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High-order finite-difference methods for Poisson's equation

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**HIGH-ORDER FINITE-DIFFERENCE
METHODS FOR POISSON'S EQUATION**

RIJKSUNIVERSITEIT TE GRONINGEN

HIGH-ORDER FINITE-DIFFERENCE METHODS FOR POISSON'S EQUATION

PROEFSCHRIFT

TER VERKRIJGING VAN HET DOCTORAAT IN DE
WISKUNDE EN NATUURWETENSCHAPPEN
AAN DE RIJKSUNIVERSITEIT TE GRONINGEN
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Aan mijn ouders

Aan Babs

Aan Paul en Mathieu

IJv'rig in schachten van natuur en taal
Zocht lang mijn speurzin met voorzichtig boren
Voor wetenschap's wolkenkrabbende Toren
Naar deugd'lijk, bouwversterkend materiaal.

Al rilde mij 't getril van 't schrille staal
Door pijnlijk merg van anders wenshende ooren,
Niets kon mijn koel geestdrift'ge vlijt verstoren:
De Toren zag ik rijzen, zaal op zaal.

En dreigde al doffe stiklucht ons te smoren,
Mij en mijn lamp, flikkerend flauw en vaal,

Niets kon mijn menschentrots zoozeer bekoren,
Als 't graven, trouw, naar nuttig mineraal -

Tot plots'ling 'k zag: alle arbeid is verloren;
Hij staat op brokk'lend zand, en zakt eenmaal.

Johan Andreas Dèr Mouw (Adwaita)

(1862 - 1919)

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