An integrated simulation tool for analyzing the Operation and Interdependency of Natural Gas and Electric Power Systems

Kwabena Addo Pambour*, Burcin Cakir Erdenerb, Ricardo Bolado-Lavinc, Gerard P. J. Dijkemaa

a University of Groningen, Energy and Sustainability Research Institute Groningen (ESRIG), Groningen, The Netherlands
b European Commission, Joint Research Centre, Institute for Energy and Transport, Ispra, Italy
c European Commission, Joint Research Centre, Institute for Energy and Transport, Petten, The Netherlands

ABSTRACT

In this paper, we present an integrated simulation tool for analyzing the interdependency of natural gas and electric power systems in terms of security of energy supply. In the first part, we develop mathematical models for the individual systems. In part two, we identify the interconnections between both systems and propose a method for coupling the combined simulation model. Next, we develop the algorithm for solving the combined system and integrate this algorithm into a simulation software. Finally, we demonstrate the value of the software in a case study on a real world interconnected gas and electric power system of an European region.

NOMENCLATURE

EU: European Union
CEI: Critical energy infrastructures
CGS: City Gate Station
GPP: Gas Fired Power Plant
LNG: Liquefied Natural Gas
PDE: Partial Differential Equation

TSO: Transmission System Operator
UGS: Underground Gas Storage
A: incidence matrix
Ad: cross-sectional area
b: line charging susceptance
c: speed of sound
CV: Control Volume, calorific value
D: inner pipe diameter
e: Euler’s number
F: residual vector
f: electric driver factor
g: gravitational acceleration
H: elevation
I: electric current
Iw: storage working inventory
J: Jacobi matrix
k: iteration step
kc: constraint handling iteration step
Ki: participation factor
L: nodal load (vector)
Lset: load set point
LElec: gas offtake of power plants
Linfj: injection rate
Lwdr: withdrawal rate
l: pipe length
le: equivalent pipe length
LP: line pack
m: number of branches
n: number of nodes or buses
P: square pressure, active power
PD: active power demand
PDs: power demand of compressor stations
PG: active power generation
PGset: active power generation set point
ΔP: square pressure drop, power imbalance
p: gas pressure (vector)
Δp: pressure drop

*corresponding author, Email: k.a.pambour@rug.nl
**INTRODUCTION**

The ongoing integration of renewable energy resources into the energy portfolio of the European Union is connected with an increased interconnection between the different critical energy infrastructures (CEI). The interdependency between natural gas and electric power systems, for instance, is expected to grow in the near future. On the electric side, the demand for flexible backup power for intermittent renewable energy sources is increasing, which can be met by gas fired power plants (GPP) connected to the gas and electric grid, while on the gas side an increased use of electric power to operate facilities in the gas system can be observed (e.g. electric driven compressors, electric power supply to LNG Terminals etc.). Moreover, the present advancement in the Power-to-Gas technology will significantly contribute to the coupling of both systems. These trends suggest the need for simulation models to examine the depth and scope of these interdependencies, how they may affect the operation of both systems and how to proactively approach the bottlenecks that may emerge. Furthermore, developing combined gas and electricity models will also facilitate the development of Risk Assessment for this type of coupled networks.

In this paper, we present an integrated simulation tool, **SAInt** (Scenario Analysis Interface for Energy Systems), for analyzing the interdependency of gas and electric power systems in terms of security of energy supply, i.e. the uninterrupted supply of energy to its customers (e.g. commercial, residential, industrial customers, public services and power generation companies) particularly in case of difficult climatic conditions and in the event of disruptions [1].

The first part of the paper, focuses on developing the mathematical models for both systems. The second part, elaborates the interconnections between both systems and derives coupling equations for the combined system, followed by a description of the algorithm for solving the resulting system of equations. Finally, the capability of the simulation tool is demonstrated by applying it to a real world instance.

**METHODOLOGY**

The operation of gas and electric power systems is increasingly interdependent, due to an increased physical interconnection between the facilities installed in both systems. A change in one system may propagate to the other system and even back to the triggering system. For instance, an increase in power generation from a gas fired power plant, will cause the gas offtake from the gas grid to increase. This, in turn, may result in an increased power offtake of electric driven compressor stations to recover the pressure and line pack level in the area affected by the gas offtake. This additional power offtake, again, will trigger an increase in power generation which has to be balanced by the power generation units. This cycle may continue until an equilibrium state is reached.

The goal of a combined gas and power system study is to find for each time step a state of the coupled system, that satisfies the
physical equations describing the behavior of the gas and power system and the coupling equations describing the link between both systems, taking into account the controls and constraints imposed by the different facilities involved in the transport process.

The first challenge that arises, when modeling the coupled gas and power system is to find a simulation model that describes the dynamic behavior of the individual system appropriately. The dynamics in gas transport systems, for instance, are much slower than the one in power systems. Electricity travels almost instantaneously and cannot be stored economically in large quantities in current power systems\(^1\) [2]. In case of a disruption, the response time of the power system is quite small and basically the transmission line flows satisfy the steady-state algebraic equations. On the contrary, natural gas pipeline flow is a much slower process, with gas velocities typically below 10 m/s (50 [ft/s]), resulting in a longer response time in case of a large fluctuation. In particular, high-pressure transmission pipelines have much slower dynamics due to the large sums of natural gas stored in the pipelines.

Considering the different characteristics of both systems, in this paper, we propose a transient model for the gas system and a steady-state AC- power flow model for the power system. We couple both models to a combined simulation model by defining coupling equations reflecting the physical interlink between both systems. In the following, we derive the gas and power system model independently. Next, we identify the interconnection between both systems and develop the coupling equations for the combined model.

**Gas System Model**

In this section, we give a brief overview of the gas system model implemented in the simulation tool **SAInt**. Furthermore, we demonstrate the accuracy of the model by comparing simulation results for a well known sample network to results obtained with a commercial software. For a more detailed description of the gas model, we refer to PAMBOUR et al. [3].

The purpose of a gas transport system is to transport natural gas from remote production sites to demand areas, where gas is needed for heating, power generation or as a feed stock for industrial production. The transport of natural gas involves the use of different types of facilities, such as pipelines, compressor stations, regulator stations and valves. These facilities are supervised and controlled by Transmission System Operators (TSO), to ensure a safe, reliable and economical operation of the transport system.

A gas network is usually described by a directed graph composed of nodes and branches. Facilities with an inlet, outlet and flow direction are modeled as branches, while connection points between these branches as well as entry and exit stations are represented by nodes. Branches, in turn, can be distinguished between active and passive branches. Active branches represent controlled facilities, which can change their state or control during operation, such as compressor stations, regulator stations and valves (s. Table 2), while passive branches, such as pipelines and resisters represent facilities or components which state is fully described by the physical equations (s. Table 1).

