A geometric approach to multi-modal and multi-agent systems
Everts, Annerosa Roelienke Fleur

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2016

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):

Copyright
Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

Take-down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): http://www.rug.nl/research/portal. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.


DISTURBANCE DECOUPLED LINEAR COMPLEMENTARITY SYSTEMS

ABSTRACT: In this chapter we study the disturbance decoupling problem for a particular class of linear complementarity systems. We rewrite the linear complementarity system as a linear multi-modal system and provide crisp necessary and sufficient conditions for such a system to be disturbance decoupled. This chapter is based on the book chapter [Everts and Camlibel, 2015], dedicated to Prof. Dr. Harry L. Trentelman on the occasion of his sixtieth birthday.

4.1 introduction

In this chapter we study the disturbance decoupling problem for a particular class of linear complementarity systems. Linear complementarity systems are nonsmooth dynamical systems that are obtained by taking a standard linear input/output system and imposing certain complementarity relations on a number of input/output pairs at each time instant. A wealth of examples, from various areas of engineering as well as operations research, of linear complementarity systems can be found in [Camlibel et al., 2004; Schumacher, 2004; van der Schaft and Schumacher, 2000; Heemels and Brogliato, 2003]. For the work on the analysis of linear complementarity systems, we refer to [Camlibel et al., 2003; Heemels et al., 2002; Camlibel et al., 2002; van der Schaft and Schumacher, 1996; Camlibel, 2007; van der Schaft and Schumacher, 1998; Heemels et al., 2000].

Particular linear complementarity systems can be written as linear multi-modal systems, namely those of index zero [Camlibel, 2001, Chapter 2]. Different from the piecewise affine systems treated in Chapter 3, the resulting polyhedral regions on which the modes are active, can now be non-solid, and together they do not cover the full state space. The linear subsystems of a linear complementarity system share a certain geometric structure. By exploiting this geometric structure, we provide a necessary and sufficient condition for disturbance decoupledness that is crisp and easily checkable.

The structure of this chapter is as follows. In Section 4.2 we start with the formulation of the linear complementarity problem, and introduce linear complementarity systems. In
Section 4.3 we first define what we mean by a linear complementarity system to be disturbance decoupled. After providing some technical auxiliary results that are, in a way, of interest themselves, we present a necessary and sufficient condition for disturbance decoupledness, which is the main result of this chapter. Finally, the chapter closes with conclusions in Section 4.4.

4.2 LINEAR COMPLEMENTARITY PROBLEM/SYSTEM

The problem of finding a vector $z \in \mathbb{R}^n_z$ such that

$$
\begin{align*}
  z &\geq 0 \quad \text{(4.1a)} \\
  q + Mz &\geq 0 \quad \text{(4.1b)} \\
  z^T(q + Mz) &= 0 \quad \text{(4.1c)}
\end{align*}
$$

for a given vector $q \in \mathbb{R}^n_z$ and a matrix $M \in \mathbb{R}^{n_z \times n_z}$ is known as the linear complementarity problem. Here, the inequalities for vectors are componentwise inequalities. We denote (4.1) by LCP$(q, M)$. It is well-known [Cottle et al., 1992, Thm. 3.3.7] that the LCP$(q, M)$ admits a unique solution for each $q$ if and only if all principal minors of $M$ are positive. Such matrices are called $P$-matrices in the literature of the mathematical programming. It is well-known (see for instance [Cottle et al., 1992, Thm. 3.1.6 and Thm. 3.3.7]) that every positive definite matrix is in this class.

When the matrix $M$ is a $P$-matrix, the unique solution $z(q)$ of the LCP$(q, M)$ depends on $q$ in a Lipschitz continuous way. In particular, for each $q$ there exists an index set $\alpha \subseteq I_{n_z}$ such that the solution $z = z(q)$ satisfies

$$
\begin{align*}
  z_\alpha &\geq 0, \\
  (q + Mz)_\alpha &= 0, \\
  z_{\alpha^c} &= 0, \\
  (q + Mz)_{\alpha^c} &\geq 0,
\end{align*}
$$

or equivalently,

$$
\begin{align*}
  z_\alpha &= -(M_{\alpha\alpha})^{-1}q_\alpha, \\
  (q + Mz)_\alpha &= 0, \\
  z_{\alpha^c} &= 0, \\
  q_{\alpha^c} - M_{\alpha^c\alpha}(M_{\alpha\alpha})^{-1}q_\alpha &\geq 0
\end{align*}
$$

where $\alpha^c$ denotes the set $I_{n_z} \setminus \alpha$.

