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To cite this article: Laura Spierdijk & Michalis Zaouras (2017) The Lerner index and revenue maximization, Applied Economics Letters, 24:15, 1075-1079, DOI: 10.1080/13504851.2016.1254333

To link to this article: http://dx.doi.org/10.1080/13504851.2016.1254333

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The Lerner index and revenue maximization

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ABSTRACT
Based on profit-maximizing behaviour, the usual interpretation of the Lerner index is that a zero value reflects competitive behaviour, while a positive value is associated with market power. We investigate to what extent the usual interpretation of the Lerner index remains valid in a setting where firms do not pursue profit maximization, but instead maximize revenues subject to a minimum-profit constraint. We show that a positive Lerner index still indicates market power, but that the magnitude of a positive Lerner index can no longer be used to determine how much market power there is. Furthermore, extra information would be required to draw conclusions about the presence or absence of market power when the Lerner index is zero or negative. We discuss the empirical implications of our results.

KEYWORDS
Lerner index; market power; revenue maximization

JEL CLASSIFICATION
L13; L21

I. Introduction

Because the degree of competition among the firms in a sector or industry has important welfare implications for both consumers and firms (e.g. Bikker 2004), the assessment of firms’ market power is the topic of many theoretical and empirical studies in the economic literature.

Measures of market power often rely on the assumption of profit-maximizing behaviour (e.g. Hay and Liu 1997; Shaffer 2004; Boone 2008; Bikker, Shaffer, and Spierdijk 2012). However, Baumol (1958) already argued that firms in oligopolistic markets are more likely to maximize revenues subject to a minimum-profit constraint rather than to pursue profit maximization; a theory for which the early literature found some empirical evidence (e.g. Amihud and Kamin 1979; Winn and Shoenhair 1988). More recently, Segerson and Squires (1995) argued that the appropriate short-run behavioural assumption for a multi-product firm is revenue maximization. Other alternative pricing strategies that have been considered in the literature are limit pricing (Milgrom and Roberts 1982) and constant mark-up pricing (Rosse and Panzar 1977).

A widely used measure of market power is the Lerner index, whose theoretical and historical foundation has been extensively discussed in the literature (e.g. Landes and Posner 1981; Elzinga and Mills 2011; Giocoli 2012). The Lerner index compares a firm’s output price with its associated marginal costs, where marginal cost pricing is referred to as the ‘social optimum that is reached in perfect competition’ (Lerner 1934, 168). The standard interpretation of the Lerner index is that a zero value reflects competitive behaviour, while a positive value is associated with market power. However, this interpretation is directly derived from profit-maximizing behaviour, as we will see later.

This leads to the fundamental question to what extent the usual interpretation of the Lerner index remains valid in a setting where firms do not pursue profit maximization. As argued by Cairns (1995), a measure of market power ‘[…] should be able to provide a meaningful summary measure of monopoly power in any situation, not just that in which the firm is maximizing profits somehow defined.’ If the common interpretation is no longer valid in the absence of profit-maximizing behaviour, this will have stark implications for empirical studies using the Lerner index. Such studies do not test whether firms actually pursue profit maximization, but rely on the standard interpretation of the Lerner index anyhow (e.g. Fernández De Guevara and Maudos 2004; Koetter, Kolari, and Spierdijk 2012).

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The supplemental data for this article can be accessed here.

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The goal of this study is to investigate whether the interpretation of the Lerner index as a measure of market power is robust to deviations from the profit-maximization paradigm. We focus on revenue maximization as alternative pricing strategy because of two reasons. First, its empirical relevance has already been pointed out by, e.g., Segerson and Squires (1995). Second, revenue maximization subject to a minimum-profit constraint encompasses several limiting cases, including sales maximization subject to a break-even constraint, and even profit maximization. This makes it a convenient and fairly general framework to analyse.

Our main results are as follows. When firms maximize revenues subject to a minimum-profit constraint, we can safely conclude that they possess market power when the Lerner index is significantly positive. However, we can no longer use the magnitude of the Lerner index to determine how much market power they have. Furthermore, additional information would be required to draw conclusions about the presence or absence of market power when the Lerner index is zero or negative. In particular, without such information, we can no longer conclude that a zero Lerner index implies the absence of market power. We show that statistical tests for profit maximization (Varian 1984; Love and Shumway 1994) can contribute to a correct interpretation of the Lerner index.

II. The Lerner index under profit maximization

We consider a profit-maximizing firm with a single-output production technology. Let \( P(q) \) denote the inverse demand function satisfying \( P'(q) \leq 0 \) for \( q > 0 \). Furthermore, let \( C(q) \) denote total production costs as a function of output, with corresponding marginal cost function \( MC(q) > 0 \) for \( q > 0 \). The Lerner index is defined as a firm’s relative mark-up of the output price over marginal cost, given the firm’s output level \( q > 0 \):

\[
L(q) = \frac{P(q) - MC(q)}{P(q)}. \tag{1}
\]

Under profit maximization, \( L(q^*) = \epsilon_d^{-1}(q^*) \). Here \( q^* \) is the profit-maximizing output quantity and \( \epsilon_d^{-1}(q^*) = -P'(q^*)q^*/P(q^*) \) the firm’s inverse price elasticity of demand evaluated in \( q^* \) (Lerner 1934, 169). We have \( L(q^*) \geq 0 \), while \( L(q) < 1 \) for any \( q > 0 \).