The topology of the network can be described by the following node-branch incidence matrix.

\[
A = [a_{ij}]^{n \times m}
\]

with

\[
a_{ij} = \begin{cases} 
+1, & \text{node } i \text{ is outlet of branch } j \\
-1, & \text{node } i \text{ is inlet of branch } j \\
0, & \text{node } i \text{ and branch } j \text{ are not connected}
\end{cases}
\]

where \(n\) is the number of nodes and \(m\) the number of branches in the network.

The gas flow in transport pipelines is inherently dynamic. Supply and demand are constantly changing and the reaction of the system to these changes are relatively slow, due to the small flow velocities (typically below 10 [m/s], approx. 50 [ft/s]) and the large volume of gas stored in transport pipelines.

The dynamic behavior of a gas system is predominantly determined by the gas flow in pipelines. In general, a gas pipeline has four basic properties, namely, capacity (i.e. the ability to store a certain volume of gas, which depends on the geometric volume and maximum pipeline pressure), resistance (i.e. force acting opposite to the gas flow direction, caused by friction between gas and the inner walls of the pipeline), inertia (force acting opposite to the gas flow acceleration) and gravity (gravitational force acting on the gas volume in sloped pipelines). Capacity and resistance are the predominant properties, while in most cases gravity and inertia play a secondary role. The pipe elements in a gas network can be segmented into a number of pipe sections, assuming each section inherits a proportional fraction of the properties of the original pipeline[4]. Fig. 1 demonstrates this for a cross section of a pipeline. As can be seen, the volumes of the pipelines are equally distributed and assigned to the inlet and outlet nodes, respectively. According to the mass conservation law, the gas density \(\rho_i\) in a nodal control volume \(V_i\) may change in time, if there is an imbalance between gas inflow and outflow to \(V_i\). If we assume isothermal flow conditions, the mass conservation law can be expressed by the following integral form of the continuity equation:

\[
\frac{V_i}{\rho_i c} \frac{d\rho_i}{dt} = \sum_{j=1}^{k} a_{ij} Q_{ij} - L_i
\]
transport of natural gas from point a to b, short term gas storage

passive devices that cause a local pressure drop (e.g. meters, inlet piping, coolers, heaters, scrubbers etc.)

\[
\Delta P_n^{t+1} = R_f \cdot (Q_n^{t+1})^2 + R_i \cdot (Q_n^{t+1}-Q_i) (2)
\]

\[
\Delta P_n^{t+1} = p_1^{2n+1} - p_2^{2n+1} e^s, \quad R_f = \frac{2 \rho_{s} \Delta p_m}{\Delta t \Delta A}
\]

\[
s = \frac{2 g (H_2 - H_1)}{c^2}, \quad R_i = \frac{16 \Delta p_{s}^2 c^2 l_e}{\pi^2 D^5}
\]

\[
l_e = \begin{cases} 
1, & H_1 = H_2 \\
\frac{e^s - 1}{s}, & H_1 \neq H_2
\end{cases}
\]

\[
p_1 - p_2 = \frac{\zeta \rho}{2} |v| v (3)
\]

Table 1: Basic passive elements in a gas network model

Figure 1: Law of conservation of mass applied to a nodal control volume in cross section of a pipeline network

with

\[
\dot{c}^2 = \frac{p}{\rho} = ZRT, \quad V_i = \frac{\pi}{8} \sum_{j=1}^{k} D_{ij}^2 \Delta x_{ij}
\]

where \(Q_{ij}\) is the flow rate from node \(i\) to \(j\) at reference conditions, \(\rho_{n}\) the gas density at reference conditions, \(L_i\) the external load on node \(i\), \(c\) the isothermal speed of sound, \(Z\) the gas compressibility factor, \(R\) the specific gas constant, \(T\) the gas temperature, \(\Delta x_{ij}\) and \(D_{ij}\) the length and diameter of pipe segment \((ij)\), respectively. The continuity equation can be expressed for each nodal control volume in the network, thus, we obtain \((n)\)

set of equations with \(2n + m\) unknown state variables \((p_i, Q_{i,j}\) and \(L_i)\). If we perform an implicit time integration on this set of equations for a time step \(\Delta t = t_{n+1} - t_n\) and order the equations in terms of known variables at time \(t_n\) and \(t_{n+1}\) (right hand side) and unknown variables at time \(t_{n+1}\) (left hand side), we obtain the following set of linear finite difference equations for the total network:

\[
\Phi P^{t+1} - A Q^{t+1} = \Phi P^n - 0.5 (L^n + L^{n+1}) (5)
\]

where \(Q\) and \(L\) are the vectors of branch flows and nodal loads, respectively, and \(\Phi\) the following diagonal matrix, describing the pressure coefficients \(\phi_i\):

\[
\Phi = \text{diag} \{ \phi_1, \phi_2, ..., \phi_n \}, \quad \phi_i = \frac{V_i}{\rho_{n} c^2 \Delta t} (6)
\]

In order to close and solve eq. (5) for the entire network including non-pipe facilities, \(n + m\) additional independent equations are needed (i.e. one equation for each branch and each node in the network), which correlate the state variables \(p_i, Q_{i,j}\) and \(L_i\). These equations a provided by the pressure drop equation for each pipe section and the equations describing the control modes of non-pipe facilities. Tables 2, 3 & 4 give an overview of the implemented control modes, constraints and their corresponding linearized equations for non-pipe facilities modeled as active branches and nodes, respectively.

The pressure drop equation for a pipe section is derived from the law of conservation of momentum as shown in Fig. 2, where the different forces acting on a control volume in a pipe section are illustrated. The resulting momentum equation yields:

\[
\frac{\partial (\rho v^2)}{\partial t} + \frac{\partial (\rho v x)}{\partial x} + \frac{\partial p}{\partial x} + \frac{2\Delta p}{2D} + \rho g \sin \alpha = 0 (7)
\]

inertia convective term pressure friction gravity
where \( v \) is the gas flow velocity and \( \lambda \) the Darcy friction factor. Eq. 7 can be reduced to the following non-linear partial differential equation (PDE), if we assume isothermal flow conditions and neglect the convective term.

