The complementarity between risk adjustment and community rating: distorting market outcomes to facilitate redistribution*

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Abstract

We analyze optimal risk adjustment in competitive health-insurance markets when insurers have better information on their customers’ risk profiles than the sponsor of health insurance. In the optimal scheme, the sponsor uses reinsurance to screen insurers with bad and good risks, in order to lower premiums for enrollees with high expected healthcare costs. We then explore the effects of adding a community-rating requirement to complement this risk-adjustment scheme. With community rating, insurers have incentives to distort contract generosities to cherry-pick low-cost consumers. However, the reduced generosity for low-cost types makes screening through reinsurance easier, allowing the sponsor to redistribute more. When costs for reinsurance are low, or the sponsor’s bias towards high-cost consumers is high, community rating dominates risk rating.

Keywords: health insurance, cherry-picking, risk adjustment, mechanism design

JEL classification: I13, D02, D47

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1. Introduction

We study a competitive health-insurance market, in which a regulator (‘the sponsor’ ) intervenes, with the dual aims to reduce distortions due to adverse selection and to achieve redistribution from healthier, low-cost consumers, to poor-health, high-cost ones. Two standard policies to achieve these goals are risk adjustment – where the sponsor taxes insurers of low-cost types and subsidizes those of higher-cost types – and a community-rating requirement, which obliges insurers to set premiums for a given contract independent of observable characteristics of the consumer buying that contract. We explore the interaction between those two policies in a setting with asymmetric information between insurers and sponsor: insurers have better information on their consumers’ health status than the sponsor, and insurers can offer policies with qualities or generosities that cannot be fully contracted on by the sponsor.

Many countries use risk-adjustment schemes to reduce adverse selection and achieve redistributional goals. *Ex-ante* risk adjustment taxes or subsidizes an insurer based on observable characteristics of its insured that provide a signal of expected health costs. By equalizing expected healthcare costs, the sponsor reduces selection incentives for insurers and brings insurance premiums for consumers of different health characteristics closer together, in that way achieving the desired redistribution. Such *ex-ante* risk adjustment requires verifiable data that is relatively easy to obtain for the sponsor of health insurance.

In practice, however, the insurer usually has more information on its insured than the sponsor, and insurers can use that information to their advantage. For instance, Brown et al. (2014) show that in Medicare Advantage, insurers succeed in enrolling customers that are relatively less costly compared to the risk adjustment payments received for them, and Geruso and Layton (2015) demonstrate how in Medicare Advantage, insurers use upcoding to increase risk-adjustment payments.\(^1\) When the insurer is better informed on its consumers’ types, the sponsor has to elicit truthful information from the insurers on their enrollees’ health costs. To do so, the sponsor can use *ex-post* risk adjustment: it compensates insurers for consumers that turn out to be costly ex post by repaying part of the realized costs. This is a form of risk sharing or reinsurrance, with the risk adjuster playing the role of the reinsurer (see e.g. Swartz, 2003; Dow, Fulton and Baicker, 2010). As this reduces the underlying cost differences, the insurance

\(^1\) Also, the sponsor may not want to use some variables correlated with expected healthcare costs for ethical reasons, think of ethnicity or religion, or because including them would decrease insurers’ incentives to reduce costs, see Van de Ven and Ellis (2000) and Van de Ven and Schut (2011, pp. 384) for a discussion.
contracts vary less with risk type.

Ex-post risk adjustment leaves no scope for insurers to game the system, because realized costs are observable to the sponsor. Instead, the downside of ex-post risk adjustment is that insurers’ incentives for cost containment are muted when the sponsor acts as a reinsurer (Dow, Fulton and Baicker, 2010). In practice, risk-adjustment systems often include ex-post components (see for instance Van de Ven et al., 2003, who describe how risk sharing is combined with ex-ante payments in various European countries). In the US, the Health Insurance Exchanges established under the Affordable Care Act combine a transitory reinsurance program, and a risk-adjustment scheme that will take current period’s diagnoses as inputs (HHS, 2012). Also the Dutch risk-adjustment system contains elements of ex-post reinsurance, alongside ex-ante risk adjustment for observed characteristics, though the explicit ex-post component is currently being gradually phased out to stimulate insurers to contain healthcare costs.

Since reinsurance is socially costly, in the presence of information asymmetry between insurers and sponsor, optimal risk adjustment will be incomplete and leave room for selection and premium differences between consumer types. To complement an imperfect risk-adjustment system, sponsors can impose premium restrictions on contracts that insurers may offer. In practice, one important restriction is a community-rating requirement, see Gale (2007) for an overview. Community rating (CR) means that insurers have to accept any customer and charge the same price to each customer for a given contract. Policy makers’ motivation for CR is to enforce solidarity among high-risk and low-risk consumers on the health-insurance market. In the absence of CR, and assuming insurers and consumers have symmetric information on their types, insurers can engage in third-degree price discrimination, also known as risk rating (RR), and charge high (low) prices to high (low) risk consumers.

While CR may increase redistribution, a drawback is that it increases selection incentives for insurers. If contract quality is not fully contractible to the sponsor, this may lead to distortions in contracts offered in the market, reducing efficiency and potentially undoing the intended redistribution. In practice, insurers have a lot of scope to distort contract generosity in ways that are hard to regulate by the sponsor. Shepard (2016) demonstrates how the insurer’s

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2 Note that providing such incentives is often the reason for having private, competitive health insurance in the first place.

3 As Geruso and McGuire (2016) argue, many ex-ante risk-adjustments have some ex-post characteristics, to the extent that they include past treatment choices. Choosing for treatment today will then influence ex-ante risk payments next year, which influences the insurer’s incentives if consumer switching rates are low.

4 This is also referred to as ‘pure community rating’. Less restrictive forms might allow for some rate differentiation according to, for instance, age.
provider network affects selection of enrollees. Decarolis and Guglielmo (forthcoming) analyze changes in contract generosity including soft measures such as customer service, or healthcare quality, in response to changing selection incentives. Carey (2017) documents how insurers use drug benefit design to select more profitable enrollees. As these dimensions are not easily contractible for the sponsor, the insurer has an advantage which can be used to game the system. In particular, the insurer tries to cherry-pick insured whose expected costs are low within their risk-adjustment class.

In this paper, we explore optimal risk adjustment when insurers have private information on consumers’ cost types, and can use distortions in contract generosities to screen consumers. We then ask how optimal risk adjustment interacts with a premium restriction: in a second-best world, can a CR requirement be an efficient complement to a risk-adjustment scheme? To do so, we take a mechanism design approach: how can the sponsor optimally elicit the insurers’ private information on their consumers’ expected costs? We consider a two-tiered contracting model with perfectly competitive insurers who offer a menu of contracts to consumers in Rothschild-Stiglitz fashion. The insurers’ incentives for attracting high- or low-cost consumers are in turn determined by the sponsor’s risk-adjustment mechanism. We show that the sponsor can use ex-post risk adjustment to screen insurers on the privately observable part of expected costs.

We find that optimal risk adjustment offers the insurer a choice whether or not to buy some reinsurance for their customers. The scheme therefore involves subjective risk adjusting as in Sappington and Lewis (1999). Paying a tax in exchange for high ex-post reinsurance is attractive for an insurer who knows his customers have high expected healthcare costs. Conversely, for an insurer who faces customers with low expected healthcare costs, the costs of reinsurance are higher than the benefits. This insurer in fact prefers to contribute to the risk-adjustment fund instead, subsidizing the high types. In this way, optimal risk adjustment targets the information advantage of the insurers vis-a-vis the sponsor, and allows the sponsor to tax low-risk types to subsidize the high-risk types.

When insurers are allowed to vary premiums for a given contract according to a consumer’s

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5See also Geruso, Layton and Prinz (2016) and Lavetti and Simon (2016).

6Van de Ven and van Vliet also suggested a risk-adjustment scheme involving subjective risk adjustment: “Let an insurer himself decide –within certain boundaries– for which patients, or for which types of care, or to what extent he wants to share the risk with the Central Fund. (…) An important advantage of such a flexible form of risk sharing would be that the additional information the insurer might have about the residual predictable risk that is not accounted for in the capitation payment, will not be employed for cream skimming, but will be reflected in the preferred form of risk sharing.” (Van de Ven and van Vliet, 1992, italics are in the original text).
observable type – a risk-rating regime, or RR – contract prices will reflect those cost differences, and insurers have the incentive to provide efficient contract generosities. By equalizing costs, risk adjustment then serves the goal of bringing prices for different consumers closer together, and in this way promotes redistribution.

The interaction of risk adjustment with CR is subtler. With a CR requirement, insurers can no longer engage in direct price discrimination. Instead, insurers have the incentive to introduce distortions in contract generosities to screen consumers: with mandatory insurance, the market will feature lower-generosity insurance plans that are cheap and attractive to low-cost consumers, as well as more generous but expensive plans that attract high-cost consumers. In this environment, risk adjustment serves two purposes: not only does it bring prices closer together, but it also reduces distortions in insurance generosities.

Conversely, CR also affects the effectiveness of risk adjustment. With CR, the reduced generosity for insurance contracts aimed at low-cost consumers decreases costs for insurers to cover this type of enrollees. In turn, this cost reduction for low-type insurance decreases the benefits of reinsurance for such insurers, allowing the sponsor to screen those insurers more easily and, through larger transfers, redistribute more towards the high types.

The sponsor now faces a trade-off in deciding whether to impose a CR requirement. On the one hand, imposing CR drives insurers to distort non-contractible generosities, in an effort to screen consumers. These distortions result in a loss of welfare. On the other hand, these distortions make risk adjustment more effective, enabling the sponsor to redistribute more from low-cost to high-cost consumers. With optimal risk adjustment, whether CR dominates RR is then determined by the moral-hazard costs of reinsurance, and the degree of bias the sponsor has towards high-cost consumers. Imposing CR is more likely to be optimal when moral hazard for insurers is limited, or when the sponsor has a large bias towards high-cost consumers.

We analyze the trade-offs of CR in a setting with optimal risk adjustment, where reinsurance is optional, and only chosen by high-cost insurers. In practice, risk-adjustment policies are usually the same for all types, but similar trade-offs arise. With constant reinsurance across insurer plans, reinsurance is more valuable to insurers with high-cost enrollees than to those with low costs. When all insurers are reinsured and pay the same transfer to the sponsor, lower-cost insurers pay more than their actuarially fair share, and in this way cross-subsidize the high types. Since CR amplifies the cost differences between the types (as low-cost consumers get suboptimal insurance), redistribution is more effective with CR also when risk adjustment is
the same for both types.\footnote{In this case, however, the magnitude of moral-hazard costs is more important, as under this non-optimal risk adjustment, also low-types are reinsured and moral-hazard costs are incurred for them.}

Our paper relates to several strands of literature. First, our model builds on the adverse selection framework of Rothschild and Stiglitz (1976). On top of this asymmetric information problem between insurers and insured, we add a second layer of asymmetric information: between insurers and the health-insurance sponsor. In Rothschild-Stiglitz, CR induces inefficient under-insurance of low-risk types. Buchmueller and DiNardo (2002) observe this theoretical prediction in the real world. They document a decrease in coverage for the (healthier) young when the state of New York imposed a CR mandate. Moreover, if insurers cannot reduce coverage, or other dimensions of generosity, sufficiently to separate high from low-cost types, the insurance market may enter a death spiral, where lower-cost types drop out of the market entirely, see Cutler and Reber (1998). Chetty and Finkelstein (2013) provide a recent overview of selection effects in insurance markets.

Second, our paper connects to the risk-adjustment literature; see Van de Ven and Ellis (2000) and Ellis (2008) for overviews of this literature. Our paper uses a mechanism-design approach in which ex-post risk adjustment is used to elicit the insurer’s private information about customer types, as in Sappington and Lewis (1999). Whereas they focus on providers who can engage in high-risk patient dumping, our model focuses on insurers who compete and offer insurance to all consumers. Selection here takes the form of reduced generosity for low-cost types. In contrast to Sappington and Lewis (1999), in our work the contract distortion is endogenous, and takes place on the second tier in a hierarchical contracting model.

Also in this strand of the literature is Glazer and McGuire (2000), who explore a model where both the sponsor and the insurers have an imperfect signal about a customer’s type. Glazer and McGuire show how this imperfect signal should be incorporated in the risk-adjustment system to get an efficient outcome. In our model, the sponsor has no signal about the types: to the sponsor these types are observationally equivalent. But the insurer does have more information, and this allows it to game the system. We show how the sponsor can address this problem by offering ex-post risk adjustment.

Finally, our model connects to the literature on contracting hierarchies. We consider a two-tiered model of insurers designing contracts for consumers, which in turn depend on risk-adjustment contracts designed by the sponsor for the insurers. Related models are those of DeMarzo, Fishman and Hagerty (2005) and Faure-Grimaud, Laffont and Martimort (1999),
who look at an intermediary supervising an agent, and ask how the principal’s arrangements with the intermediary affect those downstream contracts.

