In this chapter, we discuss the problem of realizing a consistent inflationary scenario within a supergravity framework. We discuss its relations to string theory and present its basic properties. Then, we discuss the most common and challenging obstacles, and its possible solutions, to a successful realization of inflation in supergravity. These include the well known $\eta$-problem and the dynamical restrictions arising from the interplay between the inflaton and supersymmetry breaking sectors. Remarkable simplifications arise in the case of complete orthogonality of these two sectors. An arbitrary inflaton potential can indeed be obtained when the internal Kähler manifold is flat. On the other hand, assuming a hyperbolic geometry has dire implications for inflation: the Kähler curvature controls the amount of primordial gravitational waves and the value of the scalar tilt tends automatically to the “sweet spot” of Planck, no matter the details of the superpotential. The non-trivial Kähler geometry basically induces an attractor for observations. Finally, we present a novel supergravity construction, dubbed $\alpha$-scale model, which turns out to be at the origin of the attractor mechanism. This will allow us to construct the first single superfield formulation of $\alpha$-attractors and ultimately shed light on the connection between flat and curved internal space. The novel results of this Chapter are based on the publications [iii], and [viii].
5.1 Inflation in supergravity

In the Introduction of this thesis, we have already discussed the importance of embedding the inflationary paradigm into a complete high-energy physics scenario. The UV sensitivity of inflation makes indeed its implementation into a concrete framework of quantum gravity a primary challenge to face.

String theory [5–8] offers a great arena where to construct our cosmological models of the early Universe. Its control over Planckian degrees of freedom seems indeed to provide a robust environment where to investigate inflation. However, properly realizing inflation within a concrete stringy scenario has turned out to be quite challenging. First of all, in order to bring this complex framework in contact with reality, one must find a suitable mechanism to reduce the number of spacetime dimensions from ten to four. This procedure is named compactification [105,106] of the six extra dimensions and it has become a central research topic in theoretical physics. In addition, a successful compactification usually leads to the appearance of many light moduli, namely scalar fields with no mass. These must be stabilized by means of specific mechanisms which produce the appropriate potential constraining their dynamics (famous examples are the so-called GKP [107] and KKLT [108] mechanisms). Finally, in order to have best control on the string inflationary models, one must ensure that a desirable hierarchy of scales holds. Specifically, one would like to have

\[ H < M_{KK} < M_s < M_{Pl}, \]  

(5.1)

where \( H \) denotes the Hubble scale during inflation, \( M_{KK} \) denotes the compactification Kaluza-Klein scale below which one may consider an effective 4-dimensional description of physics, \( M_s \) is the scale at which one resolves the string structure and \( M_{Pl} \) is the Planck scale.

The route towards a complete embedding of inflation in string theory is still in progress (see [21,109–112] for some reviews on this topic). Along the way, it has produced very interesting results (see e.g. [113–121]) which have shed light on the basic properties a consistent cosmological scenario should have. Questions about the fundamental behavior of the inflaton field can be often translated into questions about the geometry of the internal manifold. Understanding how the physics of the many moduli naturally arising in string theory can (or cannot) be decoupled from the inflaton dynamics becomes of utmost importance in this context (much effort in this direction has been made by works such as [122–130] and [131–133]).

The usual strategy is investigating inflation within an effective supergravity (SUGRA) description (i.e. at energies lower than \( M_s \)), whose consistency
is automatically satisfied if one assumes the hierarchy of scales Eq. (5.1). This is the limit where the strings can be simply approximated by point particles and one can use a convenient field theory description. In fact, supergravity \([\text{[134,135]}\) (see \([\text{[136]}\) for a comprehensive book on this topic), as a local extension of supersymmetry (a perfect correspondence between bosons and fermions \([\text{[137–140]}\]), was discovered independently from string theory. It was then realized that some supergravity incarnations could arise as effective limits of this complete theory of Nature.

However, string theory compactifications usually yields severe constraints on the possible internal geometries. This implies that not every supergravity model can be regarded as an effective limit coming from string theory. One can easily encounter the risk of studying a supergravity cosmological construction which has no correspondent in the UV limit.

In this Chapter, we intend to face this last issue and prove that investigating inflation within a pure supergravity context can still yield crucial insights into the general structure of an effective UV description. In fact, the sole ingredient of supersymmetry (SUSY) can yield very strong constraints on the inflationary dynamics. In addition, the form of the scalar potential and the stability of the scalar manifold can be studied in full generality. We will show the generic properties a supergravity model should have in order to reproduce successful inflationary scenarios compatible with the current observational data. We will see that it is possible to draw very general conclusions, independently from the specific details of the model at hand. The generality of these results will assume particular relevance in the light of building a consistent inflationary model within string theory.

In the following, we will start our discussion by presenting the main properties and most common problems when one tries to embed inflation in \(\mathcal{N} = 1\) supergravity\(^1\), without referring to any specific model. We will present the general form of the inflaton Lagrangian in supergravity. Both the scalar potential and the kinetic terms will be functions of fundamental geometric properties of the internal manifold where the fields are defined. We will then discuss a generic problem threatening the flatness of the inflaton potential (namely the smallness of the \(\eta\) parameter) in supergravity and its possible ways-out. We will then examine the connections between spontaneous SUSY breaking and the inflationary dynamics. This will have direct consequences on the possibility of yielding inflation by means of just a single superfield.

---

\(^1\mathcal{N}\) defines the number of supersymmetry transformations of the theory. The higher the number of supersymmetries is, the more constrained are the field content and its dynamics. If we require the absence of particles with spin higher than 2, \(\mathcal{N} = 8\) is the maximum possible value as there are no more than eight half-steps between spin -2 and 2.
Finally, we will discuss to what extent one can regard a supergravity model as an effective limit coming from string theory.

After these first general considerations, we will devote the following sections of this Chapter to some concrete models of inflation. Specifically, we will show how the geometry of the internal field space can have dire implications on the final result of inflation in terms of $n_s$ and $r$. In Sec. 5.2 and Sec. 5.3, we will see the main differences between the cases of flat and hyperbolic geometry. The latter leads naturally to the concept of cosmological attractors providing universal observational predictions. We will devote the last section to a novel supergravity construction, dubbed $\alpha$-scale supergravity, which proves to be at the origin of the attractor mechanism. This will also shed light on the link between the two benchmark geometries considered before.

5.1.1 Basics of 4D $\mathcal{N} = 1$ supergravity

The field content of a four-dimensional $\mathcal{N} = 1$ SUGRA theory is given by the graviton $g_{\mu \nu}$, the gravitino $\psi_\mu$ coupled to an arbitrary number $n$ of chiral supermultiplets\(^2\). Each of these contains a chiral spin-$1/2$ field and a complex scalar field. In the following, we discuss just the bosonic sector as the most relevant for the next sections. We will return on the fermionic sector in the next Chapter.

Scalar fields are ubiquitous in supergravity theories. Specifically, they always come in pairs (being complex scalars) and their number is not constrained. This provides a very flexible and natural setup to embed the physics of the inflaton field, as it was introduced in Sec. 3.1.5.

The dynamics of the complex scalar fields $\Phi_i$ (with $i = 1, \ldots, n$) is fully determined by two functions:

- The Kähler potential $K(\Phi_i, \bar{\Phi}_i)$, being a hermitian function of the fields $\Phi_i$ and their complex conjugates $\bar{\Phi}_i$.
- The superpotential $W(\Phi_i)$, being a holomorphic function of the fields $\Phi_i$.

The Lagrangian of the scalar fields turns out to be\(^3\)

$$\mathcal{L} = -K_{ij} \partial_\mu \Phi^i \partial^\mu \Phi^j - V,$$  \hspace{1cm} (5.2)

\(^2\)We are considering the case of vector supermultiplets playing a subdominant role, that is, the effective description is the so-called F-term supergravity. When gauge interactions become relevant, they induce an additional so-called D-term piece in the potential $V$.

\(^3\)In this thesis, we do not intend to provide the detailed derivation of the supergravity action but just the relevant formulas for a proper discussion of inflation in this framework. We refer the interested reader to the seminal papers [141,142].
where \( K_{ij} \equiv \partial_{\Phi_i} \partial_{\bar{\Phi}_j} K \) is the metric of the \textit{Kähler manifold} spanned by the fields \( \Phi_i \). Then, the geometry of this internal space is fundamentally related to the kinetic terms of the scalars. In the case of one superfield with canonical kinetic terms, a very natural choice is \( K = \Phi \bar{\Phi} \) which corresponds to a flat \textit{Kähler geometry} \((K_{\Phi \bar{\Phi}} = 1)\). However, other forms of \( K \) are allowed in supergravity. Normally, string theory compactifications yield severe constraints on the geometric properties of this internal space.

