. We will discuss their values for specific isotopes later. True $T$ violation in the SM arises from the $CP$-violating phase of the CKM matrix and the QCD $\theta$ term. These sources contribute only at the level of $O(10^{-12})$ (Herczeg and Khriplovich, 1997), much below the current experimental precision.

1. $D$ coefficient

To first order in exotic couplings, the $D_i$ coefficient can be expressed as (Jackson, Treiman, and Wyld, 1957a)

$$D_i = a_D \text{Im} \alpha_{LR},$$

from Eq. (A15), with

$$a_D = \frac{\sqrt{\frac{4\pi\rho}{\Gamma_I\rho_I^p}}}{1 + \rho^2}.$$  (57)

The $D$ coefficient can be accessed only in mixed transitions and has been measured in both neutron and $^{19}$Ne decays, which have $a_D = 0.87$ and $a_D = -1.03$, respectively. For $^{19}$Ne the best measurement is $D = 1(6) \times 10^{-4}$ (Hallin et al., 1984), and from neutron decay $D = -0.94(2,10) \times 10^{-4}$ (Mumm et al., 2011; Chupp et al., 2012).

The value of the FSI depends on the kinematics of the experiment. For $^{19}$Ne the FSI have been derived by Callan and Treiman (1967) as $D_f = 2.6 \times 10^{-4} \rho_e/\rho^\text{max}$, which is of the same order as the experimental precision. For neutron decay the FSI were also calculated in chiral perturbation theory by Ando, McGovern, and Sato (2009). Their derivation reproduces the original result of Callan and Treiman (1967). However, Ando, McGovern, and Sato (2009) included higher-order corrections, which are of the order of $O(10^{-7})$, allowing for an accurate expression for the FSI,

$$D_f = \left( \frac{0.228 \rho_e^\text{max}}{\rho_e} + 1.083 \frac{\rho_e}{\rho^\text{max}} \right) \times 10^{-3} - 5.88 \rho_e^\text{max} \times 10^{-9},$$

where the first two terms are the Callan and Treiman (1967) terms, and the last term represents the higher-order corrections. Equation (58) is accurate to better than 1%. For the current best neutron experiment the FSI are estimated at $D_f \approx 1.2 \times 10^{-5}$ (Chupp et al., 2012). The uncertainty in $D_f$ stems from the uncertainty of the $\beta$ momentum in the experiment. The $T$-violating part of the neutron $D$ measurement gives at 90% C.L.

$$|D_f| < 4 \times 10^{-4},$$  (59)

and with $a_D = 0.87$,

$$|\text{Im} \alpha_{LR}| < 4 \times 10^{-4}.$$  (60)

Given the current experimental precision, it is clear that the FSI become increasingly more important. In this respect, neutron experiments are favored over nuclei, because the FSI can be calculated with a higher precision. Eventually the accuracy to which the FSI are known will limit measurements of true $T$ violation.

2. $R$ coefficient

Neglecting quadratic non-SM couplings, the $R_i$ coefficient is given by (Jackson, Treiman, and Wyld, 1957a)

$$R_i = \frac{(a_D \mp b_D)}{|g_A|} g_T \text{Im} \alpha_L - \frac{a_D}{2g_V} g_S \text{Im} \alpha_L,$$  (61)

from Eq. (A16), where the upper (lower) sign is for $\beta^-(\beta^+)$ decay, $a_D$ is given in Eq. (57), and

$$b_D = \frac{4\lambda_f \rho^2}{1 + \rho^2},$$  (62)

with $\lambda_f$ as given in Appendix A. The $R$ coefficient can be measured in both mixed or pure Gamow-Teller transitions, where the latter limits $\text{Im} \alpha_L$. The leading contributions to the FSI are given by the Coulomb corrections calculated by Jackson, Treiman, and Wyld (1957b),

$$R_f = \frac{Z \text{amn}}{2 \rho_e} (\mp a_D + b_D).$$  (63)

The $R$ coefficient has been measured in the pure Gamow-Teller decay of $^6$Li, where $a_D = 0$ and $b_D = 4/3$. The FSI give $R_f \approx 7 \times 10^{-4}$, leading to $R_i = (0.9 \pm 2.2) \times 10^{-3}$ (Huber et al., 2003). This constrains at 90% C.L.

$$g_T |\text{Im} \alpha_L| < 3 \times 10^{-3}.$$  (64)

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The best measurement of $R$ in a mixed decay has been obtained for neutron decay, for which $a_D = 0.87$ and $b_D = 2.2$. Kozela et al. (2012) found $R = (4 \pm 12 \pm 5) \times 10^{-3}$. The FSI are calculated with Eq. (63). By using the energy distribution seen by the experimental setup one obtains $R_f \approx 6 \times 10^{-4}$ (Kozela et al., 2012). The error in $R_f$ is less than 10%. $R_f$ can be neglected given the current experimental precision. At 90% C.L.

$$-1.1 g_T \text{Im} a_L - 0.44 g_S \text{Im} A_L < 2.4 \times 10^{-2}.$$  
(65)

With the constraint given in Eq. (64) one finds at 90% C.L.

$$g_S |\text{Im} A_L| < 6 \times 10^{-2}.$$  
(66)

### 3. Alternative correlations

The measurement of the $D$ coefficient requires the detection of the recoiling nucleus instead of detecting the neutrino. This imposes strong experimental constraints on any measurement scheme. Current schemes consider atomic trapping in a magneto-optical trap, which has led to the best value for the $\beta$ correlation $a$. Measuring $D$ requires a modification of this trap technique, to allow for a polarized sample. It will be extremely challenging to achieve high statistical precision and systematic accuracy with this technique. An alternative lies in the $\beta$-\$ correlations of polarized nuclei (Curtis and Lewis, 1957; Morita and Morita, 1957), where the photon with momentum $\vec{k}$ emitted from the state populated by the $\beta$ decay. In this way one measures the correlation proportional to

$$E \vec{J} \cdot (\vec{p}_e \times \vec{k})(\vec{J} \cdot \vec{k}),$$  
(67)

when the emission is due to an $E1$ transition. The correlation coefficient $E \propto \text{Im} a_{LR}$ is nonzero only for mixed decays. Young et al. (1995) have identified $^{36}\text{K}$ as a promising candidate for such a measurement, since this isotope allows for the comparison between a mixed and a Gamow-Teller transition. The latter is insensitive to $T$ violation and can be used to test the experimental setup and reduce systematic errors. Secondary beams of high intensity can be produced, stopped, and polarized in a buffer gas allowing one to measure $\beta$-\$ correlations (Müller et al., 2013) with high precision. Correlations alternative to measuring $R$ are also possible [the $L$ and $M$ coefficients (Ebel and Feldman, 1957; Jackson, Treiman, and Wyld, 1957a)] but, similar to $R$, will always require one to measure the polarization of the $\beta$ particle, which is an inefficient process.

In radiative $\beta$ decay, it is possible to have triple-correlation coefficients without nuclear or electron spin (Braguta, Likhoded, and Chalov, 2002; Gardner and He, 2012, 2013), such as

$$K \vec{k} \cdot (\vec{p}_e \times \vec{p}_e).$$  
(68)

This coefficient has not been measured, but Dekens and Vos (2015) showed that EDMs provide extremely strong constraints on the coefficient $K$.

### B. EDM limits

Limits exist for the neutron EDM, the electron EDM, and several atomic EDMs. The best current bounds are listed in Table VI, where the limits from molecular YbF and ThO are expressed as a constraint on the electron EDM $d_e$. The last column of Table VI indicates if a connection to the triple-correlation coefficients $D$ and $R$ exists (Khriplovich, 1991; Ng and Tulin, 2012).

#### 1. Limits on $D$ from EDM limits

Any new vector interaction that contributes to $\text{Im} a_{LR}$ (and thus to $D$) also contributes to nuclear EDMs (Herczeg, 2005; Ng and Tulin, 2012). This makes it possible to translate bounds on the EDMs of the neutron and diamagnetic atoms into bounds on $\text{Im} a_{LR}$. The $D$ coefficient is $P$ even and $T$ odd, while the EDM is both $P$ and $T$ odd. Nevertheless, loop corrections, containing the $W$ boson, allow for a relation between these observables.