This paper is organized as follows. We first describe the two-tier model of perfectly competitive insurers offering incentive-compatible contracts to consumers and the sponsor offering incentive-compatible risk-adjustment contracts to insurers. We then analyze optimal risk adjustment under both RR and CR requirements. We derive when and why CR outperforms RR. We finally discuss robustness of our results to alternative modelling assumptions, and in particular verify that similar results hold in a model with imperfect competition. Proofs can be found in the appendix.

2. The model

We consider a model of a competitive health-insurance market, regulated by a sponsor who designs a risk-adjustment scheme in order to redistribute towards high-cost consumers. The model consists of two layers. The lower level is a standard Rothschild-Stiglitz market in which competitive insurers offer insurance contracts to consumers. Insurance contracts consist of a price $p$, and a generosity $g$; the high-cost consumers’ higher willingness to pay for generosity enables insurers to screen different types. The upper level of the model consists of the sponsor, who regulates this market by stipulating whether insurers are allowed to condition prices on observed consumer types, and by implementing a risk-adjustment scheme.

2.1. Consumers, insurers, and the sponsor

We use a Rothschild-Stiglitz model for the interactions between consumers and insurers. Consumers are of two possible types. A fraction $\phi \in [0,1]$ of consumers has low expected healthcare costs, while the remaining $1 - \phi$ consumers have high expected costs. High-cost and low-cost consumers need treatment with probabilities $\theta^h$ and $\theta^l$ resp., with $\theta^h > \theta^l$.

Competitive insurers offer insurance contracts $(p,g)$, where $p$ is the premium and $g \geq 0$ is an inverse measure of generosity, or a decrease in quality. Full insurance corresponds with $g = 0$, while offering insurance with $g > 0$ reduces costs to the insurer, but causes a larger disutility to the consumer. We think of $g > 0$ as customers being forced to wait too long for insurer response to queries, experiencing long waiting times before treatment, or being faced

\footnote{For ease of exposition, we do not consider $g < 0$.}
with provider networks that are overly narrow, while we define $g = 0$ to be first-best generosity. Insuring a type-$\theta$ consumer with a contract of quality $g$ costs $\theta(y - g)$. Here $y$ are expected healthcare costs conditional on illness. $y$ is equal for high-cost and low-cost consumers.

A $\theta$-type consumer who accepts a contract $(p, g)$ has utility $u(p, g|\theta)$ with the following standard and intuitive properties. First, if insurance is fairly priced, that is $p = \theta(y - g)$, then consumer surplus is maximized at efficient generosity, $g = 0$: at $g = 0$, utility falls with a marginal increase in $g$ at the rate $\theta$. Intuitively, for generosity close to 0, inefficient $g$ is only a second-order effect, and the increase in $g$ reduces utility with the probability $\theta$ that the lack of optimality is experienced. For $g > 0$, a further increase in $g$ has a first-order effect on utility. Secondly, at efficient generosity the types do not differ in their marginal utility of consumption at a given price $p$. Finally, the marginal utility of consumption is decreasing.\(^9\) Formally, these properties can be stated as

\begin{align*}
-u_p(p, 0|\theta)\theta + u_g(p, 0|\theta) &= 0 \quad (2) \\
-u_p(p, g|\theta)\theta + u_g(p, g|\theta) &< 0 \text{ for } g > 0. \\
u_p(p, 0|\theta^h) &= u_p(p, 0|\theta^l). \quad (3) \\
u_{gp}(p, g|\theta) &\leq 0. \quad (4)
\end{align*}

We assume that all consumers participate in the insurance market; either because health insurance is mandatory or because a sponsor is willing to subsidize health insurance such that everyone prefers buying insurance.

Finally, we assume that high-cost consumers have a higher willingness to pay for increased generosity (lower $g$) than low-cost ones, captured in the following single-crossing condition.\(^10\)

**Assumption 1.** Consider two insurance contracts $(p_1, g_1), (p_2, g_2)$. Then $g_1 < g_2$ if and only if

\[ u(p_1, g_1|\theta^h) - u(p_2, g_2|\theta^h) > u(p_1, g_1|\theta^l) - u(p_2, g_2|\theta^l). \quad (5) \]

\(^9\)A simple example where all these conditions hold is the case where $g$ measures any residual disutility to the consumer after falling ill, and consumer risk aversion is modelled with mean-variance utility:

\[ u = w - p - \theta g - \frac{1}{2}r\theta(1 - \theta)g^2, \quad (1) \]

where $w$ denotes the agent’s wealth and $r > 0$ is a measure of risk aversion.

\(^{10}\)See, for instance, Fudenberg and Tirole (1991, chapter 7) for a discussion of the single-crossing condition.
More generosity (lower $g$) is preferred by both types, but more so by the high-cost consumer. This will allow insurers to separate high-cost consumers from low-cost consumers in situations in which direct price discrimination is banned, as we will discuss shortly.

Insurers observe the consumers’ types, and in the absence of intervention by the sponsor, they can offer contracts with a price that reflects a type’s costs. We will refer to this as a risk-rating, or RR regime: insurers condition contracts – in particular, the price – on this observable consumer type. In the competitive equilibrium, insurers offer contracts $(p^l = \theta^l y, g^l = 0)$ to low-cost consumers, and $(p^h = \theta^h y, g^h = 0)$ to high-cost consumers. In this RR regime, consumers get first-best insurance, but high-cost consumers pay the higher price that reflects their higher expected costs.

A sponsor may wish to contract with insurers in order to redistribute from low-cost consumers to high-cost consumers. To model this preference for redistribution, we assume that the sponsor maximizes weighted welfare and attaches weight $1 - \omega$ to high-cost consumers’ utility and $\omega \in [0, \phi]$ to low-cost consumers’ utility. If $\omega = \phi$ then the sponsor maximizes total welfare. Since we assume perfect competition in the insurer market, insurer profits will be zero and the sponsor maximizes an objective

$$W = \omega u(p^l, g^l|\theta^l) + (1 - \omega) u(p^h, g^h|\theta^h). \quad (6)$$

Solidarity is captured in two ways. First, the possibility of giving a relatively higher welfare weight to high-cost consumers than their fraction in the population would justify ($\omega < \phi$), expresses a solidarity or equity motive on the part of the sponsor.11 Second, because $u_{pp} < 0$ the planner tries to keep $p^h$ and $p^l$ close together.12

If contract generosity $g$ is contractible for the sponsor, the sponsor may require insurers to offer first-best generosity $g = 0$. In addition, the sponsor may prohibit risk rating and instead impose community rating, CR. In a CR regime, any contract offered in the market must be accessible to any consumer type. If the sponsor imposes CR, insurers are not allowed to condition prices for this contract on consumers’ types, and in the competitive equilibrium

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11Fleurbaey and Schokkaert (2011) give an overview of why the sponsor might have such equity considerations. See Bijlsma, Boone and Zwart (2014) in which a similar modelling approach is used to capture solidarity.

12Another consideration motivating redistribution from low to high risks is reclassification risk, as emphasized in Handel, Hendel and Whinston (2015). In our paper, we consider an essentially static market where consumers buy insurance only once, and their health type is fixed. In a repeated game, types can be dynamic. Higher welfare weights for the high-risk types may then be viewed as the government wishing to insure consumers against transitioning to a high-risk type sometime in the future.
they will offer these $g = 0$ contracts at population average costs, $p = (\phi \theta^l + (1 - \phi)\theta^h)y$. In this case, low-cost consumers cross-subsidize high-cost consumers, which is optimal from the sponsor’s solidarity objective.

If, however, $g$ is not contractible for the sponsor, under CR insurers have an incentive to screen consumers using distortions in generosity $g$. While CR doesn’t allow insurers to charge different prices to low-cost and high-cost consumers for the same contract, insurers can engage in second-degree price discrimination by offering contracts of differing prices and generosities, $p^h, g^h$ and $p^l, g^l$, designed so that consumers of different types self-select into the contract aimed at their type.

The Rothschild-Stiglitz separating equilibrium\(^\text{13}\) under CR involves a distortion in generosity $g^l$ offered to low types, to $g_{RS} > 0$ defined by

\[
u(p^h, 0|\theta^h) = \nu(p^l, g_{RS}|\theta^h),\tag{7}
\]

in which $p^h = \theta^h y$, $p^l = \theta^l(y - g_{RS})$. In this separating equilibrium, high-cost consumers are offered a full-insurance contract but pay the full costs associated to their types, while low-cost consumers can buy insurance at lower cost, but are not fully insured since their contract generosity is distorted: $g_{RS} > 0$. The consumer incentive compatibility condition (7) makes sure that high-cost consumers cannot gain by buying the low-cost consumers’ lower-priced contract.

We see then, that CR succeeds in achieving redistribution towards high-cost consumers if contract generosity $g$ is contractible for the sponsor. Without such contractibility, however, under CR, consumers get the same deal as under RR: they get optimal insurance but pay the full costs. Low-cost consumers do worse under CR than under RR: while they do obtain the lower price associated with their lower expected costs, in the separating equilibrium these low-cost consumers obtain inefficient insurance, $g_{RS} > 0$, unlike in the RR situation where $g^l = 0$ is at the efficient level.

However, as we argued in the introduction, the examples of $g$ suggest that in reality generosity $g$ may not be contractible for the sponsor. When is the insurer’s response to queries too slow?, when is the provider network too narrow?, etc.

Without contractibility of insurance generosity, when insurers can separate consumer types through distortions in contract generosity $g$, the sponsor needs an additional tool to realize its

\(^{13}\text{We assume that the separating equilibrium exist. This is the case if the fraction of low consumer types, } \phi^l, \text{ is not too large.}\)
solidarity objectives: risk adjustment. We turn to such risk-adjustment schemes now.

2.2. Risk adjustment

To allow redistribution from low to high-cost consumers if contract generosity $g$ is not contractible, the sponsor implements a risk-adjustment scheme. We first argue that if consumer types are observable for the sponsor, the sponsor can simply risk adjust using an ex-ante system of remunerations. Then we turn to the situation of interest in which the insurers have better information than the sponsor. In this case the sponsor has to design a scheme that elicits the information on a consumer’s type from the insurer. The instrument used to elicit that information is ex-post risk adjustment: the sponsor reinsures part of the ex-post realized costs. We first consider a uniform system of ex-post risk adjustment. Then we show that the sponsor can do even better by offering menus of ex-post risk adjustments.

If the sponsor observes the consumers’ types, first-best insurance (with $g = 0$) can be achieved at equal prices to low-cost and high-cost consumers, by charging insurers payments $t^h$ and $t^l$, for each high, respectively low-cost consumer that they insure, with

$$t^h = -\phi y(\theta^h - \theta^l), t^l = (1 - \phi)y(\theta^h - \theta^l).$$

By subsidizing insurers of high-cost consumers (negative payment $t^h$) and taxing those with low-cost consumers (positive $t^l$), the sponsor brings insurance prices closer together. Note that the payments in (8) satisfy budget balance,

$$\phi t^h + (1 - \phi)t^l = 0.$$  

With the system of subsidies and taxes (8) in place, competitive insurers charge prices

$$p^h = \theta^h(y - g^h) + t^h, p^l = \theta^l(y - g^l) + t^l.$$

It is straightforward to see that in the competitive equilibrium, we achieve pooling, with $g^h = g^l = 0$ and $p^h = p^l = \phi y\theta^h + (1 - \phi) y\theta^h$. This equilibrium is irrespective of whether the sponsor allows RR or imposes CR: since costs have been equalized, insurers have no need to introduce distortions into their contracts to screen different consumer types.

Whereas such full ex-ante risk adjustment, in which prices are equalized across consumers and every type obtains optimal insurance, does not require generosity $g$ to be contractible, it
does rely on the observability of consumer types for the sponsor. Without symmetric information on consumer types between insurers and sponsors, insurers have the incentive to claim its insured are all high-cost in order to benefit from the subsidy. As asymmetric information between sponsor and insurer is the more relevant case in practice, we now turn to this case. Types are not observable to the sponsor, and it needs to offer incentives to insurers to reveal their consumers’ types truthfully. We explore how the sponsor can then achieve its aims by using ex-post risk adjustment.

2.2.1. Uniform ex-post risk adjustment

If the sponsor cannot observe consumer type, but can only contract on observed treatment costs, it can use an ex-post risk adjustment to separate insurers of low and high-cost consumers. With ex-post risk adjustment, the sponsor effectively reinsures part of the insurer’s costs. We focus in this subsection on the simple case of a uniform proportional risk adjustment, characterized by a single reinsurance rate $x$ and a contribution $t$. With proportional reinsurance $x$, the scheme will reimburse a fraction $x$ of total costs to the insurer. To fund the scheme, insurers will pay a contribution $t$ to the scheme. The zero-profit condition on the competitive equilibrium requires that prices are then

$$
\begin{align*}
  p^h &= \theta^h(y - g^h)(1 - x) + t, \\
  p^l &= \theta^l(y - g^l)(1 - x) + t.
\end{align*}
$$

(10)

At full reinsurance $x = 1$, insurers are not at all exposed to their consumers’ costs, and prices for both consumer types will therefore be equal, $p = t$. Maximizing consumer utility will drive insurers to offer first-best insurance generosity, $g = 0$. The sponsor can therefore achieve both full equalization of prices across consumer types, and efficient insurance.