The form of the F-term scalar potential is

\[
V = e^K \left( K^{ij} \mathcal{D}_i W \mathcal{D}_j \bar{W} - 3|W|^2 \right), \tag{5.3}
\]

where \( K^{ij} \) is the inverse matrix of the \textit{Kähler metric} \( K_{ij} \) and

\[
\mathcal{D}_i W \equiv \partial_i W + K_i W, \tag{5.4}
\]

is the \textit{Kähler covariant derivative}, referred to as F-term and being the order of parameter for spontaneous SUSY breaking. The potential (5.3) is always given by two opposing contributions where the negative definite term sets the AdS scale. Developing a generic mechanism which yields a positive potential \( V > 0 \) is of primary interest for cosmological applications. We will return to this issue in Sec. 5.1.3 and discuss its connection with the SUSY breaking directions.

The squared mass matrix of the scalar fields is given by

\[
m^2 = \begin{pmatrix} K^{ik} \mathcal{D}_k \partial_j V & K^{ik} \mathcal{D}_k \partial_j V \\ K^{ik} \mathcal{D}_k \partial_j V & K^{ik} \mathcal{D}_k \partial_j V \end{pmatrix}, \tag{5.5}
\]

where the \textit{Kähler covariant derivative} \( \mathcal{D} \) acts on \( \partial V \) as in Eq. (5.4) for \( W \).

The physics described by Eq. (5.2) is invariant under a \textit{Kähler transformation}, namely

\[
K \rightarrow K + \lambda + \bar{\lambda}, \tag{5.6}
\]

\[
W \rightarrow e^{-\lambda} W, \tag{5.7}
\]

where \( \lambda \) is a holomorphic function of the fields.

In order to implement single-field inflation in this framework, firstly we need to identify one of the real degrees of freedom with the inflaton field. In the simplest case of employing just one single superfield \( \Phi \), one must choose an appropriate decomposition and assure stability of the other direction. A common choice is

\[
\Phi = \frac{\phi + i\chi}{\sqrt{2}}, \tag{5.8}
\]
where the real direction may be identified with the inflationary trajectory while the orthogonal direction must be stabilized by means of an appropriate mechanism. Alternatively, one may allow for a multi-field dynamics which appears to be quite natural from a UV perspective, given the abundance of scalar fields in these scenarios. These additional degrees of freedom may be light thus participating in the inflationary process (see [143] for a review on this topic). Conversely, they might be heavy and still produce observational features, as it was shown in the series of works [122–130].

5.1.2 The $\eta$-problem in supergravity

A generic problem arising in supergravity constructions describing inflation is the so-called $\eta$-problem: due to the specific form of the scalar potential of this class of theories, as given by Eq (5.3), the second slow-roll parameter $\eta$ generically receives contributions of order one [144,145]. This can be immediately seen for the choice of a canonical Kähler potential $K = \Phi \bar{\Phi}$. Expanding the overall exponential term of the scalar potential, we have

$$V \approx (1 + |\Phi|^2 + ...) V_0(\Phi),$$

where the factor $V_0$ is determined by the superpotential. Then, the second slow-roll parameter obtains contributions such as

$$\eta \approx 1 + \frac{V''}{V_0} + ...,$$

which is order one for generic choices of $W$.

The exponential term in Eq. (5.3) plays a very dangerous role and generically spoils the flatness of the inflaton potential. The problem becomes even more severe for super-Planckian values of the inflaton field.

In order to realise slow-roll inflation, one must therefore either resort to an undesired amount of fine-tuning to cancel the order one terms, or eliminate these contributions altogether by means of a symmetry. The latter is termed natural inflation [81]. It was first employed in a supergravity context to realise chaotic inflation [146]. Instead of the canonical Kähler, the authors opted for

$$K = -\frac{1}{2} \left( \Phi - \bar{\Phi} \right)^2.$$  (5.11)

Due to the absence of the real part of the superfield $\Phi$, the Kähler potential has a shift symmetry $\Phi \rightarrow \Phi + a$ with $a \in \mathbb{R}$. This symmetry is the key to avoid the $\eta$-problem; relatedly, it prevents the inflaton potential from
blowing up for large values of the inflaton field $\text{Re}(\Phi)$. It is evident that the dangerous term $e^K$ keeps increasing exponentially in the direction $\text{Im}(\Phi)$ while it remains constant in $\text{Re}(\Phi)$. The shift symmetry is broken only by $W$, thus generating the inflaton potential.

However, the above discussion can also be misleading as it fails to take into account the following subtlety. The careful reader may have noticed that the two quoted Kähler potentials are in fact related by a Kähler transformation, and hence are physically equivalent. Yet more strikingly, the Kähler potential can even be brought to the form

$$K = \frac{1}{2} \left( \Phi + \bar{\Phi} \right)^2,$$

(5.12)

by means of an additional Kähler transformation.

The three forms of $K$ suggest symmetry protection for either none, the real or the imaginary components, respectively. How does it come about that one Kähler potential suffers from the $\eta$-problem, while physically equivalent potentials avoid it by means of a shift symmetry, which however protects different components? The answer to this apparent conundrum is that the $\eta$-problem is not only a statement about the Kähler potential, but also about the ‘naturalness’ of the superpotential. For generic choices of the superpotential, one needs a shift symmetry in $K$ to keep $\eta$ small; without that shift symmetry in the Kähler potential, one needs a carefully picked $W$ to compensate for the order-one contribution to $\eta$. These two situations can be related by a Kähler transformations and are exactly the options alluded to above, i.e. fine-tuning or symmetry. Thus the form of the Kähler potential is not the only ingredient in evading the $\eta$-problem; also the generic or fine-tuned form of the superpotential comes into play.

### 5.1.3 sGoldstino directions and inflation

A successful implementation of the inflationary paradigm in supergravity requires a mechanism that assures a positive definite potential during the whole cosmological evolution. Achieving this is not always trivial due to the delicate balance between the two contributions in Eq. (5.3). Specifically, during inflation supersymmetry must be broken as we need $\mathcal{D}_i W \neq 0$. This fact has dire consequences for inflation.

First of all, spontaneous breaking of supersymmetry at the inflationary scale naturally implies a second quasi-light field around the Hubble scale\(^5\).

---

\(^4\)In fact, the canonical $K$ does not depend on the complex phase of the field $\Phi$; however, in the origin of the orthogonal, radial direction, the phase is not a physical field.

\(^5\)This is termed quasi-single field inflation in [147], where the inflationary consequences
Any scalars other than the inflaton - of which there is always at least one, given that scalars come in pairs in SUSY models - therefore naturally acquire Hubble-scale masses. A concrete demonstration of this phenomenon can be found in [148].

Secondly, one can prove that the inflationary dynamics is highly constrained by the direction of SUSY breaking. This is defined by $\mathcal{D}_i W$ in the scalar manifold and it is usually referred to as $sGoldstino$ directions. The latter is a pair of scalar fields that is singled out by the spontaneous breaking of supersymmetry.

It is worth elaborating on the latter point as it has an interesting group-theoretical underpinning. As emphasised in the effective field theory approach to inflation [73], the inflaton can be seen as the Goldstone boson arising from the spontaneous breaking of time translation invariance: this symmetry is necessarily broken during inflation (as exemplified by the value of the spectral tilt Eq. (3.55)) giving rise to a Goldstone boson, which can be seen as either a scalar field or an additional helicity-0 component in the metric. This is analogous to the additional degrees of freedom of the $W^\pm$ and $Z^0$ vector bosons as arising in the Higgs mechanism. A slightly different reasoning applies to spontaneous breaking of SUSY. In this case, the Goldstone modes are a pair of spin-1/2 fermions, whose supersymmetric partners are spin-0 fields. These are referred to as Goldstini and sGoldstini fields, respectively. Their emergence is completely analogous to the Higgs boson itself in the spontaneous breaking of gauge symmetry: the Higgs is the gauge partner of the aforementioned Goldstone bosons. Thus, there are interesting similarities and differences between these interpretations of the inflaton and the sGoldstini scalar fields: both arise as a consequence from the spontaneous breaking of a local symmetry (i.e. time translational invariance and SUSY), in which they are Goldstone modes or the partners thereof.

Studying the trajectories of the sGoldstini fields turns out to be very important for the dynamics of the whole system. Indeed, these scalars generically correspond to unstable directions on the scalar manifold, signaling instabilities [149, 150, 150]. In addition, depending on the angle between the inflaton and sGoldstini directions one can draw very general conclusions about the inflationary dynamics [151, 152]. Specifically, one can show that, in the case these directions have a non-negligible overlap and the gravitino mass is orders below the inflationary scale, single-field, slow-roll and small field inflation cannot be realize in supergravity [152]. This follows from a general inequality that we will refer to as the geometric bound, as it involves of an additional scalar field in the complementary series of De Sitter's unitary irreps with $0 < m^2 \leq 9/4$ were investigated.
the curvature of the Kähler manifold spanned by the scalars.

