Chapter 6

Investigating charge-current-induced spin voltage signals in Bi$_2$Se$_3$

Chapter 6 provides an investigation of the measured voltage obtained from efforts to electrically probe spin–momentum locking in the topological insulator Bi$_2$Se$_3$ using ferromagnetic contacts. Upon inverting the magnetization of the ferromagnetic contacts, we find a reversal of the measured voltage. Extensive analysis of the bias and temperature dependence of this voltage has been performed, considering the orientation of the magnetization relative to the current. Our findings indicate that the measured voltage can arise due to fringe-field-induced Hall voltages, different from current-induced spin polarization of the surface-state charge carriers, as reported recently. Understanding the origin of the measured voltage is important for realizing spintronic devices with topological insulators.

6.1 Introduction

The conservation of time-reversal symmetry gives rise to spin–momentum locking in the surface states of topological insulators, where spin and momentum of the surface-state charge carriers are directly related in a perpendicularly right-handed orientation (see chapter 2 and figure 6.1b). This spin texture can be compared with that originating from Rashba spin–orbit coupling, but consists of a single Fermi circle with opposite spin texture. This direct coupling between spin and momentum in topological insulators should allow electrical injection and detection of spin currents in spintronic structures without the need of ferromagnetic layers. In addition, the time-reversal symmetry leads to robustness against elastic backscattering from nonmagnetic impurities reducing the surface state conduction dissipation. Furthermore, the surface states are ideally located within the bulk band gap such that these can be addressed independently from the non-spin-polarized bulk. With this combination of properties, topological insulators provide a new platform for spintronics [1].

In our study, we use the canonical topological insulator Bi$_2$Se$_3$ which has a bulk band gap of 0.3 eV (see section 2.3), making it suitable for potential applications at room temperature. The spin–momentum locking of the surface states of Bi$_2$Se$_3$ has been well investigated by angular-resolved photoemission spectroscopy (ARPES) measurements where 100% spin polarization has been reported [2–4]. Additionally, the existence of spin–momentum locking in Bi$_2$Se$_3$ over a large temperature range has been investigated electrically through studying spin torque ferromagnetic resonance [5–7], current-induced magnetization switching [8–11], spin pumping from ferromagnetic metals and insulators [12–17], spin-Seebeck effects [18], tunneling spectroscopy [19], and local (semi-)spin-valve measurements [20–22], with bulk states contributing to the total transport. Furthermore, experiments on electrical detection of this spin texture have been reported in which the texture was analyzed using the three-terminal potentiometric method [23]. It has been shown that by using ferromagnetic contacts charge-current-induced spin polarization in the topological insulator channel can be measured by probing the potential at that contact [19,24–34]. For several topological insulator compounds, it has been claimed in these works that the observed change in voltage upon inverting the ferromagnet’s magnetization originates from the current-induced spin polarization in the surface states.

Here, we report on our observations of electrically probing spin–momentum locking on thin films of Bi$_2$Se$_3$, using ferromagnets to detect the spin polarization for different magnetization directions with respect to the current bias. For this, we use a Hall-bar-patterned device and change the polarity of the fixed current bias in the topological insulator channel and the magnetization of the ferromagnetic detectors to obtain similar loops in the measured voltage, as shown in similar reports [19,24,25,27–34]. Additional design flexibility in our device geometry enables the decoupling of spin and current paths, offering a unique possibility to investigate additional magnetoresistance effects to the observed voltage in our topological insulator channel. Surprisingly, we find that the measured voltages for a current bias applied both along and perpendicular to the topological insulator channel exhibit features that cannot be ascribed to the spin polarization of the surface states alone. Instead, similar signals can result from Hall voltages generated by the fringe fields from the ferromagnetic detector in
close proximity to the topological insulator channel.

6.2 Theory

In this section, concepts regarding detection of spin accumulation in the surface states of topological insulators generated by applying a charge current using ferromagnetic contacts will be briefly introduced. In the second part, an analytical expression for the size of the surface-state spin polarization that can be extracted from the measured spin voltage will be provided.

6.2.1 Detection of charge-current-induced spin accumulation in topological insulators

The generation of a spin-polarized charge current in topological insulators and its detection have been investigated in several theoretical works [1, 23, 35–37, 38]. These works mainly deal with the size of the effects and the possibility of detection of such spin polarized currents. In the work by Burkov and Hawthorn [35], it has been proposed that the resistance between a ferromagnetic contact and an Ohmic contact placed on a spin–momentum-locked channel can be modulated by the ferromagnet’s magnetization, neglecting any magnetoresistance effects from the ferromagnetic layer\(^1\). Furthermore, it has been reported that the spin-relaxation time in topological insulators is on the order of the momentum-relaxation time, making it a good spin generator but an unsuitable spin conserver [1]. More specifically, Schwab et al. have reported that the transport time (i.e. the effective relaxation time) is twice the momentum relaxation time due to forbidden backscattering [37], in agreement with another theoretical work [38]. However, the backscattering is relaxed by an angular dependent scattering [36]. Similar to the device geometry proposed by Burkov and Hawthorn, Schwab et al. have introduced a tunnel junction that can act as a spin diode in the limit of a large tunneling conductance. A current-induced spin density in a similar geometry of \(\sim 5 \times 10^{14} \text{ spins/m}^2\) has been calculated by Culcer et al. [36], making it detectable by for example Kerr rotation, too. In the same work, charge-impurity scattering as well as scattering due to surface roughness on the transport has been included and found to be dependent on the charge-carrier density of the sample. Finally, Hong et al. have proposed a three-terminal potentiometric measurement in order to reduce influences from the detector contact [23]. In this work, it has been further realized that two-dimensional electron gases at the surface of a topological insulator can contribute to the spin-polarized current via the Rashba–Edelstein effect. Furthermore, a simple expression has been given for the change in the voltage at the ferromagnetic detector when a spin accumulation is present, where a maximum (angular-averaged) spin polarization of \(2/\pi\) is reported. However, as the aforementioned works all assume 100% spin polarization, the actual spin polarization is almost half of that ideal value due to the strong spin–orbit coupling in these materials [40]. Furthermore, it is assumed that the Fermi level is located inside the bulk band gap,\(^{1}\)Later, something similar was theoretically proposed for a device where both contacts are ferromagnetic [39].
Investigating charge-current-induced spin voltage signals in Bi$_2$Se$_3$