Under profit maximization we can thus distinguish competitive \( (\epsilon_d = \infty) \) from uncompetitive \( (\epsilon_d(q^*) < \infty) \) behaviour on the basis of the sign of the Lerner index. Furthermore, the value of the Lerner index is monotonically associated to market power.

Lerner (1934, 170) suggested that the relative difference between the observed price and marginal cost can always be used to assess a firm’s market power, even in the absence of profit maximization. This suggestion has been adopted by the empirical literature in many fields of study, where the Lerner index is widely viewed as a standard tool to assess a firm’s market power regardless of the firm’s objective function.

However, the equality in \( P(q^*) \geq MR(q^*) = MC(q^*) \) does no longer hold in the absence of profit maximization. Consequently, in such a scenario, the Lerner index may become zero or even negative in the presence of market power. We will investigate the implications for the interpretation of the Lerner index in the next section.

III. The Lerner index under revenue maximization

We assume that a firm maximizes revenues subject to a minimum-profit constraint (Baumol 1958; Kafoglis and Bushnell 1970). This section characterizes the optimal output of a revenue maximizing firm and considers the measurement of market power by means of the Lerner index.

Characterization of optimal output

We assume that the following conditions hold, for \( \pi_0 \geq 0 \):

Assumption 1

(i) \( \pi(0) < \pi_0 \).
(ii) \( MR(q) = P'(q)q + P(q) \geq 0 \) for \( q > 0 \).
(iii) \( MC(q) > 0 \) for \( q > 0 \).
(iv) The profit function \( \pi(\cdot) \) is concave with the unique profit-maximizing output level \( q^* \) and maximum profit level \( \pi_{\text{max}} = \pi(q^*) \).

For a given minimum-profit level \( \pi_0 \geq 0 \), the firm’s optimization problems equals
max\(_{q \geq 0} P(q)q\) \quad (2)

s.t. \(\pi(q) = P(q)q - C(q) \geq \pi_0\). \quad (3)

We notice that \(\pi_0 = 0\) corresponds to revenue maximization subject to a break-even constraint and \(\pi_{\text{max}}\) to profit maximization.

The necessary conditions for an optimal solution are summarized below, which is an extension of Kafoglis and Bushnell (1970) because it includes the cases that \(\mu(\pi_0) = 0\) and \(\mu(\pi_0) = \infty\).

**Result 1** Under Assumptions 1 (i) – (v), if there is a solution \(q^*\) to the firm’s optimization problem, it must satisfy \(\mu(q^*) > 0\) and the following first-order condition:

\[
P'(\tilde{q})\tilde{q} + P(\tilde{q}) = \frac{\mu(\pi_0)}{1 + \mu(\pi_0)} MC(\tilde{q}),
\]

for the Kuhn–Tucker multiplier \(0 \leq \mu(\pi_0) \leq \infty\).

**Proof:** The Kuhn–Tucker conditions for the firm’s optimal output \(\tilde{q}\) are

\[
P'(\tilde{q})\tilde{q} + P(\tilde{q}) = -\mu(\pi_0)[P'(\tilde{q})\tilde{q} + P(\tilde{q}) - MC(\tilde{q})]; \quad \text{[optimality]}
\]

\[
P(\tilde{q})\tilde{q} - C(\tilde{q}) - \pi_0 \geq 0; \quad \text{[feasibility]}
\]

\[
\mu(\pi_0)[P'(\tilde{q})\tilde{q} - C(\tilde{q}) - \pi_0] = 0; \quad \text{[complementary slackness]}
\]

\[
\mu(\pi_0) \geq 0; \quad \text{(8)}
\]

\[
\tilde{q} \geq 0. \quad \text{(9)}
\]

Because \(\pi(0) < \pi_0\), we must have \(\tilde{q} > 0\). We can rewrite Condition (5) as

\[
P'(\tilde{q})\tilde{q} + P(\tilde{q}) - \frac{\mu(\pi_0)}{1 + \mu(\pi_0)} MC(\tilde{q}) = 0. \quad \text{(10)}
\]

**Case 1:** If \(MR(\tilde{q}) > 0\), Equation (10) implies that \(\mu(\pi_0) > 0\). To show that \(\mu(\pi_0) = \infty\) is possible, assume that \(\varepsilon_d < \infty\) and \(\pi_0 = \pi_{\text{max}}\). To prove that \(\tilde{q} = q^*\) is a solution such that \(\mu(\pi_{\text{max}}) = \infty\), we have to verify feasibility and optimality. Evidently, \(P(q^*)q^* - C(q^*) - \pi_{\text{max}} = 0\). Furthermore, Equation (10) is satisfied for \(\tilde{q} = q^*\) and \(\mu(\pi_{\text{max}}) = \infty\) because \(MR(q^*) = MC(q^*)\) under profit maximization. It is straightforward to verify that also competitive behaviour \((\varepsilon_d = \infty, P = MC)\) yields a solution with \(\mu(\pi_0) = \infty\).