There are two extreme cases one may consider. We list them below.

**sGoldstino inflation**

When the directions of the inflaton and of the sGoldstino coincide, we refer to this scenario as *sGoldstino inflation* (this framework has been investigated by several studies such as [94, 153–164]). In the most economical scenario, this is the situation when just a single superfield is involved in the supergravity construction. In this case, the inflaton plays a double role: it drives the quasi-exponential expansion and it breaks supersymmetry at the same time. However, one cannot evade the geometric bound of [152] and, then, one needs to take a number of facts into account in order to realize a successful cosmological scenario. In addition, obtaining an arbitrary inflationary potential becomes very challenging.

Here, we present three possible settings in terms of their internal Kähler space:

* A first natural choice for $K$ is given by the shift-symmetric function (5.11) corresponding to a flat Kähler geometry. This allows for a truncation to only the imaginary part of $\Phi$ provided one takes

$$W = f(\Phi),$$

(5.13)

where the function $f$ is a real holomorphic function of $\Phi$; in other words, when expanded in terms of its holomorphic argument, all coefficients are required to be real. The mass spectrum for this model reads

$$m^2_{\text{Re}(\Phi)} = -6f'' - 6ff'' + 2f'' + 2f'f''',$$

$$m^2_{\text{Im}(\Phi)} = 4V + 4f^2 + 2f'^2 - 2f f'' + 2f'' - 2f'f''',$$

(5.14)

when evaluated at $\Phi = \bar{\Phi}$ and where primes denote derivatives with respect to the variables the function depends on. In this set-up, the imaginary part of the superfield $\Phi$ will generically give rise to a Hubble-scale field, which is stabilised at zero. This is exactly as expected, as the Kähler potential contributes order one to $\eta$, and the contribution from $W$ will generically be much smaller (think e.g. a sum of exponential - the shift symmetry of the imaginary part is mildly broken, leading to a small contribution). Hence the total $\eta$ will be order one, allowing stabilisation of this field. In contrast, the real part $\text{Re}(\Phi) = \phi$ will be light and play the role of the inflaton.
The resulting scalar potential, along $\Phi = \bar{\Phi}$, reads

$$V = f'(\Phi)^2 - 3f(\Phi)^2.$$  \hfill (5.15)

This model thus allows for a truncation to a single field. However, the form of this potential is clearly not the most general due to the appearance of both the function $f$ and its derivatives. For large field inflation with a single monomial dominating the superpotential at large field values, the negative-definite contribution dominates the scalar potential. Thus, it is impossible to realise in particular chaotic inflation with a monomial in this way (two explicit examples with polynomials of fourth order as superpotentials are given in $[157]$).

- A second natural possibility is given by taking a logarithmic rather than polynomial Kähler potential. The choice

$$K = -3\alpha \ln \left( \Phi + \bar{\Phi} \right),$$  \hfill (5.16)

leads to a hyperbolic Kähler geometry, a manifold $SU(1,1)/U(1)$, whose curvature is parameterised by $\alpha$. Thus, it is well motivated from a supergravity point of view as well as from string theory, as we will discuss below. In addition it eliminates the dangerous exponential terms arising from the overall Kähler exponential in the scalar potential. Therefore, there is no longer a compelling reason to identify the imaginary part of $\Phi$, which now enjoys the shift symmetry of $K$, with the inflaton. This is good news, as it is generically inconsistent to set the real part of $\Phi$ equal to zero in order to obtain a single field model. The fact that this is consistent in the model with the shift symmetric Kähler potential is a consequence of the square in (5.11). As the logarithmic Kähler potential no longer has this feature, the only consistent truncation is to the real part. For this one needs to take the same requirement on the superpotential (5.13) being a real function of $\Phi$.

This model, with the same $W$ as in Eq. (5.13) and along $\Phi = \bar{\Phi}$, leads to the scalar potential:

$$V = 8^{-\alpha} \Phi^{-3\alpha} \left[ \frac{(3\alpha f(\Phi) - 2\Phi f'(\Phi))^2}{3\alpha} - 3f(\Phi)^2 \right]$$  \hfill (5.17)

The choice $\alpha = 1$ is special due to the no-scale structure $[165–167]$, where the negative definite term is exactly cancelled. However, the functional form of Eq. (5.17) appears to be even more complicated than the flat correspondent Eq. (5.15).
5.1 Inflation in supergravity

- A final possibility was recently proposed by Ketov and Terada in [158,159] (see also [160,163,164] for subsequent developments) where the authors considered a logarithmic Kähler potential of the form

\[
K = -3 \ln \left[ 1 + \frac{\Phi + \bar{\Phi} + \zeta (\Phi + \bar{\Phi})^4}{\sqrt{3}} \right]. \tag{5.18}
\]

In this setup, the role of the inflaton is played by the Im(\(\Phi\)) = \(\chi\). The term with constant parameter \(\zeta\) serves to stabilize the field Re(\(\Phi\)) = \(\phi\) during inflation at \(\phi \approx 0\). The main idea is that by making \(\zeta\) sufficiently large one can make the field component \(\phi\) heavy and constrained to a very small range of its values, \(\phi \ll 1\), so it plays almost no role during inflation with the inflaton field \(\chi \gg 1\). For superpotentials

\[
W = \frac{1}{\sqrt{2}} f(-\sqrt{2}i\Phi), \tag{5.19}
\]

where \(f\) is a real function of its argument, the potential along the inflaton direction \(\phi \ll 1\) becomes

\[
V \approx [f'(\chi)]^2. \tag{5.20}
\]

For example, for \(W = \frac{1}{2}m\Phi^2\) one recovers the simplest chaotic inflation potential \(V = \frac{m^2}{2}\chi^2\) along the direction \(\phi = 0\). A numerical investigation of this scenario in [159] confirms that for sufficiently large \(\zeta\), the field \(\phi\) practically vanishes during the main part of inflation. Its evolution begins only at the very end of inflation, so the cosmological predictions almost exactly coincide with the predictions of the quadratic scenario. At the end of inflation, the field rolls down towards its supersymmetric Minkowski vacuum at \(\Phi = 0\), where \(V = 0\), \(W = 0\), and supersymmetry is restored.

However, this scenario does not lead to a pure single-field truncation and a two fields dynamics generically appears near the minimum\(^6\). We will return to this model in the next Chapter.

In conclusion, obtaining a general mechanism which assures always successful inflation, within a single superfield context, seems to be a rather

\(^6\)In order to truncate consistently the orthogonal direction to the inflaton, Re\(\Phi\), one has to ensure that its equation of motion is satisfied. It can be checked that this receives contributions from the Christoffel symbols, which do not allow us to decouple the field. However, the extra factors obtained are proportional to the slow-roll parameters and, therefore, the inflationary trajectory occurs approximately along the imaginary part of \(\Phi\). Just after inflation, we can notice a shift of the inflaton from the initial straight initial direction.
non-trivial and challenging task. However, later in Sec. 5.4, we will clarify the fundamental steps to follow in order to achieve a consistent inflationary scenario with a model involving simply one superfield. We will do this in a separate Section given the relevance of the proposed original recipe. The basic mechanism indeed involves a novel supergravity construction (named $\alpha$-scale model) firstly discovered in [94]. Surprisingly, it sheds light on the deep connection between the flat Kähler (5.11) and the hyperbolic one (5.16).

**Orthogonal inflation**

When the directions of the inflaton and of the sGoldstino are orthogonal to each other, we refer to this scenario as orthogonal inflation. This particular framework necessarily involves at least two superfields: $\Phi$, responsible for inflation, and the complex scalar $S$ breaking supersymmetry. In addition, it provides a unique way to evade the geometric bound of [152] thus allowing for remarkable flexibility.

The benchmark model is characterized by a superpotential linear in $S$ such as

$$W = S f(\Phi), \quad (5.21)$$

with $f$ being again a real holomorph function. Then, after choosing a suitable Kähler potential which allows for a consistent truncation along the direction $S = 0$ (this is usually assured if $K$ is invariant under $S \rightarrow -S$ and, then, e.g. depending on $S\bar{S}$), the scalar potential assumes the form

$$V = e^K K^{S\bar{S}} |f(\Phi)|^2. \quad (5.22)$$

The latter is always a positive function as the negative definite contributions of Eq. (5.3) disappears at $S = 0$. Along the same trajectory, the superpotential is indeed identically zero and the F-terms are

$$D_\Phi W = 0, \quad D_S W = f. \quad (5.23)$$

The latter fact sheds light on the peculiar role of the complex scalar $S$ in this construction: this field belongs to the sGoldstino supermultiplet and inflation happens in the orthogonal direction to the one defined by the sGoldstino along which supersymmetry is broken.

