Inflation and de Sitter Landscape

In this chapter, we discuss the possibility to construct a consistent and unified framework for inflation, dark energy and supersymmetry breaking. This approach is motivated by the idea that a vast landscape of string vacua may provide a possible explanation for the value of the current acceleration in our Universe. We employ an effective supergravity description and investigate the restrictions and main properties coming from the interplay between the inflationary and the supersymmetry breaking sectors. Specifically, we show that the physics of a single-superfield scenario is highly constrained due to a specific no-go theorem regarding the uplifting of a SUSY Minkowski vacuum. On the other hand, the addition of a nilpotent sector yields remarkable simplifications and allows for controllable level of dark energy and supersymmetry breaking. We study this powerful framework both in the context of flat Kähler geometry and in the case of $\alpha$-attractors. Interestingly, in the latter case, we prove that the attractor nature of the theory is enhanced when combining the inflationary sector with the field responsible for uplifting: cosmological attractors are very stable with respect to any possible value of the cosmological constant and, remarkably, to any generic coupling of the two sectors. The novel results of this Chapter are based on the publications [vi], [vii] and [ix].
6.1 Introduction and outline

Observational evidence [13, 14, 206–208] seems to point at acceleration as a fundamental ingredient of our Universe. Primordial inflation is the leading paradigm to account for the origin of the anisotropies in the CMB radiation and, then, the formation of large scale structures (as we reviewed in Ch. 2 and Ch. 3 of this thesis). These are currently observed to experience a mysterious accelerating phase, whose source has been generically called dark energy. Although the origin of both early- and late-time acceleration still represents a great theoretical puzzle, the simple assumption that the potential energy of a scalar field may serve as fundamental source has turned out to be successful in terms of investigation, extraction of predictions and agreement with the present observational data (see Ch. 3). In the simplest scenario, a scalar field slowly rolls down along its potential, driving inflation, and eventually sits in a minimum with a small positive cosmological constant of the order $\Lambda \sim 10^{-120}$, as displayed in Fig. (6.1).

![Cartoon picture of the simplest possible scenario where a single scalar field is responsible both for inflation and current acceleration of the Universe. The amount of dark energy can be controlled, following the string landscape scenario.](image)

The embedding into high-energy physics frameworks, such as supergravity or string theory, seems to be natural. On the one hand, the high energy-scale of inflation would require UV-physics control (see Ch. 5 for supergravity embeddings of the inflationary paradigm). On the other hand, the anthropic argument in a landscape of many string vacua [108, 209–213] would provide a possible explanation of the smallness of the current cosmological constant.

In an effective unified framework for inflation and dark energy, the con-
crete implementation of the idea of a de Sitter landscape would provide an enormous number of possibilities for the minimum of the scalar potential where the field eventually sits after driving inflation. Quantum corrections or interactions with other particles may certainly lead to some additional contributions to the value of the potential at the minimum. However, this should not affect the existence of a landscape of dS vacua and any possible correction to the cosmological constant (CC) would be easily faced, within a scenario with controllable level of dark energy. Therefore, we aim to construct a supergravity framework suitable for inflation with exit into de Sitter space with all possible values of the cosmological constant (see Fig. (6.1)).

Our starting point will be the models of inflation discussed in Ch. 5. A common property of these scenarios is that supersymmetry is restored at the minimum $V = 0$ after inflation ends. Then, uplifting the SUSY Minkowski vacuum seems to be the next natural step in order to consider the current acceleration. However, it has been pointed out that obtaining a de Sitter vacuum from a SUSY one is subject to a number of restrictions encoded in a recent no-go theorem [200] which make a unified picture of inflation and dark energy very challenging to achieve, especially when using just one chiral superfield [160]. Specifically, this generically yields a large Gravitino mass which is undesirable from a phenomenological point of view. We discuss the case of one single superfield in detail in Sec. 6.2, in the context of the model proposed by Ketov and Terada in [158,159] (we have already reviewed this framework in the previous chapter).

A way to overcome the issue of uplifting a SUSY Minkowski minimum and still having controllable level of SUSY breaking is to employ a nilpotent superfield $S$ [138,214–219] (we review the important properties of this construction in Sec. 6.3.1). In fact, the nilpotent field seems to be naturally related to de Sitter vacua when coupled to supergravity [220–225] (see [226] for an interesting review on this topic) and it has been used in order to construct inflationary models with de Sitter exit and controllable level of SUSY breaking at the minimum [198,202,203,227–230]. The two sectors appearing in these constructions have independent roles: the $\Phi$-sector contains the scalar which evolves and dynamically determines inflation and dark energy while the field $S$ is responsible for the landscape of vacua. However, in general, the inflationary regime is really sensitive to the coupling between the two sectors and to the value of the uplifting. One needs to make specific choices for the superpotential. We show the details and the limitation of this construction in Sec. 6.3.

Finally, in Sec. (6.4), we present special stability of $\alpha$-attractors when combined with a nilpotent sector. We prove that their inflationary predictions
are extremely stable with respect to any possible value of the cosmological constant and to any generic coupling between $\Phi$ and $S$, exhibiting attractor structure also in the uplifting sector. These scenarios simply emerges as the most generic expansion of the superpotential.

### 6.2 Single superfield inflation and dark energy

In this Section, we intend to investigate the consequences of uplifting a SUSY Minkoswki vacuum in a supergravity framework consisting of just one superfield. Specifically, we consider the class of inflationary theory proposed by Ketov and Terada (KT) [158, 159]. Following [159], one may consider a logarithmic Kähler potential of the form

$$K = -3\ln \left[ 1 + \frac{\Phi + \bar{\Phi} + \zeta (\Phi + \bar{\Phi})^4}{\sqrt{3}} \right]. \quad (6.1)$$

Notice that, within this model, the inflaton field is played by the Im$\Phi = \chi$, unlike the other supergravity constructions considered in Ch. 5. The quartic term in the argument of the logarithm is introduced in order to stabilize the field $\chi$ during inflation at $\phi \approx 0$.

As already explained in Sec. 5.1.3, this supergravity scenario allows to produce an almost arbitrary inflaton potential when $\phi \ll 1$. After inflation, the field rolls down towards a Minkowski minimum placed at $\Phi = 0$ where supersymmetry is unbroken.

This situation is valid for a large variety of superpotentials $W(\Phi)$, but not for all of them. In particular, we will show that one can have a consistent inflationary scenario in the theory with the simplest superpotential $W = c\Phi + d$, but both fields $\phi$ and $\chi$ evolve and play an important role. At the end of inflation, the field may roll to a Minkowski vacuum with $V = 0$ or to a dS vacuum with a tiny cosmological constant $\Lambda \sim 10^{-120}$. This is an encouraging result, since a complete cosmological model must include both the stage of inflation and the present stage of acceleration of the universe, and our simple model with a linear potential successfully achieves it. However, this success comes at a price: in this model, supersymmetry after inflation is strongly broken and the gravitino mass is $2 \times 10^{13}$ GeV, which is much greater than the often assumed TeV mass range.