\[
\frac{\partial p}{\partial x} = -\frac{\rho_g}{A} \frac{\partial Q}{\partial t} - \frac{\lambda}{2DA^2} |Q|^2 p - g \sin \alpha \frac{Q}{c^2} p \tag{8}
\]

where \( A \) denotes the cross-sectional area of the pipe section. Eq. 8 can be discretized using an implicit finite difference scheme as shown in Table 1, where \( l \) denotes the length, \( H_1 \) and \( H_2 \) the inlet and outlet elevation of a pipe segment, respectively. In sum, the system of equations describing the behavior of the gas system can be expressed by the following linearized matrix equation:

\[
\begin{pmatrix}
\Phi & -A & I \\
C_p & C_Q & 0 \\
K_p & K_Q & K_L
\end{pmatrix}
\begin{pmatrix}
p_{n+1} \\
Q_{n+1} \\
L_{n+1}
\end{pmatrix}
=
\begin{pmatrix}
\Phi & p^n \\
D & \rho_g A dx \\
S & F
\end{pmatrix}
\tag{9}
\]

where the first row describes the continuity equation, the second row the linearized equation for passive and active branches (s. Table 1, 2 & 4) and the third row the equations for the control mode of entry, exit stations, LNG terminals and underground gas storage facilities (s. Table 3). Eq. 9 is solved iteratively for each simulation time step \( t_{n+1} \) using the initial state \( t_n \) or the solution of a preceding time step \( t_{n-1} \) as an initial guess for the iterative linearization. We adapted the linearization methods presented by van der Hooven [5] for the steady state to the transient case. The algorithm is detailed in [3].

**MODEL BENCHMARKING**

In the following, we benchmark the accuracy of the presented gas model against results from the commercial software SIMONE for a sample network adapted from [6]. The data of the network topology, gas and pipe properties and steady state boundary conditions are given in Tables 8 and 9. In the first step, we run a steady state simulation to obtain an initial state of the network, which we then use in a second step to compute a dynamic simulation over 24 hours, using the load profile shown in Figure 18, which we multiply with the steady state load for each demand node. Figures 3 and 4 show the steady state solution, while Figure 5 illustrates the results for the dynamic simulation. As can be seen, the results obtained with SAInt are very similar to the SIMONE results, which confirms the accuracy of the simulation model. The comparison of the steady state results obtained with SAInt (s. Figure 3) and SIMONE (s. Figure 4) shows small deviations (< 0.2 [bar], 2.9 [psi]) in the nodal pressure and compressor flow rate (< 2 [kms/h], 1.62 [mmscfd]) in the area around compressor station Co3. Similar discrepancies can be made for the time plots for the dynamic simulation (s. Figure 5). The shape of the time plots for the gas supply in the source node (s. Figure 5 a)) and the nodal pressure for three selected nodes (s. Figure 5 b) c) and d)) obtained with SAInt and SIMONE are very similar, however, small deviations (< 0.2 [bar], 2.9 [psi]) can be observed for the last five simulation hours. Discrepancies for the transient model in other nodes are in the same range as the ones shown here. We consider these discrepancies (below 0.5%) as quite good results.

**POWER SYSTEM MODEL**

After presenting the model for the gas system, in this section we elaborate the model implemented for the power system. In the first part, we give an overview of the different elements involved in the operation of a power system, their function and constraints. Finally, we develop the system of equations describing the steady state power flow in power systems.

An electric power system can be divided into three subsystems operating at different voltage levels, namely, the generation (11-35[kV]), transmission (usually above 110[kV]) and distribution system (11k-400[V] or 230[V])\(^3\). The generation system produces electricity by converting primary energy sources (e.g. fossil fuels, wind, hydro etc.) to electric energy, using synchronous turbo generators, which are driven by gas, steam, water or wind turbines. The generating units inject Alternating Currents (AC) to a 3-phase transmission system at a constant voltage magnitude (\(|V|\)) and frequency (\(f\)) (usually 50 [Hz]). Voltage magnitude and frequency are typically controlled by a designated Automatic Generation Control System (AGC) [7].

In order to reduce the power losses incurred during transportation (\(\sim I^2 \cdot R\)), the output voltages of generation units are usually increased to transmission system level using step up transformers. The transmission system provides a network of in-
In this paper, we focus primarily on the high voltage electric transmission system, which is the most crucial subsystem in the power supply chain. Similar to a gas network, a power transmission system can be described by a directed graph consisting of nodes and branches, where each branch represent a transmission line or a transformer and each node a connection point between two or more electrical components, also referred to as bus. At some of the buses power is injected into the network, while at others power is consumed by system loads. In contrast to gas systems, power systems are predominantly in steady state operation or in a state that could with sufficient accuracy be regarded as steady state [8]. Thus, the 3-phase transmission system is typically modeled as a balanced per phase equivalent system using linear models for the elements involved in the transport process. A transmission line, for instance, can be described by an equivalent $\pi$-circuit as depicted in Table 5, which considers the basic properties of an actual transmission line, such as line resistance $R_{ij}$, line reactance $X_{ij}$ and line charging susceptance $b_{ij}$. Using these properties the complex inlet and outlet bus voltages $V_i$ and $V_j$ can be related to their corresponding complex current injections $I_i$ and $I_j$ through the branch admittance matrix $Y_{br}$ as shown in eq. (10) in Table 5. A similar branch admittance matrix can be expressed for in-phase and phase shifting transformers as listed in eq. (11) in Table 5.

<table>
<thead>
<tr>
<th>Facility</th>
<th>Function</th>
<th>Control Modes</th>
<th>Constraints</th>
<th>Envelope</th>
</tr>
</thead>
</table>
| Compressor Station | compensates the pressure and head losses due to friction and heat transfer by increasing the gas pressure | inlet pressure ($p_i$,set) outlet pressure ($p_o$,set) pressure ratio ($\Pi_{set}$) pressure difference ($\Delta p_{set}$) flow rate ($Q_{set}$) volumetric flow ($Q_{vol, set}$) shaft power ($PW_{ shaft}$) driver power ($PW_{d set}$) driver fuel ($Q_{f, set}$) closed (OFF) bypass (BP) | internal hard limits: $p_o \geq p_i$ & $Q \geq 0$
user defined limits: max. outlet pressure ($p_{o,max}$) min. inlet pressure ($p_{i,min}$) max. volumetric flow ($Q_{vol,max}$) max. flow rate ($Q_{max}$) max. pressure Ratio ($\Pi_{max}$) max. driver power ($PW_{d,max}$) |  |
| Regulator Station | reduces the upstream pressure to a lower downstream pressure and/or regulates the gas flow rate | inlet pressure ($p_i$,set) outlet pressure ($p_o$,set) pressure difference ($\Delta p_{set}$) flow rate ($Q_{set}$) volumetric flow ($Q_{vol, set}$) closed (OFF) bypass (BP) | internal hard limits: $p_i \geq p_o$ & $Q \geq 0$
user defined limits: max. outlet pressure ($p_{o,max}$) min. inlet pressure ($p_{i,min}$) max. volumetric flow ($Q_{vol,max}$) max. flow rate ($Q_{max}$) |  |
| Valve Station | interrupts the gas flow and shuts-off sections of the gas network for maintenance or safety reasons | closed (OFF) opened (BP) | internal hard limit: $V \leq 60$ [m/s] user defined limits: max. flow velocity ($V_{max}$) |  |