The relevant $CP$-odd dimension-6 operator is (Ng and Tulin, 2012)

$$\mathcal{L}^{(\text{eff})} = \frac{c}{\Lambda^2} \bar{u}_R \gamma^\mu d_R \bar{q} \gamma^\nu i D_\mu q + \text{H.c.},$$  
(69)

where $c$ is a complex coefficient, $\Lambda$ is the scale of new physics, $D_\mu$ is the covariant derivative, and $q$ is the Higgs doublet with $\bar{q} = c^{IJ} q^I$, where $c^{IJ}$ is the antisymmetric tensor. Figure 8 shows the energy evolution of this operator. First, electroweak symmetry breaking generates the coupling of the $W$ boson to right-handed quarks,

$$\mathcal{L}^{(\text{eff})} = \frac{g t^2}{2\sqrt{2}\Lambda^2} (\bar{c} u_R \gamma^\mu d_R W_\mu^+ + c^* \bar{d} R \gamma^\mu u_R W_\mu^-),$$  
(70)

where $\nu$ acquired its vacuum-expectation value $v/\sqrt{2}$ and $g$ is the $SU(2)_L$ coupling constant. The $W$ boson can couple to a lepton current or a quark current. At lower energy, the $W$ boson is integrated out. This generates a $P$- and $T$-odd four-quark coupling and the lepton-quark coupling $a_{LR}$ in $\beta$ decay. The effective Lagrangian is

$$\mathcal{L}^{(\text{eff})} = -\frac{c}{\Lambda^2} (\bar{u}_R \gamma^\mu d_R \bar{c} L_\mu \psi_L + V_{ud} \bar{u}_R \gamma^\mu d_R \bar{d} L_\mu \psi_L) + \text{H.c.},$$  
(71)

| Table VI. The current best EDM limits of the neutron, diamagnetic Hg, paramagnetic Tl, and molecular YbF and ThO. The neutron EDM and Hg can be connected to the $D$ coefficient (and $E$ coefficient). Other EDM measurements, except the neutron, can be connected to the $R$ coefficient. The limit from molecular YbF and ThO are expressed as a constraint on the electron EDM $d_e$. |
|-----------------|-----------------|-----------------|-----------------|
| EDM             | $e\,\text{cm}$  | Reference        | Connection to $\beta$ decay |
| $n$             | $2.9 \times 10^{-26}$ | Baker et al. (2006) | $D$               |
| $^{199}\text{Hg}$ | $2.6 \times 10^{-29}$ | Griffith et al. (2009) | $D, R$            |
| $^{205}\text{Tl}$ | $0.9 \times 10^{-24}$ | Regan et al. (2002) | $R$               |
| YbF             | $|d_e| < 10.5 \times 10^{-28}$ | Hudson et al. (2011) | $R$               |
| ThO             | $|d_e| < 8.7 \times 10^{-29}$ | Baron et al. (2014) | $R$               |
which shows that the two couplings $c$ and $a_{LR}$ have a common origin. They are related by

$$\text{Im} a_{LR} = \frac{\text{Im} c}{2 \sqrt{2} G_F \Lambda^2}. \quad (72)$$

When evolving to the QCD scale, the second term in Eq. (71) is affected by QCD renormalization. However, this has only a small numerical effect (Dekens and de Vries, 2013), which can be neglected given the uncertainties coming from the calculation of the neutron EDM.

Bounds on $\text{Im} c$ thus lead to an upper limit on $\text{Im} a_{LR}$. The dependence of the EDM on $\text{Im} c$ involves theoretical calculations at different energy scales. Especially for diamagnetic atoms such as $^{199}$Hg, differences in nuclear calculations lead to a large uncertainty in the interpretation of the bounds on atomic EDMs. Therefore, we do not consider bounds from $^{199}$Hg. No such problem occurs for the neutron, and de Vries et al. (2013) and Seng et al. (2014) estimated the link between the neutron EDM and $\text{Im} c$ as

$$d_n = -1 \times 10^{-20} \frac{\text{Im} c}{2 \sqrt{2} G_F \Lambda^2} e \text{ cm}. \quad (73)$$

This result differs by an order of magnitude from the result used by Ng and Tulin (2012), which was obtained from He and McKellar (1993) and An, Ji, and Xu (2010). de Vries et al. (2013) and Seng et al. (2014) pointed out that, due to the use of a relativistic meson-nucleon field theory, He and McKellar (1993) and An, Ji, and Xu (2010) overestimated the neutron EDM by an order of magnitude.

The current bound on the neutron EDM $|d_n| < 2.9 \times 10^{-26} e \text{ cm}$ (Baker et al., 2006) and Eq. (73) gives at 90\% C.L.

$$|\text{Im} a_{LR}| < 3 \times 10^{-6}. \quad (74)$$

This bound is at least 2 orders of magnitude below the bound obtained from $\beta$ decay. Improving this bound in $\beta$ decay requires a measurement of $D_t < 10^{-6}$, which is an order of magnitude below the contribution of the FSI.

The result above is obtained in a model-independent EFT approach, by introducing dimension-6 operators. The constraints apply to left-right symmetric models, exotic fermion models, and the $R$-parity violating minimal supersymmetric standard model (MSSM) (Ng and Tulin, 2012). Evasion of the bounds in Eq. (74) is possible only in either a strongly fine-tuned model or in a model in which the dimension-6 operators do not exist or do not contribute to either EDMs or $\beta$ decay. An example of the latter is leptoquarks (LQs). LQs are particles with both baryon and lepton numbers, which can be either vector or scalar particles depending on their spin. These were previously considered “EDM safe,” but in fact they are not (Ng and Tulin, 2012). LQs can contribute to $\beta$ decay at tree level, for example, via the exchange of scalar LQs as depicted in Fig. 9(a). Leptoquarks also contribute to EDMs, but only the $W$ exchange [Fig. 9(b)]. Ng and Tulin (2012) showed that these loop contributions are not suppressed by the light-quark masses $m_{u,d}^2$, as was previously argued (Herczeg, 2001). Therefore, the constraints from EDMs in the LQ scenario are much more stringent than previously thought.

Estimates of the limit on $D_t$ in this scenario depend on the LQ mass and on whether light right-handed neutrinos exist.
Assuming the existence of light right-handed neutrinos, Ng and Tulin (2012) found

$$\text{Im} a_{LR} = D_t/a_D < 3 \times 10^{-4} \left( \frac{300 \text{ GeV}}{m_{LQ}} \right)^2,$$

while without them

$$\text{Im} a_{LR} = D_t/a_D < 7 \times 10^{-5} \left( \frac{300 \text{ GeV}}{m_{LQ}} \right)^2.$$

Ng and Tulin (2012) conservatively took $m_{LQ} = 300$ GeV, which would give, assuming the existence of light right-handed neutrinos, $D_t < 3 \times 10^{-4}$, a limit of the same order as the current $\beta$-decay bounds. Nevertheless, improving the current $\beta$-decay limit seems a difficult task, since there are many experiments ongoing or planned that aim to improve the bounds on the neutron EDM (Ito, 2007; Altarev et al., 2014), which suggests much stronger bounds on $D_t$.

2. Limits on $R$ from EDM limits

The $R$ coefficient and the EDM are both $P$ and $T$ odd. EDM measurements in atoms and molecules limit both the electron EDM and BSM scalar and tensor electron-nucleon interactions. Kriplovich (1991) showed the relation between these electron-nucleon interactions and the electron-quark interaction of $\beta$ decay. The scalar and tensor electron-nucleon interactions are defined by

$$\mathcal{L} = \sum_{N} \frac{G_f}{\sqrt{2}} [C_S \bar{N}N \bar{e}i\gamma_5 e + C_T \bar{N}\sigma_{\mu\nu}N \bar{e}i\gamma_5 \sigma^{\mu\nu} e].$$

where $C_S$ ($C_T$) is the scalar (tensor) coupling and we have neglected pseudoscalar couplings. Kriplovich (1991) and Kriplovich and Lamoreaux (1997) showed that the limits on $C_S$ and $C_T$ can be related to both $\text{Im} A_L$ and $\text{Im} a_{\ell L}$, the couplings contributing to the $R$ coefficient.

