Testing Lorentz invariance in $\beta$ decay
Sytema, Auke

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2016

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):

Copyright
Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

Take-down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): http://www.rug.nl/research/portal. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Download date: 05-09-2020
Chapter 5

Summary, Outlook and Conclusions

5.1 Summary

The SM is a highly successful description of the interactions between the fundamental particles. Lorentz invariance is assumed to hold for the SM and this property is very well tested in general, but relatively few experiments have tested LIV in the weak interaction. This important assumption should be subject to scrutiny and is tested in this thesis.

We have shown that in $\beta$ decay, to first order and neglecting boost terms, LIV would lead to an apparent preferred direction of decay. The nuclear spin and the $\beta$ momentum dependence of the decay rate on such direction can be measured efficiently.

We have performed an experiment that searches for spin dependence, improving an earlier experiment that had limited statistical precision and indicated sources of systematic uncertainty that would have limited experiments with more yield. The experiment of this thesis has higher statistical precision while eliminating or reducing sources of systematic uncertainty.

The experiment used $^{20}$Na. This is a $\beta$ emitter with a $2^+ \rightarrow 2^+$ GT transition. The daughter nucleus $^{20}$Ne subsequently emits $\gamma$ radiation ($E_\gamma = 1.63$ MeV). The $^{20}$Na nucleus is polarized via optical pumping with a solid-state laser resulting in polarization in east (+) or west (-) direction. The polarization is determined from the SM $\beta$ asymmetry, which is obtained from the $\beta$ rates measured with detectors that are positioned east/west. The LIV signal is obtained from the $^{20}$Na lifetime $\tau$ by detecting the $\gamma$ rays. It is defined as the sidereal variation of $\Delta \tau / (\tau P_{\text{eff}})$ in Section 4.2, where $P_{\text{eff}}$ is the effective polarization.

The experiment comprised the following methods to suppress systematic uncertainty:

- the enhancement of the signal relative to the background by various
methods such as measuring the lifetime difference of $^{20}$Na polarized in +/- directions (instead of measuring the $^{20}$Na decay rate);

- the inference of the $^{20}$Na lifetime by measuring the $\gamma$ emission of $^{20}$Ne$^*$;

- a photon-detection threshold above the annihilation radiation of $E_\gamma = 0.511$ MeV;

- requiring that the LIV signal has a sidereal frequency;

- prejudice on the outcome of the experiment was limited by performing a blind analysis.

Two aspects were of particular importance in the analysis. First, the measured lifetime correlates with the time dependence of the polarization. To understand this dependence, various processes in the fiducial volume that play a role in the polarization were considered. Most important are chemical processes. These processes are described in Chapter 3.

Secondly, we found correlations between the LIV signal and a selection of environmental parameters, i.e. parameters that were not part of the parametrization of the $\gamma$ rates used to extract the LIV signal. The most important of these was found to be the difference in laser power used for the two polarization directions (+/-). These dependencies were corrected for by using the blinded data sets, after determining how the corrections affect the measurement, as described in Section 4.4. We inferred that some of the parameters vary due to temperature. This is based on the observation that the environmental parameters were correlated with each other, and one of the environmental parameters was a temperature measurement.

We obtained for spin-dependent LIV the bound $|A_{\text{LIV}}| < 2 \times 10^{-4}$ at 90% confidence level. This result stands independent of theory. In the $\chi$ tensor framework, the resulting bounds are (Section 4.5)

\[ |X_{i}^{13}| < 2 \times 10^{-4}, \]
\[ |X_{i}^{23}| < 2 \times 10^{-4}. \] (5.1) (5.2)

The $\chi$ tensor formalism can be used to relate results of different experiments involving weak decay. An overview of experiments is given in
5.1. Summary

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Measurement</th>
<th>Limit (90% C.L.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This thesis</td>
<td>$\frac{\Delta \tau}{\tau} T_{\text{eff}}(t)$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Forbidden $\beta$ decay</td>
<td>$\delta(\beta \text{ rate})(\theta_{\text{lab}})$</td>
<td>$1.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>Pion $\beta$ decay</td>
<td>$\frac{1}{2} \frac{d\Gamma}{d\tau} \Delta \Gamma$</td>
<td>$2.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Kaon decay</td>
<td>$\frac{\Delta \tau}{\tau}$ in CMB dipole</td>
<td>$2.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Muon decay</td>
<td>Michel parameter</td>
<td>$-7 \times 10^{-4} &lt; \varrho_{\exp} - \varrho_{\text{SM}} &lt; 2 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 5.1: Limits from experiments that searched for LIV. Limits have been recalculated at 90% C.L.

In fact there are 15 independent coefficients making up $\chi^{\mu\nu}$, for which bounds need to be measured.

The current bounds are mostly due to the experiment of Newman and Wiesner [13], as the analysis in Ref. [10] showed. This experiment searched for a dependence of the decay rate on the $\beta$ direction with a high-intensity source and set limits smaller than $10^{-8}$ on several components of $\chi$. This experiment measured the $\beta$ current instead of counting the individual $\beta$ particles. The other pioneering experiment searching for $\beta$-direction-dependent LIV is described in Ref. [14]. The limits in Ref. [10] are at 95% C.L.; we adopt 90% C.L. for the following limits. We assume $\chi^{\mu\nu}$ ($\chi^{\mu\nu}_i$) symmetric (antisymmetric) in $\mu \leftrightarrow \nu$. Then

\begin{align}
-3 \times 10^{-9} < 2X^3_{30} - 2X^1_{12} &< 2 \times 10^{-8}, \\
-3 \times 10^{-6} < 3X^3_{33} - X^0_{00} &< 7 \times 10^{-7}, \\
\left[ (2X^0_{20} + 2X^1_{10})^2 + (2X^2_{12} - 2X^2_{13})^2 \right]^{1/2} &< 3 \times 10^{-8}, \\
\left[ (2X^3_{20})^2 + (2X^2_{13})^2 \right]^{1/2} &< 1 \times 10^{-6}, \\
\left[ (2X^1_{12})^2 + (X^2_{22} - X^2_{11})^2 \right]^{1/2} &< 1 \times 10^{-6}.
\end{align}