Note that the potential Eq. (5.22) still depends on the two real degrees of freedom of the complex field $\Phi$ and, in order to have single-field inflation, one must truncate along a specific direction and assure stabilization of the trajectory. However, both the consistency of the final truncation and stabilization issues strictly depend on the specific form of the Kähler potential.
The simple framework, with canonical shift-symmetric Kähler in the inflaton sector coupled to $S$, was first proposed in [146] and further developed in [168,169]. On the other hand, examples of working models of orthogonal inflation with logarithmic Kähler of the form (5.16) in the inflaton sector (that is, at $S = 0$) already appeared in [170–172]. However, a full general analysis with an arbitrary superpotential, such as the one of Eq. (5.21), was first performed in [145] and, then, in the context of the $\alpha$-attractors model in [92,95,173–175] (see also [176] for related analysis).

To conclude, a model with the inflaton orthogonal to the sGoldstini fields always allows for remarkable flexibility in terms of the scalar potential. We will discuss the two important scenarios of orthogonal inflation with flat and hyperbolic Kähler geometry respectively in Sec. 5.2 and in Sec. 5.3.

5.1.4 Towards an embedding in string theory

Here we would like to discuss to what extent successful supergravity models of inflation can be implemented in string theory. Which are the typical form of $K$ and $W$ following from a string-theoretic configuration? Specifically, is it possible to realize orthogonal inflation by means of sectors naturally arising in string theory?

Let us start by discussing open string fields as candidates for inflation, the most famous case being D-brane inflation [113–115], where the position of a D-brane in the internal compact dimensions plays the role of the inflaton. However, other open strings, such as more generic matter fields, can also be considered as inflaton candidates. Matter fields (including open string moduli) in string theory obtain a Kähler potential of the form\(^7\)

\[ K = \alpha \Phi \bar{\Phi} \quad \text{or} \quad K = \alpha \left( \Phi - \bar{\Phi} \right)^2. \tag{5.24} \]

Here we have assumed that any closed string moduli have been stabilised and their vevs are taken into account in the constant $\alpha$. Hence matter fields can satisfy all symmetry requirements for a shift-symmetric or simply a canonical Kähler potential such as Eq. (5.24). Therefore, from the point of view of the Kähler potential, the matter sector alone can provide both sGoldstino and inflaton candidates. Examples of matter fields with a shift symmetry have been discussed in the context of D-brane inflation with D3/D7 in [116] and

\(^7\)When restricting to the fields $S$ and $\Phi$, which will generically be a subset of all fields in string-theoretic scenarios, we are assuming that this is a consistent procedure and will not address the subtleties of such truncations as pointed out in e.g. [177].

\(^8\)In this subsection the fields $\Phi$ and $S$ do not necessarily denote the inflaton and the sGoldstino; instead, their role should be clear from the context.
in the context of fluxbrane inflation with D7/D7 in [119]. These Kähler potentials can also arise for some matter fields in heterotic theory [178,179]. Moreover, the superpotential for matter fields generically turns out to be of the form\(^9\)

\[ W = \beta \sum_{n} \prod_{\alpha_n} i\Phi_{\alpha_n}, \tag{5.25} \]

where again, we take into account a likely dependence on any closed string moduli vevs into the constant \(\beta\). From the structure above we have the following properties:

- We generically expect to get the sum over several couplings for all the fields involved, including the sGoldstino, which is in contrast to the linear structure of (5.21).

- On the other hand, there is a simple case which can fit completely. If one is allowed to truncate the superpotential to only a single term in the sum over \(n\) in \(W\) above, then it is always possible to add a phase such that the superpotential has the form \(W = Sf(\Phi_i)\) with \(f\) real.

In conclusion, having matter fields alone in a configuration where some of these have a shift symmetry, it is possible, restricting to a single term in \(W\), to identify the relevant sectors needed for inflation. The detailed implementation of this model will be discussed in Sec. 5.2. Extra sectors in the configuration can be added to \(W\) so long a separation is possible, for example as it happened in [180].

The other possibility to consider is geometric closed string moduli in string theory. Generically these fields are the so-called dilaton \(S\), the complex structure moduli \(U\) and the Kähler moduli \(T\). These have a well known Kähler potential, which takes the form such as Eq. (5.16) where we denote \(\Phi = \{S, T, U\}\). Such fields do enjoy the shift symmetry (which in this case we take in the imaginary part) of \(\Phi\) but not the \(\mathbb{Z}_2\) symmetry \(\Phi \rightarrow -\Phi\). Therefore, there is no field which can be identified with the sGoldstino direction, as it was employed for orthogonal inflation (we remind the reader that the function \(K\) must allow for consistent truncation along \(S = 0\)). Turning to the superpotential:

- If we consider the shift symmetry to be broken only by non-perturbative effects, the superpotential turns out to be a function of \(\Phi\). For example,

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\(^9\)The somewhat unconventional \(i\) arises in the superpotential as a consequence of the choice of Kähler potential (5.24) depending on \(\Phi - \bar{\Phi}\) rather than \(\Phi + \bar{\Phi}\), as often considered in the string theory literature.
this is the case of $W \propto e^{-a\Phi}$ for $T$ in type IIB compactifications with fluxes considered widely in the literature.

- On the other hand, if the shift symmetry is broken at tree, *perturbative* level, then $W$ is a function of $i\Phi$. This is the case of the tree-level superpotentials for the $STU$-moduli generated via bulk, geometric and non-geometric fluxes.

In conclusion the closed string sector provides promising inflationary directions; however, the lack of $\mathbb{Z}_2$-symmetric Kähler potentials prevents the implementation of orthogonal inflation.

From the discussion above, an interesting hybrid emerges naturally: the case when both matter and closed string moduli play a role. The identifications of the fields with the relevant sectors of SUSY breaking and inflation are clear: a Kähler modulus is identified with the inflaton sector, while a matter field is identified with the sGoldstino sector. In this case generically we expect the Kähler potential to be of the form:

$$K = -3\alpha \ln \left( \Phi + \bar{\Phi} - S \bar{S} \right), \quad \text{or} \quad K = -3\alpha \ln \left( \Phi + \bar{\Phi} \right) - S \bar{S},$$

where $\Phi$ is identified with a closed string modulus, for example the Kähler modulus, and $S$ is identified with some matter field, for example a brane position. From a string theory perspective, natural values of curvature parameter $\alpha$ are of order one (relevant examples are $1, 2/3$ and $1/3$). We will discuss the details of such a model in Sec. 5.3.

### 5.2 Flat Kähler geometry and arbitrary inflation

In this section, we will discuss a number of two-superfield models of inflation with the common feature of the orthogonality between the sGoldstino directions and the inflaton. We have indeed already seen in Sec. 5.1.3 that this construction allows for remarkable flexibility. We will show the details below. In addition, we focus on flat Kähler geometry for the inflaton field $\Phi$ such that the metric takes the form

$$ds^2 = d\Phi \, d\bar{\Phi}.$$  

A useful parametrization for $K$ which avoids the $\eta$-problem and allows for the orthogonal separation between the inflaton and the sGoldstino was firstly introduced in [146]. Successfully realizing chaotic inflation in supergravity was
the original motivation of this investigation. This pioneering model consists of
\[ K = -\frac{1}{2} (\phi - \bar{\phi})^2 + S \bar{S}, \quad W = MS\phi, \quad (5.28) \]
in terms of a real constant \( M \). Inflation can be chosen to take place along \( \phi - \bar{\phi} = S = 0 \), while the remaining degree of freedom \( \text{Re}(\phi) = \phi \) has a quadratic scalar potential:
\[ V = M^2 \phi^2. \quad (5.29) \]
However, in this case the three truncated fields are not yet stabilised: while the imaginary part of \( \phi \) has a Hubble-scale mass in compliance with the \( \eta \)-problem, this is not the case for the \( S \)-field. To this end one can add a higher-order term to the Kähler potential,
\[ K = -\frac{1}{2} (\phi - \bar{\phi})^2 + S \bar{S} + \zeta (S \bar{S})^2, \quad (5.30) \]
which parametrises its curvature. The ensuing mass eigenvalues are
\[ m_{\text{Im}(\phi)}^2 = V + M^2, \quad m_S^2 = \zeta V + M^2. \quad (5.31) \]
For coefficients \( \zeta \) of order one this will indeed allow both \( \text{Im}(\phi) \) and \( S \) to be stabilised.

