Theoretische en numerieke behandeling van de buiging door een ronde opening.

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SUMMARY

In the present thesis a diffraction problem in three dimensions, namely the diffraction of a scalar plane wave through a circular aperture in an infinite plane screen, has been solved in a manner which allows of numerical calculation. According to two different boundary conditions, $\varphi = 0$ or $d\varphi/dn = 0$ on the screen, two functions $\varphi_1$ and $\varphi_2$ have been defined by

$\varphi_1$, $\varphi_2$ are solutions of the wave equation (1, § 1) with the character of a spherical wave diverging from the aperture,

$\varphi_1 = d\varphi_2/dn = 0$ on the screen,

$\frac{1}{ik} \frac{d\varphi_1}{dn} = \varphi_2 = 1$ in the plane of the aperture (not necessarily circular).

$\varphi_2$ is the velocity potential in the problem of sound diffraction through the aperture. According to the Babinet principle of § 3 the functions $\varphi_2$ resp. $\varphi_1$ also describe the diffraction round a plane disc (complementary screen) for the first, resp. second boundary condition, and so $\varphi_1$ is the velocity potential for the diffraction of sound round the disc. Moreover the radiation of sound by a vibrating disc is described by $\varphi_1$ (12, § 3).

In chapter II the functions $\varphi_1$ and $\varphi_2$ have been constructed for the circular aperture. For the sake of numerical calculations only the case of the diffraction of a plane wave with direction of propagation perpendicular to the screen has been investigated; thus the problem has rotational symmetry. After separation of the wave equation in spheroidal coordinates $\xi, \eta, \psi$ (15), the differential equation for the factor $X$ is

$$(1 - \xi^2)X'' - 2\xi X' + (k^2\xi^2 + \lambda)X = 0$$

The eigenvalue problem of this equation has already been investigated by various authors $5, 8, 9, 10, 11, 12)$. In this thesis the eigenfunctions $X_m(\xi)$ are given as expansions in Legendre polynomials

$$X_m(\xi) = \sum_0^\infty b_m^{(m)} P_m(\xi)$$
In analogy with the method of Ince and others in the theory of the Mathieu equation we get the eigenvalues $\lambda_m$ as the roots of a continued fraction (29). For small values of $k$ these roots can be evaluated by the expansions in powers of $k$ (44, 44a, § 8). For other values a method of successive approximation has been used (§ 10). This method gives immediately the coefficients $b^{(m)}_0$ in (23) and thus the eigenfunctions $X_m(\xi)$; they are so normalized that $X_m(\xi)$ and $P_m(\xi)$ have the same norm.

Having found the eigenfunctions of (16') we construct the corresponding $Y_m(\eta)$ (§ 6). These functions are solutions of (16') after substituting $\xi = i\eta$. Only functions $Y_m(\eta)$ can be used which at a large distance from the aperture ($\eta \to \infty$) have the character of diverging spherical waves. This condition is not fulfilled by $X_m(\eta)$. According to Strutt (9), p. 68) $\sum b^{(m)}_n Q_m(\xi)$ would be a second solution of (16'). This is found to be erroneous. We construct a solution $\Sigma_m(\xi)$ which tends to $Q_m(\xi)$ when $k \to 0$. $\Sigma_m(\xi)$ and $X_m(\xi)$ are interrelated by a formula which is the analogue of Neumann's relation between $Q_m$ and $P_m$:

$$e^{ik\xi} X_m(t) = \frac{1}{\xi - t} \int_{-\infty}^{+\infty} e^{ik\xi} X_m(t) \, dt$$

The required function of $\eta$ is then $Y_m(\eta) = \Sigma_m(i\eta)$.

In § 7 $Y_m(\eta)$ is expanded in a series of Hankel functions

$$Y_m(\eta) = C_m \sum_{n=0}^{\infty} \frac{i^n b^{(m)}_n H^{(2)}_{n+1}(k\eta)}{\sqrt{k\eta}}$$

As the series at the right diverges for $0 < \eta < 1$, it was up to now impossible to calculate the quantities $Y_{2m}(0)$ and $Y_{2m+1}(0)$. The uniform representation (35), however, allows to do so (40, § 7).