Linear complementarity systems (LCSs) are nonsmooth dynamical systems that are obtained in the following way. Take a standard linear input/output system. Select a number of input/output pairs $(z_i, w_i)$, and impose for each of these pairs a
complementarity relation of the type (4.1) at each time instant. In this chapter we will focus on the LCSs of the following form:

\[ \dot{x}(t) = Ax(t) + Bz(t) + Ed(t) \]  \hspace{1cm} (4.3a)
\[ w(t) = Cx(t) + Dz(t) + Fd(t) \]  \hspace{1cm} (4.3b)
\[ 0 \leq z(t) \perp w(t) \geq 0 \]  \hspace{1cm} (4.3c)
\[ y(t) = Jx(t). \]  \hspace{1cm} (4.3d)

Here \( x \in \mathbb{R}^{n_x} \) is the state, \((z, w) \in \mathbb{R}^{2n_z}\) are the complementarity variables, \( d \in \mathbb{R}^{n_d} \) is the disturbance, \( y \in \mathbb{R}^{n_y} \) is the output, \( \perp \) denotes orthogonality and all the matrices are of appropriate sizes.

In the sequel we will work under the following blanket assumptions:

1. The matrix \( D \) is a \( P \)-matrix.

2. The transfer matrix \( F + C(sI - A)^{-1}E \) is right-invertible as a rational matrix.

Since \( D \) is a \( P \)-matrix, \( z(t) \) is a piecewise linear function of \( Cx(t) + Fd(t) \) (see e.g. [Cottle et al., 1992]). This means that for each initial state \( x_0 \) and locally-integrable disturbance \( d \) there exist unique absolutely continuous trajectories \((x_{x0,d}, y_{x0,d})\) and locally-integrable trajectories \((z_{x0,d}, w_{x0,d})\) such that \( x_{x0,d}(0) = x_0 \) and the quadruple \((x_{x0,d}, z_{x0,d}, w_{x0,d}, y_{x0,d})\) satisfies the relations (4.3) for almost all \( t \geq 0 \).

Although LCSs are nonsmooth and nonlinear, their local linear behavior enables elegant characterizations of certain system-theoretic properties. In the next section we will study the disturbance decoupling problem for LCSs.

### 4.3 Disturbance Decoupled LCSs

We say that an LCS (4.3) is disturbance decoupled if for all initial states \( x_0 \) and all locally integrable disturbances \( d_1 \) and \( d_2 \) we have

\[ y^{x_0,d_1}(t) = y^{x_0,d_2}(t), \quad \forall t \geq 0. \]

In this section, we will investigate necessary and sufficient conditions for an LCS (4.3) to be disturbance decoupled. To do so, we first derive an alternative representation of an LCS. This representation is closely related to the piecewise affine systems
in Chapter 3 and makes the underlying switching behavior more transparent.

Since $D$ is a $P$-matrix, we can solve the LCP given by (4.3c) by employing (4.2). To simplify notation later on, we first define the following matrices for a given index set $\alpha \subseteq I_{nz}$:

\[
N_\alpha = -B_{\bullet \alpha}(D_{aa})^{-1}I_\bullet \\
A_\alpha = A + N_\alpha C \\
E_\alpha = E + N_\alpha F \\
G_\alpha = \begin{bmatrix} - (D_{aa})^{-1}C_\bullet \\
C_\bullet - D_{aa}(D_{aa})^{-1}C_\bullet \
\end{bmatrix} \\
H_\alpha = \begin{bmatrix} - (D_{aa})^{-1}F_\bullet \\
F_\bullet - D_{aa}(D_{aa})^{-1}F_\bullet \
\end{bmatrix},
\]

where a subscript $\alpha \beta$ selects rows $\alpha$ and columns $\beta$ of a matrix, for given index sets $\alpha$ and $\beta$. Furthermore, the $\bullet$ means selecting all rows or columns and $\alpha^c$ denotes the complement of $\alpha$ in $I_{nz}$.