The required uniform level of contribution $t$ that insurers pay to the sponsor is governed by budget balance. In a benchmark case in which reinsurance does not lead to any inefficiencies, total expected cost to the sponsor of reinsuring is equal to the expected transfers to the insurers. If there are no additional costs from reinsurance, budget balance then requires a contribution from the insurers equal to

$$
t = \phi x \theta^l (y - g^l) + (1 - \phi) x \theta^h (y - g^h)
\begin{align*}
  &= \phi \theta^l y + (1 - \phi) \theta^h y,
\end{align*}
$$

(12)
with \( x = 1 \) and \( g^l = g^h = 0 \). In this situation with frictionless reinsurance, even though the
sponsor cannot observe consumer types or contract on generosity \( g \), ex-post risk adjustment
achieves first-best insurance and full equalization of costs, with prices equal to the population-
average costs.

In reality though, ex-post reimbursement leads to moral hazard on the insurers’ side. If the
sponsor covers part of the realized costs, the incentives for insurers to keep healthcare costs low
are reduced. Indeed, insurers face the full effort cost to keep expenditure low, but a fraction \( x \)
of the benefits of low expenditure flows to the sponsor.

In the main text,\(^{14}\) we will include those moral-hazard costs in a reduced form by assuming
that there is an additional mark-up on total costs, \( \varepsilon \alpha(x, g|\theta) \), where \( \varepsilon \geq 0 \) is a scalar that
we use to explore comparative statics, and \( \alpha(x, g|\theta) \) measures how moral-hazard costs vary
with reinsurance rate \( x \) and generosity \( g \). We assume \( \alpha(0, g|\theta) = 0 \) (no costs in the absence
of reinsurance), and \( \alpha_x \equiv \frac{\partial \alpha}{\partial x} > 0 \) captures how more generous ex-post adjustment leads to higher
overall costs. To ease the exposition, we assume that higher costs \( \varepsilon \alpha \) have no value at all for
the insured.\(^{15}\) We then have a budget balance condition

\[
t = \phi x \theta^l(y - g^l) + (1 - \phi) x \theta^h(y - g^h) + \phi \varepsilon \alpha(x, g^l|\theta^l) + (1 - \phi) \varepsilon \alpha(x, g^h|\theta^h), \tag{11}
\]

which states that total contributions by insurers \( t \) should equal total expected costs of reinsurance,
including those costs arising from moral hazard.

In the presence of these moral-hazard costs, we can combine the budget balance condition
(11) with the zero-profit prices (10), to find competitive-equilibrium prices for uniform risk
adjustment, as summarized in the following lemma.

**Lemma 1.** With uniform ex-post risk adjustment \((t, x)\), if insurers make zero profits and the
sponsor’s budget constraint holds with equality, prices in competitive equilibrium with contract
generosities \( g^h \) and \( g^l \) are given by

\[
p^h = \text{average cost} + \text{mark-up} + \phi(1 - x) \cdot \text{wedge}, \tag{12}
\]

\[
p^l = \text{average cost} + \text{mark-up} - (1 - \phi)(1 - x) \cdot \text{wedge}, \tag{13}
\]

\(^{14}\)Appendix B presents a simple model where insurers invest effort to reduce healthcare costs that underlies
this reduced form version.

\(^{15}\)More generally, \( \varepsilon \alpha \) measures the difference between costs and benefits of the additional treatments used if
insurers are less vigilant about expenditure.
We see that a sponsor can bring prices for low-cost and high-cost consumers closer together by increasing ex-post risk adjustment $x$. At $x = 1$, the wedge term—driven by expected cost differences between types—vanishes and we again have equal prices, $p_h = p_l$. But now there is a trade-off in the form of higher overall prices caused by moral-hazard costs $\alpha$ as measured by the mark-up terms, and it may pay to reduce those prices by sacrificing some redistribution. The sponsor, optimizing weighted welfare (6), chooses the level of ex-post risk adjustment $x$ to balance these two forces.

2.2.2. Optimal risk adjustment: menus of contracts

Adding reinsurance helps in bringing prices for low-cost and high-cost consumers closer together, at a cost of introducing inefficiency through insurer moral hazard. So far, we considered a uniform ex-post risk adjustment $x$ at a uniform price $t$ for either insurer type. However, as we shall now see, the sponsor can do better by allowing for different risk-adjustment schemes for insurers, $(x_l, t_l)$ and $(x_h, t_h)$ respectively. This allows the sponsor to screen insurers, facilitating redistribution from low-cost insurers to higher-cost ones, at lower moral-hazard costs.

To find the optimal scheme, we follow a contract-theory approach. Given the sponsor’s inability to verify a consumer’s type, each insurer should find it in its private interest to truthfully reveal its consumers’ types. An insurer reveals its customer by self-selecting into the intended risk-adjustment scheme. We therefore restrict to contracts that are incentive compatible (IC).

Consider a menu of risk adjustments, $(x_l, t_l)$ and $(x_h, t_h)$, which again satisfies a budget balance condition

$$
\phi t_l + (1-\phi) t_h = \phi x_l \theta_l(y - g_l) + (1-\phi)x_h \theta_h(y - g_h) + \phi \varepsilon \alpha(x_l, g_l|\theta_l) + (1-\phi) \varepsilon \alpha(x_h, g_h|\theta_h).
$$

If each insurer gets the risk-adjustment scheme corresponding to its consumer’s type, prices
in a competitive market are given by

\[ p^h = \theta^h (y - g^h)(1 - x^h) + t^h, \]
\[ p^l = \theta^l (y - g^l)(1 - x^l) + t^l. \]

(18)  
(19)

This parallels the expressions with uniform ex-post risk adjustment (10). Similarly, with full information, the sponsor optimally chooses \( x^h = 0 = x^l \), to avoid the inefficiencies of reinsurance, and sets transfers \( t^l > 0, t^h < 0 \) as in (8) such that prices are equalized.

When insurers are privately informed on their consumers’ types, insurers can lie about their consumer’s type in order to benefit from a more generous risk adjustment. To avoid such gaming of the system, the risk-adjustment contracts need to be incentive compatible. That is,

\[ \theta^h (y - g^h)(1 - x^h) + t^h \leq \theta^h (y - g^h)(1 - x^l) + t^l, \]
\[ \theta^l (y - g^l)(1 - x^l) + t^l \leq \theta^l (y - g^l)(1 - x^h) + t^h. \]

(20)  
(21)

For an insurer, truthfully revealing its customer (left-hand side) leads to lower expected costs than lying about its type (right-hand side). Note that in these expressions the lying insurer does not adjust \( g \). In the proofs (of lemma 3 and 5 resp.) we check that, indeed, when an insurer decides to lie about the type of an customer, there is no reason to offer this customer an insurance contract with a different \( g \).

Rewriting the IC conditions in terms of prices, using competitiveness of the insurance market, we obtain

\[ IC^h_i : \quad p^h \leq p^l + (1 - x^l) \left[ \theta^h (y - g^h) - \theta^l (y - g^l) \right] \]
\[ IC^l_i : \quad p^l \leq p^h - (1 - x^h) \left[ \theta^h (y - g^h) - \theta^l (y - g^l) \right]. \]

(22)  
(23)

We see that in first-best, with \( x^h = 0 = x^l \) and equal prices \( p^h = p^l \), it is \( IC^l_i \) that is violated, while \( IC^h_i \) is slack. Since the sponsor subsidizes insurers with high-cost consumers, low-cost insurers want to pose as high types to benefit from this cross-subsidy.

To avoid such mimicking, the sponsor can use ex-post risk adjustment \( x \), which allows for screening of low-cost insurers because of the following single-crossing condition:

\[ \frac{\partial}{\partial x} \left[ \theta^l (y - g^l)(1 - x) \right] \geq \frac{\partial}{\partial x} \left[ \theta^h (y - g^h)(1 - x) \right], \]

(24)
which holds if \( g^l \geq g^h \), i.e. equilibrium contracts to high-cost consumers are at least as generous as those aimed at low-cost consumers. We will see that this condition is verified in our model, both in RR equilibria (when each type gets efficient generosity \( g = 0 \)), and in the CR case (when low-cost consumers may get distorted generosity \( g^l > 0 \), but high-cost consumers receive contract offers with efficient generosity \( g^h = 0 \)).

This single-crossing condition for insurers is the equivalent of (5) for consumers. It implies that insurers with high-cost consumers benefit more from ex-post risk adjustment than those with low-cost consumers. The intuition is that reinsurance is more valuable when expected costs are higher, as this raises expected ex-post contributions from the risk-adjustment scheme. As a result, genuine high-cost insurers have a higher willingness to pay for reinsurance. The sponsor can use this difference in willingness to pay to separate insurers, offering reinsurance to high-cost insurers at a price that is attractive to high-cost insurers but not to low-cost ones.

To restore incentive compatibility, the sponsor should therefore introduce \( x^h > 0 \) to make sure \( IC \) holds for the low-cost insurers. For a general scheme satisfying binding \( IC^l_i \) and the budget constraint, we find the analogon of lemma 1 for prices:

**Lemma 2.** Consider a menu of ex-post risk-adjustment contracts \((t^h, x^h)\) and \((t^l, x^l)\), that satisfy the sponsor’s budget constraint and the low-cost insurer’s incentive compatibility condition with equality. If insurers make zero profits, and assuming that equilibrium contracts for high-cost consumers are at least as generous as those for low-cost consumers, \( g^h \leq g^l \), then prices in competitive equilibrium are given by

\[
\begin{align*}
    p^h &= \text{average cost} + \text{mark-up} + \phi(1 - x^h) \cdot \text{wedge}, \\
    p^l &= \text{average cost} + \text{mark-up} - (1 - \phi)(1 - x^h) \cdot \text{wedge},
\end{align*}
\]

with

\[
\begin{align*}
    \text{average cost} &= \phi \theta^l (y - g^l) + (1 - \phi) \theta^h (y - g^h), \\
    \text{mark-up} &= \phi \varepsilon \alpha (x^l, g^l | \theta^l) + (1 - \phi) \varepsilon \alpha (x^h, g^h | \theta^h), \\
    \text{wedge} &= \theta^h (y - g^h) - \theta^l (y - g^l).
\end{align*}
\]

We see that for achieving solidarity, it is sufficient to introduce only ex-post risk adjustment for the contract aimed at the high-cost insurer, \( x^h > 0 \). We will verify later that in the optimal scheme, the sponsor should refrain from ex-post risk adjustment for the low-types, \( x^l = 0 \):
low-type ex-post risk adjustment only results in an increased mark-up term, reflecting the moral-hazard costs of reinsurance for low-cost insurers. Although \( x^l \) increases costs through the mark-up term, it has no benefits in terms of separating types.

Compared to the case of uniform risk adjustment, for given redistribution \( x^h = x \), the benefit of reducing the wedge between the two prices comes at a lower mark-up in lemma 2: only moral-hazard costs for the high-cost insurers are incurred. As a consequence, the optimal scheme with a risk-adjustment menu will involve higher reinsurance, \( x^h \), and hence higher redistribution, than with uniform risk adjustment.

In the remainder of this paper we focus on the case in which the sponsor can offer a menu of risk-adjustment schemes. In section 6 we will comment on what changes if the sponsor needs to restrict to a uniform scheme.

2.3. Timeline

Figure 1 below summarizes the timing of the game. First, the sponsor determines the menu of risk-adjustment contracts \( (t^i, x^i) \), and either allows RR, or forces insurers to use CR. Insurers observe the risk adjustment system and on that basis set contracts \( (p^i, g^i) \). With RR, consumers can only buy a contract aimed at their type. With CR, consumers can opt for any offered contract. After consumers have selected from contract offers that they are eligible for, they pay their chosen premiums \( p \) and insurers report their consumers’ types to the sponsor. Insurers pay the risk adjustment transfers \( t \) consistent with their reports. Finally, consumers incur health costs and payments are settled.

Figure 1: Timeline

In the main text, a stage in which insurers exert effort to reduce treatment costs is treated implicitly. We model this explicitly in appendix B.

So far we have not yet determined equilibrium generosities, \( g^h \) and \( g^l \), that insurers choose
when facing a particular risk-adjustment scheme. Clearly, these generosities will depend on the choice for either community rating, CR, or risk rating, RR. In the next sections we explore equilibrium in either case, and characterize the optimal choice of ex-post risk adjustment \( x^h \) given the insurers’ response to the sponsor’s risk-adjustment menu. It is the interaction between this optimal risk-adjustment scheme and the choice of introducing CR or RR that will be the focus of our analysis.

3. Risk rating

In this section we analyze optimal contracts in the case of RR, when insurers are allowed to explicitly price discriminate based on the consumer type \( \theta^h, \theta^l \) that they observe— but which the sponsor does not observe. Section 4 analyses CR. The main result with RR is that risk adjustment moves prices closer together and redistributes towards high-cost consumers, but does not affect the generosity of the insurance contracts.

RR without risk adjustment yields an equilibrium with the following properties. To maximize the joint insurer and consumer surplus, insurers set the efficient generosities \( g^h = g^l = 0 \). Since under RR, the insurers can separate high-cost and low-cost consumers directly, there is no reason to distort \( g \). Both consumer types get efficient health insurance but at different prices. Due to perfect competition, each type pays his expected costs, which are higher for a high-cost than for a low-cost consumer, \( \theta^h y > \theta^l y \).