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1We already presented this Kähler potential in the context of sGoldstino inflation in Ch. 5. We explicitly show this again for a matter of convenience.
In view of this result, one may wonder what will happen if one adds a tiny correction term $c\Phi + d$ to the benchmark superpotentials of the inflationary models described in [159] with supersymmetric Minkowski vacua. Naively, one could expect that, by a proper choice of small complex numbers $c$ and $d$, one can easily interpolate between the AdS, Minkowski and dS minima. In particular, one could think that for small enough values of these parameters, one can conveniently fine-tune the value of the vacuum energy, uplifting the original supersymmetric minimum to the desirable dS vacuum energy with $\Lambda \sim 10^{-120}$.

However, the actual situation is very different. We will show that adding a small term $c\Phi + d$ always shifts the original Minkowski minimum down to AdS, which does not correctly describe our world. Moreover, unless the parameters $c$ and $d$ are exponentially small, the negative cosmological constant in the AdS minimum leads to a rapid collapse of the universe. For example, adding a tiny constant $d \sim 10^{-54}$ leads to a collapse within a time scale much shorter than its present age. Thus, the cosmological predictions of the models of [159] with one chiral superfield and a supersymmetric Minkowski vacuum are incredibly unstable with respect to even very tiny changes of the superpotential. Of course one could forbid such terms as $c\Phi + d$ by some symmetry requirements, but this would not address the necessity to uplift the Minkowski vacuum to $\Lambda \sim 10^{-120}$.

While we will illustrate this surprising result using KT models as an example, the final conclusion is very general and valid for a much broader class of theories with a supersymmetric Minkowski vacuum; see a discussion of a related issue in [201]. We will show that this result is a consequence of the no-go theorem of [200] (see also [149, 177]), which is valid for arbitrary Kähler potentials and superpotentials and also applies in the presence of multiple superfields:

*One cannot deform a stable supersymmetric Minkowski vacuum with a positive definite mass matrix to a non-supersymmetric de Sitter vacuum by an infinitesimal change of the Kähler potential and superpotential.*

This no-go theorem can be understood from the role of the sGoldstino field, the scalar superpartner of the would-be Goldstino spin-1/2 field (as also emphasized in [151, 157, 197]). Since the mass of the sGoldstino is set by the order parameter of supersymmetry breaking, it must vanish in the limit where supersymmetry is restored. The only SUSY Minkowski vacua that are continuously connected to a branch of non-supersymmetric extrema therefore necessarily have a flat direction to start with: this is the scalar field that will play the role of the sGoldstino after spontaneous SUSY breaking. A corollary
of this theorem is that one cannot obtain a dS vacuum from a stable SUSY Minkowski vacuum by a small deformation. As we will see, this is exactly what forbids a small positive CC after an infinitesimal change of the KT starting point.

As often happens, the no-go theorem does not mean that uplifting of the supersymmetric Minkowski minimum to a dS minimum is impossible. In order to achieve that, the modification of the superpotential should be substantial. We will show how one can do it, thus giving a detailed illustration of how this no-go theorem works and how one can overcome its conclusions by changing the parameters of the correction term $c\Phi + d$ beyond certain critical values. For example, one can take $d = 0$ and slowly increase $c$. For small values of $c$, the absolute minimum of the potential corresponds to a supersymmetric AdS vacuum. When the parameter $c$ reaches a certain critical value, the minimum of the potential ceases to be supersymmetric, but it is still AdS. With a further increase of $c$, the minimum is uplifted and becomes a non-supersymmetric dS vacuum state. Once again, we will find that the modification of the superpotential required for the tiny uplifting of the vacuum energy by $\Lambda \sim 10^{-120}$ leads to a strong supersymmetry breaking, with the gravitino mass many orders of magnitude greater than what is usually expected in supergravity phenomenology.

This problem can be solved by introducing additional chiral superfields responsible for uplifting and supersymmetry breaking. However, this may require an investigation of inflationary evolution of multiple scalar fields, unless the additional fields are strongly stabilized [231] or belong to nilpotent chiral multiplets [161,201–203,228].

### 6.2.1 Inflation and uplifting with a linear superpotential

To understand the basic features of the theories with the Kähler potential (6.1), it is instructive to calculate the coefficient $G(\phi, \chi)$ in front of the kinetic term of the field $\Phi$. For an arbitrary choice of the superpotential, this coefficient is given by

$$G(\phi, \chi) = \frac{3(1 + 32\zeta^2\phi^6 - 8\zeta\phi^2(3\sqrt{3} + \sqrt{2}\phi))(\sqrt{3} + \sqrt{2}\phi + 4\zeta\phi^4)^2}{(\sqrt{3} + \sqrt{2}\phi + 4\zeta\phi^4)^2}. \quad (6.2)$$

This function does not depend on $\chi$. For small $\phi$ the fields are canonically normalized. $G(\phi, \chi)$ is positive at small $\phi$, while it vanishes and becomes negative for larger values of $|\phi|$ (provided $\zeta > 0$). Thus the kinetic term is positive definite only in a certain range of its values, depending on the constant $\zeta$. In this Section, we will usually take $\zeta = 1$, to simplify the comparison with [159], see Fig. 6.2.
6.2 Single superfield inflation and dark energy

![Graph showing the coefficient in front of the kinetic term for the field Φ as a function of ϕ for ζ = 1.](image)

*Figure 6.2*

The coefficient in front of the kinetic term for the field Φ as a function of ϕ for ζ = 1.

It is equally important that the expression for the potential $V$ in this theory, for any superpotential, contains the coefficient $1 + 32ζ^2 ϕ^6 - 8ζ^2 (3\sqrt{3} + \sqrt{2})$ in the denominator, so it becomes infinitely large exactly at the boundaries of the moduli space where the kinetic term vanishes (for $ζ = 1$, the boundaries are located at $ϕ ≈ ±0.15$). For large $ζ$, the domain where $G$ is positive definite becomes more and more narrow, which is why the field $ϕ$ becomes confined in a narrow interval, whereas the field $χ$ is free to move and play the role of the inflaton field. This is very similar to the mechanism of realization of chaotic inflation proposed earlier in a different context in Section 4 of [232].

We will study inflation in this class of theories by giving some examples, starting from the simplest ones. The simplest superpotential to consider is a constant one, $W = m$. In this case, the potential does not depend on the field $χ$. It blows up, as it should, at sufficiently large $ϕ$, and it vanishes at $ϕ = 0$, see Fig. 6.3. This potential does not describe inflation.

As a next step, we will consider a superpotential with a linear term

$$W = m \left(cΦ + 1\right).$$  \hspace{1cm} (6.3)

In this case, just as in the case considered above, the potential has an exactly flat direction at $ϕ = 0$, but now the potential at $ϕ = 0$ is equal to

$$V(ϕ = 0, χ) = m^2 c (c - 2\sqrt{3}).$$  \hspace{1cm} (6.4)

Thus for $c < 2\sqrt{3}$ it is an AdS valley, but for $c > 2\sqrt{3}$ it is a dS valley. But this does not tell us the whole story. At large $χ$, the minimum of the potential in the $ϕ$ direction is approximately at $ϕ = 0$, but at smaller $χ$, the
minimum shifts towards positive $\phi$. For $c \approx 3.671$, the potential has a global non-SUSY Minkowski minimum with $V = 0$ at $\chi = 0$ and $\phi \approx 0.06$. By a minuscule change of $c$ one can easily adjust the potential to have the desirable value $\Lambda \approx 10^{-120}$ at the minimum. This requires fine-tuning, but it should not be a major problem in the string landscape scenario. The full potential is shown in Fig. 6.4. In general, one would expect higher-order corrections which might slightly perturb the potential; however, we focus on the effect of the lower-order terms.