The best current limit on nucleon scalar couplings is due to the EDM limit on molecular ThO. $|C_S| < 5.9 \times 10^{-9}$ (90% C.L.) (Baron et al., 2014). The best bound on the nucleon tensor coupling $|C_T| < 1.3 \times 10^{-9}$ (90% C.L.) is derived from the EDM limit on atomic Hg (Ginges and Flambaum, 2004; Griffith et al., 2009). These couplings must be translated to quark couplings in order to compare them to the $\beta$-decay couplings in Eq. (11). At the quark level, scalar and tensor couplings in the electron-quark ($e-q$) interaction are described by (Herczeg, 2003)

$$\mathcal{L} = \frac{\mu}{\sqrt{2}} \ln \left( \frac{M_W^2}{\mu^2} \right) V_{ud} \text{Im}(2A_L + 24a_L) \times \left[ e^\ast q \slashed{u} + \frac{1}{2} e^\ast q \sigma_{\mu\nu} \slashed{e} u^{\ast} \sigma^{\mu\nu} \right],$$

where $\mu$ is the renormalization scale. Limits on the scalar electron-nucleon interaction $C_S$ thus limit both $A_L$ and $a_L$. The effective $e-d$ interaction contains only $A_L$ and gives similar constraints.

Comparing Eqs. (78) and (79) we arrive at an expression for $k_{Sd}$ and $k_{Td}$. By using $k_{Sd} < 10^{-8}$ (90% C.L.) and the conservative assumption that $\ln(\mu^2/m_W^2) = 1$ as in Kriplovich (1991), we estimate that at 90% C.L.

$$|\text{Im} A_L| < 10^{-5},$$

$$|\text{Im} a_{\ell L}| < 10^{-6}.$$
Both bounds are at least 2 orders of magnitude better than those obtained from the $R$ coefficient in $\beta$ decay.

C. Conclusion

Table VII summarizes the limits on imaginary couplings. Bounds obtained from EDMs are several orders of magnitude better than current bounds from $T$-violating $\beta$-decay coefficients. The many ongoing efforts in the EDM field will strengthen the EDM bounds even further.

The $D$ coefficient should be measured with a precision of $10^{-6}$ to improve the current EDM limits. Such a measurement is below the FSI interactions and would require precise knowledge of the FSI for the used isotope. Measurements of the $D$ coefficient are considered as part of a larger effort to measure 11 coefficients ($R_j$) in neutron decay (Bodek et al., 2011). Measurements of $D$ are also considered in nuclear decays (Behr et al., 2014; Liénard, 2014). The $E$ coefficient in Eq. (67) depends on the same BSM coupling as the $D$ coefficient and is thus subject to the same EDM constraints.

It might be possible that the connection between EDMs and $\beta$ decay is diminished in a specific new-physics model, when such a model is strongly fine-tuned. For the $D$ coefficient examples are leptoquark models. Conservatively, this model is strongly fine-tuned. For the $\beta$-decay limit of $\lambda_{i\mu} > 3 \times 10^{-4}$ (Ng and Tulin, 2012).

VI. Lorentz Violation

We now review the new field of searches for the violation of Lorentz symmetry in the weak interaction. Recently, it was found that $\beta$ decay offers unique possibilities to test Lorentz and/or CPT invariance in the weak interaction, in both the gauge and neutrino sectors. We discuss these two sectors separately.

A. Gauge sector

In the gauge sector, Lorentz violation can be studied in a general theoretical framework, developed to study allowed and forbidden $\beta$ decay and orbital electron capture (Noordmans, Wilschut, and Timmermans, 2013a, 2013b; Vos, Wilschut, and Timmermans, 2015b). This framework considers a broad class of Lorentz-violating effects on the $W$ boson, by adding a general tensor $g^{\mu\nu}$ to the Minkowski metric. At low energies, this modifies the $W$-boson propagator to

$$\langle W^\mu W^{\nu}\rangle = \frac{-i(g^{\mu\nu} + \chi^{\mu\nu})}{M_W^2},$$

where $g^{\mu\nu}$ is the Minkowski metric and $M_W$ is the $W$-boson mass. Vertex corrections are described by

$$-i\Gamma = -ig\left(g^{\mu\nu} + \chi^{\mu\nu}\right).$$

However, such vertex modifications also require the modification of the electron and neutrino spinors (Noordmans, Wilschut, and Timmermans, 2013b). We restrict ourselves to propagator corrections, for which Hermiticity of the Lagrangian implies that $\chi_{\mu\nu}(p) = \chi_{\nu\mu}(-p)$. In terms of the SME discussed in Sec. II.B, one finds, at lowest order,
where $q$ is the momentum of the W boson and $g$ is the SU(2) coupling constant.

Bounds on $\gamma$ have been derived from allowed (Bodek et al., 2014; Müller et al., 2013; Wilschut et al., 2013) and forbidden $\beta$ decay (Noordmans, Wilschut, and Timmermans, 2013a), pion decay (Altschul, 2013; Noordmans and Vos, 2014), muon decay (Noordmans et al., 2015), and nonleptonic kaon decay (Vos et al., 2014). Here we discuss allowed and forbidden $\beta$ decay.

1. Allowed $\beta$ decay

For allowed $\beta$ decay, Noordmans, Wilschut, and Timmermans (2013b) derived the Lorentz-violating differential decay rate using the modified W-boson propagator in Eq. (81). The complete expression is given in Eq. (B1). Lorentz violation gives many additional correlations, since the observables (momentum and spin) can now also couple to the tensor $\chi$. In $\beta$ decay, a variety of correlations can be used to access different (combinations of) $\chi$ components. The necessary expressions can be derived by integrating over one or more kinematic variables. Momentum-dependent terms are always suppressed by some power of a heavy mass ($M_W$ in the least-suppressed case) and can therefore be neglected given the current experimental precision. Neglecting momentum-dependent contributions to the propagator, the relation $\chi^{\mu \nu}(p) = \chi^{\nu \mu}(-p)$ implies that $\chi$ can only be real and symmetric or imaginary and antisymmetric, i.e., $\chi^{0 \mu} = \chi^{\mu 0}$, $\chi^{i \mu} = -\chi^{\mu i}$, $\chi^{00} = 0$, $\chi^{ij} = \chi^{ji}$, and $\chi^{ik} = -\chi^{ki}$. The subscripts $r$ and $s$ denote the real and imaginary parts of $\chi$, respectively. This leaves 15 independent CPT-even components of $\chi^{\mu \nu}$ that need to be measured.

With this simplification and in the absence of tensor polarizations, the decay rate is (Noordmans, Wilschut, and Timmermans, 2013b; Vos, Wilschut, and Timmermans, 2015a)

$$dW = \frac{F(\pm Z, E_e)}{(2\pi)^5} |\tilde{p}_e| E_e (E_e - E_0)^2 dE_e d\Omega_ee d\Omega_ee \tilde{\tilde{\xi}} \left[ 1 + (2a - c')\chi_{r0}^{00} + (2a - c')\chi_{r0}^{00} + 2i\tilde{\gamma}_r^{00} \frac{p_i^e}{E_e} \right]$$

where $\langle \tilde{J} \rangle$ is the expectation value of the spin of the parent nucleus $\tilde{J} = e^{i m k \tilde{\alpha}}$, and Latin indices run over spatial directions. The last line of Eq. (84) contains only the neutrino momentum or the neutrino momentum and the nucleon polarization, and can therefore mostly be ignored. In fact, the neutrino correlations give access to a similar combination of $\chi$ components as the electron correlations. The latter are considerably easier to obtain, and we further consider only the electron correlations. The coefficients $\tilde{\xi}$, $\tilde{\alpha}$, $A$, and $B$ are the standard $\beta$-decay coefficients listed in Appendix A, and the coefficient $c'$ is a modified $c$ coefficient. The coefficients with a breve (') multiply Lorentz-violating coefficients. They are given by

$$c' = \frac{\rho^2}{1 + \rho^2}, \quad \tilde{\xi} = \frac{\frac{\rho^2}{1 + \rho^2} - 2}{2} c',$$

$$\tilde{\alpha} = \frac{\frac{\rho^2}{1 + \rho^2} + 1}{2} c',$$

where the upper (lower) sign refers to $\beta^+$ decay, and $\lambda_{JF}$ and

$$\Lambda_{JF} = \langle \tilde{J} \cdot \tilde{\tilde{J}} \rangle - \frac{1}{4} J(J + 1)
$$

are the standard $\beta$-decay coefficients given in Eqs. (A6) and (A7), respectively. The coefficient $c'$ vanishes for nonoriented nuclei and for nuclei with $J' = J = 1/2$.