Finally the functions $\varphi_1$ and $\varphi_2$ have been expanded in series of normal solutions $X_m(\xi) Y_m(\eta)$

$$\varphi_1 = -\frac{1}{ik} \sum_{m=0}^{\infty} (4m + 1) b^{(m)}_1 X_{2m+1}(\xi) Y_{2m+1}(\eta)$$
$$\varphi_2 = \sum_{m=0}^{\infty} (4m + 1) b^{(m)}_0 X_{2m}(\xi) Y_{2m}(\eta)$$

For the radiation of sound the behaviour of the potential at large distance from the aperture is of special importance. In § 12, we find

$$\varphi \approx \frac{e^{-ikr}}{r} \sum_{m=0}^{\infty} \sigma_m X_m(\cos \theta)$$

where the complex of the source of oscilla-

$$\Theta_2n = \frac{\pi}{2k}$$

The energy passing the energy ratio

$$W_1 = \frac{\pi}{2\sigma}$$

For small values of

$$D_1 = \frac{8}{27\pi^2} k^4 \{1 + \eta \}$$
$$D_2 = \frac{8}{3\pi^2} \{1 + \eta \}$$

Finally the functions $\varphi_1$ and $\varphi_2$ have been tabulated for the diffraction amplitudes $\sigma_m$ for the circular disc (density $\rho$). In § 11 the curves $D_k$, $D_{1k}$, $\sigma_m$ for the circular disc with the real number

$$\Theta_2n = \frac{\pi}{2k}$$

and the energy radiated circular disc (density $\rho$) found to be

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where the complex amplitude $\sigma_m$, which gives the contribution of the source of order $m$, is

$$\sigma_m = \frac{2m + 1}{k} \frac{X_m(1)}{\Theta_m - i}$$  \hspace{1cm} (54)

with the real number $\Theta_m$ defined by

$$\Theta_{2n} = \frac{\pi}{2k} \left( \frac{X_{2n}(0)}{b_0^{(m)}} \right)^2 \quad \Theta_{2n+1} = -\frac{\pi}{2k} \left( \frac{X_{2n+1}(0)}{\frac{1}{2} kb_1^{(m)}} \right)^2$$  \hspace{1cm} (53)

The energy passing through the circular aperture, divided by the energy passing through the same area of the undisturbed wave (transmission coefficient) is

$$D_2 = \sum_{n=0}^{\infty} \frac{2}{4n + 1} |\sigma_{2n}|^2$$  \hspace{1cm} (55')

and the energy radiated per second into the half space by a vibrating circular disc (density $\rho$, frequency $\omega$, velocity amplitude $v$) is found to be

$$W_1 = \frac{\pi \nu^2 \rho \omega}{2k} D_1 = \frac{\pi \nu^2 \rho \omega}{2k} \sum_{n=0}^{\infty} \frac{2}{4n + 3} |\sigma_{2n+1}|^2$$  \hspace{1cm} (56)

For small values of $k$ we have the following expansions (§ 14)

$$D_1 = \frac{8}{27\pi^2} k^4 \{1 + 0.32 k^2 + 0.027427 k^4 - 0.004393 k^6 + \ldots\}$$  \hspace{1cm} (59a)

$$D_2 = \frac{8}{\pi^2} \{1 + 0.039160 k^2 - 0.000741 k^4 - 0.000267 k^6 + 0.000054 k^8 + \ldots\}$$  \hspace{1cm} (59b)

In § 11 the eigenvalues $\Lambda_m$ and numbers $t_m^{(m)}$ for $0 < k < 10$ have been tabulated. Fig. 1, 2 show the eigenvalues necessary for the diffraction problem in dependence of $k^2$. § 15 gives the amplitudes $\sigma_m$ for these values of $k$, from which we have constructed tables IX, X, p. 50 and fig. 3, 4 for $D_1$ and $D_2$. When $k$ tends to zero the transmission coefficient of the aperture is $8/\pi^2$.

The curves $D_{K_1}$, $D_{K_2}$, and $D_{K_2}$ give the energy transmission coefficients of the circular aperture for Kirchhoff solutions (C.f. § 2, 13). Moreover $D_{K_1}$ describes the sound radiating from the circular disc vibrating in a wall (§ 3).

For the values $k = 5$ and $k = 10$ the exact solution and the solution according to Kirchhoff are found in good agreement (§ 16). Fig. 5 shows the radiating parts $\sum \sigma_m X_m(\cos \theta) = \Psi_1 + i\Psi_2$ of the function $\varphi_2$ and $iu_{K_2}$ of the Kirchhoff solution $U_2$. The latter has no real part in contrast to the exact solution, which accordingly does not vanish in any direction.