One practice in giving bounds is putting all coefficient values to zero except for one. In that case one can read off the limits from Eqs. (5.3–5.7). We
write the result in the form \(|X_{i,r}^{\mu\nu}| < 10^x\), where we round off \(x\). We obtain

\[
|\chi_r^{\mu\nu}| < \begin{bmatrix}
10^{-6} & 10^{-8} & 10^{-8} & 10^{-8} \\
10^{-8} & 10^{-6} & 10^{-6} & 10^{-6} \\
10^{-8} & 10^{-6} & 10^{-6} & 10^{-6} \\
10^{-8} & 10^{-6} & 10^{-6} & 10^{-6}
\end{bmatrix},
\]  

(5.8)

and

\[
|\chi_i^{\mu\nu}| < \begin{bmatrix}
\cdot & \cdot & \cdot & \cdot \\
\cdot & 10^{-8} & 10^{-8} & \cdot \\
\cdot & 10^{-8} & 10^{-8} & \cdot \\
\cdot & 10^{-8} & 10^{-8} & \cdot
\end{bmatrix}.
\]  

(5.9)

Here “\(\cdot\)” indicates that there are no limits and “\(\cdot\)” indicates that there is no such entry.

Another practice in giving bounds is to allow all coefficients free so that cancellations can occur. These limits follow mostly from the combination of the analysis [9] of the forbidden \(\beta\) decay experiment [13] and pion \(\beta\) decay [56]. Specifically, the limits of Ref. [56] are

\[
\begin{align*}
&\left| -1.4X_r^{01} - 0.82X_r^{13} - 0.81X_r^{02} - 0.46X_r^{23} \right| < 2.5 \times 10^{-5}, \\
&\left| 0.81X_r^{01} + 0.46X_r^{13} - 1.4X_r^{02} - 0.82X_r^{23} \right| < 2.5 \times 10^{-5}, \\
&\left| 0.17(X_r^{11} - X_r^{22}) - 0.58X_r^{12} \right| < 2.5 \times 10^{-5}, \\
&\left| -0.30(X_r^{11} - X_r^{22}) - 0.35X_r^{12} \right| < 2.5 \times 10^{-5}.
\end{align*}
\]  

(5.10) \(\ldots\) (5.12)

We also use the limit

\[
-4.0 \times 10^{-4} < X_r^{00} - 0.404 \left( X_r^{03} + 0.25X_i^{12} \right) < 1.3 \times 10^{-3},
\]  

(5.14)

obtained from muon decay [57]. In the limits from kaon decay it is assumed that the central value is zero. The limits from kaon decay are [19]

\[
\begin{align*}
&\left| -0.97X_r^{01} + 0.22X_r^{02} - 0.11X_r^{03} \right| < 1.3 \times 10^{-3}, \\
&\left| 0.12X_r^{01} + 0.82X_r^{02} + 0.56X_r^{03} \right| < 2.9 \times 10^{-3}, \\
&\left| 0.22X_r^{01} + 0.52X_r^{02} - 0.82X_r^{03} \right| < 2.4 \times 10^{-3}.
\end{align*}
\]  

(5.15) \(\ldots\) (5.17)
5.1. Summary

If we exclude the bounds of this thesis given in Eqs. (5.1, 5.2) then the combination of the bounds of Eqs. (5.3–5.7, 5.10–5.17) leads to bounds on several of the coefficients of $\chi^\mu_\nu$,

$$
|\chi^\mu_\nu| < 
\begin{bmatrix}
10^{-2} & 10^{-3} & 10^{-3} & 10^{-2} \\
10^{-3} & 10^{-4} & 10^{-2} \\
10^{-3} & 10^{-4} & 10^{-3} \\
10^{-2} & 10^{-2} & 10^{-3} & 10^{-3}
\end{bmatrix},
$$

(5.18)

and

$$
|\chi_i^\mu_\nu| < 
\begin{bmatrix}
\cdot & \cdot & \cdot & \cdot \\
\cdot & 10^{-2} & 10^{-2} \\
10^{-2} & \cdot & 10^{-2} \\
-10^{-2} & 10^{-2} & \cdot
\end{bmatrix},
$$

(5.19)

Limits on $X_r^{11}, X_r^{22}$ could not be obtained, since only their difference $X_r^{11} - X_r^{22}$ appears in the inequalities.