Table 2: Overview of available control modes and constraints settings for active elements
<table>
<thead>
<tr>
<th>Facility</th>
<th>Control Modes</th>
<th>Constraints</th>
<th>Envelope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry Station</td>
<td>pressure (p_{set}) inflow (Q_{set})</td>
<td>internal hard limits: (L \leq 0) user defined limits: min. supply flow (Q_{\text{min}}) max. supply flow (Q_{\text{max}}) min. supply pressure (p_{\text{min}}) max. supply pressure (p_{\text{max}})</td>
<td>~</td>
</tr>
<tr>
<td>Exit Station</td>
<td>pressure (p_{set}) outflow (Q_{set})</td>
<td>internal hard limits: (L \geq 0) user defined limits: min. delivery flow (Q_{\text{min}}) max. delivery flow (Q_{\text{max}}) min. delivery pressure (p_{\text{min}}) max. delivery pressure (p_{\text{max}})</td>
<td>~</td>
</tr>
<tr>
<td>UGS</td>
<td>pressure (p_{set}) withdrawal/injection rate (Q_{set}) initial working inventory (INV) withdrawal state (WDR) injection state (INJ)</td>
<td>internal hard limits: (L^{wdr} \leq 0 &amp; L^{inj} \geq 0) user defined hard limits: max. working inventory (I_{w,\text{max}}) max. withdrawal rate (Q_{wdr,\text{max}}) max. injection rate (Q_{inj,\text{max}}) user defined limits: max. supply pressure (p_{wdr,\text{max}}) min. offtake pressure (p_{inj,\text{min}})</td>
<td></td>
</tr>
<tr>
<td>LNG Terminal</td>
<td>pressure (p_{set}) regasification rate (Q_{set}) initial working inventory (INV) arriving vessel size (VESSEL)</td>
<td>internal hard limits: (L \leq 0) user defined hard limits: max. working inventory (I_{w,\text{max}}) max. regasification rate (Q_{\text{reg,\text{max}}\text{max}}) user defined limits: max. supply pressure (p_{\text{reg,\text{max}}\text{max}})</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Overview of available control modes and constraints settings for non-pipe facilities modeled as nodes
### Table 4: Control modes for non-pipe facilities and their mathematical implementation

<table>
<thead>
<tr>
<th>Control Mode</th>
<th>Equation</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>inlet pressure ($p_{i,\text{set}}$)</td>
<td>$p_i = p_{i,\text{set}}$</td>
<td>$c_1 = 1$, $c_2 = 0$, $c_3 = 0$, $d = p_{i,\text{set}}$</td>
</tr>
<tr>
<td>outlet pressure ($p_{o,\text{set}}$)</td>
<td>$p_o = p_{o,\text{set}}$</td>
<td>$c_1 = 0$, $c_2 = 1$, $c_3 = 0$, $d = p_{o,\text{set}}$</td>
</tr>
<tr>
<td>pressure ratio ($\Pi_{\text{set}}$)</td>
<td>$\frac{p_o}{p_i} = \Pi_{\text{set}}$</td>
<td>$c_1 = -\Pi_{\text{set}}$, $c_2 = 1$, $c_3 = 0$, $d = 0$</td>
</tr>
<tr>
<td>pressure difference ($\Delta p_{\text{set}}$)</td>
<td>$p_o - p_i = \Delta p_{\text{set}}$</td>
<td>$c_1 = -1$, $c_2 = 1$, $c_3 = 0$, $d = \Delta p_{\text{set}}$</td>
</tr>
<tr>
<td>flow rate ($Q_{\text{set}}$)</td>
<td>$Q = Q_{\text{set}}$</td>
<td>$c_1 = 0$, $c_2 = 0$, $c_3 = 1$, $d = Q_{\text{set}}$</td>
</tr>
<tr>
<td>volumetric flow ($Q_{\text{vol,\text{set}}}$)</td>
<td>$Q = \frac{p_i}{Z_{i}T_{i}R_{p}}Q_{\text{vol,\text{set}}}$</td>
<td>$c_1 = -\frac{Q_{\text{vol,\text{set}}}}{Z_{i}T_{i}R_{p}}$, $c_2 = 0$, $c_3 = 1$, $d = 0$</td>
</tr>
<tr>
<td>shaft power ($PW_{s,\text{set}}$)</td>
<td>$PW_{s,\text{set}} = \frac{K_{o}Q}{c_{K}}\left[\Pi^{\varepsilon} - 1\right]$</td>
<td>$c_1 = -\frac{K_{o}Q}{p_i}\Pi^{\varepsilon}$, $c_2 = \frac{K_{o}Q}{p_o}\Pi^{\varepsilon}$, $c_3 = \frac{K_{i}}{c_{K}}\left[\Pi^{\varepsilon} - 1\right]$, $d = PW_{s,\text{set}}$</td>
</tr>
<tr>
<td>driver power ($PW_{d,\text{set}}$)</td>
<td>$PW_{d,\text{set}} = \frac{K_{o}Q}{c_{K}}\left[\Pi^{\varepsilon} - 1\right]$</td>
<td>$c_1 = -\frac{K_{o}Q}{p_i}\Pi^{\varepsilon}$, $c_2 = \frac{K_{o}Q}{p_o}\Pi^{\varepsilon}$, $c_3 = \frac{K_{i}}{c_{K}}\left[\Pi^{\varepsilon} - 1\right]$, $d = PW_{d,\text{set}}$</td>
</tr>
<tr>
<td>bypass (BP)</td>
<td>$p_i = p_o$</td>
<td>$c_1 = -1$, $c_2 = 1$, $c_3 = 0$, $d = 0$</td>
</tr>
<tr>
<td>off (OFF)</td>
<td>$Q = 0$</td>
<td>$c_1 = 0$, $c_2 = 0$, $c_3 = 1$, $d = 0$</td>
</tr>
</tbody>
</table>

For the total network, a bus admittance matrix $Y_{\text{bus}}$ containing the elements of the individual branch admittance matrices can be expressed as follows:

$$I = Y_{\text{bus}} V$$  \hspace{1cm} (12)

where $V$ and $I$ denote the vectors of complex voltages and currents at each bus, respectively.

