Theoretische en numerieke behandeling van de buiging door een ronde opening.

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
1941

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):

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SUMMARY

In the present thesis a diffraction problem in three dimensions, namely the diffraction of a scalar plane wave through a circular aperture in an infinite plane screen, has been solved in a manner which allows of numerical calculation. According to two different boundary conditions, $\varphi = 0$ or $d\varphi/dn = 0$ on the screen, two functions $\varphi_1$ and $\varphi_2$ have been defined by

$\varphi_1, \varphi_2$ are solutions of the wave equation (1, § 1) with the character of a spherical wave diverging from the aperture,

$\varphi_1 = d\varphi_2/dn = 0$ on the screen,

$\frac{d\varphi_1}{ik} = \varphi_2 = 1$ in the plane of the aperture (not necessarily circular).

$\varphi_2$ is the velocity potential in the problem of sound diffraction through the aperture. According to the Babinet principle of § 3 the functions $\varphi_2$ resp. $\varphi_1$ also describe the diffraction round a plane disc (complementary screen) for the first, resp. second boundary condition, and so $\varphi_1$ is the velocity potential for the diffraction of sound round the disc. Moreover the radiation of sound by a vibrating disc is described by $\varphi_1$ (12, § 3).

In chapter II the functions $\varphi_1$ and $\varphi_2$ have been constructed for the circular aperture. For the sake of numerical calculations only the case of the diffraction of a plane wave with direction of propagation perpendicular to the screen has been investigated; thus the problem has rotational symmetry. After separation of the wave equation in spheroidal coordinates $\xi, \eta, \psi$ (15), the differential equation for the factor $X$ is

$$(1 - \xi^2)X'' - 2\xi X' + (k^2\xi^2 + \Lambda)X = 0 \quad (16')$$

The eigenvalue problem of this equation has already been investigated by various authors $5, 8, 9, 10, 11, 12$). In this thesis the eigenfunctions $X_m(\xi)$ are given as expansions in Legendre polynomials

$$X_m(\xi) = \sum_0^\infty b_m^{(m)}P_m(\xi) \quad (23)$$
In analogy with the method of Ince and others \(^4\) in the theory of the Mathieu equation we get the eigenvalues \(\Lambda_n\) as the roots of a continued fraction (29). For small values of \(k\) these roots can be evaluated by the expansions in powers of \(k\) (44, 44a, § 8). For other values a method of successive approximation has been used (§ 10). This method gives immediately the coefficients \(b^{(mn)}_0\) in (23) and thus the eigenfunctions \(X_m(\xi)\); they are so normalized that \(X_m(\xi)\) and \(P_m(\xi)\) have the same norm.

Having found the eigenfunctions of (16') we construct the corresponding \(Y_m(\eta)\) (§ 6). These functions are solutions of (16') after substituting \(\xi = i\eta\). Only functions \(Y_m(\eta)\) can be used which at a large distance from the aperture (\(\eta \to \infty\)) have the character of diverging spherical waves. This condition is not fulfilled by \(X_m(\xi)\). According to Strutt (\(^5\)), p. 68) \(\sum b^{(mn)}_0 Q_m(\xi)\) would be a second solution of (16'). This is found to be erroneous. We construct a solution \(\Xi_m(\xi)\) which tends to \(Q_m(\xi)\) when \(k \to 0\). \(\Xi_m(\xi)\) and \(X_m(\xi)\) are interrelated by a formula which is the analogue of Neumann's relation between \(Q_m\) and \(P_m\):

\[
ed^2 \Xi_m(\xi) = \frac{1}{2} \int_{-1}^{+1} \frac{e^{ik\xi} X_m(t)}{\xi - t} dt\]

The required function of \(\eta\) is then \(Y_m(\eta) = \Xi_m(i\eta)\).

In § 7 \(Y_m(\eta)\) is expanded in a series of Hankel functions

\[
Y_m(\eta) = C_m \sum_{n=0}^{\infty} \nu b^{(mn)}_n \frac{H^{(2)}_n(k\eta)}{\sqrt{k\eta}}
\]

As the series at the right diverges for \(0 < \eta < 1\), it was up to now impossible to calculate the quantities \(Y_{2m}(0)\) and \(Y_{2m+1}(0)\). The uniform representation (35), however, allows to do so (40, § 7).