Including our bounds, Eqs. (5.1, 5.2), leads to

$$
|\chi^\mu_\nu| < 
\begin{bmatrix}
10^{-3} & 10^{-3} & 10^{-3} & 10^{-3} \\
10^{-3} & 10^{-4} & 10^{-3} \\
10^{-3} & 10^{-4} & 10^{-3} \\
10^{-3} & 10^{-3} & 10^{-3} & 10^{-3}
\end{bmatrix},
$$

(5.20)

and

$$
|\chi_i^\mu_\nu| < 
\begin{bmatrix}
\cdot & \cdot & \cdot & \cdot \\
\cdot & 10^{-3} & 10^{-4} \\
10^{-3} & \cdot & 10^{-4} \\
-10^{-4} & 10^{-4} & \cdot
\end{bmatrix},
$$

(5.21)

If we compare Eqs. (5.20, 5.21) with Eqs. (5.18, 5.19), we see that the limits obtained in this thesis have improved the bounds on $X_i^{13}$ and $X_i^{23}$ with one order of magnitude in the case of minimal constraints. The limits on the coefficients $X_r^{00}, X_r^{03} = X_r^{30}, X_r^{21} = X_r^{13}$ and $X_r^{21} = X_r^{12}$ have decreased as well. In both cases we found the limit $|X_r^{11} - X_r^{22}| < 10^{-4}$. Our numerical analysis appears to indicate that the bounds in Eqs. (5.18, 5.19) are not reliable i.e. the number of constraints is too low. With the added limits of this thesis the results are robust.
5.2 Outlook

There are various options to expand the search for violations of Lorentz invariance and for improving limits on $\chi$. These are the measurement of coefficients $\chi_{ik}^{0k}$ that have not yet been measured, the improvements of current experiments, and new experimental methods, which will be discussed in the following.

5.2.1 Missing coefficients

First, we note that the components $\chi_{ik}^{0k}$ are not yet measured. Inspecting Eq. (2.4) one observes that the terms which contain $\chi_{ik}^{0k}$ are available via the terms $\chi_{ik}^{k0}(p \times I)^k$. In Ref. [22] it is discussed how these can be accessed through the correlation of $\hat{J} \times \hat{p}_e$ and a component of $\chi$. One possible observable is the asymmetry measured with polarized nuclei. We suggest decays of pure GT transitions,

$$A_{\beta,J,GT} = \frac{W_L^1 W_R^1 - W_R^1 W_L^1}{W_L^1 W_R^1 + W_R^1 W_L^1} = -2A \left( \chi_{ik}^{0k} \epsilon^{klm} - \chi_{ik}^{lm} \right) \frac{P \hat{J} \hat{p}_e^m}{E_e}. \quad (5.22)$$

Here $W_{L,R}$ is the rate of $\beta$ particles in the opposite left ($L$) and right ($R$) directions and the nuclei polarized perpendicular $\uparrow$ ($\downarrow$) $\hat{J}$ to this direction. A measurement of a pure GT decay with $J \rightarrow J - 1$ is preferred to maximize the $\beta$ asymmetry parameter $A$.

The full expression for the $\beta$-decay rate in Ref. [9] has another term with $\chi_{ik}^{0k}$ that can be used to construct a relevant observable for our purpose. For pure F decay this is the asymmetry [22]

$$A_{\beta\nu,F} = \frac{W_L^1 W_R^1 - W_R^1 W_L^1}{W_L^1 W_R^1 + W_R^1 W_L^1} = 4\chi_{ik}^{0k} \epsilon^{klm} \frac{P_e \hat{p}_\nu^m}{E_e}. \quad (5.23)$$

$W_{L,R}$ is the rate of $\beta$ particles in the opposite left ($L$) and right ($R$) directions, where instead of the neutrino direction $\hat{p}_\nu$ the direction of the recoiling nucleus is detected in the perpendicular $\uparrow$ ($\downarrow$) direction. Because there are no limits on $\chi_{ik}^{0k}$, any measurement would set new limits.
5.2.2 Improving current experiments

5.2.2.1 Measurements that use polarized sources

For single-laser optical pumping of an alkali metal for the measurement of $\chi_{i,20}\text{Na}$ is the best isotope. Other isotopes can be considered by using techniques such as optical pumping with two lasers or polarization transfer in a buffer gas. The use of segmented $\gamma$ detectors could limit the contamination of the $\gamma$ signal by coincident summing with positrons, which reduces systematic errors. For $\beta$ decay with a polarized source [9] the ratio between LIV and SM decay rates is

$$\frac{\Gamma_{\text{LIV}}}{\Gamma_{\text{SM}}} = 2\omega_2 \cdot P \hat{J}, \quad (5.24)$$

with $\omega_2 = \bar{K}(\chi^{0k}_r - \chi^{k0}_r) - \bar{L}\chi^k_i$. The expressions for $\bar{K}$, $\bar{L}$ are given in Ref. [9]. With the assumption that $\chi^{0k} = \chi^{k0}$, we find $\omega_2 = -2\bar{L}\chi^k_i$. For a pure GT transition $\bar{L} = -\frac{1}{2}A$. This results in the asymmetry

$$\frac{W^{\leftarrow} - W^{\rightarrow}}{W^{\leftarrow} + W^{\rightarrow}} = A\bar{P}\chi^{k0}\hat{J}^k, \quad (5.25)$$

where $W^{\leftarrow}, W^{\rightarrow}$ are the decay rates for opposite polarization directions. This asymmetry also holds for mixed transitions with $A$ the $\beta$-asymmetry parameter for mixed transitions. These will be reduced in magnitude depending on the amount of F/GT mixing. Because in the experiment $A\bar{P}$ is measured from the $\beta$ asymmetry, the mixing does not need to be known. The advantage could be that effectively a higher sensitivity might be achieved.

The largest systematic errors in our measurement were related to fluctuations of the laser power and of the buffer gas conditions. These can be reduced by taking it into account in the analysis of the primary observables instead of the post-analysis as done here. They could also be reduced in a temperature-controlled environment.

The polarization in this experiment was maximally about 50% and was time-dependent. We considered an experiment in which the beam is always on. However, this complicates the experiment because then the lifetime measurement is not possible and one would have to measure the difference between production rate and decay rate. If these complications
could be overcome, an experimental design that has high, constant polarization could improve the statistical precision, but not by more than a factor four.