In this equilibrium with efficient generosities, when designing the risk-adjustment scheme, the sponsor can focus on prices. How can the risk-adjustment parameters \( t^l, t^h \) and \( x^l, x^h \) be chosen in order to increase solidarity, i.e., to lower wedge \( p^h - p^l \), while retaining sufficient incentives for insurer cost reduction? To analyse that, we first characterize the RR equilibrium in the following lemma.

**Lemma 3.** With RR, an insurance market equilibrium always exists and has \( g^h = g^l = 0 \). Further, in the sponsor’s problem, the budget constraint (17) and the low-cost insurer incentive compatibility constraint (23) hold with equality. In the optimal risk-adjustment scheme, there is no reinsurance for low-cost insurers, \( x^l = 0 \).

With this characterization, it follows from lemma 2 that prices are as specified in that
lemma, with generosities $g^l$ and $g^h$ set to zero, and $x^l = 0$,

\begin{align*}
  p^h &= \phi \theta^l y + (1 - \phi) \theta^h y + (1 - \phi) \varepsilon \alpha(x^h, 0|\theta^h) + \phi (1 - x^h)(\theta^h y - \theta^l y), \\
  p^l &= \phi \theta^l y + (1 - \phi) \theta^h y + (1 - \phi) \varepsilon \alpha(x^h, 0|\theta^h) - (1 - \phi)(1 - x^h)(\theta^h y - \theta^l y).
\end{align*}

Introducing risk adjustment does not create any reason for the insurers to distort contract generosities $g$ away from zero: because of consumer risk aversion, consumers value increased generosity (reduced $g$) more than it costs insurers to provide it. Secondly, a sponsor that values solidarity wants to reduce the wedge between high and low prices. Because getting part of their costs reimbursed ex post is more profitable for high-cost insurers, who face higher expected costs than low-cost insurers, $\theta^h y > \theta^l y$, separation can be achieved by raising $x^h$, that is, increasing ex-post risk adjustment. Of course, the disadvantage of raising $x^h$ is the inefficiency $\varepsilon \alpha(x^h)$ it induces. This will increase costs, and hence average prices, which reduces consumer utility.

Whereas $x^h$ can reduce $p^h$, it is clear from equations (25), (26) together with (28) that $x^l$ can only raise prices $p^h, p^l$, while it has no effect on the redistribution term. Hence, it is no surprise to find that in the optimal menu, the sponsor sets ex-post risk adjustment for low-cost insurers to zero, $x^l = 0$.\(^{16}\)

With $x^h = 1$, the wedge term disappears, and both types are charged the same price: $p^h = p^l$. We assume that the sponsor does not want to redistribute more than $p^h = p^l$ and hence chooses the optimal $x^h \in [0, 1]$.\(^{17}\)

It is clear that in the absence of costs to ex-post risk adjustment, when $\varepsilon = 0$ and moral hazard is absent, the sponsor will choose to fully redistribute, $x^h = 1$, so that prices for both types are equal. Conversely, if ex-post risk adjustment is very costly, that is for $\varepsilon$ high, the benefits of redistribution will never outweigh the costs of doing so, and we expect that the sponsor chooses $x^h = 0$. The following proposition summarizes how the optimal $x^h$ (with RR) changes as we vary the intensity $\varepsilon$ of moral-hazard costs.

**Proposition 1.** With RR both consumer types get efficient health insurance $g^h = g^l = 0$, the planner sets $t^h \geq t^l \geq 0$. Furthermore,

- if $\varepsilon = 0$, there is full ex-post reinsurance, $x^h = 1$;
- if the sponsor has a bias $\omega < \phi$, then there exists an $\varepsilon > 0$ such that it chooses full

---

\(^{16}\)This is a manifestation of the standard “no distortion at the top” result.

\(^{17}\)The sponsor’s objective function in (6) assumes that $p^h \geq p^l$ and hence $u^h \leq u^l$. 

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reinsurance $x^h = 1$ for all $\varepsilon < \bar{\varepsilon}$;

- if $\omega = \phi$ and reinsurance is costly, $\varepsilon > 0$, partial reinsurance obtains, $x^h < 1$;

- for $\varepsilon > 0$ sufficiently large, there is no risk adjustment, $t^h = t^l = x^h = x^l = 0$. Each consumer type pays a premium equal to expected cost $p^h = \theta^h y, p^l = \theta^l y$.

The proposition makes two main points. First, the analysis of changes in moral-hazard costs $\varepsilon$ shows when the sponsor sets full ex-post reinsurance: with $\varepsilon = 0$, full insurance $x^h = 1$ is optimal. If the sponsor’s objective function is biased towards the high-cost consumer ($\omega < \phi$), it remains optimal for the sponsor to implement $p^h = p^l$, as long as reinsurance is not too costly ($\varepsilon < \bar{\varepsilon}$). Thus, in that case the sponsor optimally sets $x^h = 1$ for small moral hazard $\varepsilon > 0$. For higher $\varepsilon > 0$, we get partial reinsurance $x^h < 1$. If insurance moral hazard is strong, the sponsor cannot use reinsurance at all, $x^h = 0$.

Second, the proposition makes clear how the risk-adjustment mechanism succeeds in separating insurer types and hence redistributing from low- to high-cost insurers. The sponsor effectively offers insurers the choice of either buying reinsurance $x^h > 0$ at a price $t^h$, or paying (a tax) $t^l < t^h$ without reinsurance ($x^l = 0$). For high-cost insurers, reinsurance is attractive. They receive ex-post risk adjustment with a probability $\theta^h > \theta^l$ and their total compensation from reinsurance is positive. For low-cost insurers, however, the cost of reinsurance exceeds the benefit. They prefer paying a tax $t^l < t^h$ instead. This tax allows the sponsor to cross-subsidize the high types. As a result, risk adjustment moves prices closer together and redistributes towards high-cost consumers.

4. Community Rating

Under CR, insurers cannot discriminate based on consumers’ observable characteristics. This can arise because the sponsor does not allow insurers to use their information on relevant characteristics when selling insurance, or because insurers do not observe such characteristics when selling the contract.

As discussed, without risk adjustment, under CR the insurers separate consumers by offering contracts that induce consumers to self-select, based on their types: equilibrium contracts $(p^l, g^l), (p^h, g^h)$ have to satisfy consumer incentive compatibility ($IC_{c^{h,l}}$) to get truthful revela-
tion by the insured consumers:

\[ u(p^h, g^h | \theta^h) \geq u(p^l, g^l | \theta^h), \]  
(\text{IC}^h_c)  

\[ u(p^l, g^l | \theta^l) \geq u(p^h, g^h | \theta^l). \]  
(\text{IC}^l_c)

In words, the high-cost consumer is (weakly) better off buying the \( h \)-contract than with the \( l \)-contract. Similarly, for the low-cost consumer and \( l \)-contract. Note that these IC constraints do not assume that two different contracts are offered (although this will be the case in equilibrium). The inequalities are satisfied if \( p^h = p^l \) and \( g^h = g^l \).

We now analyze how the equilibrium contracts that insurers offer to consumers change, when the sponsor offers a risk-adjustment scheme to the insurers. Also with risk adjustment, a separating equilibrium should involve contracts satisfying consumer incentive compatibility. These constraints introduce distortions in generosity \( g \) that we have to take into account.

Taking the risk-adjustment scheme as given, we define the CR competitive market equilibrium as in Rothschild and Stiglitz (1976):

**Definition 1.** Vector \((p^l, g^l, p^h, g^h)\) forms a CR equilibrium, given the risk-adjustment scheme \((t^l, x^l, t^h, x^h)\), if

- contracts \((p^l, g^l), (p^h, g^h)\) satisfy consumer incentive compatibility conditions \((\text{IC}^h_c)\) and \((\text{IC}^l_c)\),
- each contract that is offered earns a non-negative profit and
- it is not possible to introduce a (new) contract which makes strictly positive profits.

From the definition, we can make a number of observations on the equilibrium.

**Lemma 4.** If a CR equilibrium exists, it satisfies the following conditions

1. high types get efficient generosity, \( g^h = 0 \),
2. \((\text{IC}^h_c)\) holds with equality and
3. insurers make zero profits.

These properties of the equilibrium, familiar from the Rothschild-Stiglitz context, carry over for any risk-adjustment scheme. The intuition is that, because generosity \( g = 0 \) is optimal, see
equation (2), the cost saving of a marginal reduction in \( g > 0 \) is always less than the value the consumer attaches to such a decrease in generosity. The only reason for not setting \( g \) to 0 is that a constraint is hit. While the \( IC^h_c \) constraint bars \( g^l \) from being equal to 0, nothing stops \( g^h \) from being set efficiently, because the low-cost consumer does not want to mimic the high-cost consumer: we find \( g^h = 0 \).

With RR, lemma 3 guarantees existence of an equilibrium. It is well known that existence of a separating Rothschild-Stiglitz equilibrium is not guaranteed, as there may be a profitable deviation to a pooling contract if the proportion of low-cost consumers \( \phi \) is sufficiently high.

For the remainder of this analysis we ignore this issue. We note that the sponsor can always design a risk-adjustment scheme \((t^l, x^l), (t^h, x^h)\), such that an equilibrium exists. In particular, a (pooling) equilibrium exists by setting \( x^h = 1, x^l = 0 \). Then it is the case that \( p^h = p^l \) and \( g^h = g^l = 0 \).

We now turn to explore the sponsor’s optimization problem, (6), using the constraints that are summarized in lemma 4. The sponsor chooses a set of risk-adjustment contracts, \((t^l, x^l), (t^h, x^h)\), such that the resulting equilibrium in the health-insurance market \((p^l, g^l), (p^h, g^h)\) maximizes weighted consumer surplus. With RR we have \( x^l = 0 \) (see lemma 3) because \( x^l \) raises prices \( p^h, p^l \), while it does not affect redistribution. A similar reasoning leads to \( x^l = 0 \) with CR as well. Also, as in the RR case, the low-cost insurer’s incentive compatibility constraint and the budget constraint hold with equality:

**Lemma 5.** Assume that a CR equilibrium exists. In the optimum, insurer incentive compatibility \( IC^l_i \) and the budget constraint \( BC \) hold with equality, and there is no ex-post risk adjustment for low-cost insurers, \( x^l = 0 \).

From this lemma, we can again conclude that prices are given by lemma 2. In addition, we know from lemma 4 that high-cost generosity is efficient, \( g^h = 0 \), so that

\[
\begin{align*}
p^h &= \phi \theta^l (y - g^l) + (1 - \phi) \theta^h y + (1 - \phi) \epsilon \alpha(x^h, 0|\theta^h) + \phi (1 - x^h) (\theta^h y - \theta^l (y - g^l)), \\
p^l &= \phi \theta^l (y - g^l) + (1 - \phi) \theta^h y + (1 - \phi) \epsilon \alpha(x^h, 0|\theta^h) - (1 - \phi) (1 - x^h) (\theta^h y - \theta^l (y - g^l)).
\end{align*}
\]  

(32)  

(33)

We are now ready to analyze the sponsor’s optimization problem: substituting the results from lemmas 4 and 5 in the binding constraints, we study the optimal choice of ex-post risk adjustment \( x^h \) for high-cost insurers. Compared to the analysis under RR, we now also have to take into account that, in the CR market equilibrium, generosity \( g^l \) may be positive. As
in the case of RR, we again characterize optimal $x^h$ as a function of the cost of reinsurance, parametrized by $\varepsilon$. In particular, we are interested in how optimal risk adjustment changes in the neighborhood of the full-reinsurance point, $x^h = 1$, as we increase reinsurance costs $\varepsilon > 0$. We have the following result.

**Proposition 2.** With CR, the planner sets $t^h \geq t^l \geq 0$, $g^h = 0$, $g^l \geq 0$ such that consumer incentive compatibility $IC^h_c$ binds. Furthermore,

- if $\varepsilon = 0$, there is full ex-post reinsurance, $x^h = 1$ and low-type generosity is undistorted, $g^l = 0$;
- if the sponsor has a bias $\omega < \phi$, then there exists an $\bar{\varepsilon} > 0$ such that it chooses full reinsurance $x^h = 1$ and generosities are undistorted, $g^l = 0$ for each $\varepsilon < \bar{\varepsilon}$;
- if $\omega = \phi$ and reinsurance is costly, $\varepsilon > 0$, partial reinsurance obtains, $x^h < 1$, and the low types’ generosity, $g^l$, is distorted, $g^l_{RS} \geq g^l > 0$;
- for $\varepsilon > 0$ sufficiently large, there is no risk adjustment, $t^h = t^l = x^h = x^l = 0$. Low-type generosity distortion is at the Rothschild-Stiglitz level, $g^l = g^l_{RS} > 0$.

As in the RR case, high-cost insurers opt for reinsurance and pay less than the actuarial cost of such reinsurance through the ex-ante payment $t^h$. For low-cost insurers, ex-post insurance is less valuable since their expected costs are lower, with the consequence that they prefer paying a tax $t^l$. This again forces prices closer together than would be the case without risk adjustment, and helps achieving redistribution.