If the quadruple $(x, d, z, w)$ satisfies (4.3a)-(4.3c) for almost all $t \geq 0$ then for almost all $t \geq 0$ there exists an index set $\alpha_t \subseteq I_{nz}$ such that

\[
\dot{x}(t) = A_{\alpha_t}x(t) + E_{\alpha_t}d(t) \quad \text{when} \quad [G_{\alpha_t} \quad H_{\alpha_t}] \begin{bmatrix} x(t) \\
d(t) \end{bmatrix} \geq 0. \quad (4.9)
\]

The resulting system (4.9) is a linear multi-modal system. Different from the piecewise affine systems in Chapter 3, the underlying polyhedral regions are not all solid and their union is not equal to $\mathbb{R}^n_x$. However, the linear subsystems of (4.9) share a certain geometric structure, which we will exploit to prove the following auxiliary result concerning the subspace $\Sigma_{\gamma \subseteq I_{nz}} \langle A_{\gamma} \mid \text{im} \ E_{\gamma} \rangle$, which plays an important role in the disturbance decoupling problem later on.

**Lemma 4.1** Let $S = \Sigma_{\gamma \subseteq I_{nz}} \langle A_{\gamma} \mid \text{im} \ E_{\gamma} \rangle$. The following statements hold:

1. $\text{im} \ (N_\alpha - N_\beta) \subseteq S$ for any $\alpha, \beta \subseteq I_{nz}$.
2. $S$ is invariant under $A_\alpha$ for any $\alpha \subseteq I_{nz}$.
3. $S = \langle A \mid \text{im} \ [B \ E] \rangle$.

**Proof.** To prove the first statement, let $\Sigma_{\gamma}$ denote the linear system $\Sigma(A_{\gamma}, E_{\gamma}, C, F)$ for $\gamma \subseteq I_{nz}$. It follows from (1.7) that

\[
T^*(\Sigma_{\gamma}) \subseteq \langle A_{\gamma} \mid \text{im} \ E_{\gamma} \rangle \subseteq S.
\]
Then, we have
\[(A + N_\gamma C)T^*(\Sigma_\gamma) = (A + N_\gamma C)(A_\gamma | \text{im } E_\gamma) \subseteq \langle A_\gamma | \text{im } E_\gamma \rangle \subseteq S.\]

Let \(\tilde{\Sigma}\) denote the linear system \(\Sigma(A, E, C, F)\). It follows from (1.6) that \(T^*(\Sigma_\gamma) = T^*(\tilde{\Sigma})\) and hence that
\[(A + N_\gamma C)T^*(\tilde{\Sigma}) \subseteq S\]
for any \(\gamma \subseteq I_{nz}\). This yields
\[(N_\alpha - N_\beta)CT^*(\tilde{\Sigma}) \subseteq S \tag{4.10}\]
for any \(\alpha, \beta \subseteq I_{nz}\). Also we have
\[\text{im } (E + N_\gamma F) \subseteq \langle A_\gamma | \text{im } E_\gamma \rangle \subseteq S\]
for any \(\gamma \subseteq I_{nz}\). Thus, we get
\[(N_\alpha - N_\beta)\text{im } F \subseteq S.\]

By combining the last relation with (4.10), we obtain
\[(N_\alpha - N_\beta)(\text{im } F + CT^*(\tilde{\Sigma})) \subseteq S.\]

Since the transfer matrix \(F + C(sI - A)^{-1}E\) is right-invertible as a rational matrix, it follows from (1.9) that \(\text{im } F + CT^*(\tilde{\Sigma}) = \mathbb{R}^n_y\). Therefore, we have
\[\text{im } (N_\alpha - N_\beta) \subseteq S.\]

To prove the second statement, let \(\alpha, \gamma \subseteq I_{nz}\). Note that
\[A_\alpha \langle A_\gamma | \text{im } E_\gamma \rangle \subseteq A_\gamma \langle A_\alpha | \text{im } E_\gamma \rangle + \text{im } (A_\alpha - A_\gamma) \subseteq \langle A_\gamma | \text{im } E_\gamma \rangle + \text{im } (N_\alpha - N_\gamma).\]

