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Scherpen, J.M.A.; Schaft, A.J. van der

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Balancing of the normalized right coprime factorization of a nonlinear system

J.M.A. Scherpen* and A.J. van der Schaft
Department of Applied Mathematics, University of Twente
P.O. Box 217, 7500 AE Enschede, The Netherlands
e-mail: scherpen@math.utwente.nl

For model reduction of linear systems we often use the balanced realization of the system. Some of the ideas to balance linear systems turned out to be useful for a set up of balancing nonlinear systems. In [4] we have set up such a method to balance a stable nonlinear system. Like for linear systems we considered the minimum amount of control energy required to reach a state and the amount of output energy generated by this state. We can define the controllability and observability function to be respectively the minimum amount of control energy required to reach a state, and the amount of output energy that is generated by this state. We can find a state space transformation that brings the controllability function and observability function in a special form, which gives rise to the definition of the singular value functions. In the linear case these are the squared Hankel singular values. If the nonlinear stable system is in balanced form, we can decide whether a state component is important or not for the reduced order system by means of this control and output energy and therefore use balancing of the nonlinear system as a tool for model reduction, see [4].

This talk concerns balancing of unstable nonlinear systems and, like above, we make use of the ideas from the theory for linear systems. Balancing for unstable linear systems can be found in e.g. [1], [2] and [3]. In [2], the normalized right coprime factorization of the unstable linear system is used to obtain the (stable) state space realization of the corresponding graph operator. Then the order of the original unstable system is reduced based on the balanced realization of this graph operator. The Hankel singular values of this realization are called the Graph Hankel singular values of the original system. Similarly, balancing of the normalized left coprime representation can be done, see [3]. In [1] LQG balancing is treated and in [3] it is shown that for the original systems all of these methods result in the same reduced order system. In this talk we go into an extension of the results of [2] to nonlinear systems. For the normalized left coprime representation of a nonlinear system, for the nonlinear version of LQG balancing and the relation between this way

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of balancing and balancing of the normalized right coprime factorization of a nonlinear system we refer to [6].

Hence, balancing of an unstable nonlinear system can be done by considering the normalized right coprime factorization of the system. In order to do so, we first introduce the concept of an inner nonlinear system by considering the Hamiltonian extension of a system. We get some conditions in terms of the observability function, the input vectorfield and the output vectorfield of the system, which are equivalent with the nonlinear system being inner, and which are well known conditions in the linear case. The right factorization of an unstable nonlinear system is defined by the state space representation of two stable nonlinear systems where a serial connection of the first system and the inverse of the second system gives exactly the original unstable nonlinear system. Moreover, the factorization is called a normalized factorization if the combination of both systems is an inner system, and is called coprime if the zero-dynamics of the combination of both systems is trivial. Such a factorization can be obtained if the original system is zero-state observable and if a certain Hamilton-Jacobi-Bellman equation, known from optimal control theory, has a smooth positive definite solution. Furthermore, we can give some properties of the normalized right coprime representation, the corresponding observability function and the corresponding controllability function. The singular value functions of this representation are called the Graph singular value functions of the original system. Finally, we deal with the linearization.

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