Finally the functions \(\varphi_1\) and \(\varphi_2\) have been expanded in series of normal solutions \(X_m(\xi) Y_m(\eta)\)

\[
\varphi_1 = -\frac{i}{2} \sum_{m=0}^{\infty} (4m + 3)b^{(mn)}_m X_{2m+1}(\xi) Y_{2m+1}(\eta) \quad \text{(36)}
\]

\[
\varphi_2 = \sum_{m=0}^{\infty} (4m + 1)b^{(mn)}_m X_{2m}(\xi) Y_{2m}(\eta) \quad \text{(36')}
\]

For the radiation of sound the behaviour of the potential at large distance from the aperture is of special importance. In § 12, we find

\[
\varphi \approx \frac{e^{-ikr}}{r} \sum_{m=0}^{\infty} \sigma_m X_m(\cos \theta) \quad \text{(51', 51'')}
\]

where the complex

\[
\sigma_m = \frac{\pi}{2k}
\]

The energy passing through the aperture (transmission coefficient)

\[
W_1 = \frac{\pi}{2}
\]

For small values of \(k\)

\[
D_1 = \frac{8}{27\pi^2} k^4 (1 + o)\]

\[
D_2 = \frac{8}{\pi^2} (1 + 0.039)
\]

In § 11 the eigenvalues have been tabulated for the diffraction amplitudes \(\sigma_m\) for the circular disc (density zero). The transmission coefficients of the circular disc will be found to be

\[
\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 1
\]

The curves \(D_1, D_2\) of the circular disc will be found to be

\[
D_1 = \frac{8}{27\pi^2} k^4 (1 + 0)
\]

\[
D_2 = \frac{8}{\pi^2} (1 + 0.039)
\]

For the values \(k\) solution according (§ 16). Fig. 5 shows the function \(\varphi_2\) and the latter has no real according does no
where the complex amplitude $\sigma_m$, which gives the contribution of the source of order $m$, is

$$\sigma_m = \frac{2m + 1}{k} \frac{X_m(1)}{\Theta_m - i}$$  \hspace{1cm} (54)$$

with the real number $\Theta_m$ defined by

$$\Theta_{2n} = \frac{\pi}{2k} \left( \frac{X_{2n}(0)}{B_{2n}(0)} \right)^2 \hspace{1cm} \Theta_{2n+1} = -\frac{\pi}{2k} \left( \frac{X_{2n+1}(0)}{i \lambda b_{2n}(0)} \right)^2$$  \hspace{1cm} (53)$$

The energy passing through the circular aperture, divided by the energy passing through the same area of the undisturbed wave (transmission coefficient) is

$$D_2 = \sum_{n=0}^{\infty} \frac{2}{4n + 1} |\sigma_{2n}|^2$$  \hspace{1cm} (55')$$

and the energy radiated per second into the half space by a vibrating circular disc (density $\rho$, frequency $\omega$, velocity amplitude $v$) is found to be

$$W_1 = \frac{\pi \nu \rho \omega}{2k} \frac{Y_{2n}(0)}{Y_{2n+1}(0)} \sum_{n=0}^{\infty} \frac{2}{4n + 3} |\sigma_{2n+1}|^2$$  \hspace{1cm} (56)$$

For small values of $k$ we have the following expansions ($\S$ 14)

$$D_1 = \frac{8}{27 \pi^2} k^4 \{1 + 0.32 k^2 + 0.027427 k^4 + 0.004393 k^6 + \ldots \}$$  \hspace{1cm} (59a)$$

$$D_2 = \frac{8}{\pi^2} \{1 + 0.03916 \bar{0} k^2 + 0.000749 k^4 - 0.00026 \bar{0} k^6 + 0.000005 \bar{4} k^8 + \ldots \}$$  \hspace{1cm} (59b)$$

In $\S$ 11 the eigenvalues $\Lambda_m$ and numbers $t_m^{(m)}$ for $0 < k < 10$ have been tabulated. Fig. 1, 2 show the eigenvalues necessary for the diffraction problem in dependence of $k^2$. $\S$ 15 gives the amplitudes $\sigma_m$ for these values of $k$, from which we have constructed tables IX, X, p. 50 and figs. 3, 4 for $D_1$ and $D_2$. When $k$ tends to zero the transmission coefficient of the aperture is $8/\pi^2$.

The curves $D_{K_1}$, $D_{K_2}$, and $D_{K_3}$ give the energy transmission coefficients of the circular aperture for Kirchhoff solutions (C.f. $\S$ 2, 13). Moreover $D_{K_1}$ describes the sound radiating from the circular disc vibrating in a wall ($\S$ 3).

For the values $k = 5$ and $k = 10$ the exact solution and the solution according to Kirchhoff are found in good agreement ($\S$ 16). Fig. 5 shows the radiating parts $\sum \sigma_n X_n(\cos \beta) = \psi_1 + i \psi_4$ of the function $\varphi_3$ and $i w_{K_2}$ of the Kirchhoff solution $U_2$. The latter has no real part in contrast to the exact solution, which accordingly does not vanish in any direction.