Extending the run-time of the experiment to about a year would gain only about one order of magnitude in statistical precision, compared to the experiment of this thesis. Of course, this experiment could be improved in statistical precision by a higher source strength. However, an important finding of the analysis was that the systematic uncertainty is of the same order of magnitude as the statistical uncertainty. Even if an experiment of this type would reach a statistical precision of $10^{-8}$ on $\chi_i$, reducing the systematic uncertainty down to the same level would be a tour-de-force. It is not yet clear whether such a reduction is possible.

5.2.2.2 Measuring the anisotropy of the $\beta$-particle emission

The experiments of Refs. [13, 14] measured the $\beta$ direction. Generally one could probe $\beta$ emitters decaying with a F, GT or mixed transition. These experiments explore different combinations of coefficients of $\chi$ as

$$\frac{\Gamma_{\text{LIV}}}{\Gamma_{\text{SM}}} \bigg|_F = -2\chi^0_k \frac{p^k_e}{E_e}$$

and

$$\frac{\Gamma_{\text{LIV}}}{\Gamma_{\text{SM}}} \bigg|_{\text{GT}} = \frac{2}{3} \left( \chi^0_k + \chi^i_k \right) \frac{p^k_e}{E_e}.$$  \hspace{1cm} (5.27)

A mixed transition gives a combination of these.

A very-high-intensity source and extended measurement time are required to reach the statistical precision of $10^{-8}$. Therefore, one needs to consider availability and safety regulations. Commercially manufactured source strengths are mostly of order 1 mCi to 100 mCi.

In the experiments of Refs. [13, 14] a nucleus with a forbidden transition ($\Delta J \geq 2$) was used. The experiment of Ref. [13] measured the first-forbidden transition of $^{90}$Y by using a 10 Ci $^{90}$Sr source. The sources used in the experiment of Ref. [14] were $^{137}$Cs, which has a first-forbidden transition, and $^{99}$Tc which has a second-forbidden transition. Noordmans et al. [10] have shown that forbidden transitions have an enhancement of
the LIV part of the decay rate with a factor $\alpha Z/pR$ compared to allowed transitions. This enhancement is typically of order 10. Nonetheless, using allowed transitions of different F/GT nature the boundaries can be extracted without the ambiguities in Eqs. (5.3–5.7), which removes issues concerning finetuning. It would be preferred to use $\beta^-$ emitters that decay to the ground state of the daughter nucleus. Generally the radiation from these sources are easier to shield, in particular if their end point is below pair production. A selection of nuclei that would be useful in this respect is given in Table 5.2.

Because $\beta$ currents have to be measured when measuring very high rates, the thickness of the source should be limited. Given such a packaging, several planar sources may be placed in parallel for a measurement of the $\beta$ current. The simplicity of the measurement also allows to reduce systematic errors.

### 5.2.3 New experimental methods

There are several unexplored possibilities for LIV searches. In the following the measurement of $\gamma$ radiation of unpolarized sources, the use of electron capture (EC), and exploiting the boost in accelerator-based experiments are considered.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$T_{1/2}$</th>
<th>$J_{i}^{P}$</th>
<th>$J_{f}^{P}$</th>
<th>F/GT</th>
<th>Q(gs) (keV)</th>
<th>decay to gs (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{32}$P</td>
<td>14.3 d</td>
<td>$1^+$</td>
<td>$0^+$</td>
<td>GT</td>
<td>$1.7 \times 10^3$</td>
<td>100</td>
</tr>
<tr>
<td>$^{33}$P</td>
<td>25.4 d</td>
<td>$1/2^+$</td>
<td>$3/2^+$</td>
<td>GT</td>
<td>$2.5 \times 10^2$</td>
<td>100</td>
</tr>
<tr>
<td>$^{35}$S</td>
<td>87.4 d</td>
<td>$3/2^+$</td>
<td>$3/2^+$</td>
<td>F/GT</td>
<td>$1.7 \times 10^2$</td>
<td>100</td>
</tr>
<tr>
<td>$^{63}$Ni</td>
<td>101 y</td>
<td>$1/2^-$</td>
<td>$3/2^-$</td>
<td>GT</td>
<td>$6.7 \times 10^1$</td>
<td>100</td>
</tr>
<tr>
<td>$^{45}$Ca</td>
<td>163 d</td>
<td>$7/2^-$</td>
<td>$7/2^-$</td>
<td>F/GT</td>
<td>$2.6 \times 10^2$</td>
<td>99.998</td>
</tr>
</tbody>
</table>

Table 5.2: Selection of isotopes for an experiment that measures $\beta$ currents. $gs$ = ground state. $J_{i,f}^{P}$ are the spin-parity of the initial and final state, respectively.
5.2.3.1 Anisotropy of $\gamma$ radiation

Measuring the anisotropy of $\gamma$ radiation due to LIV seems to be a good option. If LIV influences the $\beta$ direction, then the random magnetic substate distribution of daughter nuclei of an initially unpolarized source will be altered in case of a GT transition. If the daughter nucleus emits $\gamma$ radiation the non-uniform magnetic substate distribution would become manifest as a $\gamma$ anisotropy. This measurement is exclusively sensitive to $\chi_r$. The experimental requirements have been analyzed recently [58, 59]. Suitable candidates are $^{22}$Na and $^{60}$Co. To reach a statistical precision of $1 \times 10^{-8}$, a measurement of one year using a 58 mCi source of $^{22}$Na or a 27 mCi source of $^{60}$Co is required. This is an acceptable amount in terms of radiation safety. Also here the simplicity of the measurement is important in view of systematic errors.