With CR, as opposed to RR, there is another benefit of risk adjustment, apart from redistribution. In the absence of risk adjustment, insurers use distortions of contract generosity to separate high-cost consumers from low-cost ones. Setting low-type contract generosity to $g^l = g^l_{RS} > 0$ makes sure that high-cost consumers do not pose as low-cost ones, and instead self-select in the more expensive high-cost contract. Ex-post risk adjustment reduces the cost difference between types, which translates into a smaller price difference. This smaller price difference relaxes consumer incentive compatibility, allowing insurers to reduce the distortion $g^l$ on low-cost consumer generosity.

As in the RR case, if ex-post risk adjustment does not lead to higher costs ($\varepsilon = 0$), the sponsor implements the efficient outcome with $x^h = 1$, full reinsurance. In that case, prices are equal for both consumer types, equations (32,33), and there is no remaining distortion in
generosity, $g^l = 0$. If $\varepsilon > 0$ and the sponsor maximizes total welfare ($\omega = \phi$), the cost of ex-post risk adjustment implies incomplete risk adjustment, leading to some selection incentive for insurers, who now set $g^l > 0$. However, as in the RR case, if the sponsor puts more weight on the $h$-consumers ($\omega < \phi$), a range of $\varepsilon > 0$ exists such that the sponsor implements $x^h = 1$ and $g^l = 0$. For $\varepsilon > 0$ close enough to 0, increasing $x^h$ reduces $p^h$. This reduction in $p^h$ at the expense of low-cost consumers increases welfare if $\omega < \phi$: low-cost consumers pay for the costs of ex-post risk adjustment, and this cost has a lower weight in the sponsor’s objective function.

For high values of $\varepsilon$, also $p^h$ is increasing in $x^h$ to reflect increased moral-hazard costs; from that point onward, there is incomplete reinsurance, $x^h < 1$, and in equilibrium insurers distort generosity $g^l > 0$.

If the inefficiency of ex-post reinsurance is high enough, the sponsor does not use risk adjustment at all ($x^h = x^l = 0$ and $t^h = t^l = 0$ because of (17)). The standard Rothschild-Stiglitz equilibrium then obtains, with $g^l = g_{RS} > 0$. Comparing with RR, we know that in this case without risk adjustment, welfare is higher with RR than it is with CR. Indeed, without risk adjustment, the contracts for high-cost consumers are the same under RR and CR while there is inefficient generosity for the low-cost consumers ($g^l > 0$) under CR, as insurers use generosity distortion to separate the types. In contrast, for low costs of ex-post reinsurance, in the following section we show that CR can lead to higher welfare than RR.

5. When is CR optimal?

Without risk adjustment, CR is Pareto inferior to RR, in a setting where insurers can use non-contractible quality to separate high-cost consumers from low-cost ones. The reason is that with separating contracts, high-cost consumers are not better off in CR than in RR, while there is an additional distortion $g^l > 0$ that decreases welfare for low-cost consumers.

With risk adjustment, we have a similar welfare-decreasing generosity distortion on low-cost consumers. However, the interaction between community rating and the risk-adjustment scheme creates a compensating effect that dominates when generosity distortions are sufficiently low, which is the case when reinsurance costs are not too high.

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18As discussed in section 2, in settings in which insurers have no quality dimension to screen consumers, or in which quality is contractible for the sponsor, community rating is preferable if the sponsor has a bias towards high-cost consumers.
Let $W_{RR}(\varepsilon)$ denote the optimal value of weighted welfare (6) in the RR case and $W_{CR}(\varepsilon)$ in the CR case as a function of $\varepsilon$, the scale of the moral-hazard costs of reinsurance. We then have the following proposition.

**Proposition 3.** If $\omega < \phi$ and assuming that $x_{RR}^h(\varepsilon) = 1$ is globally optimal whenever it is locally optimal. Then:

1. $W_{CR}(\varepsilon) = W_{RR}(\varepsilon)$ for $\varepsilon \geq 0$ small enough;
2. $W_{CR}(\varepsilon) > W_{RR}(\varepsilon)$ for $\varepsilon > 0$ in a middle range;
3. $W_{RR}(\varepsilon) > W_{CR}(\varepsilon)$ for $\omega > 0$ and $\varepsilon$ big enough.

As we make few assumptions on second derivatives, we cannot be sure that the optimal $x^h$ is well behaved as a function of $\varepsilon$. For our result that CR can dominate RR, we only need that $x^h$ under RR remains at its maximum level until that ceases to be locally optimal.

The intuition for this result is that increasing $g_l$ means reducing the cost of insuring a low-cost consumer. There is an advantage in that reduced cost when introducing reinsurance: the value to a low-cost insurer of mimicking a high-cost one, and buying reinsurance from the sponsor, goes down. Hence, with $g_l > 0$, the sponsor can, for the same amount of reinsurance $x^h$, levy a larger tax $t^l$ from low-cost insurers without violating their incentive compatibility condition. In other words, higher $g_l$ relaxes ($IC_l^i$) and hence allows the sponsor to reduce the price difference between contracts for both types. In this way, the sponsor achieves higher redistribution.

A sponsor that maximizes weighted welfare will therefore prefer to have some generosity distortion $g^l$ for low-type consumers, balancing the improved redistribution with the costs of inefficient insurance for low-cost consumers. While the sponsor cannot actively set $g_l > 0$ (generosity is not contractible), it can endogenously create a generosity distortion by imposing a community-rating requirement, causing insurers to choose positive $g_l$ in market equilibrium for screening purposes.

For $\varepsilon \geq 0$ small enough and $\omega < \phi$, we know from propositions 1 and 2 that the solutions under RR and CR are the same: $x^h = 1, g^l = 0$ and prices are the same as well. Hence, $W_{CR} = W_{RR}$ for these values of $\varepsilon$, and there is no need to introduce CR. Further, if $\varepsilon$ is big enough that no risk adjustment is used ($x^h = t^h = t^l = 0$), the benefit of relaxing insurer incentive compatibility is not operative. In this case, we only have the disadvantage of reduced low-cost welfare, and RR dominates CR.
The most interesting case is in between these two extremes. The proposition shows that a range of moral-hazard costs $\varepsilon$ exists such that CR dominates RR in terms of weighted welfare $W$. The enhanced redistribution of CR, with its distortive generosity $g^l > 0$, here outperforms RR which has efficient coverage for both types.

CR thus outperforms RR if the cost of reinsurance is small but not too small. In this sense, policy makers’ preference for CR can be better founded than health economists tend to acknowledge. A bias in favour of high-cost consumers because the sponsor values solidarity together with risk adjustment as an instrument to screen insurers can motivate a choice for CR.

6. Robustness

6.1. Uniform risk adjustment

Optimal risk adjustment allows the sponsor to screen insurers, by offering a choice for insurers to buy reinsurance $x^h > 0$ at price $t^h$, or to forgo reinsurance ($x^l = 0$) and paying a tax $t^l < t^h$. For genuinely high-cost insurers, reinsurance is attractively priced, but low-cost insurers prefer to pay the tax, subsidizing the high-cost market.

In practice we do not observe such risk-adjustment menus. Instead, in practice insurers are exposed to a uniform risk adjustment schedule, $x^h = x^l$, with $t^h = t^l$. As we discussed in lemma 1, also in that case prices for both types are brought closer together. The drawback of uniform risk adjustment is the increased cost of moral hazard, as now costs $\varepsilon\alpha$ are also incurred on the low-type insurer. As a result, the mark-up on prices in the uniform case grows faster with $x$. Since this is the only change, the qualitative results of our analysis hold true also in the case of uniform risk adjustment.

6.2. Imperfect competition

Our paper builds on a Rothschild-Stiglitz model with perfect competition among insurers. In practice, market power can be important in health-insurance markets, as was demonstrated for instance in Dafny (2010). Lustig (2010) and Starc (2014) find evidence for welfare effects of market power dominating those of selection in the Medicare + Choice and Medigap markets, respectively. Moreover, risk-adjustment systems can interact in interesting ways with market
power, as demonstrated for example in Mahoney and Weyl (forthcoming), who find that in
the presence of market power, risk adjustment may even decrease coverage and welfare when
insurers have no instruments to screen consumers and insurance is not mandatory.

To explore the effects of imperfect competition in our model with consumer screening
through generosity, we turn to a Hotelling duopoly model of competition in insurance con-
tracts, as in Bijlsma, Boone and Zwart (2014). Consider two profit-maximizing insurers lo-
cated at either end of a Hotelling line, with consumers located homogeneously along that line.
We assume that consumers’ types are independent of their location. Consumers incur travel
costs $s$, and choose the insurer that maximizes their utility. Each insurer $i$ offers a menu of con-
tracts, $(p^h_i, g^h_i)$ and $(p^l_i, g^l_i)$, and serve $\phi \left( \frac{1}{2} + \frac{u^l_i - u^l_j}{2s} \right)$ low-cost consumers and $(1 - \phi) \left( \frac{1}{2} + \frac{u^l_i - u^h_j}{2s} \right)$ high-cost consumers, where $u^l_i$ and $u^h_i$ are the utilities from accepting the low- and high-type contracts from insurer $i$.

The sponsor offers a risk adjustment menu $(t^l, x^l)$, $(t^h, x^h)$, with budget constraint and
insurer incentive compatibility conditions as before, so that we can introduce cost levels $c^h$ and $c^l$ which satify

$$c^h \equiv t^h + (1 - x^h)\theta(h(g^h) - g^h) = \text{average cost + mark-up + } \phi(1 - x^h) \cdot \text{wedge}$$
$$c^l \equiv t^l + (1 - x^l)\theta(h(g^h) - g^l) = \text{average cost + mark-up - } (1 - \phi)(1 - x^h) \cdot \text{wedge}$$

as in lemma 2. The difference with that situation is, of course, that, with market power, prices
are not equal to costs but rather are determined by the insurers’ profit maximization.

Let us for simplicity consider the case in which utilities are linear in prices, $u(p, g) =
-p + \theta v(g)$ with $v'(g) < 0$. We write insurer $i$’s optimization problem as

$$\max_{p^l_i, g^l_i, p^h_i, g^h_i} \phi \left( \frac{1}{2} + \frac{-p^l_i + \theta h v(g^h_i) - u^l_j}{2s} \right)(p^l_i - c^l_i) + (1 - \phi) \left( \frac{1}{2} + \frac{-p^h_i + \theta h v(g^h_i) - u^h_j}{2s} \right)(p^h_i - c^h_i)$$
$$+ \lambda(p^l_i - p^h_i + \theta h (v(g^h_i) - v(g^l_i)))$$

where $\lambda \geq 0$ is the shadow price of the consumer incentive compatibility constraint $IC^h_i$. The

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19For another approach to modelling market power when insurers compete in menus of contracts, see e.g. Lester et al. (2015).
first order conditions for $p^l, p^h$ in symmetric equilibrium can be written as

\begin{align*}
  p^h &= c^h + s - \frac{2s\lambda}{1 - \phi} \\
  p^l &= c^l + s + \frac{2s\lambda}{\phi},
\end{align*}

(34) 
(35)

With risk rating $IC^h_c$ is not relevant and $\lambda = 0$. As under perfect competition, insurers maximize joint surplus with consumers by offering first-best generosities, $g^h = 0 = g^l$, and prices are similar to those under perfect competition except for the standard market-power mark-up equal to the travel cost parameter $s$,

\begin{align*}
  p^h &= c^h + s \\
  p^l &= c^l + s.
\end{align*}

Under a CR regime, as with perfect competition, $IC^h_c$ is binding ($\lambda > 0$). This leads to a distortion in low-type generosity, $g^l \geq 0$. With full risk adjustment, $g^l$ and $\lambda$ are zero, but as we move away from $x^h = 1$, both the shadow price and the distortion in low-type generosity $g^l$ start to increase.

The sponsor’s maximization problem over weighted consumer surplus is then similar to the problem with perfect competition: $x^l$ will be set to zero for the same reasons. In the RR regime, the sponsor faces the same problem as before, with prices shifted by the constant market-power mark-up $s$. Only for the CR regime is there a qualitative difference with the perfect-competition benchmark: when moving away from full adjustment, $x^h = 1$, consumer incentive compatibility induces the firms to reduce the market-power mark-up charged to high-cost consumers, financed by increased mark-ups on the low types, keeping the average market-power mark-up equal to $s$.

The analysis of optimal choice of risk adjustment around $x^h = 1$ (i.e., for low values of moral hazard parameter $\varepsilon$) then mirrors that for the perfect competition case, propositions 1 and 2, with the market-power mark-up effect favouring the community-rating environment slightly more than in the perfect-competition case. Indeed, if $\omega = 0$, so that generosity distortions $g^l$

\footnote{Bijlsma, Boone and Zwart (2014) exploit this effect of incentive compatibility linking mark-ups for low and high types, to argue that when competition for low types is more vigorous, distortions in $g^l$ are more attractive since they leverage benefits from this low type competition to the high-cost consumers. Full risk equalization, even if costless, may then be suboptimal for a biased sponsor.}
for the low types have no impact on the sponsor’s objective, even with large moral hazard $\varepsilon$, CR is the dominating regime.