Inflation in this model happens when the field slowly moves along the nearly flat valley and then rolls down towards the minimum of the potential. It is a two-field dynamics, which cannot be properly studied by assuming that $\phi = 0$ during the process, as proposed in [158, 159]. Indeed, the potential along the direction $\phi = 0$ is exactly constant, so the field would not even move if we assumed that during its motion. However, because of the large curvature of the potential in the $\phi$ direction, during inflation this field rapidly reaches an inflationary attractor trajectory and then adiabatically follows the position of the minimum of the potential $V(\phi, \chi)$ for any given value of the field $\chi(t)$. This can be confirmed by a numerical investigation of the combined evolution of the two fields whose dynamics is shown in Fig. 6.5.

Then, the adiabatic approximation of the effective potential driving in-

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$^{2}$An understanding of this value of $c$ and its role in terms of (non-)supersymmetric branches is given in appendix A of [160].
6.2 Single superfield inflation and dark energy

The scalar potential in the theory with $W = m (c\Phi + 1)$, for $\zeta = 1$. For $c \approx 3.671$, it has a $dS$ valley, and a near-Minkowski minimum at $\chi = 0$, $\phi \approx 0.06$. Inflation happens when the field slowly moves along the nearly flat valley and then rolls down towards the minimum of the potential. It is a two-field inflation, which cannot be properly studied by assuming that $\phi = 0$ during the process.

The inflation reads

$$V(\phi(\chi), \chi) = m^2 c (c - 2\sqrt{3}) - \frac{2m^2(c - \sqrt{3})^2}{27\sqrt{3}\chi^2},$$

(6.5)

neglecting higher order terms which play no role in the inflationary plateau. The effective fall-off of $1/\chi^2$ is responsible for determining the main properties of a fully acceptable inflationary scenario.

This investigation shows that this simplest model leads to a desirable amplitude of inflationary perturbations for $m \sim 7.75 \times 10^{-6}$, in Planck units. The inflationary parameters $n_s$ and $r$ in this model are given by (at leading order in $1/N$)

$$n_s = 1 - \frac{3}{2N}, \quad r = \frac{2(c - \sqrt{3})}{\sqrt{26c(\sqrt{3}c - 6)} N^{3/2}}.$$  

(6.6)

Numerically, we find $n_s \approx 0.975$ and $r \approx 0.0014$ for $N = 60$, in excellent agreement with the leading $1/N$ approximation. We checked that the values of $n_s$ remains approximately the same in a broad range of $\zeta$, from $\zeta = 0.1$ to $\zeta = 10$. The value of the parameter $r$ slightly changes but remains in the $10^{-3}$ range. As of now, all of these outcomes are in good agreement with the data provided by Planck.
The dynamical evolution of the inflaton field (blue line) in the model with $W = m(c\Phi + 1)$, for $\zeta = 1$. The adiabatic approximation of the effective potential (dashed red line) and the contour plot of $V(\phi, \chi)$ in logarithmic scale are shown as superimposed.

There is an initial stage of oscillations before the field approaches the inflationary attractor, as well as the final stage of post-inflationary oscillations. However, during inflation, which happens between these two oscillatory stages, the field accurately follows the position of the adiabatically changing minimum of the potential $V(\phi(\chi), \chi)$.

However, this simplest inflationary model has a property which is shared by all other models of this class to be discussed in this Section: supersymmetry is strongly broken in the minimum of the potential. In particular, for $\zeta = 1$, the superpotential at the minimum is given by $W \approx 9 \times 10^{-6}$, and the gravitino mass is $m_{3/2} \sim 8.34 \times 10^{-6}$, in Planck units, i.e. $m_{3/2} \sim 2 \times 10^{13}$ GeV. This is many orders of magnitude higher than the gravitino mass postulated in many phenomenological models based on supergravity.

Of course, supersymmetry may indeed be broken at a very high scale, but nevertheless this observation is somewhat worrisome. One could expect that this is a consequence of the simplicity of the model that we decided to study, but we will see that this result is quite generic.

### 6.2.2 Inflation and uplifting with a quadratic superpotential

As a second example, we will discuss the next simplest model, defined by

$$W = \frac{1}{2}m\Phi^2. \quad (6.7)$$

This case was one of the focuses of [159] and gives rise to a quadratic inflationary potential. As we will demonstrate, perturbing such a superpotential
by means of a linear and constant term, leads to general properties which are
shared by the class discussed in the previous section.

We will start by perturbing this model via a constant term such as

\[ W = m \left( \frac{1}{2} \Phi^2 + d \right). \] (6.8)

The inflationary regime is unaffected by such correction and the scalar potential still reads \( V = \frac{1}{2} m^2 \chi^2 \), at \( \phi = 0 \). However, the vacuum of \( V(\phi, \chi) \) will move away from the supersymmetric Minkowski minimum, originally placed at \( \Phi = 0 \), but just in the \( \phi \)-direction (because the superpotential is symmetric). Then, for small parameter values, the minimum of \( \phi \) moves as

\[ \phi_0 = \sqrt{6d} - \sqrt{\frac{3}{2}} d^2. \] (6.9)

This shift immediately leads to an AdS phase which, at small values of \( d \), goes as

\[ \Lambda = -\sqrt{3} m^2 d^2, \] (6.10)

which is fully in line with the no-go theorem [200] summarized in the Introduction. These solutions do not break supersymmetry and they can be obtained by the equation \( D_\Phi W = 0 \). As \( |d| \) increases, such a SUSY vacuum moves further away from the origin and, at one point, it crosses the singular boundary of the moduli space. Then, if we search for numerical solutions within the strip corresponding to the correct sign of the kinetic terms (this means for \( |\phi| \lesssim 0.15 \)), we run into a feature which will be common also in other examples: for specific values of \( d \), the SUSY-branch of vacuum solutions leaves the fundamental physical domain \( |\phi| \lesssim 0.15 \) and it is replaced by a new branch of vacua with broken supersymmetry. This is shown in Fig. 6.6. However, as one keeps increasing the absolute value of \( d \), \( \phi_0 \) approaches a constant value which corresponds to an asymptotic AdS phase. Therefore, perturbing \( W \) by means of a constant term does not help to uplift to dS.

As second step, we include a linear correction such that the superpotential reads

\[ W = m \left( \frac{1}{2} \Phi^2 + c\Phi \right), \] (6.11)

where the coefficients are real due to the constraint on\(^3 W\).