An experiment with $^{60}$Co could result in further reduction of the limits on $\chi$ of several orders of magnitude with sources of very high activity. $^{60}$Co is the most commonly used source for sterilization by $\gamma$ radiation. $^{60}$Co sources with activities of tens of kCi up to several MCi are used for this purpose [60]. Source activities up to about 25 kCi are used in relatively compact irradiators at hospitals. Much higher source strengths are used for sterilization of products on an industrial scale at irradiation facilities. For the latter the source material is typically encased in pencils of which multiple are mounted in modules that go into a source rack.

Certainly with the industrial tools for $^{60}$Co an experiment is possible. However, practical issues need to be addressed. The source is mostly placed at a position where it might be difficult to perform the experiment. Moreover, the source itself is probably not very stable in time and position. More research would be required to validate an experimental design that suppresses sources of systematic error.

5.2.3.2 Electron-capture sources

Sources that decay exclusively with EC are an option because the emission of ionizing radiation is limited to X-ray emission, Auger electrons and internal bremsstrahlung. Therefore, considering radiation-safety requirements, higher count rates can be realized compared to $\beta$ emitters. A
5.2. Outlook

A discussion of theory and possible sources is presented in Ref. [21]. The decay rate can be measured from the current of Auger electrons.

If the nuclei are polarized one could infer the polarization by the internal bremsstrahlung [61, 62], and measure $\tilde{\chi}_i$ with a setup similar to the experimental setup presented in this thesis. Alternatively, the recoil direction of daughters of unpolarized nuclei may be measured to infer $\tilde{\chi}_i^k + \chi_r^{k0}$. In another possible experiment with polarized nuclei both the nuclear polarization and recoil direction are measured to infer $\tilde{\chi}_i^k$ from the triple correlation $\tilde{\chi}_i^k(p \times I)^k$.

The two experiments appear to be quite difficult since the requirements of high decay rates and the measurement of nuclear recoils appear to be conflicting. A 'bulk' measurement of nuclear recoils similar to that of the electron current measurement would have to be available. However, an EC polarization experiment seems feasible. As discussed in Ref. [21], the only viable isotopes for this are $^{37}$Ar and possibly $^{131}$Cs.

5.2.3.3 Accelerator-based experiments

We now consider decaying particles with high Lorentz factor $\gamma$. It is possible to exploit the boost term in accelerator-based experiments searching for LIV. Calculations have been made for pion $\beta$ decay [56] and nonleptonic kaon decay [19].

In both calculations the components of $\chi$ in the center-of-mass frame of a weakly decaying particle with relative velocity $\beta = (\beta_1, \beta_2, \beta_3)$, are expressed in terms of the components in the laboratory frame ($X^{\mu\nu}_{lab}$) by the Lorentz transformation

$$
\chi^{\mu\nu} = B^{\mu}_{\rho} B^{\nu}_{\sigma} X^{\rho\sigma}_{lab},
$$

with

$$
B = \begin{bmatrix}
\gamma & -\gamma \beta_1 & -\gamma \beta_2 & -\gamma \beta_3 \\
-\gamma \beta_1 & 1 + (\gamma - 1) \frac{\beta_2^2}{\beta_3^2} & (\gamma - 1) \frac{\beta_1 \beta_2}{\beta_3} & (\gamma - 1) \frac{\beta_1 \beta_3}{\beta_2} \\
-\gamma \beta_2 & (\gamma - 1) \frac{\beta_2 \beta_1}{\beta_3} & 1 + (\gamma - 1) \frac{\beta_2^2}{\beta_3^2} & (\gamma - 1) \frac{\beta_2 \beta_3}{\beta_1} \\
-\gamma \beta_3 & (\gamma - 1) \frac{\beta_3 \beta_1}{\beta_2} & (\gamma - 1) \frac{\beta_3 \beta_2}{\beta_1} & 1 + (\gamma - 1) \frac{\beta_3^2}{\beta_2^2}
\end{bmatrix}.
$$
Observables that measure LIV are constructed, starting with the decay rate in the center-of-mass frame. With the transformation to the laboratory frame [Eq. (5.28)] a factor $\gamma^2$ is introduced in some observables, as we show below. To determine if the observable depends on sidereal time, the expression for $X_{\text{lab}}^{\mu\nu}$ is transformed to the Sun-Centered frame (with components $X_{\text{SC}}^{\mu\nu}$) by another Lorentz transformation. For our purposes the boost from Earth’s frame to the Sun-centered frame is negligible ($\gamma \simeq 1$). Therefore, the coefficients in the laboratory frame can be expressed in terms of the coefficients of the Sun-centered frame ($X_{\text{SC}}^{\mu\nu}$) as

$$X_{\text{lab}}^{\mu\nu} = R_{\rho}^{\mu} R_{\sigma}^{\nu} X_{\text{SC}}^{\rho\sigma}.$$  \hfill (5.30)

We can express $R$ as $R(t) = R_{\text{rot}} R_t(t)$ with the rotation matrix

$$R_{\text{rot}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \zeta & 0 & -\sin \zeta \\ 0 & 0 & 1 & 0 \\ 0 & \sin \zeta & 0 & \cos \zeta \end{bmatrix},$$  \hfill (5.31)

and the time-dependent rotation matrix

$$R_t(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \Omega t & \sin \Omega t & 0 \\ 0 & -\sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$  \hfill (5.32)