The intuition for the result remains the same: distorting $g^l$, though adversely impacting low-type utility, also makes it easier to screen insurers, and hence to make the transfer of surplus from low-type to high-type consumers easier. The sponsor would like to set $g^l > 0$ to its optimal value. Since this is not feasible, the sponsor enforces CR to make sure that insurers endogenously introduce such a distortion.

6.3. Multi-dimensional types

The main result of section 5 is that –under an assumption on the inefficiency of reinsurance $\varepsilon$– the sponsor prefers to distort the market outcome to facilitate redistribution from low-cost to high-cost consumers. For this result we need that the type that gets distorted away from first best, is the type that is relatively disliked by the sponsor. In our case, the low-cost type gets distorted ($g^l > 0$) and is relatively undervalued ($\omega < \phi$) by the sponsor. The low-cost distortion helps the sponsor to separate low-cost and high-cost insurers and hence to redistribute from low-cost to high-cost consumers. Although this property holds across a range of models, we give two examples where it does not necessarily hold.

First, Bundorf, Levin and Mahoney (2012) introduce a model where consumers differ on more than one dimension, while insurers discriminate along one dimension (expected health costs or health risk) only. If price differences between plan options do not fully reflect the cost differences, some customers choose the wrong plan from a social point of view. Although in our set-up each type has its own contract, equations (34,35) also imply that $p^h - p^l < c^h - c^l$ in case insurers have market power ($s > 0$) and there is CR ($\lambda > 0$).

Assuming that the sponsor is interested in redistribution along health risk only (and not the other dimensions), it can no longer target the high-cost type. Indeed, this type is now “mixed” with the low-cost type in each contract. This implies that some low-cost consumers will benefit from the redistribution and some high-cost consumers suffer from the distortion. This makes CR less attractive for the sponsor compared to the situation where each cost type has its own contract, but not necessarily irrelevant.

Second, extending this logic, CR is no longer optimal if all high-cost types choose the “wrong” contract. Boone and Schottmüller (2017) present a model in which this can happen. The starting point of this paper is that agents differ in two dimensions: health risk and income.
Further, these two dimensions are negatively correlated; to illustrate, high health risk tends to go hand-in-hand with low income. As health is a normal good, low-income consumers can decide to choose the cheaper distorted contract while high income consumers choose the undistorted contract. CR in this case directly hurts the high-cost (low-income) type which is valued more by the sponsor. In this case, CR does not help the sponsor to redistribute towards the high-cost consumers.

7. Conclusion

We studied optimal risk adjustment in competitive health-insurance markets when insurers have better information on their customers’ risk profiles than the sponsor of the health-insurance scheme. Such an information advantage occurs if the insurer observes more consumer characteristics than a system based upon ex-ante observable characteristics of its insured can correct for and allows insurers to game the system by cherry-picking insured whose expected costs are low. The sponsor can try to reduce this inefficiency by inducing insurers to truthfully reveal their private information on their customers’ health costs through ex-post risk adjustment, which compensates insurers for consumers that turn out to be costly ex post. By offering such reinsurance, the sponsor can screen insurers and redistribute from low to high-risk consumers.

If reinsurance does not induce costs due to moral hazard, the sponsor can achieve both first-best insurance as well as solidarity by completely reinsuring insurers. In practice, however, such ex-post risk adjustment is socially costly due to moral hazard and the sponsor will optimally only offer partial reinsurance. As a result, when insurers are allowed to vary premiums for a given contract according to a consumers observable type, that is, under RR, insurers will charge high prices to high-cost consumers and low prices to low-cost consumers.

The sponsor can improve on this equilibrium by introducing a premium restriction in the form of CR. On the one hand, this restriction comes at a potential cost because it increases selection incentives for the insurers, which induces contract distortions if contract generosity is non-contractible. In health insurance, contract generosity is typically hard to contract on for the sponsor. Insurers will then distort contract generosity in order screen high-cost from low-cost consumers. On the other hand, the reduced generosity makes risk-adjustment more effective in screening insurer types: by reducing the insurers’ expected costs for low-risk consumers, CR makes reinsurance a less attractive option for these insurers. This makes it easier for the
sponsor to screen insurer types and hence allows more redistribution from low to high-risk consumers, raising welfare.

Although in our setting CR is never optimal in the absence of a motivation to screen insurers, when risk adjustment tries to elicit the insurers’ private information, CR can raise total weighted surplus if the cost of reinsurance is low or the sponsor’s preference for redistribution is sufficiently high. Thus, our analysis both provides a rationale behind the presence of CR requirements seen in reality and clarifies the subtle interaction between CR and risk adjustment, that is somewhat different from the straightforward motivation of policy-makers to enforce solidarity among high-risk and low-risk consumers on the health-insurance market.

We focused our analysis on markets in which the sponsor has no information on consumer types and the sponsor only uses ex-post risk adjustment. In reality, consumer risk characteristics may be partly observable to the sponsor. The sponsor will then optimally use ex-ante risk adjustment for these observable characteristics, adding ex-post risk adjustment to screen for differences within a given risk class that are unobservable to the sponsor. As the extent of asymmetric information will vary across risk classes, ideally the degree of reinsurance will also depend on these risk classes.
References


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A. Proofs of results

Proof of lemma 3 The proof has the following steps. First, we argue that $g^l = g^h = 0$. Then, we show that $x^l = 0$ assuming that (23) and (17) are binding. Finally, we check that (22) is satisfied as well.

Because insurers can risk-rate, insurance contracts do not have to satisfy consumer incentive compatibility constraints (as with CR in section 4). Suppose in equilibrium one of the insurers offers a contract with inefficient generosity, $g > 0$. Using prices in (18,19), we find that

$$\frac{du(p,g)}{dg} = -u_p\theta(1-x) + u_g < 0$$

(36)

because of (2): consumers value increased generosity (lower $g$) more than it costs to provide that generosity for any reinsurance $x \in [0,1]$. Thus the insurer can raise profits by reducing $g$. Hence both $h$ and $l$-consumers get offered efficient insurance contracts ($g = 0$). Note that $g^h = g^l = 0$ holds irrespective of whether the insurer reports the customer’s type truthfully to the sponsor.

Budget balance, equation (17) holds with equality. If it would be slack, it is possible to lower $t^h, t^l$ keeping $IC^l_i$ and $IC^h_i$ binding. This unambiguously increases the sponsor’s objective function, as it reduces prices. Next consider the insurer IC constraints. At least one of these is binding. Suppose not, i.e. both (22) and (23) are slack. Then we can reduce $p^h$ and increase $p^l$ (by adjusting $t^{h,l}$) in a way that satisfies budget balance, (17). This increases $W$ because $\omega \leq \phi$, $u_{pp} \leq 0$ and (23), with $g^h = g^l = 0$, implies that $p^h > p^l$. We assume here that (23) is binding and check afterwards that (22) is satisfied as well.

Because both low-cost insurer incentive compatibility (23) and budget balance (17) hold with equality, we use lemma 2 for the prices $p^h, p^l$. Then the effect of $x^l$ is only to increase prices (by the wedge term). It follows that $x^l = 0$.

We finish this proof by checking that high-type incentive compatibility (22) is satisfied. This follows from binding $IC^l_i$ and $x^h \geq 0 = x^l$. Q.E.D.

Proof of proposition 1 With $x^l = g^l = g^h = 0$ –from lemma 3– we have prices given by expressions (30) and (31). We are interested in optimization of the sponsor’s objective $W$ over
\( x^h \), so consider

\[
\frac{\partial W}{\partial x^h} = \omega \frac{\partial u(p^l, 0|\theta^l)}{\partial x^h} + (1 - \omega) \frac{\partial u(p^h, 0|\theta^h)}{\partial x^h} = \omega u_p^l \frac{\partial p^l}{\partial x^h} + (1 - \omega) u_p^h \frac{\partial p^h}{\partial x^h} \]

\[
= (1 - \phi) \varepsilon \alpha_x(x^h, 0|\theta^h)(\omega u_p^l + (1 - \omega) u_p^h) + (\theta^h - \theta^l)y(\varepsilon \alpha_x(x^h, 0|\theta^h))
\]

The first term, reflecting the mark-up term in the prices, leads to increases in both prices with \( x^h \), since \( \alpha_x > 0 \). With \( u_p < 0 \), this reduces \( W \). The second term is non-negative: with \( p^h \geq p^l \), we have \( |u_p^h| \geq |u_p^l| \), and in addition \( \omega \leq \phi \). As one raises \( x^h \), the wedge-term in prices causes a decrease in \( p^h \) and an increase in \( p^l \), increasing solidarity and hence \( W \).

Evidently, if \( \varepsilon = 0 \), the mark-up is zero, and the only effect of increasing \( x^h \) is bringing prices closer together, unambiguously increasing weighted welfare. Hence \( x^h \) takes its maximal value \( x^h = 1 \), and prices are equalized.

For positive moral-hazard costs \( \varepsilon > 0 \), and \( \omega = \phi \), then in \( x^h = 1 \), where prices are equal, the second, positive term vanishes and \( \partial W/\partial x^h|_{x^h=1} < 0 \). Hence \( x^h < 1 \).

If \( \omega < \phi \), at \( x^h = 1 \) the derivative remains positive for \( \varepsilon \) sufficiently small, so there exists \( \varepsilon > 0 \) as defined in the proposition.

Finally, equation (21) implies

\[
t^h - t^l = \theta^l y x^h \geq 0
\]

and from (17) it follows that low types are taxed to bring prices closer together,

\[
t^l = (1 - \phi)x^h(\theta^h - \theta^l)y + (1 - \phi)\varepsilon \alpha(x^h, 0|\theta^h) \geq 0.
\]

Q.E.D.

Proof of lemma 4

- Suppose to the contrary that high types receive inefficient generosity, \( g^h > 0 \). Then an insurer could offer a new, more profitable contract with slightly better generosity, as, by inequality (2), the high type consumer’s value of such lower \( g^h \) grows more strongly than the costs to the insurer. Such a new contract will certainly continue to satisfy consumer incentive compatibility \( IC^h \). If \( l \)-consumers decide to buy this contract as well, it becomes more profitable as their expected costs are lower. This profitable deviation contradicts
definition 1. Hence \( g^h = 0 \).

- Suppose by contradiction that \( IC_c^h \) is slack. We consider two cases:
  - \( g^l > 0 \): because of equation (2), a new contract with \( g^l < \tilde{g}^l \) and \( \tilde{p}^l > p^l + (g^l - \tilde{g}^l)\theta^l \)
    can be introduced which \( l \)-consumers prefer (but \( h \)-consumers do not; as \( IC_c^h \) is slack by assumption) and which leads to strictly positive profits. This contradicts definition 1.
  - \( g^l = 0 \): then we have
    \[
    u(p^h, 0|\theta^h) > u(p^l, 0|\theta^h) \quad \text{(slackness of } IC_c^h) \\
    u(p^l, 0|\theta^l) \geq u(p^h, 0|\theta^l) \quad \text{(by } IC_l^h). 
    \]
    But this is impossible because \( u_p < 0 \).

Hence, in each case there is a violation and thus \( IC_c^h \) cannot be slack. With \((IC_c^h)\) holding
with equality, assumption 5 implies that \((IC_c^l)\) is satisfied as well.

- Suppose an insurer makes positive profits in equilibrium. If positive profits are from \( h \)-consumers, an insurer can offer a new contract with slightly lower \( p^h \) and make a strictly
  positive profit. Even if \( l \)-consumers choose this contract as well, it is profitable (as \( l \)-consumers have lower expected costs than \( h \)-consumers). This contradicts definition 1.
  Next, consider the case where the insurers make a profit on the \( l \)-consumers. Then one
  can construct a new profitable contract \( \tilde{g}^l > g^l, \tilde{p}^l < p^l \) such that \( IC_c^h \)
  remains satisfied and the new contract is more attractive to \( l \)-consumers. Again this contradicts definition 1.

Q.E.D.

Proof of lemma 5

- We first show that \( IC_l^l \) binds. Assume to the contrary that \( IC_l^l \) is slack. First, suppose
  that \( g^l > 0 \). Then the sponsor can slightly increase \( t^h \) and decrease \( t^l \) without violating 
  \( IC_l^l \) (such a change cannot violate \( IC_c^h \)). Such a transfer from \( l \)-to \( h \)-consumers is in itself 
  beneficial for welfare, as welfare is biased towards \( h \)-consumers. The transfer increases
  \( p^l \) and decrease \( p^h \), which relaxes \( IC_c^h \). This, in turn, allows \( g^l \) to fall in the resulting
  equilibrium, and hence \( W \) is increased; a contradiction.
If, instead, \( g' = 0 \), we are in a pooling equilibrium, with necessarily \( p' = p^h \), or

\[
t^h + (1 - x^h)\theta^h y = p^h = p' = t' + (1 - x^l)\theta^l y < t^h + (1 - x^h)\theta^h y
\]

where the inequality represents slack \( IC^l_i \). But this cannot hold as \( \theta^l < \theta^h \).

- Next, we verify that if \( IC^l_i \) holds, an insurer cannot gain either by offering low-cost consumers a different \( g \neq g' \) and then claiming it is an \( h \)-consumer.