It follows from the definition of \(S\) and the first statement that
\[A_\alpha \langle A_\gamma | \text{im } E_\gamma \rangle \subseteq S.\]

Hence, we have
\[A_\alpha S \subseteq A_\alpha \left( \sum_{\gamma \subseteq I_{nz}} \langle A_\gamma | \text{im } E_\gamma \rangle \right) \subseteq \sum_{\gamma \subseteq I_{nz}} A_\alpha \langle A_\gamma | \text{im } E_\gamma \rangle \subseteq S.\]
4. DISTURBANCE DECOUPLED LIN. COMP. SYSTEMS

To prove the third statement, note first that \( \text{im} \, N_{\gamma} \subseteq \text{im} \, B \) for any \( \gamma \subseteq \mathcal{I}_{nz} \). Hence, we have

\[
\text{im} \, E_{\gamma} = \text{im} \, (E + N_{\gamma}C) \subseteq \text{im} \, [B \ E].
\]

This results in

\[
\langle A_{\gamma} \mid \text{im} \, E_{\gamma} \rangle \subseteq \langle A_{\gamma} \mid \text{im} \, [B \ E] \rangle
\]  \hspace{1cm} (4.11)

for any \( \gamma \subseteq \mathcal{I}_{nz} \). Since \( A_{\gamma} = A + N_{\gamma}C \) and \( \text{im} \, N_{\gamma} \subseteq \text{im} \, B \), it follows from (1.3) that

\[
\langle A_{\gamma} \mid \text{im} \, [B \ E] \rangle = \langle A \mid \text{im} \, [B \ E] \rangle.
\]

In view of (4.11), this means that

\[
\langle A_{\gamma} \mid \text{im} \, E_{\gamma} \rangle \subseteq \langle A \mid \text{im} \, [B \ E] \rangle
\]

for any \( \gamma \subseteq \mathcal{I}_{nz} \). Consequently, we obtain

\[
S = \sum_{\gamma \subseteq \mathcal{I}_{nz}} \langle A_{\gamma} \mid \text{im} \, E_{\gamma} \rangle \subseteq \langle A \mid \text{im} \, [B \ E] \rangle.
\]  \hspace{1cm} (4.12)

It follows from the fact that \( \langle A_{\gamma} \mid \text{im} \, E_{\gamma} \rangle \subseteq S \) and that

\[
\langle A \mid \text{im} \, E \rangle \subseteq S
\]

\[
\langle A - BD^{-1}C \mid \text{im} \, (E - BD^{-1}F) \rangle \subseteq S
\]

for the particular choices \( \gamma = \emptyset \) and \( \gamma = \mathcal{I}_{nz} \), respectively. We know from (1.6) that the strongly reachable subspaces of the systems \( \Sigma(A, E, C, F) \) and \( \Sigma(A - BD^{-1}C, E - BD^{-1}F, C, F) \) coincide. Let \( T^* \) denote this common strongly reachable subspace. It follows from (1.7) that

\[
T^* \subseteq \langle A \mid \text{im} \, E \rangle \subseteq S
\]

\[
T^* \subseteq \langle A_{\mathcal{I}_{nz}} \mid \text{im} \, E_{\mathcal{I}_{nz}} \rangle \subseteq S.
\]

These inclusions yield

\[
AT^* \subseteq A \langle A \mid \text{im} \, E \rangle \subseteq \langle A \mid \text{im} \, E \rangle \subseteq S
\]

\[
A_{\mathcal{I}_{nz}} T^* \subseteq A_{\mathcal{I}_{nz}} \langle A_{\mathcal{I}_{nz}} \mid \text{im} \, E_{\mathcal{I}_{nz}} \rangle \subseteq \langle A_{\mathcal{I}_{nz}} \mid \text{im} \, E_{\mathcal{I}_{nz}} \rangle \subseteq S.
\]

Using \( A - A_{\mathcal{I}_{nz}} = BD^{-1}C \), we can conclude that

\[
BD^{-1}CT^* \subseteq S.
\]  \hspace{1cm} (4.13)