$\zeta$ is the colatitude of the experiment and $\Omega$ is Earth’s sidereal rotation frequency. We have

$$R(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \zeta \cos \Omega t & \cos \zeta \sin \Omega t & -\sin \zeta \\ 0 & -\sin \Omega t & \cos \Omega t & 0 \\ 0 & \sin \zeta \cos \Omega t & \sin \zeta \sin \Omega t & \cos \zeta \end{bmatrix}.$$  \hfill (5.33)

We consider $^6$He at high $\gamma$ in a racetrack accelerator (Fig. 5.1), as was suggested for generation of intense, focused antineutrino beams in Refs. [63, 64]. $^6$He is a GT $\beta^-$ emitter ($0^+ \rightarrow 1^+, T_{1/2} = 807 \text{ ms}$) with a $\beta$-endpoint energy of $E_{\beta,\text{max}} = 3.5 \text{ MeV}$. Bunches of $^6$He are accelerated and will then
5.2. Outlook

Figure 5.1: Schematic view of the $^6\text{He}$ setup. The red (blue) curve indicates the trajectory of the $^6\text{Li}^3^+$ for forward (backward) recoil direction in the $^6\text{He}^2^+$ center-of-mass frame, relative to the $^6\text{He}^2^+$ beam direction in the laboratory frame.

coast in the racetrack at a specific value of $\gamma$ ($\approx 100$). The $^6\text{He}$ decaying in the straight sections lead to the kinematically focused neutrinos. The $^6\text{Li}$, because of its higher charge state and lower momentum is bent towards the inner radius. The $^6\text{Li}$ can be conveniently detected at the two locations indicated in Fig. 5.1. The rate at which $^6\text{Li}$ is detected is inversely proportional to the lifetime of $^6\text{He}$ in a lossless racetrack. We assume that one can observe the momentum distribution as in a spectrograph with a resolution of $\approx 10^{-5}$, as discussed below. The observables for $^6\text{He}$ decay discussed below relate to (i) the effect of the boost $\gamma$, (ii) the direction of the $^6\text{He}^2^+$ motion, and (iii) the recoil direction of the daughter $^6\text{Li}^3^+$.

(i) To demonstrate the $\gamma^2$ dependence we average over the direction of the electron in Eq. (2.4). It follows that the expression for the decay rate for a GT transition is

$$\Gamma = \Gamma_0 \left(1 - \frac{2}{3} \chi_{r00}^0 \right),$$

(5.34)

where $\Gamma_0$ is the SM decay rate. For nuclei with a F transition,

$$\Gamma = \Gamma_0 \left(1 + 2 \chi_{r00}^0 \right).$$

(5.35)
The transformation of Eq. (5.28) results in

\[
\chi^{00}_r = \frac{X^{\mu\nu}_{laboratory, r} k_\mu k_\nu}{m^2} = \gamma^2 \left[ X^{00}_{laboratory, r} + \beta_k \left( X^{0k}_{laboratory, r} + X^{k0}_{laboratory, r} \right) + \beta_i \beta_j X^{ij}_{laboratory, r} \right],
\]

(5.36)

where \( m \) is the mass of the nucleus and \( k_\mu \) is the momentum of the nucleus in the laboratory frame. Various observables in \( ^6\text{He}^{2+} \) decay can be constructed from Eq. (5.36). In the following we consider only terms up to leading order in \( X_{laboratory} \). We compare the lifetime \( \tau_{rest} \) of the particle at rest with the lifetime \( \tau' \) of the particle moving at high \( \gamma \) in the horizontal plane in the laboratory. \( \tau' \) is measured in the laboratory frame, the corresponding proper lifetime (i.e. in the center-of-mass frame) is \( \tau_\gamma = \gamma^{-1} \tau' \). The second term in Eq. (5.36) averaged over directions in the laboratory’s horizontal plane is zero. Integration of the third term in brackets in Eq. (5.36) results in a contribution to the decay rate that depends on the geometry of the accelerator. For the accelerator in Fig. 5.1 we take the ratio of a single straight section to the turn-radius to be \( n : 1 \). Integrating the third term in brackets in Eq. (5.36) over the two semicircle sections (where \( \beta = |\beta| (\cos \phi, \sin \phi, 0) \) with \( \phi \in [0, 2\pi] \)) gives \( A_1 = \pi \beta^2 (X^{11}_{laboratory, r} + X^{22}_{laboratory, r}) = \pi \beta^2 (X^{00}_{laboratory, r} - X^{33}_{laboratory, r}) \), where in the last equality we have used that \( \eta_{\mu\nu}X^{\mu\nu}_{laboratory, r} = 0 \) i.e. it is traceless. We assume the straight sections of the accelerator are parallel to the \( y \)-axis in the laboratory frame i.e. \( \beta = |\beta|(0, 1, 0) \) for one straight section and \( \beta = |\beta|(0, -1, 0) \) for the opposite straight section. Then integrating the third term over the two straight sections gives \( A_2 = 2n\beta^2 X^{22}_{laboratory, r} \). The average of the third term is then \( (A_1 + A_2)/[2(n + \pi)] \). Then

\[
\frac{\tau_\gamma - \tau_{rest}}{\tau_{rest}} = \frac{2}{3} \left( \gamma^2 - 1 \right) \left\{ X^{00}_{laboratory, r} \right. \\
+ \left. \frac{1}{2(n + \pi)} \left[ \pi \left( X^{00}_{laboratory, r} - X^{33}_{laboratory, r} \right) + 2n X^{22}_{laboratory, r} \right] \right\}.
\]

(5.37)

In this expression \( X^{22}_{laboratory, r} \) and \( X^{33}_{laboratory, r} \) have a sidereal dependence given by Eq. (5.30).