Figure 6.1: (a) Spin-density of states $\nu(E)_{\uparrow,\downarrow}$ typical for 3d-transition metal ferromagnets. The arrows for $\nu$ and those depicted inside the colored bands denote here the spin of the carrier $s$ that is opposite to the ferromagnet’s magnetization $M$ and magnetic moment of the electron $\mu$. Notice the difference in density of states at $E_F$. (b) Measurement geometry used for detection of the spin accumulation $\sigma \propto \nu_\uparrow - \nu_\downarrow$ in an $n$-type topological insulator channel (green). This accumulation is induced by a charge current $I$ yielding a net momentum of the electrons $k_e$, as explained by the shift $\delta k$ of the Fermi circle away from the center as depicted in the top right (the arrows indicate spin again). $\mu_{L,R}$ indicate the potential on the left-hand and right-hand side, respectively, and $M$ indicates the magnetization of the ferromagnet (brown). The charge current induces a splitting in the surface-state-spin potentials $\mu_\uparrow$ and $\mu_\downarrow$ in topological insulators as indicated by the blue and red line. The measured voltage $-eV$ will be measured between the ferromagnet (positive polarity, measuring the spin potential $\mu_\uparrow$ as indicated by the red and blue dots) and a normal metal at negative polarity (not shown here) that measures $\mu_{avg}$ as indicated by the black dot. Figure inspired on figure from [41].

which is not the case for $n$-type Bi$_2$Se$_3$ (section 2.3). Therefore, one has to be careful with additional effects while measuring the surface-state spin polarization.

In order to detect the proposed spin polarization in topological insulators, ferromagnetic layers are used. Ferromagnets have a finite magnetization that is determined by the difference between the majority-spin carriers (spin-down) and the minority-spin carriers (spin-up)$^2$. For 3d-transition metal ferromagnets such as Co, the bulk density of states $\nu(E)$ at the Fermi level is larger for the $d$-electron’s minority spins ($\nu(E)_\downarrow$) than for the majority spins ($\nu(E)_\uparrow$), as schematically depicted in figure 6.1a, which is important for the transport phenomena that occur within a few meV around the Fermi level $E_F$. Now, this ferromagnetic layer is used as a voltage probe to measure the spin accumulation $\sigma$ in a material, which is the difference in number between spin-up and spin-down carriers, assuming $n_\uparrow > n_\downarrow$.

An example of such a spin-accumulation channel is the topological surface state where a spin accumulation $\sigma$ is generated by an imbalance in the momentum due to

$^2$Here, the terms ‘majority’ and ‘minority’ are related to the magnetic moment $\mu$ of the charge carriers and ‘spin-up’/‘spin-down’ to the spin $s$ of the charge carriers [42], where the $s$ is indicated by an arrow in this chapter. For electrons, $\mu$ and spin $s$ are related through $\mu = -g\mu_B s/\hbar$ and are thus antiparallel to each other.
a charge current of which the measurement geometry is shown in figure 6.1b. Such a
difference in number of spins can be described by a splitting of spin-chemical poten-
tials $\mu_{\uparrow,\downarrow}$. The spins in the channel can scatter into the ferromagnet, thereby raising
the Fermi level of one of the spin species and lowering the other. Due to the imbalance
in $\nu(E)_{\uparrow,\downarrow}$ at $E_F$ in the ferromagnet, the Fermi level will be stronger influenced for
the spin species (up or down) where $\nu(E)_{\uparrow,\downarrow}$ is the smallest. When $\nu(E)_{\uparrow,\downarrow}$ is small,
changes in the spin accumulation will give a larger energy difference to account for
this imbalance with respect to the density of states of the other species. Therefore, the
ferromagnet will be sensitive to the potential $\mu_{\uparrow,\downarrow}$ with spin orientation $s$ antiparallel
to the ferromagnet’s magnetization $M$. When the magnetization is parallel (antiparallel)
to $\sigma$, the measured potential $\Delta \mu$ will be low (high) with respect to the average
potential, which leads to a positive (negative) change in the voltage $V$ at that contact
since $\Delta \mu = \mu_{\uparrow,\downarrow} - \mu_{\text{avg}} = qV$, where $q = -e$. By either inverting the ferromagnet’s
magnetization or the channel’s spin accumulation by inverting the charge current, a
change in $V$ can be obtained\(^3\). Therefore, by looking at the direction of switching, one
can determine the net orientation of the spins as well as the difference between the
number of the respective spin species, defined by the surface-state spin polarization
$P_{SS}$ (see section 6.2.2). In a real experiment such voltages are always to be measured
with respect to another (nonferromagnetic) contact and it actually depends on the
actual poling of the contacts whether the voltage increases or decreases. Another
important consideration is the direction of the current in topological insulators. In
$n$-type materials, the current is governed by electrons which flow in the opposite di-
rection with respect to the applied current that is defined for positive charge carriers.
Therefore, the polarity of the current source contacts plays an important role too. It
has to be noted that similar features are expected for $p$-type topological insulators
where both the velocity and the spin orientation of the charge carrier change sign [31].

Interestingly, the first work by Li et al. [24] shows the oppositely expected behavior
of the relative voltage levels upon changing the magnetization. It is observed that the
voltage at the ferromagnetic contact is low (high) when the spin and magnetization
are parallel (antiparallel) with respect to each other, which is opposite from what was
discussed here. A background voltage is observed because of the relative alignments of
the contacts which leads to a pickup of a longitudinal resistance related to charge
transport. Similar signals have been shown in subsequent works by the same group
[31, 41], where an additional negative sign with respect to the earlier discussion in
this section was introduced upon introducing the electrochemical potential to explain
the origin (and sign) of the observed signal [41]. In subsequent work by Tang et
al. [25], an opposite trend is detected in comparison to that by Li et al., but in
agreement with what is expected from the discussion earlier. In order to explain the
switching behavior, a model on the magnetic moment rather than the spin to explain
the switching has been provided. Such explanation has been taken over by several
other groups [27–30, 33, 34]. In the work by Ando [26], a three-terminal geometry
has been used where the center contact is shared between current source and voltage

\(^3\)Here, we assumed a simple spin polarization picture where the tunnel barrier that is used in the
experiment is considered to be a separation layer and not contributing to the (tunneling) spin polar-
ization. From this picture, $d$-electrons yielding a negative spin polarization of the bulk ferromagnet
will mainly contribute to the probing of the spin accumulation.
probe, different from the geometry as proposed by Hong et al. [23]. From the relative signs, although not accurately described, the measurements seem to follow the picture as described above: the resistance decreases when \( M \) is antiparallel to \( s \) because less states for the spins are available at \( E_F \) [43]. Discrepancies in the sign of the signal could be related to the quality and function of the grown tunnel barrier that would require concepts on tunneling spin polarization, which can induce an additional sign to the discussion in the previous paragraph.