Similarly to the previous case, the SUSY Minkowski vacuum is perturbed by such correction and, at lowest order in \( c \), it moves in the \( \phi \)-direction as

\[ \phi_0 = -\sqrt{2} c - \sqrt{\frac{3}{2}} c^2, \] (6.12)

\(^3\)Perturbing the superpotential by means of a linear term with imaginary coefficient such as \( ic\Phi \) is equivalent to adding a positive constant \( c^2 \). This is a direct consequence of the shift symmetry of the Kähler potential.
The value of the cosmological constant (left panel) in the minimum and its location $\phi_0$ (right panel) as a function of the constant term $d$ in the superpotential (6.8). The two branches of solutions (SUSY and non-SUSY), within the fundamental physical domain $|\phi| \lesssim 0.15$, are shown in different colors. At larger (positive or negative) values of the constant, both the CC and the location $\phi_0$ level off to a constant. Plots obtained for $m = \zeta = 1$.

leading to a vacuum energy given by

$$\Lambda = -\sqrt{\frac{3}{4}m^2 c^4}, \quad (6.13)$$

Then also in this case, as $|c|$ increases, such supersymmetric solutions move towards the boundary $\phi \approx \pm 0.15$ and cross it. At the same point in parameter space, a new branch of non-supersymmetric solutions appears and, remarkably, this results into a sharp increase of the scalar potential at the minimum. In fact, this very quickly gives rise to a transition from AdS to dS, as it is shown in Fig. 6.7.

The exact values for which these transitions happen are as follows. The transition from SUSY to non-SUSY vacua occurs at (calculated for $m = \zeta = 1$)

$$c = -0.118162, \quad c = 0.101918, \quad (6.14)$$

while the CC crosses through Minkowski at

$$c = -0.119318, \quad c = 0.102692. \quad (6.15)$$

Note that, at finite $c$ values, the scalar potential passes through Minkowski. In contrast to the ground state at $c = 0$, the new Minkowski vacua are non-supersymmetric, and hence can be deformed into dS without violating the no-go theorem. In fact, these non-supersymmetric Minkowski vacua are exactly the type of structures that were identified in [200] as promising starting
points for uplifts to De Sitter (although there the focus was on a hierarchy of supersymmetry breaking order parameters for different superfields). A minuscule deviation of \( c \) from (6.15) will be sufficient to obtain the physical value of cosmological constant \( \Lambda \approx 10^{-120} \).

![Figure 6.7](image-url)

The value of the cosmological constant (left panel) in the minimum and its location \( \phi_0 \) (right panel) as a function of the linear term in the superpotential. The two branches of solutions (SUSY and non-SUSY), within the fundamental physical domain \( |\phi| \lesssim 0.15 \), are shown in different colors. At larger (positive or negative) values of the coefficient \( c \), the location \( \phi_0 \) levels off to a constant while the CC approaches a quadratic shape. Plots obtained for \( m = \zeta = 1 \).

It is worthwhile to remark that the order of magnitude of the parameter \( c \), for which we get a tiny uplifting to dS, is small with respect to the coefficient of the quadratic term in the superpotential (6.11). This translates into the fact that the inflationary predictions will be basically unchanged with respect the simple scenario with a quadratic potential. In fact, the scalar potential in the direction \( \phi = 0 \) reads

\[
V(\phi = 0, \chi) = \frac{1}{2} \left(1 - \sqrt{3}c\right) m^2 \chi^2 + m^2 c^2.
\]  

(6.16)

At \( \chi \lesssim O(1) \), the field \( \phi \) no longer vanishes and starts moving towards the minimum of the potential. However, the main stage of inflation happens at \( \chi \gg c = O(0.1) \), when \( \phi \) nearly vanishes and the inflaton potential is approximately equal to \( \frac{1}{2} \left(1 - \sqrt{3}c\right) m^2 \chi^2 \). The main effect of this change of the potential is a slight change of normalization of the amplitude of the perturbations spectrum, which requires a small adjustment for the choice of the parameter \( m \):

\[
m \approx (6 + 5.2c) \cdot 10^{-6}.
\]

(6.17)

However, even though the inflationary regime is essentially unaffected by such a small correction, supersymmetry is strongly broken at the end of
inflation, just as in the theory with a simple linear superpotential, discussed in Sec. 6.2.1. This is a direct consequence of the no-go theorem discussed above and of the impossibility of uplifting the SUSY Minkowski vacuum (corresponding to \( c = 0 \)) by an infinitesimal deformation of \( W \). In particular, for values of \( c \) leading to a realistic dS phase (these values are extremely close to (6.15), corresponding to non-supersymmetric Minkowski) and for \( \zeta = 1 \), we obtain the following: for positive \( c \), the superpotential at the minimum is \(|W| \approx 3.4 \times 10^{-8} \) and the gravitino mass is \( m_{3/2} \sim 4.2 \times 10^{-8} \), in Planck units, i.e. \( m_{3/2} \sim 1.0 \times 10^{11} \) GeV; for negative \( c \), the superpotential at the minimum is \(|W| \approx 3.8 \times 10^{-8} \) and the gravitino mass is \( m_{3/2} \sim 3.2 \times 10^{-8} \), in Planck units, i.e. \( m_{3/2} \sim 7.6 \times 10^{10} \) GeV. These values are again well beyond the usual predictions of the low scale of supersymmetry breaking in supergravity phenomenology.

### 6.2.3 Discussion

In this Section we have investigated the possibility to realize a model of inflation and dark energy in supergravity. As an example, we considered the class of single chiral superfield models proposed in [159]. The models described in [159] share the following feature: The vacuum energy in these models vanishes, and supersymmetry is unbroken. One could expect that this is a wonderful first approximation to describe dS vacua with vanishingly small vacuum energy \( \Lambda \sim 10^{-120} \) and small supersymmetry breaking with \( m_{3/2} \sim 10^{-15} \) or \( 10^{-13} \) in Planck units. However, we have shown that this is not the case, because of the no-go theorem formulated in [200]. While it is possible to realize an inflationary scenario that ends in a dS vacuum with \( \Lambda \sim 10^{-120} \), these vacua cannot be infinitesimally uplifted by making small changes in the Kähler potential and superpotential. One can uplift a stable Minkowski with unbroken SUSY to a dS minimum, but it always requires large uplifting terms, resulting in a strong supersymmetry breaking with \( m_{3/2} \) many orders of magnitude higher than the TeV or even PeV range advocated by many supergravity phenomenologists.

In our investigation, we also introduced a new model, which contained only linear and constant terms in the superpotential. This superpotential is simpler than those studied in [159], but we have found that this model does describe a consistent inflationary theory with dS vacuum, which can have \( \Lambda \sim 10^{-120} \). However, just as in all other cases considered in this Section, we found that supersymmetry is strongly broken after inflation in this model. While we have analyzed only some specific cases in detail, our conclusions apply to a much wider class of models, well beyond the specific
models proposed in [159], because of the general nature of the no-go theorem of [200].

