Chapter 8

Concluding remarks

This final chapter consists of three parts. In the first section, we globally summarize the contents of this thesis and restate the major conclusions of this work. Next, we give a list of contributions of this thesis. We end with some suggestions for further research on signal sampling for data acquisition in process control.

8.1 Summary and conclusions

We observed that the standard method of conversion in A/D-converters is based upon Shannon’s sampling theorem. However, the conditions of this theorem cannot be met in practice. This was our motivation to search for methods for A/D- and D/A-conversion for which a similar theorem holds, but whose conditions can be satisfied in practical devices. We concentrated on sampling and reconstruction; quantization was not taken into account.

The sampling and reconstruction problem was formulated in an abstract context: Signals were considered as elements of a vector space, and sampling plus reconstruction, considered as a single operation, was shown to be a projection on a certain subspace of the signal space. From this point of view, the quality of a certain sampling method only depends on the dimension of the subspace, and its location in the signal space. A particular class of vector spaces, viz.; the reproducing kernel Hilbert spaces, proved to be a useful framework to derive sampling theorems. Shannon’s sampling theorem, as well as many of its extensions are covered by this framework. This abstraction is, however, very general and must be restricted to yield useful sampling and reconstruction operators.

Based on three requirements from process data acquisition, i.e., linearity, time-invariance and equidistant evaluation, the set of suitable operators was reduced. All operations satisfying these requirements have a particular structure: the general sampling scheme (GSS). This scheme consists of a linear anti-alias filter (pre-filter), an ideal sampler operating at equidistant times and a reconstruction filter (post-filter). The filters can be chosen freely, but must be carefully selected to ensure good approximation by an element of the subspace.

The recently developed multiresolution analysis (MRA) served as a framework to find filters that constitute orthogonal projections on certain subspaces. Orthogonal projection yields a best
approximation. Shannon’s sampling theorem was generalized in terms of MRA. Moreover, we formulated alternative sampling theorems based on Daubechies’ scaling functions and on spline-functions. Both fit into the general sampling scheme and have the orthogonal projection property.

The approximation quality of different schemes was experimentally evaluated for both synthetic and real-life test signals. An overall result was that the schemes with the orthogonal projection property ($L^2$-optimal schemes) have superior approximation quality for all norms. It also turned out that Daubechies’ and spline scaling functions have approximation qualities comparable to that of the sinc-function used in Shannon’s theorem. Moreover, due to faster decay of these functions, the truncation effects are less severe. Hence, there is no need to put much effort in approximation of the ideal filter; the other $L^2$-optimal schemes can produce similar results.

Sampling and reconstruction operations also occur in digital control systems. We embedded the GSS in a closed loop control system and studied the effects of different filters on the time-domain performance of the overall system. The standard situation in control systems is characterized by no pre-filter and a zero-order hold reconstruction method. This corresponds in the GSS to B₀-spline post-filtering. The experiments revealed that, compared to this standard situation, all other filters affected the performance in a negative way. The major reason for this is the extra delay caused by these filters. In other words, early knowledge of the process state is more important than having an optimal representation of it at a later time. However, in a noisy environment anti-alias filtering has to be provided in the control loop. In this case, the B₀-spline pre-filter turned out to be a good alternative to the traditionally used first-order lag filter.

We investigated the hardware-implementability of the GSS. Daubechies filters can only be implemented by digital hardware. This implies that the scheme is to be surrounded by traditional converters operating at high rates. This causes a major drawback with respect to the costs of a circuit, but this may be paid off by reductions in storage/transmission bandwidth due to a better (less redundant) representation. The sampling scheme with spline filters of low-order can be realized in analog circuitry. We presented the design of an A/D-converter based on B₀-spline pre-filtering: the local integral sampling method. The quality of the acquired data, which has been evaluated based on measurements taken by a hardware prototype, was satisfactory.

A particular application of advanced sampled data processing was found in gas-liquid-chromatography (GLC). Here signal reconstruction is not carried out explicitly. However, the general sampling scheme with a B₂-spline post-filter offered comprehensive representation, which considerably facilitates the processing of chromatograms. We developed a generic, GSS-based software package for the processing of chromatographic data based on this representation, which does not contain explicit assumptions about signal structure. It can compete with dedicated software, and can easily be adapted to process other signal types.

### 8.2 Contributions of this thesis

- Sampling and reconstruction have been generalized as an approximation operator. We have shown that all sampling and reconstruction operators, that satisfy the three conditions (viz.; linearity, time-invariance and equidistant evaluation), are covered by the general sampling scheme. Every element of the signal space is approximated by an element of
a subspace. Such a subspace is completely determined by the post-filter in the general sampling scheme.

- We have shown the relation between the general sampling scheme and multiresolution analysis, and concluded that any scaling function of an MRA can be used as a filter in the GSS. This fact was recently confirmed by several others [Wal92, AU92, JS94].

- The general sampling scheme was tested with different filters. We are not aware of any such comparison in the literature.

- We introduced an elegant analysis method for Multirate Sampled Data Systems, which fully takes place in the Z-domain. The analysis can be considered as the discrete-time equivalent of the method for Sampled Data Systems [FPW90].

- We have justified the statement that, in general, better signal reconstruction of optimal sampling and reconstruction schemes does not contribute to better performance of a control system.

- We have shown that the local integral sampling method, which has been applied in process measurement for a long time, is a member of a family with optimal properties. It is relatively easy to implement using VFC devices. This implementation has advantages over sampling with point measurements. Because of the inherent filtering provided by this scheme, extra anti-alias filtering is not necessary.

- The methods developed in this thesis may also be useful for general signal representation problems. We have shown that a representation in terms of B-splines allows efficient processing of gas-chromatographical data. This also holds for other signal processing applications which involve integration or differentiation of signals.

## 8.3 Suggestions for further research

- We have seen that, for certain signals, the GSS with spline and Daubechies’ filters yields results comparable to those of Shannon’s scheme. Recall that the GSS constitutes a projection on a subspace determined by the post-filter. Hence, for the signals that have been tested, the distance to each of the subspaces was almost the same. However, this does not necessarily hold for different kinds of signals. Therefore, for many application cases it may be necessary to establish which of the subspaces fits the type of signals involved best.

- We mentioned that the GSS with the Daubechies’ filters must be implemented in digital hardware. This involves extra traditional converters operating at high rates. Whether or not this is feasible, must further be investigated.

- So far, quantization has not been dealt with explicitly. However, estimates of the quantization error are known for Shannon’s sampling [Jer77, BSS88]. In order to complete the comparison, error estimates of the other schemes must be derived. On the other hand, in all experiments quantization was inevitable, and consequently the results include quantization errors. We, therefore, believe that the conclusions drawn will not have to be altered
dramatically if the data are quantized using a smaller number of bits. In order to justify this statement, the GSS simulations should be repeated for such an explicit quantization using a fixed number of bits.

- Both Shannon’s sampling theorem as well as GSS’s with $L^2$-optimal filters are embedded in multiresolution analysis. In fact, the sampling theorems are obtained by omission of the fine scales in the MRA decomposition. The overall contents of these fine scales constitute exactly the aliasing error. Having this decomposition of the aliasing error, one could use information on the fine scales if this error grows too large. This would turn the scheme into an adaptive one. The usefulness of such an approach still needs to be investigated.