The in- and outflow of electric power to the power system is modeled by generation units, loads, shunt capacitors and reactors connected to the buses of the power system. Table 6 shows a list of these components, their function and constraints. The steady state analysis of a power system involves the determination of the voltage magnitudes $|V_i|$, voltage angles $|\delta_i|$, active power $P_i$ and reactive power $Q_i$ supply at each bus $i$, considering the constraints imposed by the different facilities and components in the power system. These state variables are computed from the power flow balance equation, derived from Kirchhoff’s Current Law (KCL) applied to each bus. The power balance equation for a bus $i$ yields the following two non-linear
Figure 3: Steady state solution for the reference network obtained with SAInt

Figure 4: Steady state solution for the reference network obtained with SIMONE
equations for the active and reactive power balance:

\[ P_i(\delta, |V|) = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\delta_i - \delta_j - \theta_{ij}) \]  \hspace{1cm} (13)

\[ Q_i(\delta, |V|) = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\delta_i - \delta_j - \theta_{ij}) \]  \hspace{1cm} (14)

where \( P_{G,i}, Q_{G,i} \) are active and reactive power generation at bus \( i \), respectively, \( P_{D,i}, Q_{G,i} \) active and reactive power demanded at bus \( i \), respectively, and \( Y_{ij} = |Y_{ij}|(\cos(\theta_{ij}) + jsin(\theta_{ij})) \) the elements of the bus admittance matrix describing the branch connection between bus \( i \) and any connected bus \( j \). The solution of the power flow equations (13 - 16) requires additional boundary conditions, which are provided by the set points for generation units and loads. In the traditional power flow analysis, each bus is classified depending on the prescribed boundary conditions into the following three bus types:

1. Slack-Bus (Reference Bus):
   - Voltage magnitude \(|V|\) and voltage angle \(\delta\) are specified and active power \(P\) and reactive power \(Q\) are computed. A slack bus is usually connected to a generation unit with terminal voltage control. At least one slack bus is needed as a voltage angle reference and also for balancing the active power losses not covered by other generation units.

2. PV-Bus (Generation Bus):
   - Active power \(P\) and voltage magnitude \(|V|\) are prescribed and voltage angle \(\delta\) and reactive power \(Q\) are computed. Buses connected to generation units with terminal voltage control are specified as PV-Buses. If the reactive power limit of a PV-Bus is violated during computation the PV bus is changed to a PQ-Bus and the reactive power is set to the next closest feasible working point.

3. PQ-Bus (Load Bus):
   - Active power \(P\) and reactive power \(Q\) are prescribed and voltage magnitude \(|V|\) and voltage angle \(\delta\) are computed. Buses with purely load connections are usually classified as PQ-Buses.

In a real power system, a single slack bus, that balances the active power of the total system does not exist. Thus, to model the power generation and the balancing of the power system more
realistically, we introduce the concept of distributed slack bus generation discussed in [9, 10, 11], which enables the balancing of the power system by regulating the active power output of a selected number of generation units. For each generation unit, we specify an active power generation set point $P^\text{gen}_i$ and a participation factor $K_i$, describing the flexibility of the generation unit to regulate a fraction of the required additional generation $\Delta P$ for balancing the power system. The additional generation can be expressed as follows:

$$\Delta P = \sum_{i=1}^{n} P^\text{gen}_{G,i} - P^\text{D,i} - P^\text{loss}$$  \hspace{1cm} (17)

where $P^\text{loss}$ is the total power loss of the power system. The active power balance equation (13) can be modified as follows, while the reactive power balance equation (16) remain unchanged:

$$P^\text{gen}_{G,i} + K_i \cdot \Delta P - P^\text{D,i} - P_i(\delta, |V|) = 0$$  \hspace{1cm} (18)

$$\sum_{i=1}^{n} K_i = 1$$  \hspace{1cm} (19)

The resulting system of non-linear power flow equations is solved iteratively using a Netwon-Raphson approach:

$$\mathbf{J}(\mathbf{x}) \cdot \Delta \mathbf{x} = -\mathbf{F}(\mathbf{x})$$  \hspace{1cm} (20)

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta \mathbf{x}$$  \hspace{1cm} (21)

where $\mathbf{J}$ is the Jacobi matrix, $\mathbf{F}$ residual vector and $\mathbf{x} = [\delta, |V|, \Delta P]^T$ the solution vector. The algorithm for solving eq. (16, 18, 20, 21) is detailed in [7, 8, 12].

### Table 5: Basic elements in an electric network model

<table>
<thead>
<tr>
<th>Facility</th>
<th>Equations</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission Line (π-Model)</td>
<td>$[I_i] = \begin{bmatrix} (y_{ij} + b_{ij}) &amp; -y_{ij} \ -y_{ij} &amp; (y_{ij} + b_{ij}) \end{bmatrix} [V_i]$ \hspace{1cm} (10)</td>
<td>flow of active and reactive power is limited by thermal capability of the transmission line</td>
</tr>
<tr>
<td>Transformer &amp; Phase Shifter</td>
<td>$[I_i] = \begin{bmatrix} I_i^2 &amp; -I_i^2 \ -I_i^2 &amp; I_i^2 \end{bmatrix} [V_i]$ \hspace{1cm} (11)</td>
<td>heating limits</td>
</tr>
</tbody>
</table>

### INTERDEPENDENCY OF GAS AND ELECTRIC POWER SYSTEMS

Gas and electric power systems are physically interconnected at a number of facilities. The most significant interconnections between both systems are as follows:

1. Gas demand for power generation at gas fired power plants connected to the gas and electric power system. The gas offtake for power generation can be expressed as follows:

$$L^{\text{Electric}}_i = \frac{P^\text{gen}_{G,i} + K_i \cdot \Delta P}{\eta_T \cdot CV}$$  \hspace{1cm} (22)

where $\eta_T$ is the thermal efficiency of the power plant and $CV$ the caloric value of the fuel gas extracted from the pipeline.