First, consider \( g > g' \). The \( l \)-insurer costs become \( \theta^l (y - g) < \theta^l (y - g') \). Hence, if deviation to the high-type risk-adjustment contract \((t^h, x^h)\) is not profitable at \( g' \), it is certainly not profitable at \( g > g' \).

Second, consider offering a contract \( g < g' \) (in case \( g' > 0 \)), and lying to the sponsor. To make this deviation contract \((p, g)\) attractive for the \( l \)-consumer, it needs to be the case that \( u(p, g|\theta^l) \geq u(p', g'|\theta^l) \). Then a binding \((IC^h_c)\) together with assumption 5 implies that \( u(p, g|\theta^h) > u(p^h, 0|\theta^h) \): \( h \)-consumers value the increased generosity even more and buy this contract as well. We find that, with risk adjustment choice \((x^h, t^h)\), total costs of serving this mixture of consumers exceed total costs of serving \( l \)-consumers at \( g' \),

\[
t^h + \phi(1 - x^h)\theta^l(y - g) + (1 - \phi)(1 - x^h)\theta^h (y - g) \geq t^h + (1 - x^h)\theta^l(y - g').
\]

(39)

so that if truthfully revealing type dominates lying at \( g' \) (\( IC^l_i \) holds), then it certainly does so at \( g < g' \).

- Next, if (17) is slack, it would be possible to lower \( t^h, t' \) and \( g' \), keeping \( IC^l_i \) and \( IC^h_c \) binding. This unambiguously increases the sponsor’s welfare.

- With (23) and (17) binding, lemma 2 gives us prices \( p^h, p' \).

- Next, we show that \( x^l = 0 \). The effect of increasing \( x^l \) from \( x^l = 0 \) to a positive value directly increases both prices since \( \alpha_x > 0 \), and as in the RR case, this harms both types’ welfare. The difference with the RR case is that now also \( g' \) changes to make sure \( IC^h_c \) binds with equality. We need to verify that the changes in \( g' \) do not make \( x^l > 0 \) optimal.

We consider therefore any \( x^l > 0 \) and the associated mark-up \( \phi \varepsilon \bar{\alpha}^l \) with \( \bar{\alpha}^l \equiv \alpha(x^l, g'|\theta^l) > 0 \) and \( g'(\bar{\alpha}^l) \) consistent with binding \( IC^h_c \). We show that a shift from \( x^l = 0 \), where \( \alpha^l = 0 \)
with \( g' = g'(0) \), to positive \( x \) where \( \alpha' = \alpha > 0 \) with \( g' = g'(\alpha') \) always decreases both \( u^h \) and \( u^l \), so that the sponsor’s welfare decreases for any \( \omega \).

We use figure 2. The curves show indifference curves for the \( l \) and \( h \)-consumer through the low type’s contract \((p^l, g^l)\). We have that \((p^h, g^h = 0)\) and \((p^l, g^l)\) should always be on the same \( h \)-consumer’s indifference curve, by binding \( IC^h \). Furthermore, prices are determined by \( g^l \) and \( \alpha^l \) as in lemma 2

\[
\frac{\partial p^h}{\partial \alpha^l} = \phi \varepsilon \quad (40)
\]

\[
\frac{\partial p^h}{\partial g^l} = -\phi \theta^l + \phi (1 - x^h) \theta^l < 0 \quad (41)
\]

\[
\frac{\partial p^l}{\partial \alpha^l} = \phi \varepsilon \quad (42)
\]

\[
\frac{\partial p^l}{\partial g^l} = -\phi \theta^l - (1 - \phi) (1 - x^h) \theta^l \begin{cases} < 0 \\ \geq -\theta^l \end{cases} \quad (43)
\]

Consider two cases: one in which equilibrium \( g^l(\alpha^l) \) decreases as \( \alpha^l \) grows, and one in which \( g^l \) increases with \( \alpha^l \). We show that in both cases, as we increase mark-up \( \alpha^l \), \( W \) is decreasing. Hence, \( \alpha^l = 0 \) (which is the case when \( x^l = 0 \)) is optimal.

Case 1. Suppose first that \( \frac{dg^l}{d\alpha^l} < 0 \). Then

\[
\frac{dp^h}{d\alpha^l} = \frac{\partial p^h}{\partial \alpha^l} + \frac{\partial p^h}{\partial g^l} \frac{dg^l}{d\alpha^l} > 0,
\]

so \( p^h \) increases with rising \( \alpha^l \), and hence high types are worse off (\( u^h \) decreases: indifference curve shifts upward in figure 2). Since \( g^l \) decreases by assumption, the new intersection \((p^l, g^l)\) in the figure will be in region \( A \) in the figure, and clearly \( u^l \) also decreases. Hence, in this case, the increase in \( \alpha^l \) reduces both \( u^h \) and \( u^l \) and thus \( W \).

Case 2. Suppose instead that \( \frac{dg^l}{d\alpha^l} \geq 0 \). Then

\[
\frac{dp^l}{d\alpha^l} = \frac{\partial p^l}{\partial \alpha^l} + \frac{\partial p^l}{\partial g^l} \frac{dg^l}{d\alpha^l} \geq -\theta^l \frac{dg^l}{d\alpha^l} \quad (44)
\]

From equation (2): \( \frac{dp^l}{dg^l} = -u_g / u_p \cdot -\theta^l \). The indifference curve \( u^l \) has slope less (i.e. more negative) than \( -\theta^l \): an increase in \( g^l \) should be compensated by a fall in \( p^l \) that is
bigger than $\theta_l$. However, $|\partial p^l / \partial g^l| \leq \theta_l$ by equation (43) and $u^l$ falls (indifference curve shifts upward). As $g^l$ increases in this case, $(p^l, g^l)$ is in region B and again, both $u^h$ and $u^l$ decrease. Therefore, also in this case, $W$ falls with $\alpha_l$.

Hence, as we increase the mark-up from $l$-insurer moral hazard, $\alpha_l$, from zero to any positive value, and update $g^l$ along that path to keep $IC^h_c$ binding with equality, $W$ decreases. Hence the sponsor should set $x^l = 0$.

- Finally, we check that (22) is satisfied. With $g^h = 0$, binding $IC^l_i$ implies that (22) is equivalent to $x^h \geq x^l$. Since $x^l = 0$, this holds.

\textit{Q.E.D.}

\textbf{Proof of proposition 2} With CR, changing $x^h$ also implies changing $g^l$, as both are connected through the binding consumer incentive compatibility constraint ($IC^h_c$). To find the welfare effects of changing $x^h$, we therefore first derive how $g^l$ changes with $x^h$ along ($IC^h_c$) (holding with equality):

$$
\dot{u}^h = \frac{\partial p^l}{\partial x^h} d x^h + \frac{\partial p^l}{\partial g^l} d g^l + \dot{u}^l d g^l = u^h_p \left( \frac{\partial p^h}{\partial x^h} d x^h + \frac{\partial p^h}{\partial g^l} d g^l \right)
$$

(45)

where we used shorthand

$$
\dot{u}^h = u(p^l, g^l | \theta^h), \quad u^h = u(p^h, 0 | \theta^h).
$$
From the expressions for prices in the CR case, equations (32,33) we find

\begin{align}
\frac{\partial p^h}{\partial x^h} &= (1 - \phi)\varepsilon x^h - \phi(\theta^h y - \theta^l(y - g^l)) \\
\frac{\partial p^h}{\partial g^l} &= -\phi \theta^l + (1 - x^h) \theta^l \\
\frac{\partial p^l}{\partial x^h} &= (1 - \phi)\varepsilon x^h + (1 - \phi)(\theta^h y - \theta^l(y - g^l)) \\
\frac{\partial p^l}{\partial g^l} &= -\phi \theta^l - (1 - \phi)(1 - x^h) \theta^l
\end{align}

Using this and evaluating at \(x^h = 1, g^l = 0\), where prices are equal, we find

\begin{align}
\frac{\partial W}{\partial x^h} &= (-u_p)\left[-(1 - \phi)\varepsilon x^h + (\phi - \omega)\Delta \theta y\right] \\
\frac{\partial W}{\partial g^l} &= (-u_p)(\phi - \omega)\theta^l \\
dg^l \bigg/ dx^h &= -\frac{\Delta \theta y}{\theta^h} < 0
\end{align}

From this we derive

\begin{align}
\left.\frac{dW}{dx^h}\right|_{x^h=1,g^l=0} &= (-u_p) \left[-(1 - \phi)\varepsilon x^h + (\phi - \omega)\Delta \theta y \left(1 - \frac{\theta^l}{\theta^h}\right)\right]
\end{align}

Hence, at \(\varepsilon = 0\), \(x^h = 1\) is optimal, while with \(\phi - \omega\) positive, \(x^h = 1\) remains optimal for \(\varepsilon\) small but positive. For \(\varepsilon\) big enough, (53) turns negative and \(x^h < 1\) becomes optimal.

At the opposite extreme, for \(\varepsilon\) high enough, the increase in prices through moral-hazard costs \(\varepsilon \alpha^h_x\) at \(x^h = 0\) is higher than any potential gain from changes in \(g^l\) or redistribution, since the latter contributions to welfare do not scale with \(\varepsilon\).

**Q.E.D.**

**Proof of proposition 3** First note that \(\varepsilon = 0\) implies \(W_{RR}(0) = W_{CR}(0)\) because in this case \(x^h = 1, p^l = p^h\) and \(g^l = g^h = 0\) under both RR and CR. Further, propositions 1 and 2 imply that for \(\omega < \phi\) there exists \(\bar{\varepsilon}\) such that \(W_{RR}(\varepsilon) = W_{CR}(\varepsilon)\) for each \(\varepsilon \in [0, \bar{\varepsilon}]\).

Looking at the point \(x^h = 1\), where in both regimes prices are equal and \(g^l = 0\), we have

\begin{align}
\left.\frac{\partial W_{RR}}{\partial x^h}\right|_{x^h=1} &= (-u_p)\left[-(1 - \phi)\varepsilon x^h + (\phi - \omega)\Delta \theta y\right] \\
&> (-u_p)\left[-(1 - \phi)\varepsilon x^h + (\phi - \omega)\Delta \theta y \left(1 - \frac{\theta^l}{\theta^h}\right)\right] = \left.\frac{\partial W_{CR}}{\partial x^h}\right|_{x^h=1}
\end{align}
Thus, there exist values of $\varepsilon$ such that $\partial W_{RR}/\partial x^h|_{x^h=1} \geq 0 > \partial W_{CR}/\partial x^h|_{x^h=1}$. For these values of $\varepsilon$ we have $x^h_{RR}(\varepsilon) = 1 > x^h_{CR}(\varepsilon)$. With a slight abuse of notation, we write welfare as $W(\varepsilon,x^h(\varepsilon))$. Then we have for these values of $\varepsilon$ that

$$W_{CR}(\varepsilon,x^h_{CR}(\varepsilon)) > W_{CR}(\varepsilon,1) = W_{RR}(\varepsilon,1) = W_{RR}(\varepsilon,x^h_{RR}(\varepsilon)) \tag{55}$$

For $\varepsilon$ high enough, proposition 2 implies that $x^h_{CR} = 0$ and $g^i_{CR} = g^i_{RS} > 0$, so that we are in the situation without any risk adjustment. Then we know that $u^i_{CR} = u^i_{RR}$ while $u^i_{CR} < u^i_{RR}$. Hence, $\omega > 0$ implies that $W_{CR}(\varepsilon) < W_{RR}(\varepsilon)$ for such high values of $\varepsilon$. \textit{Q.E.D.}

B. General ex-post schemes

In this appendix we generalize two aspects of the main text. First, above we work with a reduced form model where ex-post risk adjustment (reinsurance) increases healthcare expenditures. Here we provide an explicit framework which implies this result. Second, above we derive our main results, propositions 1–3, for the case of proportional risk adjustment where the insurer gets a proportion $x$ of costs reimbursed by the sponsor. In general, this reimbursement scheme does not need to be linear. Below we show how the main equations of the proofs generalize to non-linear reinsurance schemes.

For this we introduce the following notation. For each expression we provide the equivalent in terms of proportional reimbursement as used above.

We assume that an insurer’s expenditure $y - g$ is contractible for the sponsor, but not $g$ itself. If an insured needs treatment and the insurer spends $z = y - g$, then the insurer receives ex post reimbursement $r(z,x)$. The reimbursement function $r$ is assumed to be smooth, satisfies $r(0,x) = 0$ and $r_z \in [0,1]$: expenditure $z$ is neither taxed nor subsidized more than one-for-one. The index $x \in [0,1]$ parameterizing this family of reimbursement functions indicates the generosity of the ex post scheme, $r_x \geq 0$. We normalize $r(z,0) = 0$ and $r(z,1) = z$. Further, $r_{xz} \geq 0$, implying that a more generous scheme raises marginal reimbursement $r_z$. In the main text we work with $r(z,x) = xz$. An example of a more general scheme is to reimburse only costs above a threshold $z^*$. This could be optimal if high costs are less elastic with respect to insurer effort than low costs.