Subsequently, a new development has build on this model to generate other inflationary potentials in a similar manner, see e.g. [168]. This has culminated in a model by Kallosh, Linde and Rube (KLR) [169], which consists of a prescription of how to build a class of supergravity models allowing for a completely arbitrary inflaton potential \( V(\phi) \). Similar to the previous model, it consists of two complex scalar fields \( \phi \) and \( S \). The role of both fields will be identical as before; the real part \( \text{Re}(\phi) = \phi \) will be the inflaton field while \( \text{Im}(\phi) \) and \( S \) will be essential in order to stabilise the inflationary trajectory, along which such fields will vanish. However, the Kähler and superpotential are generalised to the following:
\[ K = K \left( (\phi - \bar{\phi})^2, S \bar{S}, S^2, \bar{S}^2 \right), \quad W = S f(\phi), \quad (5.32) \]
where, similarly to the previous cases, \( f(\phi) \) is an arbitrary but real holomorphic function of the variable \( \phi \).

The Kähler potential can be an arbitrary function of the arguments as indicated, and, as a consequence, it is separately invariant under the following transformations:
\[ S \rightarrow -S, \quad \phi \rightarrow -\phi, \quad \phi \rightarrow \phi + a, \quad a \in \mathbb{R}. \quad (5.33) \]
A first natural choice for $K$ is Eq. (5.30). On the other hand, the prescription Eq. (5.32) allows for more complicated functional forms, provided they satisfy Eq. (5.33). Some examples with a logarithmic $K$ are given in [169, 171]. Notice that the Kähler curvature is still flat along the inflaton trajectory $\text{Im}\Phi = 0$, although the 2-dimensional internal space may not satisfy Eq. (5.27).

Amongst the main novelties of such a model is that a completely general inflationary potential can be generated from a supergravity model. Moreover, given $K$ and $W$, one does not need to perform long calculations without knowing whether the final form of the potential will be actually suitable for inflation or not. Within this model, the form of the inflaton potential will always be

$$V(\phi) = f(\phi)^2,$$

which is a completely general positive function of $\phi$. This functional freedom is guaranteed by the symmetries of the Kähler potential $K$ and by the linearity of $W$ in $S$.

In the above derivation we have set the three fields that are not protected by the shift symmetry, i.e. $S$ and $\Phi - \bar{\Phi}$, equal to zero. The consistency of this truncation can be seen from the full mass matrix, which gives rise to the following eigenvalues:

$$m^2_{\text{Im}(\Phi)} = f^2 \left(1 - K_{\Phi \Phi S \bar{S}} - \frac{1}{2} \phi^2 \ln(f)\right),$$

$$m^2_S = -(K_{S SSS} \pm |K_{S SSS} - K_{S S}|) f^2 + (\phi f)^2.$$  

(5.35)

Thus, for suitably chosen Kähler manifolds with the right sectional curvature, the mass of these components is indeed Hubble-scale and hence they are stabilised at their origin.

### 5.3 Hyperbolic Kähler geometry and attractors

In this Section, we would like to turn to the other maximally symmetric possibility for the Kähler geometry of the inflaton field. This is the hyperbolic space of the Poincaré disc or half-plane. The metric of the unit disc reads

$$ds^2 = 3\alpha \frac{d\Psi d\bar{\Psi}}{(1 - \Psi \bar{\Psi})^2},$$

(5.36)

defined for $\Psi \bar{\Psi} < 1$. The usual Kähler potential associated with this space is

$$K = -3\alpha \ln \left(1 - \Psi \bar{\Psi}\right).$$

(5.37)
Its curvature is given by

\[ R_K = -K^{-1}_{\bar{\psi}} \partial_{\psi} \partial_{\bar{\psi}} \ln K_{\psi \bar{\psi}} = -\frac{2}{3\alpha}, \tag{5.38} \]

that is constant and negative as expected (this space having hyperbolic geometry) and depending just on the parameter \( \alpha \).

It is possible to go to the half plane representation of this geometry\(^\text{10}\) by means of a change of variables such as

\[ \Phi = \frac{1 + \Psi}{1 - \Psi}, \tag{5.39} \]

which leads to the metric

\[ ds^2 = 3\alpha \frac{d\Phi \, d\bar{\Phi}}{(\Phi + \bar{\Phi})^2}, \tag{5.40} \]

defined for \( \Phi + \bar{\Phi} > 0 \). The corresponding Kähler potential for this space is the one already given in Eq. (5.16). The curvature, being an invariant, remains the same as in Eq. (5.38). An interesting analysis regarding the properties of such a geometry, its possible representations and connections to physics is performed in [181].

Throughout the following, we will mainly make use of the half-plane co-ordinates \( \Phi \) as a matter of convenience. We will present supergravity models of inflation that admit consistent truncation at \( \Phi = \bar{\Phi} \). Along this line, the relation between the geometric field and the canonical normalized field \( \varphi \) is

\[ \Phi = \bar{\Phi} = e^{-\sqrt{\frac{2}{3\alpha}} \varphi}. \tag{5.41} \]

This relation simply reflects the dramatic effects of the non-trivial geometry of the hyperbolic Kähler manifold. This indeed induces a boundary in moduli space (located at \( \Phi = 0 \)) where the theory attains a conformal [182] or a scale symmetry [183] (depending on the parameter \( \alpha \)). Inflation takes place as the inflaton moves away from this boundary, leading to universal cosmological predictions in terms of \( n_s \) and \( r \). In canonical coordinates, the boundary is indeed pushed at \( \varphi = \infty \). Then, any generic expansion around this boundary often corresponds to a scalar potential which is an exponential fall-off from de Sitter such as

\[ V = V_0 + V_1 \, e^{-\sqrt{\frac{2}{3\alpha}} \varphi} + \ldots, \tag{5.42} \]

\(^{10}\) The difference between the unit disk representation and the one in terms of the half plane coordinates can be visually appreciated, respectively, in the picture of the front cover and the bookmark of this thesis.
where the dots represents subleading terms, irrelevant for values of $\alpha$ of order one. This expansion automatically yields universal values for the spectral index and tensor to scalar ratio, which read

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{12\alpha}{N^2},$$

(5.43)

at large values of the number of e-folds $N$. The predictions (5.43) provide an excellent fit with the latest Planck data and, for specific values of $\alpha$, simply correspond to the ones of the Starobinky model [184] (together with its supergravity implementations [170–172, 185–187]) and Higgs inflation [188].

Remarkably, the Kähler curvature (5.38) plays a fundamental role in this construction and, essentially, becomes a measurement of the amount of primordial gravitational waves we are currently looking for in the sky [92, 173, 189] (the previous works [145, 172] already pointed out how sensitive $r$ is to the curvature $R_K$ and, then, to value of $\alpha$). As $\alpha$ varies from infinity (i.e. flat curvature) to order one or smaller, the inflationary predictions go from completely arbitrary (in the flat case) to the very specific values above\(^\text{11}\). Turning on the curvature therefore “pulls” all inflationary models into the Planck dome in the $(n_s, r)$ plane, similarly to what is shown in Fig. 5.1.

The phenomenon described above, which intimately relates geometric properties of the hyperbolic Kähler manifold to universal observational predictions, was first discovered in [92, 173, 182] and it is referred to as $\alpha$-attactors. However, some working examples were already found in previous investigations such as [145, 170–172], where simple and natural choices of the superpotential (such as monomial or polynomial forms) lead to an inflationary regime such as the one of Eq. (5.42). The common feature of all these supergravity constructions is the complete orthogonality between the sGoldstino $S$ and the inflaton $\Phi$. This is indeed very useful in order to kill the negative term in the scalar potential Eq. (5.3). In the context of orthogonal inflation, the first analysis with varying Kähler curvature and general superpotential $W$ was performed in [145].

It is important to note that the final result may be different depending on how the sGoldstino enters the Kähler potential. The field $S$ may have a simple canonical Kähler or appear inside the argument of the logarithm. We have already outlined these two common choices in Eq. (5.26) and we will present the details in the following subsections.

\(^{11}\)Similar attractor behaviour has been noticed also in the context of models of inflation with non-minimal coupling to gravity [91] (see also [190]). Interestingly, one can find a common origin for the attractor phenomenon observed in both contexts. This is due to a pole of order two in the kinetic term of the inflaton field [93] (see also [191, 192])
Hyperbolic Kähler geometries as well as non-minimal couplings to gravity or non-canonical kinetic terms generically induce an attractor for observations. The specific details of the model (in this case, we show monomial chaotic models of inflation) get washed out and the observational predictions all converge to the “attractor point” denoted by the yellow star. This evolution is normally regulated by a specific parameter of the model, in the supergravity case being the value of the Kähler curvature.