Since there is no evidence of low scale supersymmetry at LHC as yet, one could argue that the large scale of supersymmetry breaking is not necessarily a real problem. However, it would be nice to have more flexibility in the model building, which would avoid this issue altogether. One way to get dS uplifting with small supersymmetry breaking, without violating the no-go theorem, is to add other chiral multiplets (e.g. Polonyi fields), and to strongly stabilize them to minimize their influence on the cosmological evolution, see e.g. [231]. In certain cases, one can make the Polonyi fields so heavy and strongly stabilized that they do not change much during the cosmological evolution and do not lead to the infamous Polonyi field problem which bothered cosmologists for more than 30 years [233–237]. A more radical approach, which allows to have a single scalar field evolution is to use models involving nilpotent chiral superfields [161,201–203,228], which have an interesting string theory interpretation in terms of D-branes [230]. This framework will be investigated in the next two sections.

6.3 Arbitrary inflation and de Sitter landscape

In this Section, we intend to present how the addition of a nilpotent sector allows us to evade the restrictions presented above in Sec. 6.2 and yield remarkable simplifications, within a unified cosmological scenario of inflation and dark energy. After reviewing the main properties of the nilpotent superfield $S$, we show how to construct a general class of inflationary models with de Sitter exit and controllable level of SUSY breaking at the minimum. The Kähler geometry of these scenarios is flat thus allowing for arbitrary inflaton potential, along the line of the general model presented in Sec. 5.2. Finally, we comment on the relation between the supersymmetry breaking directions and the fermionic sector of the supergravity action.

6.3.1 The nilpotent superfield

In the 1970s Volkov and Akulov (VA) [138,214] proposed to identify the neutrino with the massless Goldstino arising from supersymmetry breaking. They derived the corresponding action which is invariant under non-linear supersymmetry transformations (see the recent investigations [238–240]). However, this idea was soon abandoned after the discovery of neutrino oscillations.

Later in [215–219], it was shown that VA Goldstino can be expressed in the form a constrained superfield (see also the recent works [241,242]).
Specifically, it can be represented by a chiral multiplet $S$ with the nilpotency condition $S^2 = 0$. We detail this below.

The unconstrained off-shell chiral superfield has the form

$$S(x, \theta) = s(x) + \sqrt{2} \theta \chi^s(x) + \theta^2 F^S(x),$$

where $s(x)$ is the scalar part, $\chi^s(x)$ is a fermion partner and $F^S(x)$ is an auxiliary field. It was shown in [219] that the nilpotent superfield $S^2(x, \theta) = 0$ depends only on the fermion $\chi^s$, the VA goldstino, and an auxiliary field $F^S$. It does not have a fundamental scalar field, that is

$$S(x, \theta)|_{S^2(x, \theta)=0} = \frac{\chi^s \chi^s}{2 F^S} + \sqrt{2} \theta \chi^s + \theta^2 F^S,$$

since $s(x)$ is replaced by $\frac{\chi^s \chi^s}{2 F^S}$. For the nilpotent off-shell superfields the rules for the bosonic action required for cosmology turned out to be very simple. Namely, one has to calculate potentials as functions of all superfields as usual, and then declare that the scalar part of the nilpotent superfield $s(x)$ vanishes, since it is replaced by a bilinear combination of the fermions. No need to stabilize and study the evolution of the complex field $s(x)$.

### 6.3.2 Arbitrary inflation, dark energy and SUSY breaking

Now we turn to the unified cosmological scenario, presented in [203], which allows to obtain general inflaton potential and controllable level of dark energy and SUSY breaking.

The Kähler potential and superpotential are of the form

$$K = -\frac{1}{2} \left( \Phi - \bar{\Phi} \right)^2 + S \bar{S}, \quad W = f(\Phi) + g(\Phi)S,$$

where $f$ and $g$ are real holomorphic functions of their arguments and $W$ has the the most general form, provided $S$ is nilpotent. Indeed, due to the nilpotency of $S$ and holomorphicity of the superpotential, $W(\Phi, S)$ in Eq. (6.20) is the most general form of the superpotential depending on $\Phi$ and $S$. This is analogous to the fact that an arbitrary function of a single Grassmann variable $\theta$ can be expanded into a Taylor series which terminates after 2 terms, $F(\theta) = a + b\theta$, since $\theta^2 = \theta^3 = \ldots = \theta^n \ldots = 0$. In our case we have $S^2 = S^3 = \ldots = S^m \ldots = 0$.

Within this class of models, the real part of the field $\Phi$ plays the role of the inflaton, rolling down along $S = 0$ and $\Phi = \bar{\Phi}$, and drives a potential which reads

$$V = g(\Phi)^2 + f'(\Phi)^2 - 3f(\Phi)^2.$$
Note that the last two terms are exactly the ones appearing in (5.15), that is, for a single superfield model (see Sec. 5.1.3).

After inflation, the journey of \( \text{Re}\Phi \) ends into a minimum placed at \( \Phi = 0 \), provided the functions \( f \) and \( g \) satisfy

\[
f'(0) = g'(0) = 0 .
\]

The values of \( f \) and \( g \) at the minimum will allow for a wide spectrum of possibilities in terms of supersymmetry breaking and cosmological constant, along the lines of the string landscape scenario. Supersymmetry is spontaneously broken just in the nilpotent direction\(^4\), namely

\[
D_S W_{\text{min}} = g(0) = M , \quad D_\Phi W_{\text{min}} = 0 , \tag{6.23}
\]

where we have introduced \( M \) as SUSY breaking parameter. Further, the gravitino mass is given by \( m_{3/2} = f(0) \). The value of the cosmological constant is equal to

\[
\Lambda = g^2(0) - 3f^2(0) = M^2 - 3m_{3/2}^2 . \tag{6.24}
\]

The vacuum is stable if the masses of both directions, as given by

\[
m^2_{\text{Re}\Phi}(\Phi = 0) = f''(0)^2 + Mg''(0) - 3m_{3/2}^2 f''(0) ,
\]

\[
m^2_{\text{Im}\Phi}(\Phi = 0) = f''(0)^2 - Mg''(0) - m_{3/2}^2 f''(0) + 2(M^2 - m_{3/2}^2) , \tag{6.25}
\]

are assured to be positive.

However, the generality of Eq. (6.21) does not assure always a viable inflationary scenario. The negative term can be dominating at large value of the inflaton field and not give rise to inflation. In the framework defined by Eq. (6.20), a successful choice for the functions \( f \) and \( g \) is given by [202, 203]

\[
f(\Phi) = \beta \ g(\Phi) , \tag{6.26}
\]

with \( \beta \) being some constant. The specific relation (6.26) leads to a situation where the negative contribution in (6.21) is exactly canceled when the minimum (6.24) is Minkowski and, then, by fine-tuning \( \beta = 1/\sqrt{3} \). Then, the scalar potential turns out to have the simple form

\[
V = [f'(\Phi)]^2 . \tag{6.27}
\]

\(^4\)This allows for a simplification of the fermionic sector of the supergravity action. Specifically, in the unitary gauge, the gravitino interacts just with the fermion of the nilpotent field leading to a simple version of the super-Higgs mechanism [202, 203].
Allowing for a small cosmological constant $\Lambda \sim 10^{-120}$ (then, having a tiny deviation of $\beta$ from $1/\sqrt{3}$) does not change effectively the inflationary predictions. Other possible choices for $f$ and $g$ are discussed in [201, 203].