2. Electric power demand for electric driven compressors installed in gas compressor stations and underground storage facilities. The electric power consumed by the compressor station can be described by the following expression describing the driver power:

$$P^\text{CS}_{D,i} = f \cdot \frac{\kappa^s}{\kappa - 1} \cdot \eta_{\text{ad}} \cdot \eta_m \cdot \left[ \frac{P_2}{P_1} \frac{\kappa - 1}{\kappa} - 1 \right]$$  \hspace{1cm} (23)

where $f$ is a factor describing the fraction of total driver power provided by electric drivers, $\eta_{\text{ad}}$ the average adiabatic efficiency of the compressors, $\eta_m$ the average mechanical efficiency of the installed drivers, $P_2$ the outlet...
<table>
<thead>
<tr>
<th>Facility</th>
<th>Function</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>connection point between transmission lines, transformers, loads, capacitors, reactors,</td>
<td>upper and lower limit on reactive power $Q_G$ and active power $P_G$ injection restricted by reactive power capability curve of the generation unit $P_{G,min} \leq P_G \leq P_{G,max}$ &amp; $Q_{G,min} \leq Q_G \leq Q_{G,max}$ (i.e. operating region is restricted by field current heating limit, stator current heating limit, end region heating limit)</td>
</tr>
<tr>
<td>Generation</td>
<td>injects electric power into the power system, by converting primary energy sources (oil, gas, coal, wind, hydro etc.) to electric energy: bus voltage $V_i$ and frequency $f_i$ at buses connected to generation units are typically controlled at a specific set point $V_G$, $f_G$</td>
<td></td>
</tr>
<tr>
<td>Load</td>
<td>represents consumption of electric power by large customers directly served from the transmission grid or the total power consumption from the local distribution grid connected to the transmission system at the respective substation</td>
<td>upper and lower limits on voltage magnitude $</td>
</tr>
<tr>
<td>Shunt Capacitor/Reactor</td>
<td>shunt reactors a placed locally to control the steady state over-voltages at buses under light load conditions, while shunt capacitors are used to boost a bus voltage in a stressed system</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Basic components in an electric network model

Pressure, $p_1$, $Z_1$, $T_1$ the inlet pressure, compressibility factor, temperature, respectively, $R$ the gas constant, $\kappa$ the isentropic exponent.

3. Power supply to LNG Terminals for operating LNG tanks and low and high pressure pumps. We capture this interaction by assuming a linear relation between the regasification rate $L_{irr}$ and the power consumption of the terminal:

$$P_{LNG}^{D_i} = P_{LNG}^{D_i,0} + c \cdot L_{irr}$$  \hspace{1cm} (24)

where $P_{LNG}^{D_i,0}$ is the fixed power supply to the LNG Terminal, $c$ the coefficient of the flow rate dependent power supply.
The coupling of the gas and power system is established through the developed coupling equations. If we integrate these equations into the developed gas and power system model, we obtain the following modified set of equations describing the combined gas and electric power system.

\[
\begin{pmatrix}
\Phi \\
C_p \\
K_p
\end{pmatrix} A \begin{pmatrix}
C_Q \\
K_Q
\end{pmatrix}
\begin{pmatrix}
I \\
0
\end{pmatrix}
\begin{pmatrix}
\Phi \\
C_p \\
K_p
\end{pmatrix} \begin{pmatrix}
Q^{n+1} \\
L^{n+1}
\end{pmatrix} = \begin{pmatrix}
\Phi \\
C_p \\
K_p
\end{pmatrix} D \begin{pmatrix}
\Phi \nL^{Elv} \\
D \\
S
\end{pmatrix} \begin{pmatrix}
I \\
0
\end{pmatrix}
\begin{pmatrix}
\Phi \\
C_p \\
K_p
\end{pmatrix} \begin{pmatrix}
Q^{n+1} \\
L^{n+1}
\end{pmatrix} (25)
\]

\[
P^{ref}_{G,j} + K_i \cdot \Delta P - P_{D,j} - P^{LNG}_{D,j} - P^{CS}_{D,j} - P_i(\delta_j,|V|) = 0 (26)
\]

The algorithm for solving the combined simulation model is illustrated in the flow diagram in Figure 6. The simulation starts from an initial state, which can be obtained from a combined steady state computation or any other terminal state of a (combined) transient computation. The solution for each simulation time step is obtained iteratively by solving the system of equations for both systems (eqs. (20 & 25)) separately and in parallel using multiple threads. The boundary conditions for both models are updated each iteration step using the coupling equations for the interconnected facilities (eq. (22, 23 & 24)). If the solution for a time step violates any constraints from Tables 2, 3, 5 & 6 a constraint and control handling algorithm, which tries to find a feasible working point for the affected facility, is invoked and the time integration is repeated with the new settings.

**MODEL APPLICATION**

The models presented in this paper where implemented in SAInt (Scenario Analysis Interface for Energy Systems), an integrated simulation software for performing steady state and dynamic simulations on gas and electric power systems. The software was developed with the objective of analyzing the operation and interdependency of critical energy infrastructures in terms of security of energy supply. SAInt was developed in MS Visual Studio .NET using the object oriented programming languages VB.NET, C# and C++ and can be used as a standalone gas simulator, standalone power system simulator or as a combined gas and power system simulator. It is divided into two separate modules, namely, SAInt-API (Application Programming Interface) and SAInt-GUI (Graphical User Interface). The API, is the main library of the software and contains all solvers and classes for instantiating the different objects comprising a gas and electric power system. The API is independent of the GUI and can be used separately in any other .NET environment (e.g. MS Excel, IronPython etc.). SAInt-GUI is the graphical interface, which enables a visual communication between the API and the user. The GUI uses the classes and solvers from the API to perform the simulation tasks requested by the user. To extend the functionality of the software, the API has been linked with MATLAB using the Matlab COM Automation Server. This enables the API to communicate with the Matlab Command Window and execute Matlab scripts. This link has been used to establish a communication between the matlab-based open source power flow library MATPOWER [13] and the API. This allows the execution of newton power flow and optimal power flow with MATPOWER [13] and the visualization of the obtained results using SAInt-GUI.

A SAInt project is divided into a static network model, which includes all topological and static properties of the network and a scenario, which is the definition of a case study to be performed on the static network models. The following simulation models are currently implemented in SAInt:

1. Steady state gas flow simulation
2. Transient gas flow simulation
3. Steady state AC-Power Flow (AC-PF) simulation (using distributed slack bus algorithm)
5. Combined steady state gas and steady state AC-PF simulation (with distributed slack bus algorithm)
6. Combined transient gas and steady state AC-PF simulation (using distributed slack bus algorithm)

In the following, we apply SAInt for a case study on the Bulgarian and Greek gas and electric transmission system. The topology and basic properties of both networks are depicted in Figures 7 & 8. The networks are interconnected through 14 gas fired power plants (7 located in the south of Greece), two electric driven compressor stations (CS.1 & CS.3, both located in Bulgaria) and one LNG Terminal (located in the south of Greece).