(ii) The term dependent on \( \beta_k \) in Eq. (5.36) can be obtained by comparing the proper lifetimes (\( \tau^\pm \)) of \( ^6\text{He} \) moving in opposite directions (\( \pm \beta \)),...
indicated in Fig. 5.1. The asymmetry is

\[
\frac{\tau^+ - \tau^-}{\tau^+ + \tau^-} = \frac{2}{3} \gamma^2 \beta_k \left( X_{\text{lab},r}^{0k} + X_{\text{lab},r}^{k0} \right) = \frac{4}{3} \gamma^2 \beta_k X_{\text{lab},r}^{0k},
\]  

(5.38)

and also has a sidereal dependence. The second equality in Eq. (5.38) assumes that \( X_{\text{lab},r}^{\mu\nu} \) is symmetric in \( \mu \leftrightarrow \nu \).

(iii) Components of \( \chi_r^{0k} \) and \( \tilde{\chi}_i^k \) are accessed by measuring the recoil of the daughter \( ^6\text{Li}^{3+} \). From measuring the rigidity of the \( ^6\text{Li} \) nuclei one may determine if it moved in the forward or backward direction relative to \( ^6\text{He} \), as shown in Fig. 5.1. From the nuclear recoil we can determine \( p_e^l \) in the expression [cf. Eq. (2.4)]

\[
\Gamma = \Gamma_0 \left[ 1 - \frac{2}{3} \chi_r^{00} + \frac{2}{3} \left( \chi_r^{k0} + \tilde{\chi}_i^k \right) \frac{p_e^l}{E_e} \right].
\]

(5.39)

For convenience we take \( p_e \simeq -p_{\text{recoil}} \). \( \chi_r^{0k} \) and \( \tilde{\chi}_i^k \) are expressed in terms of coefficients in the lab frame as

\[
\chi_r^{k0} = \gamma \left( X_{\text{lab},r}^{k0} + \beta_m X_{\text{lab},r}^{km} \right) + \frac{\gamma^2}{\gamma + 1} \beta_k \left( \beta_l X_{\text{lab},r}^{00} + \beta_l \beta_m X_{\text{lab},r}^{lm} \right)
- \gamma^2 \beta_k \left[ X_{\text{lab},r}^{00} + \beta_l \left( X_{\text{lab},r}^{0l} + X_{\text{lab},r}^{l0} \right) + \beta_l \beta_m X_{\text{lab},r}^{lm} \right];
\]

(5.40)

\[
\tilde{\chi}_i^k = \gamma \tilde{X}_{\text{lab},i}^k + \frac{\gamma^2}{\gamma + 1} \tilde{X}_{\text{lab},i}^{kl} \beta_l \beta^k - \gamma \epsilon^{klm} \beta^l \left( X_{\text{lab},r}^{0m} - X_{\text{lab},r}^{m0} \right).
\]

(5.41)

The fraction of forward to backward decaying particles would not only depend on the leg of the racetrack where the decay took place, but would also vary with sidereal time. We denote \( \Gamma_{\uparrow\downarrow}^{\uparrow\downarrow} \) the decay rate in the \( \uparrow \) (\( \downarrow \)) \( ^6\text{He} \) direction for a forward (+) or backward (−) electron emission. For simplicity we assume that the \( ^6\text{He} \) velocity is \( \beta = |\beta|(0,1,0) \) for the \( \uparrow \) direction and \( \beta = |\beta|(0,-1,0) \) for the \( \downarrow \) direction. We construct the asymmetry

\[
A_d = \frac{\Gamma_{\uparrow\downarrow}^{\uparrow\downarrow} \Gamma_{\downarrow\uparrow}^{\downarrow\uparrow} - \Gamma_{\downarrow\uparrow}^{\uparrow\downarrow} \Gamma_{\uparrow\downarrow}^{\downarrow\uparrow}}{\Gamma_{\uparrow\downarrow}^{\uparrow\downarrow} \Gamma_{\downarrow\uparrow}^{\downarrow\uparrow} + \Gamma_{\downarrow\uparrow}^{\uparrow\downarrow} \Gamma_{\uparrow\downarrow}^{\downarrow\uparrow}}
- \frac{2}{3} \left[ (2\gamma^2 - 1) X_{\text{lab},r}^{02} + \tilde{X}_{\text{lab},i}^2 \right].
\]

(5.42)

In Eq. (5.42), it is assumed that \( X_{\text{lab},r}^{\mu\nu} \) is symmetric in \( \mu \leftrightarrow \nu \). The terms \( X_{\text{lab},r}^{02} \) and \( \tilde{X}_{\text{lab},i} \) in \( A_d \) have a sidereal dependence with an angular frequency of \( 2\Omega \) [Eq. (5.30)].
The term $p_e/E_e$ in Eq. (5.39) is obtained by measuring the $^6$Li recoil. In the calculation leading to Eq. (5.42) it was assumed the electron is ultra-relativistic i.e. $|p_e|/E_e \simeq 1$. The $^6$Li recoil measurement could be done by placing a silicon strip detector at a central $\rho_{Li} = 2/3 \rho_{He}$ as indicated in Fig. 5.1, where $\rho$ is radius of curvature of the particle trajectory. The relative difference in $^6$Li momentum $p$ one would need to measure is maximally $\Delta p/p = 2E_{\beta,\text{max}}/\left(\beta m_{He}\right) \simeq 1 \times 10^{-3}$. A higher precision of $10^{-4}$ to $10^{-5}$ would be sufficient to detect the separation due to the momentum difference between the forward and the backward nuclear recoil. More research is required to determine what limits on $\chi$ could be obtained with such an experiment.