This construction is quite flexible in terms of observational predictions allowing for any possible value of $n_s$ and $r$. Nonetheless, the generality of such construction relies on the relation (6.26) and turns out to be really sensitive with respect to any other generic coupling between the inflaton and the nilpotent sector. Moreover, the negative contribution of Eq. (6.21) is balanced just if one assumes the observational evidence of a negligible cosmological constant. A generic de Sitter landscape would yield important corrections to such construction.

6.3.3 Fermionic sector after the exit from inflation

Now we will describe the fermionic sector of the theory. The generic mixing term of the gravitino with the goldstino $v$ can be expressed as a combination of fermions from chiral multiplets $\chi^i$ such as

$$\bar{\psi}^\mu \gamma_\mu v + h.c. = \bar{\psi}^\mu \gamma_\mu \sum_i \chi^i e^{\frac{K}{2}} D_i W + h.c. \quad (6.28)$$

In case of our two multiplets, we have that the inflatino $\chi^\phi$ as well as the $S$-multiplet fermion $\chi^s$ form a goldstino $v$, which is mixed with the gravitino as

$$\bar{\psi}^\mu \gamma_\mu v = \bar{\psi}^\mu \gamma_\mu \left( \chi^\phi e^{\frac{K}{2}} D_\phi W + \chi^s e^{\frac{K}{2}} D_S W \right). \quad (6.29)$$

Therefore, the local supersymmetry gauge-fixing $v = 0$ leads to a condition

$$v = \chi^\phi e^{\frac{K}{2}} D_\phi W + \chi^s e^{\frac{K}{2}} D_S W = 0. \quad (6.30)$$

This leads to a mixing of the inflatino $\chi^\phi$ with the $S$-multiplet fermion $\chi^s$. The action has many non-linear in $\chi^s$ terms and therefore the fermionic action in terms of a non-vanishing combination of $\chi^\phi$ and $\chi^s$ is extremely complicated. For example, a non-gravitational part of the action of the fermion of the nilpotent multiplet is given by

$$\mathcal{L}_{VA} = -M^2 + i \partial_\mu \bar{\chi}^s \bar{\sigma}^\mu \chi^s + \frac{1}{4M^2} (\bar{\chi}^s)^2 \partial^2 (\chi^s)^2 - \frac{1}{16M^6} (\bar{\chi}^s)^2 (\bar{\chi}^s)^2 \partial^2 (\chi^s)^2 \partial^2 (\bar{\chi}^s)^2, \quad (6.31)$$

as shown in [219]. In supergravity there will be more non-linear couplings of $\chi^s$ with other fields.

In our class of models where the only direction in which supersymmetry is spontaneously broken is the direction of the nilpotent chiral superfield and
\[ D_\Phi W = 0 \text{ the coupling is} \]

\[
\bar{\psi}^\mu \gamma_\mu \chi^s e^{K/2} D_S W|_{\text{min}} + h.c. = \bar{\psi}^\mu \gamma_\mu \chi^s M + h.c. \tag{6.32}
\]

and the goldstino is defined only by one spinor

\[ v = \chi^s M. \tag{6.33} \]

The inflatino \( \chi^\phi \), the spinor from the \( \Phi \) multiplet does not couple to \( \gamma^\mu \Phi_\mu \) since \( D_\Phi W|_{\text{min}} = 0 \). In this case we can make a choice of the unitary gauge \( v = 0 \), when fixing local supersymmetry. Since \( M \neq 0 \) it means that we can remove the spinor from the nilpotent multiplet

\[ \chi^s = 0. \tag{6.34} \]

The corresponding gauge is the one where gravitino becomes massive by ‘eating’ a goldstino. The unitary gauge is a gauge where the massive gravitino has both \( \pm 3/2 \) as well as \( \pm 1/2 \) helicity states. In our models the fully non-linear fermion action simplifies significantly since it depends only on inflatino.

All complicated non-linear terms of the form \( \frac{1}{M^2} (\chi^s)^2 \partial^2 (\bar{\chi}^s)^2 \) and higher power of fermions as well as mixing of the inflatino \( \chi^\phi \) with \( \chi^s \) disappear in this unitary gauge.

In particular, the fermion masses of the gravitino and the inflatino, at the minimum, are simply

\[ m_{3/2} = W_0 = f(0), \quad m_{\chi^\phi} = e^{K/2} D_\Phi W = f''(0) - f(0) = f''(0) - m_{3/2}. \tag{6.35} \]

Here we have presented the masses of fermions without taking into account the subtleties of the definition of such masses in the de Sitter background. This was explained in details for spin 1/2 and spin 3/2 in [243] in case including \( \Lambda > 0 \). For example, the chiral fermion mass matrix \( m^{ij} = D^i D^j e^{K/2} W \) is replaced by \( \hat{m} \equiv \hat{m} + \sqrt{\Lambda/3} \gamma_0 \).

### 6.4 Attractors and de Sitter landscape

In this Section, we provide a unified description of cosmological \( \alpha \)-attractors and late-time acceleration. As in the case of flat geometry, previously discussed in Sec. 6.3, our construction involves two superfields playing distinctive roles: one is the dynamical field and its evolution determines inflation and dark energy, the other is nilpotent and responsible for a landscape of vacua and supersymmetry breaking.
We prove that the attractor nature of the theory is enhanced when combining the two sectors: cosmological attractors are very stable with respect to any possible value of the cosmological constant and, interestingly, to any generic coupling of the inflationary sector with the field responsible for uplifting. Finally, as related result, we show how specific couplings generate an arbitrary inflaton potential in a supergravity framework with varying Kähler curvature.

6.4.1 Uplifting flat $\alpha$-attractors

In the single superfield framework defined by

$$K = -\frac{1}{2} \left( \Phi - \Phi \right)^2, \quad W = f(\Phi),$$

(6.36)

inflationary models with observational predictions given by (5.43) and in excellent agreement with Planck were found in [94]. We have reviewed these models in the previous chapter but we recall here some basics for convenience. These are defined by

$$f(\Phi) = e^{\sqrt{3}\Phi} - e^{-\sqrt{3}\Phi} F \left( e^{-2\Phi/\sqrt{3}\alpha} \right),$$

(6.37)

where $F$ is an arbitrary function having an expansion such as $F(x) = \sum_n c_n x^n$ with

$$x \equiv e^{-2\Phi/\sqrt{3}\alpha}.$$  

(6.38)

This class of models, being characterized by exponentials as building blocks of the superpotential, manifestly exhibits its attractor nature through the insensitivity to the structure of $F$. While the constant term $c_0$ would yield a de Sitter plateau $V = 12c_0$, the first linear term would define the inflationary fall-off typical of $\alpha$-attractors, such as

$$V = V_0 + V_1 e^{-\sqrt{\frac{2}{3\alpha}} \varphi} + ...,$$

(6.39)

at large values of the canonical scalar field $\varphi = \sqrt{2} \text{Re}\Phi$, with $V_0 = 12c_0$ and $V_1 = 16c_1$, the latter being negative. Higher order terms would be unimportant for observational predictions.