To summarize, the changes in voltages observed in the works by Li et al. show an opposite trend in comparison to subsequent papers. Our work, as will be discussed in this chapter, follows the trend as observed by Li et al. However, we cannot rule out artifacts due to the presence of fringe fields. In the work by Li et al., the ferromagnetic detector is placed on top of the channel and not over the channel as in most of the other works where effects at the edges do become very crucial. Furthermore, it has been shown that the spin voltage signals can change sign upon changes in the charge carrier density [30], indicating that Rashba–Edelstein effects can play a considerable role, too. The interplay between topological surface states, 2DEGs, and bulk states is thus an important point of consideration for the origin of charge-current-induced spin voltage signals.

### 6.2.2 Estimation of the charge-current-induced spin polarization

To derive an expression for the charge-current-induced spin polarization extracted from the observed change in voltage, we start out with the expression for the electron-current density in 3D:

\[
J_{3D} = -e \int \frac{d^3k}{(2\pi)^3} v(k),
\]

which reduces in 2D to:

\[
J_{2D} = -e \int \frac{d^2k}{(2\pi)^2} v(k).
\]

Here, \( v(k) \) is the velocity of the charge carriers. For the derivation, we assume that the surface state with a linear dispersion is the only contributing charge-transport channel, i.e. contributions from the bulk states or from 2DEGs due to the band bending at the surface are not included. In the end of this section, corrections for such contributions will be introduced by defining a conduction ratio. Since only effects close to the Fermi circle are investigated, the components of \( v(k) \) can be approximated by the Fermi velocity \( v_F \): \( (v_x, v_y) = (v_F, v_F) \). Upon applying a bias in the \( x \) direction, the Fermi circle is shifted with respect to the zero bias position over a length of \( \delta k \), as depicted in figure 6.1b\(^4\). From this, we can define \( d\delta k_x \) and \( d\delta k_y \):

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\(^4\)Such a shift due to the collective electron motion can be visualized by a canting of the Dirac cone as shown in literature for graphene [44] as well as for Bi\(_2\)Se\(_3\) [32]. Here, it has to be realized that the Dirac cone is hollow and therefore this linear canting occurs.
Theory

\[ dk_x = \delta k \cos \phi, \quad (6.3) \]

\[ k_y = k_F \sin \phi \Rightarrow dk_y = k_F \cos \phi \, d\phi, \quad (6.4) \]

where \( k_F \) is the Fermi wavevector and \( \phi \) is the angle (in momentum space) between the total wavevector of a nonequilibrium charge carrier and the \( k_x \) axis. Using these definitions in the expression for the current density, we obtain:

\[ J_x = \frac{2eV_F}{(2\pi)^2} \int_{-\pi/2}^{\pi/2} \delta k k_F \cos^2 \phi \, d\phi = \frac{eV_F \delta k k_F}{4\pi}, \quad (6.5) \]

where a factor of 2 was included for the contribution to the current density by the removal of states at \(-k_x\) and addition of states at \(+k_x\). In order to calculate the associated voltage with the induced current density, the linear dispersion relation of the topological surface states is included in the expression for the current density:

\[ E = \hbar k_F \Rightarrow \frac{dk}{dE} = \frac{1}{\hbar v_F} \Rightarrow \Delta k = \frac{\Delta E}{\hbar v_F} \Rightarrow J_x = \frac{e\Delta E k_F}{2\hbar}. \quad (6.6) \]

where \( \hbar \) is the Planck constant (\( \hbar = h/2\pi \)). The spin orientation revolves around the Fermi circle and to calculate the spin potential \( \mu(\phi) \) associated with the \( x \) projection of the spin, we have:

\[ \mu(\phi) = \Delta E \cos \phi. \quad (6.7) \]

By integrating over the angle \( \phi \), one obtains:

\[ qV_{\text{spin}} = \int_{0}^{2\pi} \Delta E \cos \phi \cos \phi \, d\phi = \pi \Delta E. \quad (6.8) \]

Switching the magnetization of the ferromagnetic detector, changing spin selectivity, would thus lead to a voltage difference of \( 2\pi \Delta E/e \). Substituting (6.8) into expression (6.6) for the current density:

\[ J_x = \frac{e^2 k_F \Delta V}{4\pi \hbar} \Rightarrow \Delta V = \frac{\hbar}{e^2 k_F W} I, \quad (6.9) \]

where \( W \) is the channel width. In this expression, we have assumed pure surface-state conduction in the topological insulator channel and 100% spin polarization of the surface-state carriers as well as the magnetization of the ferromagnetic layer. We correct for this by introducing the ferromagnet’s spin polarization \( P_{\text{FM}} \), surface-state spin polarization \( P_{\text{SS}} \) and conduction ratio \( \eta = n_{\text{SS}}/n_{\text{total}} \). For the implemented conduction ratio, it has been assumed that the mobility is the same for the different channels and that those do not interact. From expression 6.9, it is clear that the change in voltage due to spin–momentum locking in the surface states should scale linear with the applied current.
6.3 Results and Discussion

In this study, we have used thin films of Bi$_2$Se$_3$ of 20 quintuple layers (QL) which have been grown by MBE on Al$_2$O$_3$(0001) substrates in a custom-designed SVTA MOS-V-2 MBE system at a base pressure lower than 5×10$^{-10}$ Torr [45]. A basic investigation with STM shows a triangular growth pattern indicating high-quality growth of Bi$_2$Se$_3$ (figure 6.2a and discussed in section 2.4). Cross-sectional measurements on these triangles reveal a step height of 1 QL. A typical plot for the sheet resistivity $\rho_{xx}$ versus temperature for this film (figure 6.2b) shows a decrease in resistivity upon decreasing temperature, indicating electron–phonon scattering to be the dominant scattering mechanism. Combining this data with standard (low-field) Hall measurements yields a temperature-independent bulk charge-carrier density of 1.25×10$^{19}$/cm$^3$ (placing the Fermi level in the bulk conduction band, see the value of $k_F$ below) and a mobility of 385 and 815 cm$^2$/Vs at 300 and 1.5 K, respectively$^5$.

Figure 6.2: (a) STM topography image taken at Room Temperature (RT) showing triangular growth spirals ($I_T = 1$ nA, $V_T = 1$ V). (b) Resistivity $\rho_{xx}$ vs temperature measurement for a 20 QL Bi$_2$Se$_3$ film. (c) Device structure with Ti/Au contacts (bright yellow) on the Bi$_2$Se$_3$ channel (green), etched structure (dark) and ferromagnetic contacts (brown).

