De topologische grondslagen der meetkunde van het aantal
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SUMMARY

It is the object of this dissertation to give a foundation of enumerative geometry, in particular the symbolical calculus of Schubert, with the aid of topological methods. Van der Waerden had noticed that Lefschetz's theory of intersection numbers gives a rigorous definition of the fundamental notion "multiplicity" in algebraic geometry. Somewhat earlier Lefschetz himself had shown how topological considerations may be used to give the solution of difficult algebraic geometric problems.

Van der Waerden's original article (Math. Ann. 102 (1929)) is very concise and after a careful perusal it leaves many questions without an answer. Therefore we do not consider a more detailed treatment of the foundation of enumerative geometry on a topological basis to be superfluous.

The work has been divided in two parts. To prevent ambiguities as regards terminology we have, in the first part, given an outline of topology, notwithstanding the fact that an extensive treatment of topological notions and theorems which are useful to attain the purpose in question, may be found in Lefschetz's book "Topology". However, this book makes rather difficult reading by its aiming at generality and consequently it is not very suitable for gaining an insight into topological questions in a fairly quick manner.

The first Chapter contains a few topological definitions and propositions, e.g. neighbourhood of a point in a space, topological transformation, complex, etc.

The second Chapter deals with the combinatorial homology theory of complexes with a generality which is required for the subsequent parts of the work.

The third Chapter gives an explanation of practical methods in order to find homology characters of complexes and in particular of so-called S-complexes which are of great interest in studying algebraic varieties occurring in enumerative geometry.

Chapter IV contains an account of methods by which topological invariance of combinatory characters can be ascertained. Also a proof of the invariance of dimensionality has been given, based on the properties of local complexes according to Seifert and Threlfall.
The local complex is also used to define the manifold, whose most fundamental properties have been mentioned. The Chapter closes with a rigorous proof of a lemma proposed by Ehrmann, which is of great importance for the solution of the so-called problem of characteristics in enumerative geometry.

Chapter V is wholly devoted to Lefschetz's theory of intersections. We were able to slightly simplify the original treatment of this topic by restricting ourselves to closed orientable manifolds which fulfil the requirements of the following considerations.

A link between topological methods and analytical aids to investigate complexes is given in Chapter VI as the theory of (topological) varieties; by a variety we understand a complex on which continuous differentiable functions can be defined. Also the orientation scheme as given by Poincaré has been put on a firm basis besides analytical methods in order to obtain intersection numbers in terms of parametric representations.

The first Chapter of the second part, Chapter VII, contains an account of the general theory of algebraic varieties, including the proof of the theorem that an algebraic variety can be broken up into a finite number of simplexes, according to Van der Waerden. The more important subject of this Chapter is the so-called symbolic calculus, first introduced by Schubert in an intuitive way. This calculus has been used by Schubert and his successors in a rather reckless manner, for its foundation on the enumeration principle ("Prinzip der Erhaltung der Anzahl") turned out to be unsatisfactory, as the criticism of Study and Kohn has clearly shown.

We believe that the peculiarities and possibilities of this calculus are sufficiently illustrated when we identify Schubert-symbols with homology classes defined by chains connected with algebraic varieties. Supported by the theory of intersections, it is possible to construct an arithmetic for these classes, which were styled by us "conditions" on historical grounds; thus the desired rigour in operating with the calculus can be obtained. For peculiar varieties, $S$-varieties, a terminology introduced by Scharlau has been made to fit into the new conceptions.

The scope of the work did not permit to consider adequately such an extensive subject as enumerative geometry. We, therefore, confined the discussion to some examples which were able to illustrate the above theory very well. Much more information can be found in a paper by Ehrmann (Ann. of Math. (2) 35 (1933)). Nevertheless it must be stated here that this article deals only with the problem of the characteristics; the much more interesting problem of calculating products of conditions has not been considered by Ehrmann.

Chapter VIII is devoted to simple and multiple $n$-dimensional com-plex projective space and the notion: topologic Lefschetz in this: some classical theorem intersection of algebraic variety principle, the determinant matrices with applications.

In Chapter IX the space has been studied; generalisations of the dimensional projective with some theorems.

In the last Chapter of the work we have tried to consider adequately such an extensive subject as enumerative geometry. We, therefore, confined the discussion to some examples which were able to illustrate the above theory very well. Much more information can be found in a paper by Ehrmann (Ann. of Math. (2) 35 (1933)). Nevertheless it must be stated here that this article deals only with the problem of the characteristics; the much more interesting problem of calculating products of conditions has not been considered by Ehrmann.
plex projective spaces; the latter ones have been investigated by using the notion: topological product. The application of the general theory of Lefschetz in this subject proved to be unnecessary. We have discussed some classical theorems, e.g. Bézout's theorem concerning the intersection of algebraic varieties, the Pieri-Chasles correspondence principle, the determination of the degree of varieties defined by matrices with application to Jacobians.

In Chapter IX the variety of rays in an \( n \)-dimensional projective space has been studied. The famous Schubert formulas which are generalisations of the Halphen formulas for line-complexes in three-dimensional projective space were developed. This Chapter winds up with some theorems on connexes and varieties consisting of pairs of rays.

In the last Chapter an investigation has been made concerning varieties built up of line-elements; by a line-element is understood a combination of a ray and a point belonging to the ray. We have also derived formulas analogous to Schubert formulas in Chapter IX; as special cases they include the incidence formulas of Schubert and Pieri. Another application is a theorem first proved in a different way by Schuh; this theorem concludes the Chapter.