The effect of Lorentz violation in $\beta$ decay can already be studied by measuring the dependence of the decay rate as a function of the direction of the emitted $\beta$ particles. The modified Fermi decay rate integrated over neutrino energy and direction and summed over electron spin is

$$dW_F = dW_0 \left( 1 + 2\chi_{r0}^{00} - 2\chi_{r0}^{00} \frac{p_i^e}{E_e} \right),$$

while for Gamow-Teller transitions of randomly oriented nuclei

$$dW_{GT} = dW_0 \left( 1 - \frac{2}{3} \chi_{r0}^{00} + \frac{2}{3} \gamma_{r0}^{00} + \frac{4}{3} \gamma_{r0}^{ij} \frac{p_i^e}{E_e} \right),$$

where

$$dW_0 = \frac{1}{8\pi^3} p_i e_e (E_0 - E_e)^2 F(\pm Z, E_e) dE_e d\Omega_ee.$$
The component $\tilde{\chi}_i$ can also be accessed by measuring the Gamow-Teller decays of polarized nuclei as a function of the spin direction,

$$dW_{\text{GT}} = dW_0 \left( 1 - \frac{2}{3} \sqrt{\chi^0} + \lambda_{\text{GT}} \frac{\langle J^i \rangle}{J} \right).$$

As an example of a mixed decay, one has for the neutron $a = -0.11$, $A = -0.12$, $B = 0.98$, and $\tilde{Y} = L = \chi^2 / (1 + 3\lambda^2) = 0.27$. Integrated over the neutrino direction\(^6\)

$$dW = dW_0 \left[ 1 - 0.21\chi^0(0.21\chi^0 + 0.55\chi^1) \frac{p^i_{\tilde{e}}}{{E_e}} + \left( \frac{\langle J^i \rangle}{J} \right) \right]$$

$$- 0.12\chi^0 \left( \frac{\tilde{p}\cdot \langle \tilde{J} \rangle \times (\tilde{p}\times \tilde{J})}{JE_e} \right).$$

Equation (84) depends on SM parameters, which are
dependence on SM coefficients can be avoided by measuring asymmetries that do not depend on the accuracy of the SM
corrects Eq. (38) in Noordmans, Wilschut, and

$$A_{\beta} = \frac{W^1_L W^1_R - W^2_L W^2_R}{W^1_L W^1_R + W^2_L W^2_R} = 2P\beta(A\chi^0 e^{-ik} + B\chi^1 k)\tilde{p}\cdot j,$$

where $W_{LR}$ is obtained by measuring the $\beta$ particles in the opposite $\tilde{p}$ direction, while the nuclei are polarized in the

$$\chi^enu \equiv R^\mu R^\nu X^\nu.$$

The transformation matrix is

$$R(\zeta, \Omega) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \zeta \cos \Omega & \cos \zeta \sin \Omega & -\sin \zeta \\
0 & -\sin \Omega & \cos \Omega & 0 \\
0 & \sin \zeta \cos \Omega & \sin \zeta \sin \Omega & \cos \zeta
\end{pmatrix},$$

where $\zeta$ is the colatitude of the experiment and $\Omega$ is Earth’s sidereal rotation frequency. In the laboratory frame, $\hat{x}$ points in the north to south direction, $\hat{y}$ points west to east, and $\hat{z}$ is perpendicular to the Earth’s surface. The coefficients $\chi^0$ and $\chi^1$ can be transformed to $X^\nu$ and $X^\mu$, respectively. This transformation shows that the asymmetries $A_{\beta}$, $A_{\chi}^0$, and $A_{\chi}^1$ can oscillate with the rotational frequency of the Earth. These sidereal variations of the signal are a unique signature of Lorentz violation and can therefore be separated from other limits on BSM physics. A generic example of how sidereal oscillations can be observed is shown in Fig. 11, for $X^0_L = 0.1$. This example also shows that if the $\beta$ particles are detected parallel ($\parallel$) to the Earth’s rotation axis, the asymmetry will have no sidereal dependence (thick line). The top curve shows the case where the $\tilde{p}$ particles are detected in the east-west ($\perp$) direction. It has no offset because it is measured perpendicular to the Earth’s rotation axis. The black line gives the asymmetry for $\tilde{p}$ particles detected in the up-down ($\parallel \perp$) direction perpendicular to the Earth’s surface ($\perp$ direction in the lab frame). It shows a sidereal oscillation on a constant offset. Detection of the $\tilde{p}$ particles perpendicular to the rotation axis is preferred, since an offset could be the result of systematic errors in the measurement.

Tensor contributions involving $\chi^{ik}$ lead to terms that may oscillate with twice the Earth’s rotational frequency. Figure 12 illustrates three possible scenarios for an asymmetry that depends on $\chi^{ik} j^i \tilde{p}^k$. Line (1) shows the modulations when the polarization is in the up-down direction, while the $\beta$ particles are detected in the west-east direction. Line (2) shows the modulations in the same polarization direction, but when the $\beta$

\(^{6}\)This formula corrects Eq. (38) in Noordmans, Wilschut, and Timmermans (2013b) (see also Appendix B).
measure the lifetime variation, the latter with a constant offset. Line (2) shows the asymmetry when the fast particles are detected perpendicular to the Earth’s rotation axis. The top curve shows the asymmetry when the fast particles are detected in the east-west direction ($\hat{\mathbf{z}}$) and the black line shows when they are detected perpendicular to the Earth’s surface ($\hat{\mathbf{j}} \perp \hat{\mathbf{s}}$). Both show a sidereal variation, the latter with a constant offset.

In allowed $\beta$ decay, Lorentz violation was for the first time tested in polarized $^{20}$Na (Müller et al., 2013), by measuring the spin asymmetry $A_{\gamma}$ [Eq. (93)]. $^{20}$Na first decays with a $\beta^{+}$ transition $2^+ \to 2^+$ Gamow-Teller transition, followed by a $\gamma$ decay of the daughter nucleus. The parity-odd $\beta$ decay was used to determine the polarization $P$ by measuring the $\beta$ asymmetry (Müller et al., 2013). The parity-even $\gamma$ decay was used to measure the lifetime $\tau_{\gamma}^{1(\bar{1})}$ and to determine the $\gamma$ asymmetry $A_{\gamma}$ [Eq. (94)]. Such a measurement would measure the so-far unconstrained coefficients $\chi_{i}^{0}$.

2. Forbidden $\beta$ decay

"Forbidden" (slow) transitions are suppressed with respect to allowed transitions, because the lepton pair carries away angular momentum. Theoretically, the simplest of these transitions are the unique first-forbidden transitions ($\Delta J = 2$), since they depend on only one nuclear matrix element. Because Lorentz violation includes rotational violation, it also implies the violation of angular-momentum conservation. Forbidden $\beta$ decays are then more sensitive to rotational invariance violation in the weak interaction. In the 1970s, two experiments were performed with this motivation. Newman and Wiesner (1976) searched for anisotropies in the angular distribution of $\beta$ particles in first-forbidden $^{90}$Y decay. Ullman (1978) searched for sidereal modulations of the count rates for first-forbidden $^{137}$Cs $\beta$ decay and second-forbidden $^{99}$Tc $\beta$ decay. The strongest bounds were found in the experiment by Newman and Wiesner (1976). In this experiment the $\beta$-decay distribution of $^{90}$Y from a high-intensity source was measured in a rotating setup. Schematically, the setup is depicted in Fig. 13. The rotation of the setup allowed for the determination of three decay asymmetries.
\[ \delta_{NS} = 2 \frac{W_N - W_S}{W_N + W_S}, \quad \delta_{EW} = 2 \frac{W_E - W_W}{W_E + W_W}, \]  
\[ \delta_{2s} = 2 \frac{W_N + W_S - W_E - W_W}{W_N + W_S + W_E + W_W}, \]  
\[ \delta = a_0 + a_1 \sin(\Omega t + \phi_1) + a_2 \sin(2\Omega t + \phi_2) \]  
\[ \chi^\mu_r = \begin{pmatrix} 10^{-6} & 10^{-7} & 10^{-7} & 10^{-8} \\ 10^{-7} & 10^{-6} & 10^{-6} & 10^{-6} \\ 10^{-7} & 10^{-6} & 10^{-6} & 10^{-6} \\ 10^{-8} & 10^{-6} & 10^{-6} & 10^{-6} \end{pmatrix} \]  
\[ \chi^\mu_i = \begin{pmatrix} \times & \times & \times & \times \\ - \times & 10^{-8} & 10^{-7} & \\ - & 10^{-8} & \times & \\ -10^{-7} & 10^{-7} & \times \end{pmatrix}. \]  

These are the strongest constraints on \( \chi^\mu_r \). The only coefficients not constrained by forbidden decays are \( \chi^0_r \). These coefficients can be studied in allowed \( \beta \) decay by considering Eq. (94) or equivalent correlations. The bounds on \( \chi \) were also translated into bounds on the SME parameters (Noordmans, Wilschut, and Timmermans, 2013a), providing strong direct bounds on the SME parameters \( k_{\phi \phi} \) and \( k_{\phi \mu} \) defined in Eq. (16).