**Case 2:** If \(MR(\tilde{q}) = 0\), then we must have \(\mu(\pi_0) = 0\) because of Equation (10) and \(MC(\tilde{q}) > 0\). With \(\mu(\pi_0) = 0\), Condition (7) implies that the minimum-profit constraint is not binding. Because \(MR(\tilde{q}) = 0\) and \(MC(\tilde{q}) > 0\), this case thus excludes profit maximization and competitive behaviour for which \(MR = MC > 0\).

An implication of Result 1 is that \(MC(\tilde{q}) < P(\tilde{q})\) and \(P(\tilde{q}) < P(q^*)\) for \(\mu(\pi_0) < \infty\).

**The Lerner index and market power**

Given the optimal output level \(q^*\) under revenue-maximizing behaviour subject to a minimum-profit level, we can rewrite the associated Lerner index on the basis of Equation (10) as

\[
L(q) = \varepsilon_d^{-1}(q) - \frac{1}{1 + \mu(\pi_0)} \frac{MC(q)}{P(q)}. \quad \text{(11)}
\]

It is readily seen that \(L(q) \leq \varepsilon_d^{-1}(q)\), with strict inequality for \(\mu(\pi_0) < \infty\). We thus observe that \(L(q) \leq 0\) under competitive conditions \((\varepsilon_d = \infty)\). More specifically, we find \(L(q) < 0\) for \(\varepsilon_d = \infty\) and \(0 \leq \mu(\pi_0) < \infty\), while \(L(q) = 0\) for \(\varepsilon_d = \infty\) and \(\mu(\pi_0) = \infty\). Both \(L(q) < 0\) and \(L(q) > 0\) can arise under uncompetitive conditions. Using a similar continuity argument as Bikker, Shaffer, and Spierdijk (2012), it follows that \(L(q) > 0\) is also possible under uncompetitive conditions. The online appendix with supplementary material provides specific examples to illustrate that \(L(q) < 0, L(q) = 0\) and \(L(q) > 0\) are indeed possible under uncompetitive conditions, and that \(L(q) \leq 0\) under competitive conditions.

**Empirical implications**

The main implication of our results is that, under revenue maximization subject to a minimum-profit constraint, we can only use the Lerner index as a one-sided test for market power in the following sense. Given an empirical estimate of \(L\) (denoted \(\hat{L}\)), we distinguish two cases. If statistical tests show that \(\hat{L}\) is significantly positive, we conclude that there is market power. However, we can no longer use the magnitude of a positive Lerner index to determine how much market power there is. This happens
because the Lerner index’ competitive benchmark value is no longer 0 (instead, non-positive values may arise in competitive cases). If \( \hat{L} \) is not significantly different from 0 or significantly negative, we can draw no conclusions about the degree of market power since \( L \leq 0 \) can occur under both competitive and uncompetitive conditions. Additional information would be required in this case. In both cases, statistical tests for profit maximization (Varian 1984; Love and Shumway 1994) can contribute to a correct interpretation of the Lerner index; see Table 1.

**Table 1. Conclusions on the basis of \( L \) and the outcome of a profit maximization test.**

<table>
<thead>
<tr>
<th>Test (( \hat{H}_0 ): profit max.)</th>
<th>Value of ( L )</th>
<th>Market power present?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject</td>
<td>( L &gt; 0 )</td>
<td>Yes, but unclear how much</td>
</tr>
<tr>
<td>Reject</td>
<td>( L \leq 0 )</td>
<td>Unknown; additional information needed</td>
</tr>
<tr>
<td>Fail to reject</td>
<td>( L &gt; 0 )</td>
<td>Yes, and there is more market power the closer ( L ) to 1</td>
</tr>
<tr>
<td>Fail to reject</td>
<td>( L = 0 )</td>
<td>No</td>
</tr>
<tr>
<td>Fail to reject</td>
<td>( L &lt; 0 )</td>
<td>Unlikely to occur because ( L \in [0,1) ) under profit maximization</td>
</tr>
</tbody>
</table>

**Acknowledgment**

Laura Spierdijk gratefully acknowledges financial support by aVidi grant (452.11.007) in the Vernieuwingsimpuls-program of the Netherlands Organization for Scientific Research.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

**Funding**

Laura Spierdijk gratefully acknowledges financial support by the Vidi grant [452.11.007] in the Vernieuwingsimpuls-program of the Netherlands Organization for Scientific Research.

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