Analogous observations can be made for other particles that decay via the weak interaction. The connection with $\chi$ is better established in the treatment of leptonic and semileptonic decays than that of nonleptonic decays [19]. Specifically, in the expression for the decay rate of nonleptonic decays a cancellation of the amplitude contributions of the “tree” and “penguin” diagrams can occur resulting in partial cancellation of the LIV term. In the following we point out two other experiments that exploit the boost term: weak decays at the Large Hadron Collider beauty (LHCb) experiment, and $K$ beams.

At the LHCb experiment there is abundant production of various long-lived particles, that decay weakly in the active volume. LHCb measures the high-rapidity region in which the decaying particles have a large Lorentz boost $\gamma (\beta \simeq 1)$ along the beam direction. The lifetime measured for weakly-decaying particles will have a term quadratic in $\gamma$ due to LIV as in Eq. (5.36). Sidereal variations should be used to reduce systematic errors. As in Eq. (5.37), the lifetime of the particle at rest can be compared with the proper lifetime at high $\gamma$. Other observables may be constructed. If such observable contains the particle decay rate rather than the lifetime, the total number of incoming particles may be required in addition to relative production rates.

The decay $K^+ \rightarrow \mu^+\bar{\nu}$ is attractive, since it is a leptonic process and the kaon can have a large $\gamma$ factor. The branching ratio is about 64%. The kaon is produced at several experiments at the CERN Super Proton Synchrotron. For instance, at the NA48/2 experiment [66] simultaneous $K^+$
5.3 Conclusions

and $K^-$ beams are used. Selected kaons have a central momentum of 60 GeV/c and a momentum spread of $\pm 3.8\%$ produced at zero angle, corresponding to a central value of $\gamma = 1.2 \times 10^2$. At the entrance of the decay volume, the average positive (negative) kaon flux is $8.6 \times 10^5$ ($4.8 \times 10^5$) /s. We may consider measurements of the coefficients of Eq. (5.36) with this data. It has not yet been established what would be the resulting precision on $\chi_{\text{r}00}$.

In this section, we have given various options to expand the search for violations of Lorentz invariance in weak interactions. It was indicated to which extent these searches could lead to improved limits on $\chi$. The list of options presented here is not exhaustive, many other approaches could be considered. For instance, weak decays in astrophysical processes may be opportune for LIV searches. Moreover, further theoretical developments may guide towards new experiments.

5.3 Conclusions

In the experiment of this thesis a test of LIV in the weak decay was performed by using polarized $^{20}$Na nuclei. The dependence of the decay rate of $^{20}$Na on the nuclear spin was determined from the relative difference in the decay rate for opposite nuclear polarization directions. This experiment sets bounds on the relative variation of the decay rate difference at sidereal frequency at 90% C.L. of $2 \times 10^{-4}$. The result was interpreted in the $\chi$ tensor framework [9], with 90% C.L. of $|\tilde{X}_{1}^i, \tilde{X}_{2}^i| < 2 \times 10^{-4}$.

The reduction of systematic uncertainty was of crucial importance in the setup and analysis of the experiment. The analysis of our experiment showed that the systematic uncertainty of the measurement is of the same order as the statistical uncertainty. It is not clear that the systematic uncertainty could be reduced sufficiently in an experiment with improved statistical precision. The main reason is the complexity of the measurement; the correlation between the $^{20}$Na lifetime and properties of many experimental components, such as the laser setup, had to be considered.

We have discussed possibilities for new, innovative searches of LIV in $\beta$ decay. The coefficients $\chi_{\text{r}0k}^i$ are hitherto not measured and could be
obtained from an experiment that measures the polarization, plus either the \( \beta \) particles or the nuclear recoil perpendicular to the polarization direction.

The best limits on components of \( \chi \), of order \( 10^{-8} \), would be difficult to improve with the current experiments. The experiments that measured LIV dependent on the \( \beta \) direction used very high intensity sources. Availability and safety regulations have to be considered. To overcome the systematic errors one needs to pursue the simplest measurements.

We have shown there are possibilities for new experimental approaches to improve the current limits. In particular, an experiment measuring the anisotropy of \( \gamma \) radiation due to LIV was investigated \[58,59\]. In this experiment the source has randomly-oriented nuclei initially. The orientation of the daughter nuclei due to LIV would be measured as a \( \gamma \) anisotropy. The initial transition is a GT decay. Sources of \( ^{22}\text{Na} \) or \( ^{60}\text{Co} \) less than 1 Ci would set limits on components of \( \chi \) of order \( 10^{-8} \). Reduction of these limits with several orders of magnitude could be obtained by use of a very-high-intensity \( ^{60}\text{Co} \) source of an irradiation facility.

Leptonic and semileptonic decays at particle accelerators have a \( \gamma^2 \) enhancement of LIV. Here, too, a high number of weak decays is a requirement for improvement of the current limits on the components of \( \chi \). Experiments at neutrino facilities and \( \beta \)-beam factories should be considered. The CERN Super Proton Synchrotron is a typical example of an accelerator for high-intensity secondary beams with high \( \gamma \). Kaon and \( ^{6}\text{He} \) beams are attractive possibilities, but there may be other particles that should be given consideration.

In this chapter we have shown that there are many opportunities for searches of LIV in \( \beta \) decay. In particular, the coefficients \( \chi_{i}^{0k} \) are not measured, and the limits on other coefficients of \( \chi \) can still be significantly improved. The present experiment is just one of many results by which experiments can constrain the parameter space of new physics. In this respect, \( \beta \) decay is as relevant as it was when Lee and Yang suggested that parity might be violated.