3. Conclusion and outlook

We discussed the efforts to search for Lorentz violation in the weak interaction in forbidden and allowed \( \beta \) decays. The bounds from forbidden \( \beta \) decay are several orders of magnitude stronger than the current bounds in allowed \( \beta \) decay, due to the intense sources that were used (Newman and Wiesner, 1976; Ullman, 1978). In allowed \( \beta \) decay, Lorentz-violating effects are not enhanced and matching the statistical precision of forbidden \( \beta \) decay experiments would require long-running experiments with high-intensity sources. An interesting alternative lies in orbital electron capture, where it is possible to use such high-intensity sources (Vos, Wilschut, and Timmermans, 2015b).

Allowed \( \beta \) decay offers various correlations in which Lorentz violation could be probed. Observables can be chosen such that they give direct constraints on \( \chi \) compared to the combination of coefficients constrained by forbidden decays. Two relatively simple experiments that probe the \( \beta \) asymmetry in Fermi and Gamow-Teller decays, \( A_\beta \) and \( A_{\text{GT}} \), respectively, give direct bounds on \( \chi^{00}_r \) and \( \chi^i_r \). These asymmetries could be studied parallel to the efforts to measure the \( \beta \)-spectrum shape discussed in Sec. IV.D (Vos, Wilschut, and Timmermans, 2015a). Another interesting possibility is to exploit the \( \gamma^2 \) enhancement of decay asymmetries by considering fast-moving nuclei (Altschul, 2013; Vos et al., 2014; Vos, Wilschut, and Timmermans, 2015a). The total decay rate in the rest frame of the nucleus is proportional to \( \chi^{00}_r \) [see Eqs. (86) and (87)]. For a fast-moving nucleus, the expression can be related to the Sun-centered frame with a boost. If the nucleus is moving ultrarelativistically in the \( \hat{v} \) direction,  
\[ \chi^{00}_r = \gamma_s^2 (X^{TT}_r + 2X^{TL}_r p^L + X^{LK}_r p^L p^K), \]  
where \( \gamma_s \) is the Lorentz-boost factor and \( T, L, \) and \( K \) are coordinates in the Sun-centered reference frame. This relation was, for example, used to extract bounds of \( O(10^{-4}) \) from pion decay (Altschul, 2013). For allowed \( \beta \) decay, \( \beta \)-beam...
facilities, currently considered for producing neutrino beams (Lindroos and Mezzetto, 2010), could be exploited.

So far the coefficients $\chi^\nu_{ij}$ remain unconstrained. In Fermi decays, this coefficient can be constrained by measuring the correlation $\chi^\nu_{ij} (\hat{p}_v \times \hat{p}_e)$. The coefficients can also be constrained by measuring the polarized $\beta$ asymmetry $A_{Jp}$ in Eq. (94). Such an asymmetry could probably be explored in the neutron-decay measurement pursued by Bodek et al. (2014).

B. Neutrino sector

A different possibility to study Lorentz violation in $\beta$ decay lies in the neutrino sector of the SME (Kostelecký and Mewes, 2004, 2012). Most interesting for $\beta$ decay are the modified versions of $a^\nu$ and $c^\nu$ defined in Eq. (15).

Unlike the gauge sector, the neutrino sector has been studied extensively in several experiments. Strong bounds exist from neutrino oscillations and time-of-flight measurements (Kostelecký and Russell, 2011). However, there are four operators that do not show up in oscillations and have no effect on the neutrino group velocity. These operators are called “countershaded” (Kostelecký and Tasson, 2009).

Recently, Díaz, Kostelecký, and Lehnert (2013) showed that $\beta$ decay has a unique sensitivity to these operators. The four countershaded coefficients are denoted by $a_{ij}^{(3)}$. The operators are dimension 3 and CPT odd. These coefficients modify the neutrino dispersion relation and the available phase space of the neutrino, which affects $\beta$ decay in two ways, in the $\beta$ end point and in the correlation coefficients.

1. End point in $\beta$ decay

The $\beta$-spectrum end point is very sensitive to the neutrino phase space and to the neutrino mass (see also Sec. IV.C). Independent of the neutrino mass, the countershaded neutrino coefficients also shift the end point, as can be seen from the modified decay rate (Díaz, Kostelecký, and Lehnert, 2013; Díaz, 2014)

$$\frac{dW}{dT} \sim (\Delta T + \delta T_{LV})^2 - \frac{1}{2} m_e^2,$$  \hspace{1cm} (104)

where $\Delta T = T_0 - T_e$, $T_e = E_e - m_e$ is the electron kinetic energy, and $T_0$ is the end-point energy for $m_e = 0$. $\delta T_{LV}$ is the Lorentz-violating modification, which depends on sidereal time. Independent of the neutrino mass, a bound on the countershaded coefficients can be set by using the available data of the Troitsk (Kraus et al., 2005) and Mainz (Aseev et al., 2011) experiments; see Díaz, Kostelecký, and Lehnert (2013). Since these experiments collected data over a long period of time, all the oscillations average out and only the time-averaged Lorentz-violating coefficients can be constrained. Therefore, only two of the four countershaded coefficients could be bounded. Conservatively, the analysis of Díaz, Kostelecký, and Lehnert (2013) gives bounds of order $\mathcal{O}(10^{-8})$ GeV. These limits improve and complement previous limits. A dedicated analysis of the data of the Troitsk, Mainz, or the expected KATRIN (Otten and Weinheimer, 2008) experiments could improve these results. If the data analysis also takes into account the sidereal time, bounds on all the countershaded coefficients could be set.

2. Correlation coefficients

The Lorentz-violating neutrino coefficients of Eq. (15) also modify the neutrino spinor solutions. Near the end point, this modification can be neglected because the phase space dominates. However, the derivation of the complete modified decay rate requires both the modified spinors and the phase-space modification. The modified neutrino phase space is $d^3\tilde{p}_v = (E_v^2 - 2E_v a_{ij}^{(3)}) dE_v d\Omega_v$. The modification of the spinors requires the replacement of $\tilde{p}_v$ by $\tilde{p}_v = (\tilde{p}_v + a_{ij}^{(3)} \tilde{p}_e)$, where $a_{ij}^{(3)}$ is the isotropic component. The modified neutrino-decay rate is

$$\frac{dW}{d\Omega_v d\Omega_e dT} = F(Z, E) |\tilde{p}_e| E_v (E_v^2 + 2E_v \delta T_{LV})$$
$$\times \left( 1 + a\tilde{p}_v + \frac{A}{J} \frac{\tilde{p}_e}{E_e} + B \frac{\tilde{J}}{J} \frac{\tilde{p}_e}{E_e} \right).$$ \hspace{1cm} (105)

The neutrino coefficients modify the decay rate in a similar way as $\chi$ does, since there are now additional correlations between $\tilde{J}$ and $\tilde{p}_e$ and $a_{ij}^{(3)}$.

The countershaded coefficients could, for example, affect the $\beta$-$\nu$ correlation. The $\beta$-$\nu$ correlation can be measured as an asymmetry, defined by

$$\tilde{a} = \frac{N_+ - N_-}{N_+ + N_-},$$ \hspace{1cm} (106)

where $N_+$ ($N_-$) is the number of decays in which the neutrino and electron are emitted (anti)parallel. The Lorentz-violating neutrino coefficients modify this correlation coefficient to (Díaz, 2014)

$$\tilde{a} = a |\tilde{p}| + \sqrt{\frac{3}{\pi}} (a^2 + a |\tilde{p}|) (a_{ij}^{(3)})_{10},$$ \hspace{1cm} (107)

where the coefficients should be transformed to the Sun-centered frame and would depend on the sidereal frequency of the Earth.

No experiment has searched for these variations, but Díaz, Kostelecký, and Lehnert (2013) estimated that a 0.1% measurement of $a$ would limit the countershaded coefficients at the level of $10^{-8}$ GeV. Similar, for a 0.1% measurement of the correlation coefficient $B$, the limits are estimated at $\mathcal{O}(10^{-6})$ GeV. A dedicated experiment measuring either $a$ or $B$ would thus provide interesting new bounds on Lorentz-violating parameters in the neutrino sector. Note that $\chi$ and $a_{ij}^{(3)}$ have a similar influence on the decay rate. In a dedicated experiment both coefficients might influence the asymmetry. A measurement of Eq. (106) might also be sensitive to $\chi^\nu_{ij}$ and $\tilde{\chi}$, depending on the experimental setup.