$^5$In chapter 5 it is found that the low-field data does not suffice, but the mobility found here corresponds well to that of the high mobility channel found in that chapter. However, we attributed this to the bulk channel and therefore the mobility of the surface states is expected to be lower. This is usually compensated by a larger charge-carrier density such that the conduction channels contribute equally.
Results and Discussion

The advantage of using Bi$_2$Se$_3$ thin films over single crystals (with a random shape and size) is its design flexibility that allows for the investigation of parasitic effects on the measured voltage. In this work, the devices are first patterned with Ohmic contacts using a combination of DUV lithography and EBL techniques. The Ohmic contacts consist of Ti(5)/Au(70) deposited by electron-beam evaporation. Thereafter, Hall bar structures with a channel width of 1 µm have been realized using EBL and Ar plasma dry etching. In the last step, spin detector contacts on the topological insulator channel are fabricated by growing tunnel barriers of TiO$_x$(2) deposited by evaporating Ti, followed by in-situ oxidation in an O$_2$ atmosphere of 300 mTorr$^6$. The ferromagnetic layer is a Co(35) layer and is capped by Au(5). The ferromagnetic contacts have typical lateral dimensions of 1–3 µm. The values for the resistance–area product ($RA$) of the spin contacts are in the range of 3–100 kΩµm$^2$. The resulting device structure is as shown in figure 6.2c.

In our measurements, a current $I_e$, that indicates the direction of flow of the electrons, is sent through the topological insulator channel along the $+x$ direction such that an imbalance in the momentum is created. Due to spin–momentum locking, this imbalance in momentum leads to a net spin polarization of the surface-state charge carriers $\sigma$ perpendicular to $I_e$ as shown in figure 6.3a. The ferromagnetic contact with magnetization $M$ is used to probe voltages $V_1$ and $V_2$ in the topological insulator channel with respect to different Ohmic contacts designed outside the current path to minimize charge-related effects. In this geometry, there is ideally no net charge current flowing through the detector such that only the (spin) potential at the channel is measured and barrier effects do not play a role. The magnetization of the ferromagnetic contact, and therefore the spin sensitivity, can be inverted by application of a magnetic field $B$ along the $y$ axis. The measurements are done for three different geometries, labeled A, B, and C in which the relative orientation between the spin polarization and magnetization is varied. The measurements are performed in a flow-cryostat system with magnetic fields up to 1 T, using ac modulation techniques. The results presented here are representative of multiple devices.

In the first measurement geometry, labeled as geometry A, the voltage between the ferromagnetic contact ($RA = 3$ kΩµm$^2$) and the Ohmic contacts is measured at 4 K sourcing an ac current bias of +100 µA. The voltage is recorded while sweeping a magnetic field $B$ from positive (in the $+y$ direction) to negative ($-y$, indicated by trace) and back (retrace) as shown in figure 6.3. Voltage $V_1$ shows a clear switch at $\sim 15$ mT due to magnetization inversion of the Co contacts. The voltage difference $\Delta V$ between the magnetization directions might be indicative of a spin polarization in the topological insulator channel, as reported in related works [19, 24–34]. Upon inverting the current bias to −100 µA, thereby inverting the spin polarization in the channel, an opposite switch in the voltage is obtained, indicating that the spin polarization in the channel is indeed reversed in accordance with the surface-state spin texture (figure 6.3c). Studying the different orientations of the magnetization $M$
Investigating charge-current-induced spin voltage signals in Bi$_2$Se$_3$

Figure 6.3: (a) Measurement geometry A in which measured voltages $V_1$ and $V_2$ upon application of a current $I_e$ along the $+x$ direction are indicated. (b) Voltage $V_1$ vs magnetic field $B$ using the conventions drawn in (a) for a current $I_e$ in the $+x$ direction at a bias of 100 µA. Measured at $T = 4$ K. (c) Same measurement but at an inverted bias of $-100$ µA. The insets indicate the different orientations of spin polarization $\sigma$, magnetization $M$ and the direction of flow of the electrons $I_e$.

and the surface-state spin polarization $\sigma$ (inset figure 6.3b), the measured changes in the voltage cannot be explained by the spin–momentum locking of the surface-state charge carriers as earlier discussed (see section 6.2.1), but is similar to that reported by works by Li et al. [24,31,41]. The background signal that is observed in the measurements originates from charge-related effects and therefore does not contribute to magnetization-dependent effects in the measured voltage. The additional small jumps in the signal are probably due to instabilities of the ferromagnetic layer and scale with bias. Furthermore, magnetization-dependent voltage signals were measured for ferromagnetic contacts with $RA = 19$ kΩµm$^2$ (discussed in section 6.6.1), which show similar features as in figure 6.3 and therefore magnetoresistance effects due to the ferromagnetic contacts can be excluded.

To rule out any artifacts related to the geometry of the contacts, voltage $V_2$ was measured with respect to the opposite Ohmic contact (figure 6.12). This voltage shows a similar switching behavior but with a different magnitude of $\Delta V$ and background signal, while the Ohmic contact properties are the same. Subtracting $V_1$ from $V_2$ reveals a residual switching behavior, which indicates that the voltage at the Ohmic contacts is affected by the magnetization switching of the ferromagnetic contacts, see also figure 6.12. The difference between $V_1$ and $V_2$ might be due to a slight asymmetry in the design of the ferromagnetic contact on both sides of the Hall bar. Any extracted $\Delta V$ is thus very sensitive to the relative alignment of the contacts.
Results and Discussion

Assuming that the voltage difference $\Delta V$ arises due to current-induced spin polarization of the surface-state charge carriers, the polarization $P_{SS}$ can be calculated via equation (6.9), where for $\eta$ similar conductivities for bulk and surface are the assumed. With $k_F = (3\pi^2 n_{3D})^{1/3} = 0.072 \, \text{Å}^{-1}, W = 1 \, \mu\text{m}$, a fixed $\eta = 0.1$ (extent of the top and bottom surface states is taken to be 2 QL [40]) and $P_{FM}$ ranging from 3 to 30%, we obtain values for the surface-state spin polarization of $P_{SS}$ ranging from 1.5 to 15%. Values of $P_{SS} \approx 15\%$ for low $P_{FM}$ are comparable to the values previously reported for Bi$_2$Se$_3$ [20, 24, 27], whereas $P_{SS} \approx 1.5 \%$ (high $P_{FM}$) is in the range of those extracted for the counterdoped compounds [25, 26, 28] but both values are always lower than the theoretical limit for electrical transport [23, 40]. Similar values of $P_{SS}$ have been obtained for contacts with higher $RA$ showing a slight increase in $\Delta V$, see section 6.6.1. Contributions from bulk states or from other parallel surface-state channels (for example two-dimensional electron gases [50]) can change the value of $P_{SS}$ significantly.

