Summary

A photonic crystal is a periodic dielectric structure with a lattice constant of the order of the wavelength of the electromagnetic wave that one wants to control. In photonic crystals, the magnetic permeability is equal to one and the electrical permittivity periodically varies in space and is positive. Typical for a photonic crystal is that electromagnetic waves in a certain frequency range and/or with a certain polarization cannot propagate along certain directions in the crystal. This forbidden frequency range is called a stopgap. If the propagation of the electromagnetic wave is forbidden for any crystalline direction and any polarization, for a certain frequency range, then this forbidden frequency range is called a photonic band gap. The photonic band gap in a photonic crystal is similar to the electronic band gap in a semiconductor. In a semiconductor the periodic arrangement of atoms in the crystal lattice causes a range of energies for which the electrons cannot propagate through the material. Analogous to semiconductor technology a photonic crystal could be used to produce integrated optical circuits, which would have a lot of interesting applications.

Photonic crystals show many intriguing optical phenomena such as negative refraction, superprism, ultrarefractive phenomena, focusing of light and the localization of light. This makes that electromagnetic wave propagation in photonic crystals is not always easy to understand. However, the idea of controlling light by means of photonic crystals has led to many proposals for novel devices such as optical switches, microlasers, solar cells, and so on, and has motivated many researchers to investigate a plethora of ideas. In this thesis we study various aspects of the electromagnetic wave propagation in two-dimensional photonic crystals, but we mainly focus on their ability to mimic some aspects of negative-index materials that is materials with simultaneously negative permeability and permittivity and thus a negative refractive index, and their ability to control the spontaneous emission of light.

The propagation of light in a photonic crystal is described by the time-dependent Maxwell equations. These equations can only be solved analytically for very simple cases. For most practical applications they are solved numerically. A method that is often used for this purpose is the finite-difference time-domain (FDTD) method, based on an algorithm due to K. S. Yee in 1966. In the FDTD technique the time-dependent Maxwell equations are solved
by propagating the electromagnetic fields in the time domain and by modeling the interaction of the electromagnetic fields with the dielectric medium. Apart from a discretization in time, the FDTD technique also requires a spatial discretization. The grid spacing must be sufficiently small to resolve both the smallest electromagnetic wavelength and the smallest geometrical feature in the model. Yee’s method is fast, flexible and easy to implement but is conditionally stable, meaning that it is numerically stable if the time-step divided by the spatial mesh size is small enough. This puts limitations on the time-intervals that can be studied. Another drawback of the method is that it is not suitable to calculate spectra as the method does not conserve the energy of the electromagnetic field. To overcome both limitations, De Raedt and co-workers developed in 1999 a systematic approach to solve the time-dependent Maxwell equations with unconditionally stable numerical schemes based on the Lie-Trotter-Suzuki product-formulas. In this thesis we use this unconditionally stable numerical scheme to investigate the electromagnetic wave propagation in photonic crystals having translational invariance with respect to the \( z \)-direction.

In 1967, Veselago has shown theoretically that a flat plate of thickness \( D \) made from a negative-index material with a refractive index equal to \(-1\) and situated in vacuum can focus electromagnetic waves from a point source \( P \) positioned at a distance \( L < D \) from one side of the plate to a point \( P' \) located at a distance \( D - L \) from the other side of the plate. Also inside the plate an image is formed. Due to the relation \( L + L' = D \), the distance between \( P \) and \( P' \) is always equal to \( 2D \). Hence, to increase the distance between \( P \) and \( P' \) there are two possibilities: Increasing \( D \) or applying the principle of image transfer by a cascaded stack. In 2000, Pendry pointed out that these slabs make perfect lenses or superlenses, since both propagating and evanescent waves contribute to the resolution of the image. Because Pendry also demonstrated, by means of simulations, that such a lens with subwavelength resolution and operating at the frequency of visible light can be physically realized in the form of a thin slab of silver, a lot of research has been performed on negative-index materials and superlenses since 2000. In 2000, Notomi has shown theoretically that two-dimensional photonic crystals near the band gap frequency behave as if they have an effective refractive index. For some frequency regions this effective refractive index can be negative. As a result, in these frequency regions photonic crystals can exhibit similar phenomena as observed in negative-index materials, including negative refraction and imaging by a planar surface. Since 2003 various experiments have been performed that confirm this theoretical observation. The first theoretical demonstration of negative refraction and subwavelength imaging in three-dimensional photonic crystals was made in 2002. In 2005 such a three-dimensional photonic crystal was really produced and negative refraction and subwavelength imaging in the microwave regime was experimentally confirmed.

We have studied the possibility to observe negative refraction in triangular and square lattice photonic crystals and to focus light by means of thin slabs of these photonic crystals in great detail. We have shown that for some frequencies a photonic crystal slab consisting of a

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dielectric medium with circular holes drilled in a triangular lattice configuration behaves in many respects as a slab made from a homogeneous effective medium with an effective refractive index of approximately -1 and thus operates as a negative-index lens. Since the refractive index slightly deviates from -1 for different angles of incidence, the image of a point source, formed behind the photonic crystal slab, is not circular but elongated in the direction of propagation. Hence, as opposed to the image formation with a flat negative index material lens with a refractive index equal to -1, the image formation is not perfect. The spatial resolution in the direction of propagation is larger than the wavelength $\lambda$ of the electromagnetic wave, but the spatial resolution of the image in the direction perpendicular to the direction of propagation is smaller than $\lambda/2$. Hence, only in the direction perpendicular to the direction of electromagnetic wave propagation, a subwavelength resolution of the image can be achieved. Another important difference between the flat photonic crystal lens and the flat negative-index lens with a refractive index of -1 is that the photonic crystal lens causes an enormous loss in light intensity. Most of the light is reflected at the photonic crystal lens surface, while for the negative-index lens no reflection occurs. Despite of this, we have shown that image transfer by a cascaded stack consisting of two and three triangular lattice photonic crystal slabs separated by air layers, is feasible. Hence, this provides a method to increase the source-image distance. Namely, taking one photonic crystal slab and increasing its thickness is no option for this purpose, since it leads to worsening spatial resolutions of the image that is formed behind the slab. Although triangular lattice photonic crystals can be created that can be used to construct a flat lens acting in many respects as a flat negative-index lens, this is only for frequency ranges in the upper photonic bands. It has been suggested that for square lattice photonic crystals the same phenomena can be observed for frequencies in the lowest band, but we have shown that these square lattice photonic crystals cannot be used for the purpose of lensing.

In the last chapter of this thesis, we focused on another interesting optical property of photonic crystals, namely their ability to control the spontaneous emission of defects inside the photonic crystal. We discovered a simple, efficient procedure to compute the spontaneous emission rate from short-time FDTD data of the electromagnetic field energy. By using the unconditionally stable method to compute the emitted energy, we are guaranteed that, in the absence of external currents, the algorithm conserves the energy exactly. This ensures that the time-dependence of the emitted energy is due to the presence of the source only. We applied this procedure to two-dimensional photonic crystals, but the method applies to microcavities of arbitrary geometry. For comparison, we also calculated the local radiative density of states employing the unconditionally stable FDTD method that is without solving the eigenvalue problem and integrating over the (first) Brillouin zone. We demonstrated that both methods yield the same predictions about the enhancement or suppression of the spontaneous emission rate by photonic crystals. The spontaneous emission rate strongly depends on the location of the source in the photonic crystal and on the frequency of the emitted light. Emission en-
hancement as well as emission suppression can be observed. Even if the photonic crystal has no photonic band gap, the emission of a point source can be almost completely suppressed.