C. Conclusion

To summarize, $\beta$ decay offers a unique way to study Lorentz violation in both the gauge and neutrino sectors. The large variety of correlations allows for direct measurements of...
different components of $\chi$, while in the neutrino sector $\beta$ decay allows for the study of countercharged coefficients.

In the gauge sector, strong bounds on the order of $10^{-6}$–$10^{-8}$ exist from forbidden $\beta$-decay experiments. Unconstrained are the coefficients $\chi^{i0}$, which can be accessed in $\beta$ decay by considering the interaction of $\chi$ with two observables [Eq. (94)]. Improving the existing bounds requires high statistics and precise knowledge of the system-observables [Eq. (94)]. Improving the existing bounds requires high statistics and precise knowledge of the systematic uncertainties. Beneficial for this would be to exploit the $\gamma^2_\chi$ enhancement of boosted $\beta$ decay or to consider electron capture. The real and imaginary parts of $\chi$ can be constrained by measuring the asymmetries in Eqs. (91) and (92), respectively. Such an effort could be combined with measurements of the Fierz-interference term.

Further, we discussed the possibilities to improve constraints on Lorentz violation in the countercharged neutrino sector. In that sector no dedicated experiment has been performed so far, but using available data from tritium gives bounds of the order of $10^{-8}$ GeV. The parameters not constrained so far could be bound in $\beta$-decay correlation experiments. Lorentz violation gives a unique signal compared to other BSM physics when searched for in a dedicated experiment. Estimates for 0.1% measurements of the coefficients $a$ and $B$ gives a constraint on Lorentz violation of $10^{-8}$ GeV, which shows the potential for these future experiments.

VII. SUMMARY AND DISCUSSION

In this review we addressed the current status and role of nuclear and neutron $\beta$ decay in the search for physics beyond the SM. In these searches, the statistical precision is becoming increasingly important. However, systematic errors, despite improved detection methods, and higher-order corrections such as FSI, still appear to be the main limits. In the meantime, thanks to the evolution of EFT methods, constraints obtained in other fields weigh in, establishing bounds on the scalar and tensor contributions. This is illustrated in Figs. 6 and 7, where measurements at the LHC (Sec. IV.B) and limits from the neutrino mass (Sec. IV.C) give constraints that outperformed the $\beta$-correlation measurements in the right-handed sector. This is quantified in Table IV.

The study of fundamental aspects of $\beta$ decay will be most fruitful in the study of left-handed scalar (Sec. IV.A.1) and tensor currents (Sec. IV.A.2), as these appear linearly in most observables via the Fierz-interference term. Fortunately, these interactions can be studied in parallel to precision studies of SM parameters (Sec. III.B). For example, extracting the CKM matrix element $V_{ud}$ from superallowed Fermi transitions has, as a by-product, the most strict limit on left-handed scalar interactions. Lacking still is a similar limit on tensor contributions. An interesting option to obtain such a bound could come from measuring the detailed shape of the $\beta$ spectrum in Gamow-Teller transitions. Also the potential of mirror transitions, both for obtaining tensor limits and for obtaining a value for $V_{ud}$ independent of the superallowed Fermi transitions, has been recognized. In Table V we indicate the precision required to impose new bounds on left- and right-handed scalar and tensor currents. Measuring the Fierz-interference term in $\beta$ decay remains competitive in determining bounds on left-handed coupling constants. In contrast, Table V shows that, for right-handed couplings, the limits from the LHC and the limits derived from the neutrino mass are by far superior to the best bounds derived from the $\beta$-correlation $a$, and future experiments in $\beta$ decay are unlikely to reach this precision.

Concerning the most fundamental measurement of $T$ violation, we discussed in Sec. V the strong bounds on the triple-correlation coefficients $D$ and $R$ derived from the limits on permanent EDMs. These bounds are summarized in Table VII. Not only are the bounds from EDMs several orders of magnitude stronger than those of $\beta$ decay, but the EDM limits also have a large potential to improve faster than those from $\beta$ decay. One reason is that EDMs can be measured in stable or long-lived particles, but also because of the widely perceived urgency for improved limits in this sector.

A new twist to the discussion of symmetry violations in $\beta$ decay has been added, since $\beta$ decay also offers an interesting sensitivity to Lorentz violation in the weak interaction. In Sec. VI, we reviewed these limits for the first time. Because the discrete symmetries $C$, $P$, and $T$ are each violated in the weak interaction, this interaction is a promising portal to search for new physics when considering $CPT$ violation and thus Lorentz violation. The familiar $\beta$-decay correlations are now extended to include correlations between spin and momentum and a Lorentz-violating background tensor. Consequently, spin and momentum appear to have preferred directions in absolute space, resulting in unique signals that can be distinguished from other BSM searches.

In weak decays Lorentz violation has been parametrized with the complex tensor $\chi$. The bounds on most components of this tensor are of the order of $10^{-6}$ to $10^{-8}$ (Sec. VI.A.2). Fine-tuning between the tensor components allows one to weaken these bounds. Relatively simple new experiments can improve these bounds using very strong sources, also removing the possibility of fine-tuning. Obtaining sufficient high counting statistics is the main challenge. The searches for Lorentz violation can be expanded in a parallel effort with the more traditional searches. Alternatively, one can study $\beta$ decay in flight, exploiting the $\gamma^2_\chi$ enhancement. In this respect there may be as yet unexplored possibilities related to semileptonic decays in high-energy physics. Because this field of research is relatively unexplored, both experimentally and theoretically, the best approach may still emerge.

Improvements in theory and experimental techniques, as well as new radioactive-beam facilities, provide new possibilities to study fundamental aspects of $\beta$ decay, both in the search for exotic interactions and in the search for Lorentz violation. These studies should be done by considering also the other searches in high-energy physics and precision physics at low energies. Nuclear and neutron $\beta$ decay will remain an important topic on the research agenda.

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APPENDIX A: DECAY COEFFICIENTS

Our formalism can be linked to the original work of Lee and Yang (1956) and Jackson, Treiman, and Wyld (1957a), where

\[ \mathcal{L}_{\text{eff}} = \bar{p} n \bar{\xi} (C - C_5) \xi \]

\[ + \bar{p} \gamma^\mu n \bar{\xi} \gamma_\mu (C - C_5) \xi \]

\[ + \bar{p} \gamma^\mu [n \bar{\xi} \gamma_\mu (C - C_5) \xi] \]

\[ + \frac{1}{2} \bar{p} \bar{\sigma}^{\mu\nu} n \bar{\xi} \sigma_{\mu\nu} (C - C_5) \xi. \]  

(A1)

This notation can be related to our couplings in Eq. (11) by using the normalized couplings

\[ C_i = \frac{G_F}{\sqrt{2}} V_{ud} C_i. \]  

(A2)

and

\[ \tilde{C}_V = g_V (a_L + a_R), \]

\[ \tilde{C}_V' = g_V (a_L - a_R), \]

\[ \tilde{C}_A = -g_A (a_L' + a_R'), \]

\[ \tilde{C}_A' = -g_A (a_L' - a_R'), \]

\[ \tilde{C}_S = g_S (A_L + A_R), \]

\[ \tilde{C}_S' = g_S (A_L - A_R), \]

\[ \tilde{C}_T = 2 g_T (a_L + a_R), \]

\[ \tilde{C}_T' = 2 g_T (a_L - a_R). \]  

(A3)

For simplicity we have defined

\[ a_L \equiv a_{LL} + a_{LR}, \]

\[ a_R \equiv a_{RR} + a_{RL}, \]

\[ a_L' \equiv a_{LL} - a_{LR}, \]

\[ a_R' \equiv a_{RR} - a_{RL}, \]

\[ A_L \equiv A_{LR} + A_{LL}, \]

\[ A_R \equiv A_{RL} + A_{RR}. \]  

(A4)