<table>
<thead>
<tr>
<th>Generation Bus</th>
<th>$K_i$</th>
<th>$\eta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEN.97</td>
<td>0.121</td>
<td>0.4</td>
</tr>
<tr>
<td>GEN.98</td>
<td>0.121</td>
<td>0.45</td>
</tr>
<tr>
<td>GEN.116</td>
<td>0.121</td>
<td>0.3</td>
</tr>
<tr>
<td>GEN.136</td>
<td>0.061</td>
<td>0.57</td>
</tr>
<tr>
<td>GEN.144</td>
<td>0.061</td>
<td>0.45</td>
</tr>
<tr>
<td>GEN.152</td>
<td>0.121</td>
<td>0.46</td>
</tr>
<tr>
<td>GEN.153</td>
<td>0.121</td>
<td>0.46</td>
</tr>
<tr>
<td>GEN.154</td>
<td>0.121</td>
<td>0.55</td>
</tr>
<tr>
<td>GEN.155</td>
<td>0.091</td>
<td>0.45</td>
</tr>
<tr>
<td>GEN.156</td>
<td>0.061</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Table 7: Participation factors and thermal efficiency assigned to generation buses connected to GPPs

For the case study, we increase the interconnection between both models by assuming five additional compressor stations are powered by electric driven compressors (CS.2, CS.7, CS.8,
Start

$t_0$

Start time integration for time step $t_{n+1} = t_n + \Delta t$ and set $k = 1$ & $k_c = 1$

Make an initial approximation for the state variables $p$, $Q$, $L$, $\delta$, $|V|$, $\Delta \rho$ using the values from the previous time step

Set boundary conditions for new time step $t_{n+1}$

Assemble system matrices and vectors in eq. (20 & 25) using the (computed) values for the state variables

Linearize eq. (2) for the gas system. Run a new iteration $k = k + 1$

Solve eq. (20 & 25) in parallel for iteration step $k$ and determine residual $\|\text{Res}\|$ for both systems. Calculate the corrected solution vector $\mathbf{x}$ for the power system using eq.(21)

For each link between the gas and power system calculate the power and gas demand according to eq. (22, 23 & 24) using the computed state variables and assign the new values to the right hand side of eq. (20 & 25)

$\|\text{Res}\| \leq \epsilon$?

$|\text{Res}| \leq \epsilon$?

$\Delta t < \ell_{\max}$?

$k \leq k_{\max}$?

Constraints violated?

$\ell_c \leq k_{c,\max}$?

$I_{\ell_c} < k_c + 1$ & $k = 1$

End: converged solution

End: no convergence

End: infeasible solution

Invoke the constraint and control handling algorithm for the affected facility set $k = k + 1$ & $k = 1$

Check if the converged solution violates any constraints from Tables 2, 3, 5 & 6

$k_c \leq k_{c,\max}$?

$k_c \leq k_{c,\max}$?

$k_c \leq k_{c,\max}$?

yes

no

yes

no

yes

no

yes

no

yes

no

yes

no

yes

no

yes

no

yes

no

yes

no

End: infeasible solution

Figure 6: Flow chart for the combined simulation
Figure 7: Model of the Bulgarian-Greek gas transmission system plotted in the graphical user interface of SAİnt. Map shows results of a steady state computation for the coupled system. Diameter of the circles representing demand (red) and supply (green) nodes correspond to the magnitude of the steady state loads in logarithmic scale, as can be seen from the legend in the bottom left corner. Colors of the pipe elements correspond to the pressure levels indicated in the color bar on top. Pipe arrows indicate gas flow direction. Labels describe a selected number of characteristic facilities in the gas system.
Figure 8: Model of the Bulgarian-Greek power transmission system plotted in the graphical user interface of SAInt. Map shows results of a steady-state computation for the coupled system. Diameter of the circles representing load (red) and generation (green) buses correspond to the magnitude of active power in logarithmic scale, as can be seen from the legend in the bottom left corner. Colors of the line elements correspond to the voltage levels indicated in the color bar on top. Transmission line arrows indicate flow direction of electric current. Labels describe a selected number of generation buses (green circles) connected to gas fired power plants.
CS.9 & CS.10). The dynamic simulation of the combined model is initiated from the steady state solution shown in Figures 7 & 8 using the load profile from Figure 9 for all city gate stations, the profiles shown in Figures 10 & 11 for all load and generation buses, respectively, and the simulation and gas properties listed in Table 8. Furthermore, we assign to a number of gas power plants connected to generation buses a participation factor $K_i$ and the thermal efficiency listed in Table 7. Thus, these generation units are expected to regulate their active power injections to balance the power system. In addition, we consider the electric power offtake for operating the LNG Terminal Revythoussa located in the south of Greece. Moreover, we assign an outlet pressure control ranging between 40-54 [barg] (580-783 [psig]) to all compressor stations in the gas system. Simulation results are illustrated in Figures 12 - 17 and
are discussed in the following.

Figure 12 shows the time evolution of the total active power generation and total power offtake. As can be seen, the shape of both curves is similar to the profile assigned to load buses (s. Fig. 10). The difference between the total power generation and total load curve is equal to the total power loss of the power system, which is caused by the resistances of the transmission lines. The electric power offtake of the gas grid increases right from the start of the simulation and reaches its maximum around 2:00 (s Fig. 13, top). This increase is a result of the overall increase in gas offtake from CGS and GPPs compared to the steady state loads, as can be seen in the time evolution of the flow balance and line pack shown in Figure 17. The compressor stations require more driver power to maintain their outlet pressure set point. However, the power offtake of the gas grid is marginal compared to the total load of the power system. (s Fig. 13, top). The total gas offtake of the GPP units shows larger fluctuations than the assigned generation profile (s. Fig. 13, bottom), because of the assigned participation factors to the GPP units. The GPP units regulate their power generation to balance power demand and losses (s. Fig. 14- 16).

**CONCLUSION**

In this paper, we presented an integrated simulation tool for a coupled simulation of gas and power systems taking into account the characteristics, the control and constraints of both systems. The integrated model consists of a transient model for the gas system and a steady state AC-Power flow model for the power system, using a distributed slack bus approach for
balancing power demand and power losses. The accuracy of the transient gas model was confirmed by benchmarking simulation results against the commercial software SIMONE. The model of the individual systems were coupled through coupling equations describing the power offtake from electric driven compressor stations and LNG Terminals installed in the gas system and connected to buses of the power network, and the gas offtake for electric power generation in gas fired power plants connected to the gas transport system. The resulting system of equations describing the state change of the combined system between two consecutive time steps is solved iteratively by adapting the boundary conditions expressed by the coupling equations at each iteration step. The algorithm was implemented in an integrated simulation tool SAInt and the value of the tool was demonstrated in a case study on an actual combined gas and electric power system of an European region.

In the coming future, we intend to extend the simulation tool to allow the simulation of disruptions and the possibility to define mitigation strategies. By doing this, we intend, to assess how these strategies may reduce the impact of disruptions on both systems.