On the other hand, we readily have

\[
\text{im} \, E \subseteq \langle A \mid \text{im} \, E \rangle \subseteq S
\]

\[
\text{im} \, (E - BD^{-1}F) \subseteq \langle A - BD^{-1}C \mid \text{im} \, (E - BD^{-1}F) \rangle \subseteq S.
\]
Combining these two inclusions results in
\[ BD^{-1}\text{im } F \subseteq S. \]
Together with \((4.13)\), this implies that
\[ BD^{-1}(\text{im } F + CT^*) \subseteq S. \]
It follows from the blanket assumption and \((1.9)\) that
\[ \text{im } F + CT^* = \mathbb{R}^{ny}. \]
Thus, we get
\[ \text{im } B \subseteq S. \]

From the second statement of Lemma 4.1, we know that the subspace \( S \) is \( A_\alpha \)-invariant for any \( \alpha \subseteq \mathcal{I}_{nz} \). In particular, the choice of \( \alpha = \emptyset \) implies that \( S \) is \( A \)-invariant. Since \( \langle A \mid \text{im } B \rangle \) is the smallest \( A \)-invariant subspace that contains \( \text{im } B \), we have
\[ \langle A \mid \text{im } B \rangle \subseteq S. \tag{4.14} \]
As we readily have
\[ \langle A \mid \text{im } E \rangle \subseteq S, \]
the inclusion \((4.14)\) implies that
\[ \langle A \mid \text{im } B \rangle + \langle A \mid \text{im } E \rangle = \langle A \mid \text{im } [B \ E] \rangle \subseteq S. \]
Together with \((4.12)\), this proves that
\[ S = \langle A \mid \text{im } [B \ E] \rangle. \]

Now we are ready to present necessary and sufficient conditions for an LCS to be disturbance decoupled.

**Theorem 4.2** An LCS of the form \((4.3)\) is disturbance decoupled if and only if
\[ \langle A \mid \text{im } [B \ E] \rangle \subseteq \ker J. \]

**Proof.** Necessity: Let \( \gamma \subseteq \mathcal{I}_{nz} \). Note that
\[ \begin{bmatrix} G_\gamma & H_\gamma \end{bmatrix} = \begin{bmatrix} -(D_{\gamma\gamma})^{-1} & 0 \\ -D_{\gamma'\gamma}(D_{\gamma\gamma})^{-1} & I \end{bmatrix} \begin{bmatrix} C_{\gamma} \bullet & F_{\gamma} \bullet \\ C_{\gamma'} \bullet & F_{\gamma'} \bullet \end{bmatrix}. \tag{4.15} \]
Since $F + C(sI - A)^{-1}E$ is right-invertible as a rational matrix by the blanket assumption, $[C\ F]$ is of full row rank. So must be the matrix $[G_\gamma\ H_\gamma]$ due to (4.15). Then, one can find $x_0$ and $d$ such that

$$
\begin{bmatrix}
G_\gamma & H_\gamma
\end{bmatrix}
\begin{bmatrix}
x_0 \\
d
\end{bmatrix} > 0.
$$

Let $e \in \mathbb{R}^{n_z}$. Clearly, there exists a sufficiently small $\mu > 0$ such that

$$
\begin{bmatrix}
G_\gamma & H_\gamma
\end{bmatrix}
\begin{bmatrix}
x_0 \\
d + \mu e
\end{bmatrix} > 0.
$$

Now define

$$
d_1(t) = d \quad \text{and} \quad d_2(t) = d + \mu e
$$

for all $t \geq 0$. Let $x_i(t)$ denote the trajectory $x^{x_0,d_i}(t)$ for $i = 1, 2$. Since $x_i$ and $d_i$ are continuous, there exists an $\epsilon > 0$ such that

$$
\begin{bmatrix}
G_\gamma & H_\gamma
\end{bmatrix}
\begin{bmatrix}
x_i(t) \\
d_i(t)
\end{bmatrix} > 0
$$

holds for all $t \in [0, \epsilon)$. Thus, the trajectories $x_1$ and $x_2$ satisfy

$$
\dot{x}_i(t) = A_\gamma x_i(t) + E_\gamma d_i(t)
$$

for all $t \in [0, \epsilon)$ and $i = 1, 2$. As the system is disturbance decoupled, we have that