We write \( \alpha_L \) and \( \alpha_R \) as in Eq. (11), because \( \sigma_{\mu\nu} = (i/2)\epsilon_{\mu\nu\rho\sigma} \sigma^\rho \sigma^\sigma \). The coefficients \( a_{x5}, A_{x5} \) and \( \alpha_e \) are related to the \( e \) coefficients in Cirigliano, Gonzalez-Alonso, and Graesser (2013) and Naviliat-Cuncic and Gonzalez-Alonso (2013) by using

\[ \{a_{LL}, a_{LR}, a_{RR}, a_{RL}, A_{LL} + A_{LR}, A_{RL}, A_{RR}, \alpha_L, \alpha_R\} \]

\[ = \{1 + \epsilon_L, \epsilon_R, \bar{\epsilon}_L, \bar{\epsilon}_R, \bar{\epsilon}_S, \bar{\epsilon}_S, 2\epsilon_T, 2\bar{\epsilon}_T\}. \]  

(A5)

A full list of correlation coefficients in allowed \( \beta \) decay including Coulomb corrections is given in Jackson, Treiman, and Wyld (1957a, 1957b). Here we give the most important decay coefficients in terms of couplings defined in Eq. (9). We emphasize that only \( b, B, \) and \( N \) depend linearly on scalar and tensor couplings. We define \( \lambda = |g_A|/g_V > 0 \) and neglect Coulomb interactions. The spin factors are

\[ \lambda_{J,j} = \begin{cases} 1, & J \to J' = J - 1, \\ \frac{j+1}{j-1}, & J \to J' = J, \\ \frac{j-1}{j+1}, & J \to J' = J + 1, \end{cases} \]  

(A6)

and

\[ \Lambda_{J,J'} = \begin{cases} 1 - \frac{2j-1}{j+1}, & J \to J' = J - 1, \\ \frac{2j-1}{j+1}, & J \to J' = J, \\ \frac{2j-1}{(j+1)(2j+3)}, & J \to J' = J + 1, \end{cases} \]  

(A7)

where \( J \) and \( J' \) are the spin of the initial and final nucleus, respectively. In the following equations, \( M_F \) and \( M_G \) are the Fermi and Gamow-Teller matrix elements, the upper (lower) sign refers to \( \beta^- (\beta^+) \) decay, and \( \gamma = \sqrt{1 - a^2 Z^2} \), with \( Z \) the atomic number of the daughter nucleus and \( a \) the fine-structure constant.

Because of our normalization of the couplings in Eq. (11) we define

\[ \tilde{\xi} = \frac{G_F^2 V_{ud}^2}{2} \xi. \]  

(A8)

Neglecting Coulomb interactions, the decay coefficients are (Jackson, Treiman, and Wyld, 1957a, 1957b)

\[ a_\xi = 2 g_V^2 |M_F|^2 \left\{ |a_L|^2 + |a_R|^2 + \frac{g_S^2}{g_V^2} |A_L|^2 + |A_R|^2 \right\} + 2 g_T^2 \frac{a_\xi^2}{3} \left\{ |a_L|^2 + |a_R|^2 + \frac{4 g_T^2}{g_A^2} (|a_L|^2 + |a_R|^2) \right\}. \]  

(A9)

Neglecting Coulomb interactions, the decay coefficients are (Jackson, Treiman, and Wyld, 1957a, 1957b)

\[ a_\xi = 2 g_V^2 |M_F|^2 \left\{ |a_L|^2 + |a_R|^2 - \frac{g_S^2}{g_V^2} |A_L|^2 + |A_R|^2 \right\} + 2 g_T^2 \frac{a_\xi^2}{3} \left\{ -|a_L|^2 - |a_R|^2 + \frac{4 g_T^2}{g_A^2} (|a_L|^2 + |a_R|^2) \right\}. \]  

(A10)

\(^7\)Using our definition of \( \gamma_5 \) and neglecting pseudoscalar couplings.
\begin{align}
\beta_\xi &= \pm 2g_\xi \gamma \left\{ 2|\mathcal{M}_F|^2 \frac{g_S}{g_V} \text{Re}(A_L a_L^* + \text{Re}(A_R a_R^*)) - 4 \frac{g_T}{g_A} \lambda^2 |\mathcal{M}_{GT}|^2 \text{Re}(a_L a_L^*) + \text{Re}(a_R a_R^*) \right\}, \\
\alpha_\xi &= \pm 2g_\xi \lambda^2 \lambda_{FJ} |\mathcal{M}_{GT}|^2 \frac{-|a_L'|^2 - |a_R'|^2 + 4 \frac{g_T}{g_A} |a_L|^2 + |a_R|^2}{1}, \\
\lambda_\xi &= \pm 2g_\xi ^2 \lambda^2 |\mathcal{M}_{GT}|^2 \lambda_{FJ} \left\{ 4 \frac{g_T}{g_A} |a_L|^2 - |a_R|^2 - |a_L'|^2 - |a_R'|^2 \right\} + 2g_\xi ^2 \lambda_{FJ} |\mathcal{M}_F| |\mathcal{M}_{GT}| \frac{\sqrt{J}}{J+1} \frac{4 \frac{g_T g_S}{g_A g_V} \text{Re}(A_L a_L^*) - \text{Re}(A_R a_R^*)}{1}, \\
\gamma_\xi &= \pm 2g_\xi ^2 |\mathcal{M}_{GT}|^2 \lambda_{FJ} \left\{ -4 \frac{g_T}{g_A} m_e \frac{\gamma}{E_e} \left[ \text{Re}(a_L a_L^*) - \text{Re}(a_R a_R^*) \right] + 4 \frac{g_T}{g_A} \left[ |a_L|^2 - |a_R|^2 \right] \right\}, \\
\delta_\xi &= \pm \frac{4g_T}{g_A} \left[ |a_L a_L^*| - |a_R a_R^*| \right], \\
\epsilon_\xi &= \pm \frac{4g_T}{g_V g_A} \left[ |a_L a_L^*| - |a_R a_R^*| \right], \\
\zeta_\xi &= 2g_\xi ^2 |\mathcal{M}_{GT}|^2 \lambda_{FJ} \left\{ \frac{m_e \gamma}{E_e} \left[ |a_L'|^2 + |a_R'|^2 + 4 \frac{g_T}{g_A} \left[ |a_L|^2 + |a_R|^2 \right] \right] + 4 \frac{g_T}{g_A} \left[ \text{Re}(a_L a_L^*) + \text{Re}(a_R a_R^*) \right] \right\}, \\
\eta_\xi &= \pm \frac{4g_T}{g_A} \left[ |a_L| a_L^* \left[ \text{Re}(A_L) + \text{Re}(A_R a_R^*) \right] \right], \\
\nu_\xi &= \pm \frac{4g_T}{g_A} \left[ |a_L a_L^*| - |a_R a_R^*| \right], \\
\xi_\xi &= \pm 2|\mathcal{M}_F|^2 \frac{g_S}{g_V} \left\{ |a_L'|^2 - |a_R'|^2 + 4 \frac{g_T}{g_A} \left[ |a_L|^2 + |a_R|^2 \right] \right\} + 2|\mathcal{M}_{GT}|^2 \frac{g_T}{g_A} \lambda^2 \left\{ 4 \frac{g_T}{g_A} \left[ |a_L|^2 - |a_R|^2 \right] - |a_L'|^2 - |a_R'|^2 \right\}. 
\end{align}

The longitudinal electron polarization is (Jackson, Treiman, and Wyld, 1957a, 1957b)

\begin{equation}
P = \frac{G \frac{\omega}{\gamma} - 1 + b^{m_e}(\frac{\gamma}{E_e})}{1 + b^{m_e}(\frac{\gamma}{E_e})},
\end{equation}

with

\begin{equation}
G_\xi = \pm 2|\mathcal{M}_F|^2 \frac{g_S}{g_V} \left\{ \frac{g_T}{g_A} \left[ |a_L|^2 - |a_R|^2 \right] - |a_L'|^2 + |a_R'|^2 \right\} + 2|\mathcal{M}_{GT}|^2 \frac{g_T}{g_A} \lambda^2 \left\{ 4 \frac{g_T}{g_A} \left[ |a_L|^2 - |a_R|^2 \right] - |a_L'|^2 - |a_R'|^2 \right\}. 
\end{equation}

The neutron lifetime [Eq. (40)] depends on $V_{ud}$, which is extracted from the $0^+ \rightarrow 0^+$ superallowed Fermi decays. However, the extracted value of $V_{ud}$ might also depend on new physics. Taking into account this possibility,

\begin{equation}
\tau_n = K \left\{ 1 - \frac{2 \sqrt{2} a_L}{\sqrt{V_{ud}}} (\frac{m_e}{E_e})^{0^+ \rightarrow 0^+} + \frac{2 \sqrt{2}}{g_A} A_L^2 + \frac{2 \sqrt{2}}{g_A} A_R^2 \right\}^{1/2},
\end{equation}

where $(m_e/E_e)^{0^+ \rightarrow 0^+}$ is the inverse average energy of the superallowed decays, calculated by Pattie, Hickerson, and Young (2013). The constant $K$ is (Pattie, Hickerson, and Young, 2013)
\[ K \equiv \frac{2\pi^3}{m_e^2 f_a (1 + \Delta_{RC}) G_F^2 V_{ud}^2} = (1.9342 \pm 0.002) \times 10^{-4}, \]  
(A21)

where \( f_a = 1.6887(2) \) is the statistical rate function (Towner and Hardy, 2010) and \( \Delta_{RC} \) are the SM electroweak corrections (Czarnecki, Marciano, and Sirlin, 2004).