ACKNOWLEDGMENT

We extend our gratitude to our colleague Dr. Nicola Zaccarelli from the Joint Research Centre - Institute for Energy and Transport, in Petten, Netherlands, for providing the GIS-Data for the presented gas and electric model. We would also like to thank Dr. Tom van der Hoeven for providing the LateX template and for the productive discussions and suggestions, which has significantly improved the presented simulation tool.

REFERENCES


**AUTHOR BIOGRAPHY**

Kwabena Addo Pambour was born in Accra, Ghana, and grew up in the city of Essen, Germany. He studied Mechanical Engineering and Business Administration at RWTH Aachen University in Germany. In 2010 and 2011 he graduated with a Master of Science (Dipl.-Ing.) in Engineering and a Master of Business Administration (Dipl.-Wirt.-Ing.), respectively. After his studies, he was employed by the European Commission (EC) - Joint Research Centre (JRC) Institute for Energy and Transport (IET) in Petten, Netherlands, as a scientific researcher to conduct research in the area of security of energy supply. Since July 2014, he works as a Customer Support Engineer for the gas software company LIW ACOM, situated in Essen, Germany. Mr. Pambour is currently developing his PhD thesis at University of Groningen, Netherlands, where he is inscribed as an external PhD Researcher since December 2014. He has expertise in modeling and simulation of energy systems and in software development. He is the developer of the simulation software SAInt, which is currently used by the European Commission to assess critical energy infrastructures in terms of security of supply.

Dr. Burcin Cakir Erdener was born in Ankara, Turkey, in 1981. She received a B.Sc. degree in Industrial Engineering from Gazi University, Ankara, Turkey, in 2002 and a M.Sc. and Ph.D. in Industrial Engineering from Gazi University, Ankara, Turkey, in 2006 and 2014, respectively. In 2006, she joined the Department of Industrial Engineering, Baskent University, Ankara, as a research assistant. In 2012, she worked in a project for European Commission Joint Research Centre Institute for Energy and Transport, Petten, The Netherlands for one year. Since December 2013, she has been working for European Commission Joint Research Centre Institute for Energy and Transport, Ispra, Italy as a post-doctoral researcher. Her current research interest includes analysis of networks of critical infrastructures including reliability and security analysis of the networks and interactions between different critical infrastructures.

Dr. Ricardo Bolado-Lavin holds a BS in Physics (Complutense University of Madrid, 1987), a Diploma in Nuclear Engineering (CIEMAT, 600-hour course, 1988), and a PhD (Technical University of Madrid, 2004). Since 2010 Dr. Bolado leads the Security of Gas Supply group within the Energy Security, Systems and Market Unit of the Institute for Energy and Transport of the European Commission (DG-JRC). Within this group, he has led the development of hydraulic models for the EU gas transport network to analyze security of gas supply scenarios. Dr. Bolado is the PhD co-director of Mr. Pambour. Before working in the Security of Gas Supply area, he worked in the area of Probabilistic Safety Assessment for High Level Radioactive Waste Repositories and Nuclear Power Plants. He developed these activities at the Institute for Energy and Transport and at the Technical University of Madrid.

Prof. Dr. Gerard P. J. Dijkema is a full professor at University of Groningen since August 2014. He studied Chemical Engineering at University of Twente and acquired his PhD in Engineering Sciences from Delft University of Technology in 2004. Prof. Dijkema is the PhD director of Mr. Pambour. His research interests revolve around modeling the evolution of energy infrastructures and industrial networks as socio-technical systems.

**APPENDIX**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>time step</td>
<td>$\Delta t$</td>
<td>900</td>
<td>[s]</td>
</tr>
<tr>
<td>total simulation time</td>
<td>$t_{max}$</td>
<td>24</td>
<td>[h]</td>
</tr>
<tr>
<td>gas temperature</td>
<td>$T$</td>
<td>288.15</td>
<td>[K]</td>
</tr>
<tr>
<td>dynamic viscosity</td>
<td>$\eta$</td>
<td>1.1 x 10^{-5}</td>
<td>[kg/m · s]</td>
</tr>
<tr>
<td>pipe roughness</td>
<td>$k$</td>
<td>0.012</td>
<td>[mm]</td>
</tr>
<tr>
<td>standard pressure</td>
<td>$p_n$</td>
<td>1.01325</td>
<td>[bar]</td>
</tr>
<tr>
<td>standard temperature</td>
<td>$T_n$</td>
<td>273.15</td>
<td>[K]</td>
</tr>
<tr>
<td>relative density</td>
<td>$d$</td>
<td>0.6</td>
<td>[-]</td>
</tr>
<tr>
<td>calorific value</td>
<td>$CV$</td>
<td>41.215</td>
<td>[MJ/sm^3]</td>
</tr>
</tbody>
</table>

Table 8: Input parameter for the dynamic simulation of the sample network and the combined model

![Figure 18: Relative Load profile assigned to demand nodes of the sample network](image-url)
<table>
<thead>
<tr>
<th>Node</th>
<th>Pipe inlet</th>
<th>Pipe outlet</th>
<th>L [m]</th>
<th>D [m]</th>
<th>QSET ([km^3/s])</th>
<th>PSET [bar-g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>24000</td>
<td>0.7</td>
<td>50 (supply)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>25000</td>
<td>0.7</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4</td>
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<td>0.7</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>30000</td>
<td>0.7</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>5</td>
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<td>2</td>
<td>40000</td>
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<td>0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>22</td>
<td>45000</td>
<td>0.7</td>
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</tr>
<tr>
<td>7</td>
<td>2</td>
<td>5</td>
<td>70000</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>6</td>
<td>60000</td>
<td>0.6</td>
<td>28</td>
<td></td>
</tr>
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<td>9</td>
<td>22</td>
<td>21</td>
<td>52000</td>
<td>0.7</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>21</td>
<td>30000</td>
<td>0.6</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>4</td>
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<td>40000</td>
<td>0.7</td>
<td>39</td>
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</tr>
<tr>
<td>12</td>
<td>4</td>
<td>11</td>
<td>35000</td>
<td>0.6</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>12</td>
<td>25000</td>
<td>0.7</td>
<td>0</td>
<td></td>
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<tr>
<td>14</td>
<td>12</td>
<td>13</td>
<td>70000</td>
<td>0.6</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>12</td>
<td>11</td>
<td>30000</td>
<td>0.7</td>
<td>45</td>
<td></td>
</tr>
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<td>16</td>
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<td>10</td>
<td>50000</td>
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</tr>
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<td>17</td>
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<td>60000</td>
<td>0.6</td>
<td>120</td>
<td></td>
</tr>
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<td>18</td>
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<td>100000</td>
<td>0.6</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>14</td>
<td>15</td>
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<td></td>
</tr>
<tr>
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Table 9: Input data for the reference network taken from [6]