Our device design offers the flexibility to source the current along the $y$ axis and measure the voltage $V_1$ and $V_2$ between the ferromagnetic contact and lateral Ohmic contacts as shown in figure 6.4a (labeled as geometry B). Such an investigation allows us to know more about the origin of the measured voltage in figure 6.3, which is attributed to spin–momentum locking in related reports [19,24,25,27–31,33,34,41]. In this measurement geometry, the spin polarization of the surface-state charge carriers is oriented perpendicular to the magnetization of the Co contacts, implying that these contacts should not detect any spin polarization in the topological insulator channel. Surprisingly, we see a clear change in $V_1$ and $V_2$ for this measurement geometry, too (figure 6.4b and 6.4c).

Upon investigating the bias dependence of $\Delta V$ with applied current for geometry B (figure 6.5a), it is observed that the extracted values are on the same order of magnitude as that for the original geometry A. The voltage difference $\Delta V$ scales linearly with current as can be expected from equation (6.9). Furthermore, the background voltage scales linearly with current bias (not shown here) as expected from the Ohmic background. The similarities in the magnitude and trend of the measured voltages in both geometries A and B indicate a different origin than due to spin–momentum locking. The observed isotropy of the measured voltages for different current directions further rules out its origin due to hexagonal warping in the topological insulator [51,52] or effects related to a tilted magnetization of the ferromagnetic detector.

Furthermore, for the ferromagnetic contact with $RA = 3 \, \text{kΩ} \mu\text{m}^2$, the temperature dependence of $\Delta V$ for geometries A and B has been plotted in figure 6.5b. We find a weak temperature dependence of $\Delta V$ up to 250 K, similar to other reports. [19,24,27,31,41]. If we assume $\Delta V$ to arise due to surface states in the topological insulator, we can infer that the surface-state spin polarization does not change appreciably up to 250 K while beyond 250 K its detection is mostly limited due to a change

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7Using $n_{TSS} = k_F^2/4\pi$ would yield a Fermi level located at $\sim +0.3 \, \text{eV}$ above the conduction band edge which is unlikely in comparison to usual ARPES measurements. Therefore, it is clear that the bulk is contributing to the transport.

8However, it was theoretically proposed that these Rashba–Edelstein contributions are an order of magnitude smaller [40], as later experimentally observed [41]. Nevertheless, this contribution has been shown to be able to dominate the signal [30].
Investigating charge-current-induced spin voltage signals in Bi$_2$Se$_3$

Figure 6.4: (a) Alternative geometry B in which the current is biased along the $y$ axis and voltages $V_1$ and $V_2$ are measured in the horizontal extent at both sides of the ferromagnetic contact. (b) and (c) Measurements of respectively $V_1$ and $V_2$ vs magnetic field $B$ at a bias of $-100 \mu A$ and at $T = 4$ K. The insets indicate the different orientations of spin polarization $\sigma$, magnetization $M$ and the direction of flow of the electrons $I_e$. In the contact’s properties. The small decrease in $\Delta V$ for both geometries cannot be explained fully by the temperature-dependent resistivity of Bi$_2$Se$_3$ (figure 6.2b) and suggests a possible decrease in detection efficiency of the ferromagnetic contact with increasing temperature. The wide temperature range over which the generated

Figure 6.5: (a) Current bias dependence on the voltage difference $\Delta V$ for $V_1$ in geometry A and geometry B at $T = 4$ K. (b) Temperature dependence of $\Delta V$ for $V_1$ in geometries A and B at $I = 25 \mu A$. Dashed lines are a guide to the eye.
spin polarization can be detected is surprising when compared to reports on the counterdoped compounds, where signals disappear at low temperatures [25,26,28].

In yet another measurement geometry (geometry C, figure 6.6), the sample is aligned with respect to the magnetic field such that the magnetization is directed along the channel. In this measurement geometry, where the magnetization of the ferromagnetic is rotated by 90° as compared to geometry A, the ferromagnetic contacts should not detect a spin polarization in the topological insulator channel. However, we do observe a clear voltage difference $\Delta V$, as shown in figures 6.6b and 6.6c, when the magnetization of the ferromagnetic is switched. This differs from other reports [19,24]. The linear background as observed in figure 6.6 is due to an unintended misalignment of the device with respect to the external magnetic field. This results in an out-of-plane field component leading to a Hall voltage (for a misalignment of 0.5°, we find this to be 9 µV in good agreement with the slope in the voltage measured at 0.4 T).

The similarities in the observed signals for all different measurement configurations raise questions on their origin. Rashba spin–orbit coupling or spin Hall effects can be excluded, since similar spin textures with the momentum perpendicularly locked to the spin orientation should not be observed in the alternative geometries B and C. However, the fringe fields arising due to the proximity of the ferromagnetic layer to the topological insulator channel could mimic a similar (bulk-mediated) voltage

![Figure 6.6](image_url)

**Figure 6.6:** (a) Alternative geometry C where the current is biased along the $x$ axis and voltages $V_1$ and $V_2$ are measured in the vertical direction with a rotated magnetization of the detector contact. (b) Measurement of $V_1$ vs magnetic field $B$ at a bias of +100 µA and $T = 4$ K. (c) Same measurement but at an inverted bias of −100 µA and $T = 4$ K. The insets indicate the different orientations of spin polarization $\sigma$, magnetization $M$ and the direction of flow of the electrons $I_e$. 

117
for all the different measurement configurations as shown in figures 6.3, 6.4, and 6.6, as also suggested in another work [32]. In order to illustrate this, we calculate the magnetic fields in the topological insulator channel for the particular shape of the ferromagnetic contacts including tunnel barrier (as in figure 6.2c) which is schematically shown in figure 6.7a. In figure 6.7b, we find a strong out-of-plane ($B_\alpha$) component at the edges of the channel which is on the order of several 100 mT. Such values are important to take into consideration for similar works that have the ferromagnetic contacts deposited partially on top of the topological insulator channel and thus encountering a large step height [25–28, 30, 33, 34]9. Furthermore, we find $B_y$ fields in the $-M$ direction (as displayed in figure 6.7a) on the order of 100 mT. The fringe fields in the $z$ direction are present for all different measurement geometries and will lead to the development of local Hall voltages perpendicular to the current due to small geometrical misalignment of the contacts. As shown in section 6.6.2, we find a considerable current spread in the topological insulator channel in the vicinity of both the Ohmic and ferromagnetic contacts which could amplify the magnetoresistive effects.