$$
Jx_1(t) = Jx_2(t)
$$

for all $t \in [0, \epsilon)$. Since $d_1$ and $d_2$ are constant, we obtain

$$
J(A_\gamma x_0 + E_\gamma d) = J(A_\gamma x_0 + E_\gamma (d + \mu e))
$$

by differentiating and evaluating at $t = 0$. This results in

$$
JE_\gamma e = 0.
$$

By repeating the differentiation and evaluation at $t = 0$, we get

$$
JA_\gamma^k E_\gamma e = 0
$$

for all $k \geq 0$. Since $e$ is arbitrary, we have

$$
JA_\gamma^k E_\gamma = 0
$$
for all $k \geq 0$. Consequently, one gets
\[
\langle A_\gamma | \text{im } E_\gamma \rangle \subseteq \ker J.
\]
Thus, we have
\[
\sum_{\gamma \in \mathcal{I}_{nz}} \langle A_\gamma | \text{im } E_\gamma \rangle \subseteq \ker J.
\]
It follows from the third statement of Lemma 4.1 that
\[
\langle A | \text{im } [B \ E] \rangle \subseteq \ker J.
\]

**Sufficiency:** It is enough to show that
\[
x^{x_0,d_1}(t) - x^{x_0,d_2}(t) \in \langle A | \text{im } [B \ E] \rangle, \quad \forall t \geq 0
\]
for any initial state $x_0 \in \mathbb{R}^{n_x}$ and all locally-integrable disturbances $d_1$ and $d_2$. To do so, let $\mathcal{V} := \langle A | \text{im } [B \ E] \rangle$ and let $v \in \mathcal{V}^\perp$. From (4.3a), we have
\[
v^T (\dot{x}^{x_0,d_1}(t) - \dot{x}^{x_0,d_2}(t)) = v^T A (x^{x_0,d_1}(t) - x^{x_0,d_2}(t))
\]
for almost all $t \geq 0$. Define
\[
\zeta(t) := v^T (x^{x_0,d_1}(t) - x^{x_0,d_2}(t)).
\]
From (4.16) and $A^T$-invariance of $\mathcal{V}^\perp$, we get
\[
\frac{d^k \zeta}{dt^k}(t) = v^T A^k (x^{x_0,d_1}(t) - x^{x_0,d_2}(t))
\]
for $k \geq 0$. The Cayley-Hamilton theorem implies that there exist real numbers $c_i$ with $i = 0, 1, \ldots, n - 1$ such that
\[
\frac{d^n_x \zeta}{dt^n_x}(t) + c_{n-1} \frac{d^{n-1}_x \zeta}{dt^{n-1}_x}(t) + \cdots + c_1 \frac{d \zeta}{dt}(t) + c_0 \zeta(t) = 0.
\]
Since
\[
\frac{d^k \zeta}{dt^k}(0) = 0
\]
for $k \geq 0$, we get $\zeta(t) = 0$ for all $t \geq 0$. Consequently, we have
\[
x^{x_0,d_1}(t) - x^{x_0,d_2}(t) \in (\mathcal{V}^\perp)^\perp = \mathcal{V} = \langle A | \text{im } [B \ E] \rangle
\]
which completes the proof. \hfill \blacksquare
4. DISTURBANCE DECOUPLED LIN. COMP. SYSTEMS

4.4 CONCLUSIONS

In this chapter we studied a class of nonsmooth and nonlinear dynamical systems, namely linear complementarity systems of index zero. These systems belong to the larger family of linear multi-modal systems, and are closely related to the piecewise affine dynamical systems in Chapter 3, for which the disturbance decoupling problem has already been solved. In this chapter we have shown that the linear subsystems of a linear complementarity system share certain geometric structure. By exploiting this geometric structure, we provided a necessary and sufficient condition for a linear complementarity system to be disturbance decoupled. Compared to the conditions for general piecewise affine systems in Chapter 3, this condition is crisper and more insightful.

Future research possibilities are weakening the technical blanket assumptions and studying disturbance decoupling problem under different feedback schemes. In the next chapter we will study the disturbance decoupling problem for a general class of linear multi-modal systems, and as a special case we study another class of linear complementarity systems, namely passive-like LCSs.