5.3.1 Model with $K = -3\alpha \ln(\Phi + \bar{\Phi} - S\bar{S})$

Here, we follow the analysis done in [145]. This supergravity construction is characterized by the following choices for the Kähler potential and superpotential:

$$K = -3\alpha \ln(\Phi + \bar{\Phi} - S\bar{S}), \quad W = Sf(\Phi),$$  \hspace{1cm} (5.44)

where $f$ is still a real holomorphic function of its argument and the corresponding Kähler manifold is $SU(2,1)/U(2)$. This model allows for a consistent truncation to the inflationary trajectory at $\text{Im}(\Phi) = S = 0$\textsuperscript{12}. However, this does not imply that this truncation is also stable. For this, one needs to consider the mass spectrum of such fields. In order for effective single-field behaviour, these will have to be super-Hubble. We will later check, in explicit examples, to what extent this condition can be met. Moreover, in the case of Eq. (5.44), the shift symmetry is enhanced to the three-dimensional

\textsuperscript{12}Note that this is valid more in general for any Kähler potentials of the form $K = K(\Phi + \bar{\Phi}, SS, S^2, \bar{S}^2)$
Heisenberg group\textsuperscript{13}, which acts in the following way
\[ \Phi \to \Phi + ia + \overline{b}S + \frac{1}{2}|b|^2, \quad S \to S + b, \quad a \in \mathbb{R}, b \in \mathbb{C}. \] (5.45)

The scalar potential as function of the fields $\Phi$ and $S$ reads
\[ V = \frac{|S|^2}{3\alpha} \left( X^{1-3\alpha}|S|^2 + X^{2-3\alpha} \right) \left| f' + \frac{3\alpha f}{X} \right|^2 + \frac{X^{1-3\alpha}}{3\alpha} |f|^2 \left( 1 + \frac{3\alpha|S|^2}{X} \right)^2 \]
\[ + \frac{X^{1-3\alpha}}{3\alpha} |S|^2 \left( 1 + \frac{3\alpha|S|^2}{X} \right) \left[ \tilde{f} \left( f' + \frac{3\alpha f}{X} \right) + f \left( \tilde{f}' + \frac{3\alpha \tilde{f}}{X} \right) \right] \]
\[ -3 \frac{|S|^2|f|^2}{X^{3\alpha}}, \] (5.46)

where, for convenience of notation, we have defined a compact variable $X$ such as
\[ X \equiv \Phi + \overline{\Phi} - SS. \] (5.47)

Thus at $S = 0$ the potential is simply
\[ V = \frac{X^{1-3\alpha}|f|^2}{3\alpha}. \] (5.48)

In terms of real and imaginary parts for $\Phi$ and the field $S$, the masses of the fields read:
\[ m^2_{\text{Re}(\Phi)} = \frac{2}{3\alpha} \left[ X^{1-3\alpha} \left( 3\alpha - 2 + \frac{1}{3\alpha} \right) f^2 + X^{2-3\alpha} \left( \frac{1}{\alpha} - 2 \right) ff' + \frac{X^{3-3\alpha}}{6\alpha} (ff'' + f'^2) \right], \]
\[ m^2_{\text{Im}(\Phi)} = \frac{2}{3\alpha} \left[ X^{1-3\alpha} \left( 1 - \frac{1}{3\alpha} \right) f^2 - \frac{X^{2-3\alpha}}{3\alpha} ff' + \frac{X^{3-3\alpha}}{6\alpha} (f'^2 - ff'') \right], \]
\[ m^2_{S} = \frac{X^{1-3\alpha}}{3\alpha} \left( 3\alpha - 2 - \frac{1}{3\alpha} \right) f^2 + \frac{X^{2-3\alpha}}{3\alpha} \left( \frac{2}{3\alpha} - 2 \right) ff' + \frac{X^{3-3\alpha}}{9\alpha^2} f'^2. \] (5.49)

Clearly, for a successful single-field inflationary model, the first of these has to be light whereas the latter three degrees of freedom need to be stabilised, either around or above the Hubble scale.

\textsuperscript{13}The symmetry of the Heisenberg invariant Kähler potential has also been employed to solve the $\eta$-problem in [193, 194]. However, that scenario differs in an important way from the present: in that case, the field $S$ is identified as the inflaton, whereas a third superfield is added to play the role of the sGoldstino. The inflationary predictions of that set-up are thus unrelated to ours.
Example with $\alpha = 1$

A first interesting example of the model above is given by the Cecotti model of inflation\(^{14}\). This is found for $\alpha = 1$ and the following potentials:

\[
K = -3 \log \left( \Phi + \bar{\Phi} - S \bar{S} \right), \quad W = 3 MS(\Phi - 1).
\]

(5.50)

In terms of a canonically normalised scalar field $\varphi$, this yields the scalar potential

\[
V = \frac{3}{4} M^2 \left( 1 - e^{-\sqrt{2} \varphi / \sqrt{3}} \right)^2.
\]

(5.51)

Inflation takes place at large $\varphi$. In this limit the three masses become

\[
m^2 = \{0, 4H^2, -2H^2\},
\]

(5.52)

where $H^2 = V/3$. This has been demonstrated to be equivalent to Starobinsky’s $R + R^2$ model of inflation [184]. The relations of this model to superconformal supergravity have been discussed in [171]. In this reference it was also been pointed out that the $S$ field is not stable with this Kähler choice; to this end one could add a stabilising term $\beta (S \bar{S})^2 / (\Phi + \bar{\Phi})$ to the argument of the logarithm leading to

\[
m^2_S = (-2 + 4\beta) H^2,
\]

(5.53)

which is finite along the whole inflationary trajectory and positive for an appropriate choice of $\beta$. Instead, we find that the imaginary part of $\Phi$ is stable with the present Kähler potential and hence poses no problems for inflation.

Example with $\alpha = 1/3$

A generalisation of previous example arises when one allows for a different curvature of the Kähler manifold, i.e. including the parameter $\alpha$. For simplicity we will keep the same superpotential. Following the same line of reasoning, one ends up with a scalar potential for a canonically normalised scalar field $\varphi$ that reads

\[
V = \frac{2^{1-3\alpha}}{\alpha} \left[ e^{(3-3\alpha)\varphi / \sqrt{6\alpha}} - e^{(1-3\alpha)\varphi / \sqrt{6\alpha}} \right]^2.
\]

(5.54)

\(^{14}\) A similar set-up with an identical Kähler potential (5.50) was also used to embed the Starobinsky model in supergravity [185]. However, in that case the inflaton was identified with one of the directions of $S$, while the $\Phi$ field was argued to be stabilised by other means.
For generic values of $\alpha$ this will lead to an exponential potential for large $|\varphi|$. There are only two exceptions to this behaviour: the first is for $\alpha = 1$ discussed above, while the second is for $\alpha = 1/3$. Interestingly, this value is also consistent with string theory and leads to inflation for large and negative $\varphi$. The scalar potential becomes

$$V = 9M^2 \left(e^{\sqrt{2} \varphi} - 1\right)^2,$$

and thus can be obtained from the Starobinsky potential by a sign flip and stretching in the $\varphi$ direction. Nevertheless, in this case one needs to stabilise even along the $\text{Im}\Phi$ direction as the three masses (5.49) asymptote to

$$m^2 = \{0, 0, -6H^2\}.$$  

Having a viable single-field scenario translates into adding a term $-\gamma S\bar{S}(\Phi - \bar{\Phi})^2/(\Phi + \bar{\Phi})^2$, together with the same term stabilising $S$ in the case $\alpha = 1$, to the argument of the logarithm. With these choices, the mass spectrum turns to be finite along the inflaton direction and takes the following values:

$$m^2 = \{0, 12\gamma H^2, (-6 + 12\beta)H^2\}.$$  

Interestingly, a value of $\beta > 1/2$ leads to positive mass of the field $S$, independently of the parameter $\alpha$. Moreover, this model leads to the following spectral index and tensor-to-scalar ratio for different numbers of e-foldings:

$$N = 50 : \quad n_s = 0.961, \quad r = 0.0015,$$

$$N = 60 : \quad n_s = 0.967, \quad r = 0.0011.$$  

Comparable to Starobinsky’s, these are also comfortably consistent with the Planck results.

**Original version of $\alpha$-attractors**

From the previous example, it is clear that one cannot allow for arbitrary values of the parameter $\alpha$ by keeping the simple superpotential as in Eq. (5.50). Dangerous exponential terms may indeed ruin the flatness of the scalar potential and spoil inflation. The solution to this problem was found by Kallosh, Linde and Roest in the context of superconformal $\alpha$-attractors [92]. It consists in allowing for an $\alpha$-dependence of the superpotential such as

$$W = S\Phi^{(3\alpha - 1)/2} f(\Phi),$$

while still keeping the same $K$ as in Eq. (5.50). Then, any generic expansion of $f$ around $\Phi = 0$ translates into a scalar potential being an exponential
fall-off from a de Sitter plateau in terms of $\varphi$ such as Eq. (5.42), typical of $\alpha$-attractors.

