Some notes on Bayesian time series analysis in psychology

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Summary

In this thesis, I focus on the use of time series analysis in psychological research. I focus on two main issues. The first one pertains to the effect of different estimators and data properties on the estimation of the autocorrelation. Further, I study the challenges of empirical data and how to link the psychological theory with the statistical models. For the last part I use the Bayesian dynamic model, which is able to handle the challenges present in the empirical data sets I use.

In **Chapter 2** I discuss various estimators of the autoregressive model for univariate data. I compare their performance in estimating the autocorrelation in short time series. In Study 1, under correct model specification, I compare the frequentist r_1 estimator, C-statistic, ordinary least squares estimator (OLS) and maximum likelihood estimator (MLE), and a Bayesian method, considering flat (B_f) and symmetrized reference (B_{sr}) priors. In a completely crossed experimental design I vary lengths of time series (i.e., T = 10, 25, 40, 50 and 100) and autocorrelation (from -0.90 to 0.90 with steps of 0.10). The results show the lowest bias for the B_{sr}, and the lowest variability for r_1. The power in different conditions is the highest for B_{sr} and OLS. For $T = 10$, the absolute performance of all measurements is poor, as expected. In Study 2, I study robustness of the methods through misspecification by generating the data according to an ARMA(1,1) model, but still analysing the data with an AR(1) model. I use the two methods with the lowest bias for this study, i.e., B_{sr} and MLE. The bias gets larger when the non-modelled moving average parameter becomes larger. Both the variability and power show a dependency on the non-modelled parameter. The differences between the two estimation methods are negligible for all measurements.

In **Chapter 3** I estimate a time series model for multiple individuals using multilevel models. I compare two estimation methods for the autocorrelation in Multilevel AR(1) models, namely MLE and Bayesian Markov Chain Monte Carlo, earlier denoted as B_{sr}. Furthermore, I examine the difference between modeling fixed and random individual parameters. To this end, I perform a simulation study with a fully crossed design, in which I vary the length of the time series (10 or 25), the number of individuals per sample (10 or 25), the mean of the autocorrelation (-0.6 to 0.6 inclusive, in steps of 0.3) and the standard deviation of the autocor-

relation (0.25 or 0.40). I found that the random estimators of the population autocorrelation show less bias and higher power, compared to the fixed estimators. As expected, the random estimators profit strongly from a higher number of individuals, while this effect is small for the fixed estimators. The fixed estimators profit slightly more from a higher number of time points than the random estimators. When possible, random estimation is preferred to fixed estimation. The difference between MLE and Bayesian estimation is nearly negligible. The Bayesian estimation shows a smaller bias, but MLE shows a smaller variability (i.e., standard deviation of the parameter estimates). Finally, better results are found for a higher number of individuals and time points, and for a lower individual variability of the autocorrelation. The effect of the size of the autocorrelation differs between outcome measures.

In **Chapter 4** I use the Bayesian Dynamic Model (BDM) to compare different models for a univariate, multi-individual dataset posing several challenges, such as missing data and non-normally distributed observed data. I represent the complex structure of intensive longitudinal data of multiple individuals with a hierarchical BDM. This BDM is a generalized linear hierarchical model where the individual parameters do not necessarily follow a normal distribution. The model parameters can be estimated on the basis of relatively small sample sizes and in the presence of missing time points. I present the BDM and discuss the model identification, convergence and selection. The use of the BDM model is illustrated using data from a randomized clinical trial to study the differential effects of three treatments for panic disorder. The data involves the number of panic attacks experienced weekly (73 individuals, 10 to 52 time points) during treatment. Presuming that the counts are Poisson distributed, the BDM considered involves a linear trend model with an exponential link function. The final model included a moving average parameter, and an external variable (duration of symptoms pre-treatment). Our results show that cognitive behavioral therapy is less effective on the reduction of panic attacks than serotonin selective re-uptake inhibitors or a combination of both. Post-hoc analyses revealed that males show a slightly higher number of panic attacks at the onset of treatment than females.

In **Chapter 5** I use the BDM for the analysis of multivariate, multi-individual emotional data with missing data points. In emotion dynamic research one distinguishes various elementary emotion dynamic features, which are studied using intensive longitudinal data. Typically, each emotion dynamic feature is quantified separately, which hampers the study of relationships between various features. Further, the length of the observed time series in emotion research is limited, and often suffers from a high percentage of missing values. In this chapter I propose a vector autoregressive Bayesian dynamic model, that is useful for emotion dynamic research. The model encompasses six elementary properties of emotions, and can be applied with relatively short time series, including missing data. The individual elementary properties covered are: within person and innovation variability, inertia, granularity, cross-lag correlation and the intensity. The model can be applied to both univariate and multivariate time series, allowing to model the relationships between emotions. Further, it may model multiple individuals jointly. One may include external variables and non-Gaussian observed data. We illustrate the usefulness of the model with an empirical example of three emotions of three individuals (47 to 70 measurements), with missing time points within the series.

Finally, in **Chapter 6**, I apply the vector autoregressive BDM to bivariate affect data where I compare several models to find the best fitting one. Affect research often focuses on studying various elementary features of the individual dynamics of positive affect (PA) and negative affect (NA). The features studied are typically quantified separately, hampering the study of their mutual relations. In this chapter I consider six elementary affect features to characterize the dynamics of PA and NA: within person and innovation variability, inertia, cross-lag regression, intensity and the co-occurrence of affect. To facilitate the study of these features, I propose to use a vector autoregressive Bayesian dynamic model. This model encompasses parameters that quantify the six affect features of multiple individuals at once. The model can be applied to data typically encountered in dynamic affect research, i.e., bivariate, relatively short time series of multiple individuals, even in the presence of missing time points. Furthermore, the model allows for the inclusion of external variables. I illustrate the usefulness of the model with an empirical example using relatively short time series (53 to 71 measurements) of bivariate affect data for 12 individuals. I compare three models to find whether a weekly cycle is present in the data and whether there is autoregression present in the white noise. I compare the models with regard to the white noise, the innovation noise and a log-likelihood criterion. The model estimates provide insight into the individual dynamics of PA and NA, and their interindividual differences and similarities.

In this thesis I studied the effect of different estimators and data properties on the estimation of the autocorrelation. I concluded that the MLE and the B_{sr} are the best estimators of those used in this thesis. I showed that the estimation is improved when the time series are longer, more individuals are involved, the absolute autocorrelation is larger and the standard deviation of the autocorrelation is smaller.

Furthermore, I studied how the BDM handles practical issues in empirical data and how it can link psychological hypotheses with statistical models. The BDM was capable of handling all practical issues encountered in one model, i.e., missing values, the inclusion of external variables and the use of observed scores with non-normally distributed residuals. Finally, the BDM is capable of handling the psychological research questions posed for the data sets analyzed in this thesis, for example by comparing models to see whether the hypothesized effects are indeed found in the data.