The SM expressions can be obtained by setting \( a_{LL} = 1 \) and neglecting all other couplings. Defining \( \rho \equiv |g_A| M_{GT}/g_Y M_F \), the remaining SM expressions are

\[
a_{SM} = 1 - \frac{\rho^2/3}{1 + \rho^2}, \\
A_{SM} = \mp \lambda_f j \rho^2 + 2 \delta_{fJ} \sqrt{J/(J+1)} \rho, \\
B_{SM} = \pm \lambda_f j \rho^2 + 2 \delta_{fJ} \sqrt{J/(J+1)} \rho, \\
G_{SM} = 1,
\]
(A22a-b-c-d)

while all other coefficients vanish. For neutron decay, \( \rho = \sqrt{3}|g_A| \) and \( J = J' = 1/2 \).

1. Linear terms in \( B \)

The \( B \) coefficients contain terms linear in exotic couplings. Neglecting quadratic couplings, we can write

\[
B^x = \pm \lambda_f j \rho^2 + 2 \delta_{fJ} \sqrt{J/(J+1)} \rho \left( \frac{m_Y}{E_e} \right) b_B \xi, \\
\]
(A23)

where

\[
b_B \xi = -\rho^2 \delta_{fJ} \sqrt{J} \left( \frac{J}{J+1} \right) \frac{2 g_S}{g_Y} \text{Re} A_L + \frac{4 g_T}{|g_A|} \text{Re} A_T.
\]
(A24)

Most \( B \) measurements measure

\[
\tilde{B} = \frac{B_{SM} + b_B \left( \frac{m_Y}{E_e} \right)}{1 + b_B \left( \frac{m_Y}{E_e} \right)}. \\
\]
(A25)

For pure Gamow-Teller transitions, with \( \rho \to \infty \), \( \tilde{B}_{GT} = \pm \lambda_f j \), and the linear dependence cancels. For neutron decay and assuming real \( B_{SM} \) is 1 and

\[
\tilde{B} = B_{SM} + \left( \frac{m_Y}{E_e} \right) (\gamma b_B - b B_{SM}) \\
= \frac{2 (\lambda + \lambda^2)}{1 + 3 \lambda^2} + \frac{m_Y}{E_e} \left( -\lambda - 2 \lambda^2 + 3 \lambda^3 \right) \left[ \frac{2 g_S}{g_Y} A_L + 4 \frac{g_T}{|g_A|} \right] \\
= 1 + \left[ 0.1 \frac{g_S}{g_Y} A_L + 0.2 \frac{g_T}{|g_A|} \right] \left( \frac{m_Y}{E_e} \right) A_{SM}. \\
\]
(A26)

For comparison, for neutron decay, the Fierz-interference term is

\[
b_{neutron} = \frac{2 \frac{g_S}{g_Y} A_L - 12 \frac{g_T}{|g_A|} A_L}{1 + 3 \lambda^2} \\
= 0.35 \frac{g_S}{g_Y} A_L - 3.3 \frac{g_T}{|g_A|} A_L. \\
\]
(A27)

For the measured \( \tilde{A} \) coefficient in neutron decay, with \( A_{SM} = -0.11 \),

\[
\tilde{A}_{neutron} = A_{SM} + A_{SM} m_Y \left( 0.35 \frac{g_S}{g_Y} A_L - 3.3 \frac{g_T}{|g_A|} A_L \right). \\
\]
(A28)

So for neutron decay, \( B \) actually has a reduced sensitivity to scalar and in particular tensor terms compared to, for example, \( A \).

APPENDIX B: LORENTZ VIOLATION

The Lorentz-violating \( \beta \)-decay rate including Coulomb corrections and electron spin, to first order in \( \chi^{\mu \nu} \), is

(Noordmans, Wilschut, and Timmermans, 2013b)

\[
dW = \frac{1}{(2\pi)^3} E_e p_e (E_0 - E_e)^2 F(\pm Z, E_e) \xi dE_e d\Omega_e d\Omega_e \left\{ \left( 1 \mp \frac{\vec{p}_e \cdot \vec{J}}{E_e} \right) \left[ \frac{1}{2} \left( 1 + \frac{B \frac{\vec{p}_e \cdot \vec{J}}{E_e}}{J} \right) \right] + \left( 1 + \frac{(E_e - m_e)(\vec{p}_e \cdot \vec{A}_e)}{E^2_e - m_e^2} \right) \frac{p_e^x}{E_e} \mp \frac{\gamma m_e \vec{p}_e \times \vec{A}_e^\prime}{E_e} \frac{m_e^2}{E_e} \sqrt{1 - \gamma^2 (\vec{p}_e \times \vec{A}_e)^2} \right\} \times \left[ \frac{1}{J} \left( J^k \right)^{j^l} \frac{3}{2} \frac{\vec{p}_e \cdot j}{E_e} j^l + \frac{1}{2} \frac{p_e^l}{E_e} + w_3^1 + T_{jk} \frac{p_e^l}{E_e} + T_{ik} \frac{p_e^j}{E_e} + T_{ij} \right] - S_{2mk} \frac{J^m}{J} j^k + S_{3} \frac{J^m}{J} \frac{p_e^j}{E_e} \right\} \right\}. \\
\]
(B1)

where \( \gamma = \sqrt{1 - \alpha^2 Z^2} \). The Lorentz-violating constants are\(^8\)

\(^8\)Note the sign error in \( w_3 \) in Noordmans, Wilschut, and Timmermans (2013b).
\[
t = (a - \frac{1}{2}c')^{\mu^0}, \quad w^i_j = -x^{i} r^{j} + y^{i} r^{j} - z^{j} r^{i}, \quad w^3_2 = K(\chi^{00} - \chi^{00}) - L \chi^{0^+}, \quad w^3_3 = -x^{i} r^{0} + y^{i} r^{0} + z^{0} r^{i},
\]

\[
T_{1m}^{ij} = \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k m}, \quad T_{2j}^{i j} = \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j} - \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j}, \quad T_{3j}^{i j} = (x + y) \epsilon^{j i} \epsilon^{k j} - \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j},
\]

\[
T_{4 j}^{i j} = \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j} - \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j}, \quad S_{1m}^{i j} = \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j} - \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j}, \quad S_{2m}^{i j} = \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j} - \frac{1}{2} \bar{\epsilon} \epsilon^{j i} \epsilon^{k j},
\]

where \( r \) and \( i \) denote the real and imaginary parts of \( \chi^{\mu\nu} \), respectively, \( \tilde{\chi} = e^{\mu l k} \chi^{\mu l k} \), and \( p^j \) denotes the electron momentum in the \( j \) direction. \( a, A, B, \) and \( \xi \) are the standard \( \beta \)-decay coefficients, given in Eq. (A22); the other coefficients are

\[
\begin{align*}
\lambda_{j,j} & = (1 + x) \lambda_{j,j}, \\
\tilde{\gamma} & = \frac{1}{3} \left( 1 - x \right) \left( 1 + \frac{2}{3} \lambda_{j,j} \right),
\end{align*}
\]

\[
K = -y \sqrt{\frac{J}{J + 1}} \delta_{j,j}, \quad L = \frac{1}{2} \frac{p^2}{1 + p^2} \lambda_{j,j},
\]

where the upper (lower) signs refer to \( \beta^{-} (\beta^{+}) \) decay. The coefficient \( \lambda_{j,j} \) is given in Eq. (A6) and

\[
\Lambda_{j,j} = \frac{(\tilde{\gamma} \cdot \tilde{\gamma})^2 - \frac{1}{4} J(J + 1)}{J(2J - 1)},
\]

with \( \Lambda_{j,j} \) given in Eq. (A7).

**REFERENCES**


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