This scenario can be naturally embedded in the construction discussed in the previous section. A first step would be simply choosing (6.37) as function $f$ in Eq. (6.20). In fact, this represents a valid alternative to the specific choice (6.26): it yields always a balance of the negative term in (6.21), independently of the value of the uplifting at the minimum, and, interestingly, it decouples the functional forms of $f$ and $g$. As second step, one may notice
that, given the form of the scalar potential Eq. (6.21), any generic expansion such as
\[ f(x) = \sum_n a_n x^n, \quad g(x) = \sum_n b_n x^n, \]  
(6.40)
with \( x \) given by Eq. (6.38), would give rise to a fall-off from de Sitter analogous to Eq. (6.39) with
\[ V_0 = b_0^2 - 3a_0^2, \quad V_1 = 2b_0b_1 - 6a_0a_1, \]  
(6.41)
and, then, yield the universal predictions (5.43).

It is remarkable that the attractor structure of the theory is enhanced when combining the inflaton with the nilpotent sector. The inflationary regime is very stable with respect to any deformation of the superpotential and any value of the uplifting.

Within this construction, the condition (6.22) of a minimum placed at \( \Phi = 0 \) (\( x = 1 \)) translates into
\[ \sum_{n=1}^{\infty} n a_n = 0, \quad \sum_{n=1}^{\infty} n b_n = 0. \]  
(6.42)

Interestingly, the value of the cosmological constant at the minimum is given by
\[ \Lambda = \left( \sum_n b_n \right)^2 - 3 \left( \sum_n a_n \right)^2, \]  
(6.43)
and then as a sum of the coefficients of the expansions (6.40) which, separately, determine the gravitino mass and the scale of supersymmetry breaking, such as
\[ m_{3/2} = \sum_n a_n, \quad M = \sum_n b_n. \]  
(6.44)

Stability of the inflationary regime in the imaginary direction is always assured, for any value of \( \alpha \), as the condition is simply
\[ |b_0| > |a_0|. \]  
(6.45)

In fact, the mass of \( \text{Im} \Phi \) turns out to have a natural expansion at small value of \( x \) (large values of \( \varphi \)) such as
\[ m_{\text{Im} \Phi}^2 = 2(b_0^2 - a_0^2) + \frac{4}{3\alpha} [b_0b_1(3\alpha - 1) - a_0a_1(3\alpha + 1)] x + \ldots, \]  
(6.46)
that is an exponential deviation from a constant plateau. Interestingly, this is the typical functional form of the scalar potential of \( \alpha \)-attractors, where
higher order terms do not play any role. During inflation, the \( \text{Re}\Phi \) moves along a valley of constant width. This phenomenon can be appreciated below in Fig. 6.9, for a specific example. Stability at the minimum is model dependent since, generically, the infinite tower of coefficients \( a_n \) and \( b_n \) contribute to the masses.

![Figure 6.8](image)

**Figure 6.8**
Scalar potential of the model defined by Eq. (6.47) with \( \alpha = 1 \) and uplifting equal to \( \Lambda = \{0, 0.1, 0.3, 0.5\} \).

The simplest example of such class of models is given by the following choice:

\[
\begin{align*}
 f &= a_0 + a_1 x + a_2 x^2, \\
 g &= b_0.
\end{align*}
\]

(6.47)

In fact, this is a minimum in order to have a deviation from de Sitter typical of \( \alpha \)-attractors, which comes from the linear term, and a non-trivial solution of Eq. (6.42) to have a minimum placed at the origin, thanks to the quadratic contribution. Higher order terms will not affect neither the inflationary energy nor the characteristic fall-off, as it is clear from Eq. (6.41). The scalar potential, for \( \alpha = 1 \) and different amount of uplifting, is shown in Fig. 6.8. Stability occurs along the full inflationary trajectory and also at the minimum where both directions of \( \Phi \) turn out to be stable, as it is shown in Fig. 6.9. Analogous results hold for other values of \( \alpha \).

The addition of higher order terms both in \( f \) and \( g \) would allow for more flexibility in terms of separation of the physical scales. In fact, whereas the inflationary regime would be absolutely insensitive to high order contributions, the coefficients of these terms turn out to be fundamental in determining the scale of SUSY breaking, the gravitino mass and the cosmological constant, as given by Eq. (6.43) and Eq. (6.44).
6.4 Attractors and de Sitter landscape

6.4.2 Uplifting geometric $\alpha$-attractors

The appealing property of the original formulation of $\alpha$-attractors, as discovered in [92, 173, 182], is the unique relation between the Kähler geometry and the observational predictions (5.43). In particular, the logarithmic Kähler potential fixes the spectral tilt while its constant curvature

$$R_K = -\frac{2}{3\alpha},$$

(6.48)

determines the amount of primordial gravitational waves. However, these original models require always the presence of a second superfield.

Figure 6.9
Masses of the real and imaginary part of the field $\Phi$ for the model defined by Eq. (6.47) with $\alpha = 1$ and uplifting equal to $\Lambda = \{0, 0.1, 0.3, 0.5\}$. Both scalar parts are massive at the minimum. During inflation, at large values the $\varphi$, the mass of $\text{Re}\Phi$ goes to zero while the mass of $\text{Im}\Phi$ approaches a constant value as defined by Eq. (6.46).
Single superfield geometric formulations have been discovered in [94, 162]. As shown in [94], they originate from a natural deformation of the well-known no-scale constructions and they are defined by

\[ K = -3\alpha \ln \left( \Phi + \bar{\Phi} \right), \quad W = \Phi^n - \Phi^n + F(\Phi), \quad (6.49) \]

with power coefficients equal to

\[ n_\pm = \frac{3}{2} (\alpha \pm \sqrt{\alpha}) \], \quad (6.50) \]

and \( F \) having general expansion \( F(\Phi) = \sum_n c_n \Phi^n \) which encodes the attractor nature of these scenarios.

This class gives rise to the flat \( \alpha \)-attractors of the previous section in the limit \( \alpha \to \infty \) and, then, when the curvature becomes flat, as shown in [94]. The procedure is the following: one performs a field redefinition such as \( \Phi \to \exp(-2\Phi/\sqrt{3\alpha}) \), an appropriate Kähler transformation and, in the singular limit, one obtains canonical and shift-symmetric \( K \) and \( W \) equal to (6.37), with \( F \) constant. On top of this, one adds exponential corrections which returns the desired inflationary behavior.