\footnote{Note that we do not exclude the possibility where $x^h_{CR}$ drops discontinuously from 1 to some $x^h_{CR} < 1$.}
Define $R(x, g|\theta)$ as the expected costs, net of ex ante payment $t$ and premium income $p$, for an insurer of insuring a $\theta$-consumer with a contract $(p, g)$, when the insurer faces reinsurance $r(z, x)$. With proportional reimbursement we have $R(x, g|\theta) = (1 - x)\theta(y - g)$.

Let $C(x, g|\theta)$ denote the sponsor’s expected costs of financing an ex post scheme $r(.)$ with generosity $x$. By our normalization of $x$, we have that $C(0, g|\theta) = 0$. In the main text we have $C(x, g, |\theta) = x\theta(y - g) + \varepsilon\alpha(x, g|\theta)$.

The sponsor then chooses a function $r(z, x)$ and offers a menu $(t^l, x^l), (t^h, x^h)$ to insurers, where the insurer truthfully reveals the type $(l, h)$ of its customer. For our analysis the function $r(.)$ is exogenous; given this function, we derive the optimal $t^i, x^i$.

Truthful revelation of the insurer’s private information $\theta$ requires the risk adjustment scheme to be incentive compatible. The equivalent of equations (20, 21):

$$t^h + R^h \leq t^i + \hat{R}^i$$
$$t^i + R^l \leq t^h + \hat{R}^h$$

where we use short-hand notation: $R^l = R(x^l, g^l) = R(x^l, g^l|\theta^l)$, $R^h = R(x^h, g^h) = R(x^h, g^h|\theta^h)$ and for an $l$-insurer who claims to be $h$: $\hat{R}^h = \hat{R}^h(x^h, g^l) = R(x^h, g^l|\theta^l)$. Similarly, an $h$-insurer who claims to be $l$ has expected costs $\hat{R}^l = \hat{R}^l(x^l, g^h) = R(x^l, g^h|\theta^h)$.

**B.1. Model of insurer effort**

This section introduces a simple model where insurers invest effort to keep health expenditures low. We assume the function $R(.)$ to have the following property

$$R^h_x(x, 0) < \hat{R}^h_x(x, g') < 0$$

To see the intuition for this and following equations, we use proportional reinsurance. As $x$ increases, the cost reduction is bigger for customers with higher expected costs. With proportional reimbursement we get $\frac{d}{dx}(1 - x)\theta^h(y - g^h) < \frac{d}{dx}(1 - x)\theta^l(y - g')$ because $\theta^h > \theta^l$ and $g' \geq g^h = 0$.

We derive the following equations that we need to generalize our three propositions in the
First, \( R^h - \hat{R}^h \geq 0 \) because \( \theta^h > \theta^l \), \( g^l \geq g^h \) and –as in the main text– an \( l \)-insurer claiming to be \( h \) has no incentive to change \( g^l \). Even if an \( l \)-insurer mimicks an \( h \)-insurer, expected costs are lower for the \( l \)-insurer. Second, an increase in generosity \( x \) increases the sum of costs \( R + C \) with \( \varepsilon \alpha_x \). Third, suppose that there is no ex post transfer for \( l \)-consumers: \( x^l = 0 \). If an \( l \)-insurer claims to be \( h \), part of her costs are reimbursed ex post which lowers ex post costs compared to the case where it truthfully reveals the customer’s type. With proportional reimbursement this becomes: \( \theta^l(y - g^l) \geq (1 - x^h)\theta^l(y - g^l) \). Fourth, assuming \( x^l = 0 \), a small increase in \( g^l \) reduces the costs of an \( l \)-insurer by the probability \( \theta^l \) that \( g^l \) is saved by the insurer: \( \frac{d(\theta^l(y - g))}{dg} = -\theta^l \). Finally, when an \( l \)-insurer claims to be \( h \), part of the expenditure is reimbursed ex post, hence the effect of an increase in \( g \) is smaller. With proportional reimbursement \( |\frac{d}{dg} (1 - x^h)\theta^l(y - g)| \leq |\frac{d}{dg} \theta^l(y - g)| = \theta^l \).

We define \( \bar{x}^h \) as the level of ex-post risk adjustment such that the \( l \)-insurer’s information rent, \( R^h - \hat{R}^h \), disappears. That is, at \( x^h = \bar{x}^h \), the costs of an \( h \)-insurer equals the costs of an \( l \)-insurer claiming to have an \( h \)-consumer:

\[
R^h(\bar{x}^h, 0) - \hat{R}^h(\bar{x}^h, g^l) = 0
\]

and

\[
|\hat{R}^h_{\bar{g}^l}(\bar{x}^h, g^l)| \leq \theta^l
\]

With proportional ex post reimbursement, we find \((1 - \bar{x}^h)[\theta^h y - \theta^l(y - g^l)] = 0 \) and \( \hat{R}^h_{\bar{g}^l}(\bar{x}^h, g^l) = 0 < \theta^l \) if \( \bar{x}^h = 1 \).

For the analysis here, these are the aspects of ex-post risk adjustment that are important. The following model yields the required results.

Let \( Y \) denote the interval of expenditures \( y \), once an agent falls ill. Insurers can invest
effort \( e \) to reduce expenditure. Let \( F(y|e) \) denote the distribution function of \( y \), which is the same for both types \( l, h \). Then comparing two effort levels \( e_1 > e_2 \), we assume that \( F(y|e_2) \) first order stochastically dominates \( F(y|e_1) \). That is, \( F(y|e_2) \leq F(y|e_1) \). Expected expenditure (conditionally on needing health care) is given by

\[
\bar{y}(e) = \int_Y y dF(y|e) \tag{65}
\]

First order stochastic dominance implies that \( \bar{y}_e(e) \leq 0 \). The expected ex-post reimbursement can be written as

\[
\bar{r}(x, g, e) = \int_Y r(y-g,x) dF(y|e) \tag{66}
\]

As \( r_z \geq 0 \), first order stochastic dominance implies that \( \bar{r}_e \leq 0 \). Further, \( r_z(z, x) \in [0, 1] \) implies that \( \bar{r}_g(x, g, e) \in [-1, 0] \).

We assume that insurers’ effort cost \( \psi(e) \) is incurred once an insured needs treatment. What we have in mind is the following: once a patient falls ill, the insurer can check whether treatments are necessary, try to guide the patient to a cheaper provider etc. The insurer chooses effort \( e \) that minimizes total costs:

\[
R(x, g|\theta) = \min_e \theta[\bar{y}(e) - g - \bar{r}(x, g, e) + \psi(e)] \tag{67}
\]

Let \( e(x, g) \) denote the effort level that solves this minimization problem. In case of an interior solution, we have

\[
\bar{y}_e(e(x, g)) - \bar{r}_e(x, g, e(x, g)) + \psi_e(e(x, g)) = 0 \tag{68}
\]

It follows that \( de/dx \leq 0 \). This can be seen as follows:

\[
[\bar{y}_{ee}(e(x, g)) - \bar{r}_{ee}(x, g, e(x, g)) + \psi_{ee}(e(x, g))] \frac{de}{dx} = \bar{r}_{ex}(x, g, e(x, g)) \tag{69}
\]

The expression in square brackets is positive (second order condition) and the right hand side is non-positive (first order stochastic dominance with \( r_{xz}(z, x) \geq 0 \)). Hence, we find \( de/dx \leq 0 \): as the ex-post reimbursement becomes more generous, insurers invest less effort.

The cost for the sponsor of implementing ex-post insurance with generosity \( x \) is given by

\[
C(x, g|\theta) = \theta \bar{r}(x, g, e(x, g)) \tag{70}
\]
Total cost can then be written as

$$R(x, g|\theta) + C(x, g|\theta) = \theta(\bar{y}(e(x, g)) - g + \psi(e(x, g)))$$  \hspace{1cm} (71)$$

Hence we find that

$$R_x(x, g|\theta) + C_x(x, g|\theta) = \theta(\bar{y}_e(e(x, g)) + \psi_e(e(x, g)) \frac{de}{dx}$$  \hspace{1cm} (72)$$

$$= \theta \bar{r}_e(x, g, e(x, g)) \frac{de}{dx} \geq 0$$  \hspace{1cm} (73)$$

where the second equality follows from (68); the inequality follows from $\bar{r}_e \leq 0$ and $de/dx \leq 0$ derived above. Writing $\alpha_x(x, g|\theta) = \theta \bar{r}_e de/dx \geq 0$, we find equation (59).

The following inequalities derive equation (58):

$$R^h(x, 0) = R(x, 0|\theta^h) = \theta^h[\bar{y}(e(x, 0)) - \bar{r}(x, 0, e(x, 0)) + \psi(e(x, 0))] \geq$$  \hspace{1cm} (74)$$

$$\theta^h[\bar{y}(e(x, 0)) - \bar{r}(x, 0, e(x, 0)) + \psi_e(e(x, 0))] \geq$$  \hspace{1cm} (75)$$

$$\theta^h[\bar{y}(e(x, 0)) - g^l - \bar{r}(x, g^l, e(x, 0)) + \psi_e(e(x, 0))] \geq$$  \hspace{1cm} (76)$$

$$\theta^h[\bar{y}(e(x, g^l)) - g^l - \bar{r}(x, g^l, e(x, g^l)) + \psi(e(x, g^l))] = R(x, g^l|\theta^l) = \hat{R}^h(x, g^l)$$  \hspace{1cm} (77)$$

where the first inequality follows from $\theta^h > \theta^l$, the second from $g^l \geq 0$ and $\bar{r}_e \in [-1, 0]$, the third inequality from the fact that $e(x, g^l)$ minimizes the insurer’s cost with an $l$-insured.

With a similar reasoning we can prove (60):

$$R^l(0, g^l) = \theta^l[\bar{y}(e(0, g^l)) - g^l + \psi(e(0, g^l))] \geq$$  \hspace{1cm} (78)$$

$$\theta^l[\bar{y}(e(0, g^l)) - g^l - \bar{r}(x, g^l, e(0, g^l)) + \psi(e(0, g^l))] \geq$$  \hspace{1cm} (79)$$

$$\theta^l[\bar{y}(e(x, g^l)) - g^l - \bar{r}(x, g^l, e(x, g^l)) + \psi(e(x, g^l))] = \hat{R}^h(x, g^l)$$  \hspace{1cm} (80)$$

Finally,

$$R_y(x, g|\theta^l) = \hat{R}^h_y(x, g) = -\theta^l(1 + \tilde{r}_g(x, g, e)) \geq -\theta^l$$  \hspace{1cm} (81)$$

because $\tilde{r}_g \in [-1, 0]$; which proves (62). Further, $\bar{r}(0, g, e) = 0$ for all $g \geq 0$; hence $R_y(0, g|\theta^l) = -\theta^l$: equation (61).
B.2. Generalizing the three main results

In this section we show how the three propositions above can be proved with the more general reinsurance set-up in this appendix. The previous version of the paper—available on request—gives the complete proofs with this set-up, here we focus on the main equations of each proof.

For proposition 1, the main equation is (37). With more general reinsurance, we write this equation as

$$\frac{\partial W}{\partial x} h = (1 - \phi) \varepsilon \alpha(x, 0)[\theta]^h (\omega u_p + (1 - \omega) u_p^h) - (\hat{R}_x^h - \hat{\hat{R}}_x^h)(|u_p^h| \phi(1 - \omega) - |u_p^h| \omega(1 - \phi))$$

where $-(\hat{R}_x^h - \hat{\hat{R}}_x^h) > 0$ giving us a similar expression as (37). Hence, the important feature is that an increase in $x$ reduces the information rent $R^h - \hat{R}^h$. The argument follows that in the proof above by evaluating $x^h$ close $\bar{x}^h$ instead of 1.

For proposition 2, we write equation (53) as

$$\frac{dW}{dx} h \bigg|_{x^h = \bar{x}^h, g'} = 0 = (-u_p) \left[ -(1 - \phi) \varepsilon \alpha^h_x - (\phi - \omega)(R_x^h - \hat{R}_x^h) \left( 1 - \frac{\hat{R}_g^h + \theta^h}{\theta^g + \hat{R}_g^h} \right) \right]$$

where, again, $-(R_x^h - \hat{R}_x^h) > 0$ and $0 \leq \hat{R}_g^h + \theta^h \leq \theta^h + \hat{R}_g^h$ by equation (62). So also here we find that $x^h = \bar{x}^h$ is optimal in case $\varepsilon = 0$ and remains optimal for $\varepsilon > 0$ but small.

Finally, proposition 3; we can write equation (54) as

$$\frac{\partial W_{RR}}{\partial x^h} h \bigg|_{x^h = \bar{x}^h} = (-u_p) \left[ -(1 - \phi) \varepsilon \alpha^h_x + (\phi - \omega)R_x^h - \hat{R}^h_x \right]$$

where the inequality follows from $(\hat{R}_g^h + \theta^h)/(\theta^h + \hat{R}_g^h) \in (0, 1)$ because of (64). We then write equation (55) as

$$W_{CR}(\varepsilon, x_{CR}(\varepsilon)) > W_{CR}(\varepsilon, \bar{x}^h) = W_{RR}(\varepsilon, \bar{x}^h) = W_{RR}(\varepsilon, x_{RR}(\varepsilon))$$