Furthermore, we have modeled the magnetic field at the interface between the ferromagnetic contact and the channel due to the triangular growth spirals at the Bi$_2$Se$_3$ (figure 6.2a). As shown in figures 6.7c and 6.7d, out-of-plane magnetic fields $B_z$ on the order of 50 mT can develop locally at the edges of the triangular steps. Due to the asymmetry of these triangular features a net magnetic field will act on the charge carriers. This will give rise to Hall voltages which we calculate to be on the order of several tens of $\mu$V for the device structure used and is thus comparable to the measured voltage signals. Note that in this model we neglect any irregularities in the ferromagnetic layer which could significantly enhance the fringe fields [54]. Upon inverting the magnetization of the ferromagnetic layer, the direction of the fringe fields is inverted which inverts the isotropic local Hall effect and manifests itself as a switch in the measured voltage for all geometries.

### 6.4 Conclusions

In summary, our detailed investigation of the measured voltages in different measurement geometries along with the calculations of the fringe-field-induced voltage make us believe that it is nontrivial to relate the measured voltage to spin polarization of topological insulator surface states alone. Our findings clearly highlight the necessity of a careful analysis of the observed voltage in electrical measurements with topological insulators for future spintronic devices.

### 6.5 Outlook

The uncertainty about the exact sign of the spin voltages and the seemingly small differences in signal between metallic and insulating films [55], especially with respect

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9Such height differences could further lead to shorts between the ferromagnetic contact and the topological insulator channel. Only the work by Dankert et al. showed proper tunneling characteristics [27] that could be related to the weak temperature dependence of the spin voltage [53].
Figure 6.7: (a) Schematics of fringe fields (black line) arising due to the proximity of ferromagnetic contact with magnetization $M$ (brown) on topological insulator channel (green). Magnetic field components parallel as well as perpendicular to the magnetization are present (indicated in red). (b) Calculation of the out-of-plane component $B_z$ at the interface between ferromagnetic contact (outer square) and topological insulator channel (inner cross structure, colored false green). At the edges we find magnetic fields around 0.7 T. (c) Calculation of the $B_z$ component at the interface for a triangular step feature with sides of 200 nm typical for the thin films used. (d) Calculated $B_z$ along trace arrow indicated in (c) indicating an asymmetry in the magnetic field.

To spin-pumping measurements and spin torque ferromagnetic resonance, cast doubts on the origin of such signals. As became clear from our work, spurious effects such as local Hall voltages can play an important role in mimicking the detected voltage that supposed to originate from the spin accumulation in topological surface states. In order to remove such effects, nonlocal device schemes where spin and charge currents are disentangled can be an important step towards electrically detecting the spin polarization in topological insulators\textsuperscript{10}. Vaklinova et al. have reported on such a geometry for the first time by combining the spin-generating properties of Bi$_2$Te$_2$Se and the spin-preserving properties of graphene [53]. However, an investigation of the sign of the spin polarization was not made and the sign is difficult to deduce from the undetailed measurement geometry. Furthermore, the employed device structure

\textsuperscript{10}Besides this detection method, it has been recently proposed that the stray fields coming from the spin accumulation can be detected and disentangled from Oersted fields by modern sensing techniques [56]. Furthermore, spin-polarized four-probe STM could provide a good alternative where artifacts can be removed [57].
could lead to limited reproducibility due to a random ordering of the stack in the device.

An idea for an alternative geometry where spins can be injected/detected non-locally has been proposed by K. S. Das, member of the Physics of Nanodevices group. The geometry is based on a metallic nonlocal spin valve [58, 59] where spins from a permalloy (Ni_{80}Fe_{20}, Py) injector travel diffusively via an Al channel to the Py detector without a moving charge current. Now, in between the ferromagnetic contacts a spin-absorbing strip can be fabricated below or on top of the Al channel, which has given insights in the origin of spin Hall effects [60–63]. Using a topological insulator as such an absorber, a voltage in the strip will be generated upon injecting spin due to spin–momentum locking in the surface states. The advantage of such a geometry is that spins can be injected from the Py contact and detected by the topological insulator and vice versa such that reciprocity can be checked, similar to the work by Liu et al. [19]. Furthermore, such spin valves show a good reproducibility in the channel’s spin-relaxation times and the (temperature-dependent) ferromagnet’s spin polarization. The first steps towards realization of such a geometry are made, but the fabrication recipe still needs optimization.

As a proof of principle, a nonlocal metallic spin valve has been fabricated on a bare sapphire substrate that also serves as a substrate for MBE-grown Bi_{2}Se_{3}. After defining the big marker pattern by DUV lithography and electron-beam evaporation, the actual spin valve is designed by EBL. First, the ferromagnetic contacts are defined with different aspect ratios as to tune the coercive field of the contacts. Thereafter, layers of Ti(5) for adhesion and Py(20) have been deposited by electron-beam evaporation. In the second EBL step, the Al channels including contact leads to the channel as well as the ferromagnetic contacts are defined. Afterwards, the Py strips are cleaned by a 30-s Ar-ion etching procedure prior to the deposition of a thick Al(80) layer to ensure transparent interfaces. The result of such a device with different separations between the ferromagnet contacts is shown in figure 6.8a.

In regular spin-valve measurements, a current is sourced between the ferromagnetic contact and one end of the Al channel while the voltage is probed at the other ferromagnetic contact with respect to the other end of the Al channel, as shown in figure 6.8a. Depending on the relative magnetization of the ferromagnetic contacts, that can be controlled by an external magnetic field \( B \), the detected voltage is either high or low probing the different spin channels. As shown in figure 6.8b, such switches in the nonlocal resistance \( R_{NL} = V_{NL}/I \) are on the order of 1 m\( \Omega \) at room temperature. Unclear behavior in the bias dependence is observed that can be due to nonlinear responses such as thermal effects. This suspicion is confirmed by the large background resistance that can be related to thermal effects [64], which can be due to the relatively high thermal conductivity of Al_{2}O_{3}. In later measurements, such large backgrounds have not been observed which might hint at an effect related to the ‘cleanliness’ of the fabrication steps of the particular device, too. Furthermore, from the contact-separation dependence a rough estimation of the spin-diffusion length of 700±400 nm is found, which is twice as large as that found for a very similar spin valve on a Si/SiO_{2} substrate [65][11].