The original version of $\alpha$-attractors [92] was formulated in terms of disk coordinates $\Psi$ and stability was proved for any value of the curvature. Later, it was shown in [173,195] that the same physics can be described by means of half-plane coordinates $\Phi$.

5.3.2 Model with $K = -3\alpha \ln(\Phi + \bar{\Phi}) - S\bar{S}$

A remarkable simplification arises when the field $S$ enters the Kähler potential as a simple canonical sector. The framework defined by

$$ K = -3\alpha \ln \left( \Phi + \bar{\Phi} \right) - S\bar{S}, \quad W = S\Phi^{3\alpha/2}f(\Phi). \quad (5.60) $$

yields indeed always a scalar potential which attains a plateau at infinite values of the canonical inflaton $\varphi$ and has exponential drop-off at finite values. This happens at $\Phi = \bar{\Phi}$ and for any generic expansion of the function $f$ in the superpotential. The curious power of $\Phi$ in Eq. (5.59) becomes an overall factor which can be gauged away by means of a Kähler transformation thus yielding a Kähler potential which is invariant under a shift of the canonical inflaton [196]. Interestingly this case, where the Kähler (5.60) parametrizes a manifold $SU(2,1)/U(1) \times U(1)$, leads generically to an improved stability of the system [197] (see also Ch. 6). It was first analyzed in full generality by [95,174,175] (the case $\alpha = 1$ was previously investigated by [176]). The field $S$ has canonical kinetic terms and its directions may be stabilized by means of higher order terms in the Kähler potential. Alternatively it may be identified as a nilpotent superfield. We will discuss this latter case in the next Chapter.

5.4 $\alpha$-Scale supergravity and attractors

In the previous sections, we have studied the stringent implications of the geometric properties of the internal Kähler manifold on the physics of inflation. Specifically, we have shown how a non-trivial hyperbolic geometry yields dire observational consequences and forces the cosmological parameters $n_s$ and $r$ to converge towards the universal values (5.43). However, we must note that all the models presented above (including the original formulation of $\alpha$-attractors [92,173]) employ the orthogonal separation between the inflationary and the supersymmetry breaking directions. Then, one would like to answer a very natural question: is the attractor phenomenon independent from the field responsible for supersymmetry breaking? Answering
5.4 $\alpha$-Scale supergravity and attractors

this question is certainly very important in order to prove the generality of the attractor mechanism, independently of the other fields involved. This is a fundamental step towards a proper string theory realization where many moduli appear naturally.

In this following we present evidence for the universality of $\alpha$-attractors: while the previous models contain two chiral supermultiplets and employ a separation between the inflaton and the sGoldstino, we demonstrate that the same phenomenon can be achieved in a model containing just one superfield. The economical framework of realizing inflation in single-superfield models has been discussed in [153, 154, 157–161, 198], but these do not include a variable Kähler geometry and hence lack the parameter $\alpha$.

Our construction also highlights a novel approach to Minkowski and de Sitter model building. Whereas the classic no-scale supergravity [165–167] yields two flat Minkowski directions, one of these can be lifted to a stable direction by deforming the Kähler curvature and allowing for a more general monomial dependence of the superpotential. Interestingly, a combination of these structures leads to a De Sitter plateau. This turns out to be stable only for such $\alpha$-deformed supergravities with a smaller Kähler curvature than the one corresponding to a combination of no-scale constructions. Remarkably, generic deformations of these De Sitter plateaus lead to inflationary regimes with prediction (5.43). This sheds light on the fundamental origin of the attractor phenomenon.

Finally, analogous results emerge in the singular limit $\alpha \to \infty$ where the Kähler geometry becomes flat. However, in this case the natural ingredients providing Minkowski or dS solutions and inflationary deformations will be exponentials, peculiar to this geometry.

5.4.1 No-scale supergravity and de Sitter

Our starting point will be the no-scale structure for a supergravity with a single chiral superfield. In this case the Kähler potential reads

$$K = -3 \ln \left( \Phi + \bar{\Phi} \right), \quad (5.61)$$

describing a manifold SU(1,1)/U(1) and invariant under a shift of Im($\Phi$), while the superpotential is independent of the superfield and hence constant. This model is characterized by a Minkowski vacuum in any point in field space, as it is shown in Fig. 5.2. The negative definite contribution to the scalar potential, proportional to the square of the superpotential, is cancelled by the positive definite term, proportional to the square of the order
parameter of supersymmetry breaking

\[ D_\Phi W = \partial_\Phi W + K_\Phi W. \]  

(5.62)

Note that only the latter term of this contribution is non-vanishing due to the constancy of the superpotential. The resulting no-scale model necessarily has a flat direction along the imaginary part of \( \Phi \), as this does not appear in either \( K \) or \( W \).

By a field redefinition, one can bring this simple no-scale model to a different form. In particular, in order to leave the Kähler potential invariant, one can combine an inversion of the holomorphic field \( \Phi \) with a specific Kähler transformation, defined as in Eq. (5.6), with \( \lambda = -3 \ln \Phi \) in this case. Under these transformations, a constant superpotential becomes cubic instead. While this model has the same scalar potential and is therefore also of the no-scale type, it receives contributions from both terms in \( D_\Phi W \).

Remarkably, one can combine the constant and cubic superpotentials to move away from no-scale models and generate a non-vanishing cosmological constant. In particular, the superpotential

\[ W = 1 - \Phi^3, \]  

(5.63)

leads to a scalar potential with a flat direction along \( \Phi = \bar{\Phi} \), where the original Minkowski vacuum is shifted to \( V = \frac{3}{2} \) (while the combination with opposite sign leads to AdS), as one can see in Fig. 5.3. However, this De Sitter solution turns out to be unstable: the mass of the imaginary direction is given by \( m_{\text{Im}\Phi}^2 = -2 \).
5.4 α-Scale supergravity and attractors

5.4.2 α-Scale supergravity and stable de Sitter

In order to improve on the previous instability, we will consider the logarithmic Kähler potential of the form\(^{15}\)

\[
K = -3\alpha \ln \left( \Phi + \bar{\Phi} \right). \tag{5.64}
\]

This still parametrizes a symmetric geometry \(SU(1,1)/U(1)\), whose curvature is given by Eq. (5.38).

A single monomial superpotential \(W = \Phi^n\) will give a scalar potential equal to

\[
V = \frac{8^{-\alpha} [(2n - 3\alpha)^2 - 9\alpha]}{3\alpha} \Phi^{2n - 3\alpha}, \tag{5.65}
\]

along the real direction \(\Phi = \bar{\Phi}\). Note that a constant potential corresponds to \(2n = 3\alpha\) which, for any value of \(\alpha\), leads always to AdS [172]. In contrast, a vanishing scalar potential corresponds to one of the following solutions

\[
n_{\pm} = \frac{3}{2} \left( \alpha \pm \sqrt{\alpha} \right), \tag{5.66}
\]

displayed in Fig. 5.4. These are the counterparts of the constant and cubic superpotentials of the previous section, corresponding to \(\alpha = 1\). We will refer to the above model as \(\alpha\)-scale supergravities for the following reason.

\(^{15}\)We already presented this form of \(K\) in Eq. (5.16), in the context of sGoldstino inflation. We explicitly show this again for a matter of convenience.
Similarly to the standard no-scale model, the real part of \( \Phi \) has flat direction. On the other hand, the mass of the imaginary part gets a dependence on the field and, along \( \text{Im} \Phi = 0 \), reads
\[
m^2_{\text{Im} \Phi} = \frac{2^{2-3\alpha}(\alpha - 1)}{\alpha} e^{-\sqrt{6}\varphi},
\]
where the sign of the power depends on the choice of one of the solutions (6.50). This result assures stability of the Minkowski vacuum for \( \alpha \geq 1 \) [199].

\[
\text{Figure 5.4}
\text{The monomial powers } n_{\pm} \text{ for the } \alpha \text{-scale models as function of } \alpha.
\]

Following the previous construction, one obtains a de Sitter plateau along the real direction by considering a pair of monomials,
\[
W = \Phi^{n_-} - \Phi^{n_+}, \quad V = 3 \cdot 2^{2-3\alpha}.
\]
While this generically leads to terms with irrational powers, these are integers when \( \alpha \) is a perfect square. Moreover, the more general choice with \( 9\alpha \) a perfect square yields still integer powers multiplied by an overall phase which can be gauged away by means of a Kähler transformation.