In order to uplift the SUSY Minkowski minimum of these scenarios, one can add a nilpotent field which breaks supersymmetry and yields a non-zero cosmological constant. The geometric analogous of the flat case, discussed in the previous section, is given by

\[ K = -3\alpha \ln (\Phi + \bar{\Phi}) + S \bar{S}, \quad W = \Phi^{3\alpha} \left[ f(\Phi) + g(\Phi)S \right]. \quad (6.51) \]

In fact, along the real axis \( \Phi = \bar{\Phi} \) and at \( S = 0 \), this supergravity model yields a scalar potential

\[ V = 8^{-\alpha} \left[ g(\Phi)^2 - 3f(\Phi)^2 + \frac{4\Phi^2 f'(\Phi)^2}{3\alpha} \right], \quad (6.52) \]

which, when expressed in terms of the canonical field \( \varphi = -\sqrt{3\alpha/2} \ln \Phi \), coincides with the one obtained in the flat case Eq. (6.21), up to an overall constant factor. Furthermore, Eq. (6.51) reduces to Eq. (6.20) in the

---

\(^5\) No scale models, as originally proposed in [165, 166], represent a good starting point in order to produce consistent inflationary dynamics (see e.g. [170, 172, 176, 185, 244, 245]). However, the geometric models of this section emerge from a different construction which naturally leads to stable de Sitter solutions and have scale depending on the parameter \( \alpha \) (see [94] for explicit derivation). The no scale symmetry is intimately related to a specific value of the Kähler curvature (6.48) and it is restored just in the limit \( \alpha \to 1 \).
flat singular limit. The Kähler potential \((6.51)\) parameterizes a manifold \(SU(2, 1)/U(1) \times U(1)\) and related analysis with similar settings are performed in [175, 176].

The correspondence between the scalar potentials of the flat and the geometric construction (for the single superfield case it was proven in [94]) is remarkable as it allows to identically assume the whole set of results, from Eq. \((6.40)\) to Eq. \((6.46)\), found and described in the previous section, provided one identifies

\[
x \equiv \Phi.
\]  

(6.53)

The functions \(f\) and \(g\) can be assumed to have generic expansion \((6.40)\) and the inflationary behavior will be of the form \((6.39)\). However, in this case, the fall-off will be governed by the curvature of the Kähler manifold which depends on the parameter \(\alpha\). The minimum, placed at \(\Phi = 1\), provided

\[
f'(1) = g'(1) = 0,
\]  

(6.54)

will have uplifting equal to \((6.43)\), gravitino mass and SUSY breaking scale given by \((6.44)\) and, again, supersymmetry broken just in the \(S\) direction, as given by

\[
D_S W_{\text{min}} = g(1) = M, \quad D_\Phi W_{\text{min}} = 0.
\]  

(6.55)

Remarkably, the condition on the stability of the inflationary trajectory turns out to be the same of the previous section. At large value of the canonical field \(\varphi\), the mass of \(\text{Im}\Phi\) is positive when Eq. \((6.45)\) is satisfied, independently of the value of \(\alpha\). This represents a considerable improvement with respect to the single superfield case defined by \((6.49)\) which is stable just for \(\alpha > 1\) [94]. Furthermore, the mass of \(\text{Im}\Phi\) approaches a constant value during inflation as given by \((6.46)\), up to an overall constant.

6.4.3 General inflaton potential from curved Kähler geometry

We have so far developed a general framework in order to obtain inflation together with controllable level of uplifting and SUSY breaking at the minimum when the Kähler geometry is curved and defined by Eq. \((6.51)\). We have proven that generic expansion of \(f\) and \(g\) gives rise to \(\alpha\)-attractors with cosmological predictions extremely stable.

On the other hand, also in this context, it is possible to make the specific choice \((6.26)\) and consider the geometric analogous of the class of models introduced in [202, 203] and reviewed in Sec. 6.3. Then, the Kähler potential
and the superpotential read
\[
K = -3\alpha \ln \left( \Phi + \Phi \right) + S \bar{S}, \quad W = \Phi^{\frac{2}{3\alpha}} f(\Phi) \left( 1 + \frac{S}{\beta} \right). \tag{6.56}
\]
The choice \(\beta = 1/\sqrt{3}\) gives rise to a scalar potential with a Minkowski minimum. Along \(\Phi = \bar{\Phi}\) and \(S = 0\), one has (up to an overall constant factor)
\[
V = \frac{2}{3\alpha} \Phi^2 f'(\Phi)^2, \tag{6.57}
\]
which, in terms of the canonical scalar field \(\varphi\) reads
\[
V = f'(e^{-\sqrt{\frac{2}{3\alpha}}\varphi})^2, \tag{6.58}
\]
where primes denote derivatives with respect to the variables the function depends on. Then, one can implement an arbitrary inflaton potential, independently of the value of the Kähler curvature which is parametrised by \(\alpha\). Related results for the case \(\alpha = 1\) were obtained in [176]. In the case of a flat Kähler geometry the works [169, 202, 203] developed analogous constructions.

Within this setup, one can implement even a quadratic potential \(V = \frac{1}{2}m^2\varphi^2\) by choosing
\[
f(\Phi) = \frac{3\alpha m}{4\sqrt{2}} \ln^2(\Phi). \tag{6.59}
\]

The properties at the minimum remain the same as in the flat case of Sec. 6.3. Then, a small deviation of \(\beta\) from the value \(1/\sqrt{3}\) yields the desirable tiny uplifting which reproduces the current acceleration of the Universe.

### 6.4.4 Discussion

In this Section, we have provided evidences for the special role that \(\alpha\)-attractors would play in the cosmological evolution of the Universe. In the simple supergravity framework consisting of two sectors (one containing the inflaton and the other controlling the landscape of possible vacua), any arbitrary expansion of the superpotential would yield automatically such inflationary scenarios. We have obtained these results both in the case of a flat Kähler geometry, as given by Eq. (6.20), and in the case of the logarithmic Kähler as defined by Eq. (6.51) where the geometric properties of the Kähler manifold determines the observational predictions. In this latter case, the overall factor \(\Phi^{\frac{2}{3\alpha}}\) in \(W\) can be removed by means of an appropriate Kähler transformation (this choice makes the shift symmetry of the
canonical inflaton $\varphi$ manifest even in the case of a logarithmic Kähler potential, as pointed out in [196]). However, one would lose immediate contact with string theory scenarios as the form of $K$ would change consequently. In this respect, polynomial contributions to the superpotential, typically arising from flux compactification, would be possible if
\[ \alpha = \frac{2}{3^n} \]  
with $n$ integer. In particular, the simple choice $n = 1$ would give
\[ K = -2 \ln (\Phi + \bar{\Phi}) + S\bar{S}, \]
\[ W = (a_0\Phi + a_1\Phi^2 + \ldots) + (b_0\Phi + b_1\Phi^2 + \ldots) S, \]  
where dots stand for higher order terms in $\Phi$ (see [133] for a recent analysis of this class of models in the context of supplementary moduli breaking supersymmetry). Then, the minimal addition of a nilpotent sector with canonical $K$ to the class proposed in [94] leads to a simplification of the original superpotential (6.49) and enhancement of stability of the inflationary trajectory, which now occurs for any value of $\alpha$ (see [196] for a discussion on the connection between curvature and stabilization).

We have shown that cosmological $\alpha$-attractors are absolutely insensitive with respect to any value of the cosmological constant and to the coupling between $\Phi$ and $S$. The plateau and the fall-off turn out to be extremely stable with respect to generic deformations of the superpotential (similar stability can be observed in some examples of [198]). These scenarios would arise naturally in any possible Universe, independently of the amount of dark energy. In this regard, cosmological attractors seem to be fundamentally compatible with the idea of Multiverse and landscape of vacua.