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[11] This is under the assumption that \( \lambda_{N} \ll d_{FM-FM} \). The polarization of the Py contacts is unknown and results for larger \( d_{FM-FM} \) are needed to make a proper fit.
Figure 6.8: (a) Device geometry and measurement setup for nonlocal spin valve measurements on a bare sapphire substrate. Per device six nonlocal spin valves are designed with the ferromagnets (brown) having different separation over the Al channel (blue). (b) Current-bias dependence of nonlocal resistance $R_{\text{NL}}$ as a function of magnetic field at room temperature (RT). A typical spin valve is clearly observed. The data for $I = 1$ mA has been taken earlier before the data for the other biases such that a shift is present. (c) Change in the nonlocal resistance $\Delta R_{\text{NL}}$ as a function of the separation between the ferromagnetic contacts $d_{\text{FM-FM}}$. An exponential fit (red line) yields a value for spin relaxation length $\lambda_N$ of $700^{+400}_{-200}$ nm.

The next step is to include Bi$_2$Se$_3$ as spin absorber along the Al channel with large-area, MBE-grown Bi$_2$Se$_3$ films as starting point. In order to isolate thin Bi$_2$Se$_3$ strips ($\sim$100 nm), etch lines have been used where fine lines in the resist are designed by EBL and underlying Bi$_2$Se$_3$ is removed by reactive-ion etching very similarly as described in section 6.3. This etching includes the areas of Bi$_2$Se$_3$ below the ferromagnetic contact and the Al channel. In order to protect the strips against the Ar-ion etching step, a small window has been created by EBL in which Al has been deposited. The subsequent steps of fabricating the ferromagnetic contacts and the Al channel are performed in a similar way as the first device described here. However, the adhesion of the small Bi$_2$Se$_3$ strips, as one can see from figure 6.9, as well as the Al pattern is
poor and remains a challenge at the moment of writing this thesis.

The fabrication issues have yielded a very low number of working devices, where there was no contact possible with the Bi$_2$Se$_3$ strips at all. In one spin valve fabricated from the MBE-grown Bi$_2$Se$_3$ films where there was no absorber strip in between, a change in the nonlocal resistance of only $\sim 50 \mu\Omega$ was measured at $T = 4$ K. This could be related to the poor Al adhesion to the etched substrate or to the remaining Bi$_2$Se$_3$ that can be of large influence such that a regular spin valve hardly shows any signal either. Since both Al$_2$O$_3$ and Bi$_2$Se$_3$ have a large thermal conductivity (see chapter 4) and Bi$_2$Se$_3$ has a considerably large Seebeck coefficient [66], thermal effects might play an important role in the functioning of the devices here, which can be an interesting direction in studying topological insulators (see section 7.2.2). If the fabrication issues can be overcome such that a proper interface is created between the absorber and channel, this geometry can be very powerful in that it automatically excludes many effects that cannot be ruled out by local geometries as used earlier. One solution to avoid the etching step in the recipe that potentially damages the topological insulator strip is to create a spin valve using the shadow evaporation deposition technique such that transparent interfaces between ferromagnet and channel are created.

As mentioned earlier, there is a lot of uncertainty about the quality of the tunnel barriers used in the different experiments which can be detrimental for the origin of the observed signals. Furthermore, it has been seen that two-dimensional electron gases can play a role in the transport. In order to improve on both these issues, our collaborators have grown an In$_2$Se$_3$ capping layer directly after the growth of Bi$_2$Se$_3$. Such a capping layer prevents doping at the surface as well as that due to the band gap of $\sim 1.3$ eV [67] it can serve as a tunnel barrier. From investigations by

![Figure 6.9: (a) Zoom-in at a single spin valve with ferromagnets (brown) and channel and contact leads (blue) fabricated on the Bi$_2$Se$_3$ (green) samples. The non-false-colored parts indicate the substrate as well as the realized Bi$_2$Se$_3$ strip in line with the arrow at the bottom. (b) Similar image of another junction but now the Bi$_2$Se$_3$ strip is clearly broken.](image)
Appendix

Burema [68], tunneling characteristics have been observed for a 10 QL capping layer (10 nm), as shown in figure 6.10a. However, from the extracted tunneling parameters using the Simmons model, it is seen that the effective thickness of the barrier is 10% of the grown thickness and can be partially related to the inhomogeneity of the layer as found by atomic force microscopy (figure 6.10b). Furthermore, a barrier height of $\sim 1.6$ eV has been observed at low temperatures, which is in good agreement with values from literature [67]. Such barrier characteristics could be for example enhanced by growing an additional oxide barrier on top of the capping layer.

![Figure 6.10](image_url)

Figure 6.10: (a) Temperature-dependent $I-V$ characteristics of 10 QL In$_2$Se$_3$ capping layer on top of Bi$_2$Se$_3$ with the top contact consisting of Ti(5)/Au(70). (b) AFM image of In$_2$Se$_3$ capping layer grown on top of Bi$_2$Se$_3$.

6.6 Appendix

6.6.1 Additional Measurements

For a ferromagnetic contact with $RA = 19 \text{k}\Omega \mu\text{m}^2$ with dimensions of $3 \times 3.5 \mu\text{m}^2$, similar results in the original geometry A as described in section 6.3 are obtained but with higher $\Delta V$ and lower signal-to-noise ratio indicating that the tunnel barrier influences the measurements. For a ferromagnetic contact with an $RA$ of $100 \text{k}\Omega \mu\text{m}^2$, the measured signals have been too noisy to observe any switches in the voltage.

Furthermore, $V_1$, $V_2$ and the difference signal $V_2 - V_1$ similar to those showed in figure 6.3 are plotted for a current bias of 10 $\mu$A in figure 6.12. One can observe the linear slope in $V_2$ which is clearly absent in $V_1$. The difference signal $V_2 - V_1$ shows a residual switch in the voltage around zero magnetic field, indicating that the fringe field from the ferromagnetic layer affects the potential at the Ohmic contacts too, which might be important for geometries where the Ohmic contact is in the transverse direction with respect to the ferromagnetic contact [24, 31, 33, 41]. Such effects could be studied by using a geometry as reported by Pham et al. [69], where one could shift the relative positions of the ferromagnets to enhance/suppress such fringe field effects at the points where the ferromagnets are in close contact. This method assumes that the fringe fields at the other side of the ferromagnetic contact are minimized by elongation of the contacts, by a reduced thickness of the channel, or by a supporting layer of for example Si$_3$N$_4$ as used by Li et al. [24].
Investigating charge-current-induced spin voltage signals in Bi$_2$Se$_3$

Figure 6.11: Measurements for $V_1$ using a contact with $RA = 19 \ \text{k}\Omega \mu\text{m}^2$ (geometry A).

(a) 

(b) 

(c) 

(d) 

Figure 6.12: Measurement of (a) $V_1$, (b) $V_2$ and (c) the differential signal $V_3$ at a current bias of 10 $\mu$A and $T = 4$ K. (d) Zoom-in of (c).