Remarkably, unlike the standard case \( \alpha = 1 \), the mass of \( \text{Im} \Phi \) gets also some field dependent contributions and, along the real axis, reads
\[
m^2_{\text{Im} \Phi} = -\frac{4V}{3\alpha} \left[ 1 - (\alpha - 1) \sinh^2 \left( \frac{3}{2}\varphi \right) \right] .
\]
Such a solution for the mass of the imaginary component allows to identify regions of stable de Sitter vacua, as one can appreciate in Fig. 5.5. In particular, for \( \alpha > 1 \), the field dependent terms dominate in the limit of large \( |\varphi| \), leading to a positive mass for the imaginary component. Just a small region around the symmetric point \( (\varphi = 0) \) leads to an instability, which disappears in the limit \( \alpha \to \infty \).
5.4 \(\alpha\)-Scale supergravity and attractors

5.4.3 Single superfield \(\alpha\)-attractors

Once we know how to construct de Sitter in this context, we can add corrections to the superpotential (5.68) in order to reproduce a consistent inflationary dynamics. Deviations from the positive plateau are given by higher powers \(n\) of \(\Phi\) \((n_- < n_+ < n)\). In full generality, one can consider a deformation of the form

\[
W = \Phi^{n_-} - \Phi^{n_+} F(\Phi),
\]

with \(F\) being a general function with an expansion \(F(\Phi) = \sum_n c_n \Phi^n\). The corresponding scalar potential, in terms of the geometric field \(\Phi\), reads

\[
V = \frac{2^{2-3\alpha} (\Phi F'(\Phi) + 3\sqrt{\alpha} F(\Phi)) \left(\Phi^{3\sqrt{\alpha}+1} F'(\Phi) + 3\sqrt{\alpha}\right)}{3\alpha},
\]

along the real axis, where primes denote derivatives with respect to \(\Phi\). In the inflationary regime, close to \(\Phi = 0\), only the first non-constant term is relevant: the scalar potential approximates an exponential fall-off from a de Sitter plateau such as Eq. (5.42) at large values of the canonical field \(\varphi\).

The inflationary scenario emerging from this construction is therefore the one typical of the \(\alpha\)-attractors: the Kähler geometry, described by eq. (5.64), determines unequivocally the observational predictions which, on the other hand, will be insensitive to specific changes in the superpotential. Moreover, the predicted values for the spectral tilt and tensor-to-scalar ratio are (5.43), in the limit of large number of e-foldings \(N\).
To demonstrate the stability and vacuum structure with an explicit example, we take

\[ F(x) = 1 + 3\sqrt{\alpha} - 3\sqrt{\alpha}x. \]  

(5.72)

These coefficients have been chosen to have a quadratic expansion around the Minkowski minimum at \( \Phi = 1 \). Both the scalar potential along the real axis as well as the mass of the imaginary direction is shown in Fig. 5.6 for different values of \( \alpha \). This model is fully stable for \( \alpha > 1 \) while the Kähler curvature leads to an instability along the imaginary direction when \( \alpha \leq 1 \). Finally, its observational predictions superimposed on the confidence levels released by Planck2015 [13] are given in Fig. 5.7. These interpolate between the \( \alpha \)-attractor values (3.55) and (3.56), and those of a linear scalar potential.

The above approach leads to a supersymmetric Minkowski minimum. Uplifting this vacuum by means of supersymmetry breaking to include a non-zero cosmological constant is strongly constrained [200]: generically this cannot be done with a small deformation and, within one single superfield,
leads to an undesirable large gravitino mass \([160]\). An additional nilpotent sector can elegantly solve the issue of the separation of the physical scales \([95, 162, 175, 198, 201–203]\). Nevertheless, as the two sectors prove to be independent from each other and play distinct roles \([95]\), it remains fundamental to construct a consistent inflationary dynamics in a single superfield context.

### 5.4.4 Flat Kähler limit

In the singular limit \(\alpha \to \infty\) the Kähler geometry becomes flat. One could wonder whether there is a similar \(\alpha\)-scale model as well as de Sitter uplift in this limit. Indeed this is the case: upon a field redefinition \(\Phi \to \exp(2\Phi/\sqrt{3\alpha})\) and a Kähler transformation with \(\lambda = \frac{3}{2} \alpha \ln 2 + \sqrt{3\alpha} \Phi\), the Kähler potential \((5.64)\) yields

\[
K = -\frac{1}{2} \left( \Phi - \bar{\Phi} \right)^2,
\]

in the singular limit \(\alpha \to \infty\). Note that \(K\) has become shift-symmetric in the inflaton field \(\text{Re} \Phi\) \([146]\). This naturally provides a solution to the so-called \(\eta\)-problem \([144]\), whereas, for finite values of \(\alpha\), the latter is mitigated by the logarithmic form \((5.64)\) \([145]\).

Under the same operations, the monomial superpotential turns into (mod-
Inflation and Attractors in Supergravity

One can check that this leads to a vanishing scalar potential along the line \( \Phi = \bar{\Phi} \). Moreover, a linear combination of the two exponentials, such as \( W = \sinh(\sqrt{3} \Phi) \), leads to a constant and positive value of \( V \) (while a \( \cosh \), instead, leads to AdS). The mass of the orthogonal imaginary component of \( \Phi \) is equal to the \( \alpha \to \infty \) limit of (5.69).

The above construction can be perturbed to have deviations from de Sitter and produce a consistent inflationary dynamics. A first guess could be to include the same deformation in the polynomial (5.70) and take the \( \alpha \to \infty \) limit. However, in this case the field dependence of this function is washed out: for finite values of the constants \( c_n \), the resulting superpotential reads

\[
W = e^{\sqrt{3} \Phi} - e^{-\sqrt{3} \Phi} F(1),
\]

(5.75)

leading to a constant scalar potential.

A more natural possibility, given the exponential ingredients of the above superpotential, would be to take

\[
W = e^{\sqrt{3} \Phi} - e^{-\sqrt{3} \Phi} F \left( e^{-2\Phi/\sqrt{3} \alpha'} \right),
\]

(5.76)

where we have parametrized the additional dependence in terms of a new parameter \( \alpha' \). Remarkably, when truncating to the real axis, the scalar potential arising from this supergravity model with a flat Kähler geometry is identical to (5.71) of the supergravity model with a curved Kähler geometry, provided one identifies \( \alpha = \alpha' \). The specific choice (5.72) for \( F \) in this case leads to the identical predictions of Fig. 3; however, interestingly, this model proves to be stable for any positive value of \( \alpha \).

It therefore turns out to be possible to represent the same single-field inflationary potential by means of curved or flat Kähler geometry. Only the former has the attractive interpretation of the robustness of \( \alpha \)-attractors arising from a non-trivial Kähler geometry; the same dynamics arises in the flat case by the peculiar non-polynomial form of \( W \).

The first example of such a model \([153, 154, 161]\) fits perfectly into the recipe given above: it has a superpotential

\[
W = \sinh(\sqrt{3} \Phi) \tanh(\sqrt{3} \Phi),
\]

(5.77)

corresponding to the choice \( F(x) = (3 - x)/(1 + x) \) for the case of \( \alpha' = 1/9 \). The same inflationary potential can also be embedded in a logarithmic Kähler structure \([198]\).
5.4.5 Discussion

In this Section we have outlined a strikingly simple route to construct single superfield models with stable de Sitter solutions. Generic deformations of these models yield an inflationary trajectory fully consistent with Planck. The key quantity in this set of models, similar to the original $\alpha$-attractors, is the curvature of the Kähler manifold (3.56). This quantity determines both the (in)stability of such constructions as well as the inflationary predictions of the deformed models.

Remarkably, this provides a realization of $\alpha$-attractors employing a single superfield, in contrast to the two-field model of [92, 173]. This suggests that the phenomenon of Kähler curvature leading to the inflationary predictions (3.55) and (3.56) is universal, and applies to a much larger set of Kähler geometries than $SU(1,n)/U(n)$ with $n = 1, 2$.

Given the prominence of no-scale models in the literature, it would be interesting to study other possible applications of the $\alpha$-generalization reviewed in this Section and originally proposed in [94]. An example could be the no-scale inflationary constructions of [172, 176, 185, 204, 205]. Moreover, while we have focused on single superfield models, it is straightforward to generalize this construction to multi-fields:

\[ K = \sum_i -3\alpha_i \log(\Phi_i + \bar{\Phi}_i), \quad W = \prod_i \Phi_i^{n_i}, \quad (5.78) \]

where we have suppressed other fields with a different dependence. The condition for Minkowski is

\[ \sum_i \frac{(2n_i - 3\alpha_i)^2}{3\alpha_i} = 3. \quad (5.79) \]

Remarkably, also in the multi-field case, the interference of superpotential terms with flat Minkowski vacua leads to a de Sitter phase, proving the generality of such a feature. It would be very interesting to investigate the stability and inflationary aspects of such constructions.

Finally, our construction invites investigations of string theory scenarios leading to (5.78). Many moduli contribute with a factor $\alpha_i = 1/3$ to the Kähler potential, while flux compactifications yield polynomial contributions to the superpotential. It would be of utmost interest to realize this in a concrete setting.