Subsequent measurements after the main results as presented above are performed using a modified geometry where the ferromagnetic contacts are elongated in such a way.
way to avoid stray fields interfering with the charge current, see figure 6.13a. Although, the yield of working contacts is low and the tunnel barrier performance is poor, it has been possible to obtain results for \( V_1 \) and \( V_2 \), see figure 6.13c and 6.13d, using the encircled contact in figure 6.13a. The changes in voltage are lower compared to those discussed above, which is related to the lower applied current bias and the larger cross section that yield a low current density. Interestingly, it is observed that the background for \( V_1 \) and \( V_2 \) are distinctly different from each other as well as that a clear anisotropic magnetoresistance (AMR) signal is present at zero magnetic field, most probably due to transparent barrier between Bi\(_2\)Se\(_3\) and the ferromagnet, which seems to be present at low current biases in another (dubious) work, too [34]. Removing this contribution leads to a peculiar result in which the voltage switches twice, where the second switch occurs at large fields of \( \sim 0.3 \) T (figure 6.13e). The exact origin is unclear, and the statistics are low, but it might be that the ferromagnetic layer is unstable such that the low voltage state switches at some point. Temperature-dependent studies show that the behavior of the voltage, except for the charge-current background, hardly change upon increasing temperature (not shown here) in contrast to the results in section 6.3. This suggests that the measured voltages are purely related to magnetoresistance effects from the ferromagnetic layer rather than to probing

Figure 6.13: (a) Device structure (100×100 \( \mu \)m\(^2\)) with Ti/Au contacts (bright yellow) on the Bi\(_2\)Se\(_3\) channel (green), etched structure (dark) and ferromagnetic contacts (brown). (b) Measurement geometry in which measured voltages \( V_1 \) and \( V_2 \) upon application of a current \( I_e \) along the \(-x\) direction are indicated. The ferromagnetic contact (lead) used is encircled in (a). (c) Voltage \( V_1 \) vs magnetic field \( B \) using the conventions drawn in (b) at a DC current bias of 25 \( \mu \)A and at \( T = 4.2 \) K. (d) \( \Delta V_2 \) vs \( B \) where a constant background voltage of –750 \( \mu \)V was subtracted. In blue a fit of the Lorentzian AMR is shown. (e) Residual signal \( \Delta V_2^* \) after subtracting the AMR fit.
of a spin accumulations, which is expected to be more temperature dependent.

6.6.2 Modeling of current spread

In order to estimate the current density at the edges of the ferromagnetic contact, we modeled the current density $J_x$ around the cross section, as shown in figure 6.14. The extracted FWHM is $1.3 \, \mu m$ indicating that the current density around edges of the ferromagnetic (having a total width of $3 \, \mu m$), where $B_z$ is large, is not negligible and can lead to the enhancement of Hall voltages. Furthermore, we find a considerable current spread in the topological insulator channel in the vicinity of both the Ohmic and ferromagnetic contacts which might influence the measured voltages as in section 6.6.1. The presence of a $J_y$ component experiencing a $B_z$ field will give rise to additional voltages in the $x$ direction.

Figure 6.14: (left) Current density spread at cross section of Hall bar. (right) Current density along trace arrow indicated in the left schematics. Extracted FWHM is $1.3 \, \mu m$.

6.6.3 Analytical expression for fringe fields

Consider a ferromagnet placed at $(0,0,0)$ with edges $(\pm x_{FM}, \pm y_{FM}, \pm z_{FM})$ of which we want to know the magnetic field at a point $P$ with coordinates $(x_P, y_P, z_P)$, see figure 6.15.

The ferromagnet is defined to have a magnetization $M = M\hat{y}$ with $M_{Co}$ for Co given by:

$$M_{Co} = 1.72\mu_B N = 1.72\mu_B \frac{\rho N_A}{A_{Co}} = 1.44 \times 10^6 A/m. \quad (6.10)$$

Every volume element $d\tau$ with respect to point $P$ is indicated with vector $r = (x - x_P, y - y_P, z - z_P)^{12}$. The magnetic field $B$ can be calculated from the vector potential $A$ via $B = \nabla \times A$ with $A$:

\footnote{The direction of the vector $r$ is opposite to what has been defined in [70], which will yield a minus sign in the final calculated magnetic field.}
Figure 6.15: Studied ferromagnetic slab with magnetization $\mathbf{M} = M\hat{y}$ and dimensions $3\times3\times0.035$ µm$^3$. The magnetic field is calculated at observation point $P$ from volume elements $d\tau$ at a position $r$ with respect to that point.

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{M} \times \hat{r}}{r^2} d\tau,$$

(6.11)

where $\mu_0$ is the permeability constant and $r^2 = (x - x_P)^2 + (y - y_P)^2 + (z - z_P)^2$. Plugging in the defined vectors in equation (6.11):

$$\frac{\mathbf{M} \times \hat{r}}{r^2} = \frac{M (z - z_P)}{r^3} \hat{x} - \frac{M (x - x_P)}{r^3} \hat{z}.$$  

(6.12)

Now, the curl of the vector potential $\nabla \times \mathbf{A}$ can be calculated:

$$\nabla \times \mathbf{A} \big|_x = \left\{ -\frac{\partial}{\partial y} \left[ \frac{M (x - x_P)}{r^3} \right] \right\},$$

(6.13a)

$$\nabla \times \mathbf{A} \big|_y = \left\{ -\frac{\partial}{\partial x} \left[ \frac{M (x - x_P)}{r^3} \right] - \frac{\partial}{\partial z} \left[ \frac{M (z - z_P)}{r^3} \right] \right\},$$

(6.13b)

$$\nabla \times \mathbf{A} \big|_z = \left\{ -\frac{\partial}{\partial y} \left[ \frac{M (z - z_P)}{r^3} \right] \right\}.$$  

(6.13c)

In this way, we reach to an analytical expression for $\mathbf{B}$ that has to be solved numerically:
For a slab with dimensions (±x_{FM}, ±y_{FM}, ±z_{FM})=(±1.5 \times 10^{-6}, ±1.5 \times 10^{-6}, ±17.5 \times 10^{-9}) (in meters) and with \( P = (0, 0, -19.5 \times 10^{-9}) \), a magnetic field of around 10 mT is found from numerical calculation with Mathematica. Notice that the position of \( P \) is 2 nm below the ferromagnet’s center to take into account the grown tunnel barrier. Towards the edges a magnetic field of around 800 mT is found, which is in good correspondence with the simulations as shown in figure 6.7.

6.7 References

References

Investigating charge-current-induced spin voltage signals in Bi$_2$Se$_3$


