Statistics and dynamics of the perturbed universe
Lemson, Gerard

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Chapter 6

Dynamical effects of the cosmological constant: the evolution of aspherical structures\(^1\)

1 Introduction

Recently it has become clear that in the theory of large scale structure formation a non-vanishing cosmological constant may prove to be useful and perhaps necessary. Peebles (1984) finds a value of \(\Omega_0 = 0.2\) from tests based on theories of the homogeneous as well as the inhomogeneous universe. Assuming a positive value of \(\Lambda\) can make these observations consistent with the requirements of inflation and may furthermore help to keep fluctuations in the cosmic microwave background within the limits. Others use the extended dynamical dominance of radiation in a universe with non-vanishing \(\Lambda\) to provide the standard cold dark matter (CDM) model with more power on large scales (e.g., Silk & Vittorio, 1987). This might for instance help restoring the consistency of this model with the APM angular correlation function (Maddox et al., 1990).

On the other hand, the evolution of structure as calculated using cosmological N-body codes is seen to be little affected by the introduction of a non-vanishing cosmological constant (Davis et al., 1985, Martel, 1991a); only a small adjustment of the initial conditions is needed to produce apparently very similar structures at present, irrespective of the cosmology. In an attempt to understand this behaviour, Lahav et al. (1991) calculated the dynamical effects of a cosmological constant on structure formation in the linear approximation and in the non-linear spherical approximation (see also Martel 1991b). They showed that in these simple cases one should not expect significant deviations from the well known behaviour in standard backgrounds with \(\Lambda = 0\). Furthermore, any possible differences were marginalized by competing cosmological effects.

Apart from their density contrasts, large scale structures as observed in the universe are conspicuous for their shapes, be they described as sheetlike or filamentary. Standard theory actually predicts that at some point during the gravitational collapse a highly flattened or

filamentary structure will appear (Zel’dovich, 1970). A simple model for this behaviour is provided by the collapsing homogeneous ellipsoid. This model has been applied by Lin, Mestel & Shu (1965) and Lynden-Bell (1964) who showed that small initial asphericities will be amplified in the cause of its collapse, leading to highly flattened or filamentary structures. It has been used further by Icke (1973) to calculate the formation of galaxies inside clusters and by White & Silk (1979) to model the local supercluster. These authors generalized the earlier model to include the influence of the background, turning it into a better approximation of the growth of small fluctuations in a homogeneous universe (see also Barrow & Silk, 1981).

Simple modeling of structures with a mild initial anisotropy as is for instance expected for CDM-peaks (Bardeen et al., 1986) shows that these will collapse along their major axes soon after collapse along the minor axis ("pancaking"). The fact that we observe the large scale structure to be highly anisotropic at present would seem to imply that we see it near the time of collapse. Dekel (1983) however discovered a mechanism which in principle might stabilize flattening without requiring dissipation or virialization. When one assumes that after pancaking a collapsed system is allowed to oscillate freely around the plane of symmetry, one can show that in favourable cases the occurrence of an adiabatic invariant determines that the flattening will actually increase as long as the expansion in the other directions continues.

In the present paper the work of Lahav et al. (1991) will be extended by calculating the effect of a non-vanishing cosmological constant on the development and further evolution of anisotropy within the model of homogeneous ellipsoidal collapse as defined by Icke (1973) and White & Silk (1979). The most interesting effects appear to take place in between the linear and the highly non-linear regimes. This is also the regime where in the real universe highly anisotropic large scale structures are found (Geller & Huchra, 1990; Haynes & Giovanelli, 1988) and it bridges the gap between the linear and the highly non-linear regimes considered by Lahav et al. (1991).

The qualitative behaviour of collapse in a $\Lambda > 0$ Lemaître universe is similar to that in an open universe without cosmological constant, but since the deceleration of the minor axis is increased it allows for lower amplitude fluctuations to collapse. A non-vanishing cosmological constant induces prolonged expansion of the major axes, compared to similar structures in an Einstein-de Sitter universe, giving rise to stronger anisotropies. The resulting flattening may persist without requiring the mechanism induced by Dekel’s adiabatic invariant. This last scenario is seen to be relevant only for those overdensities which are initially strongly anisotropic.

No assumptions on the shape and amplitude of the initial fluctuation spectrum will be made; the fairest comparison of the different cosmologies is to normalize fluctuations to those presently observed in the real universe. We then find that the influence of the cosmological constant all but vanishes in the following sense: by simply adjusting the initial over-density, objects can be made which have the same over-density and axial ratio at present and for a great part of their evolution irrespective of the cosmological background. Since in more realistic models the effects of a non-vanishing cosmological constant should still be separated from possible other effects such as dynamical relaxation through fragmentation, relaxation through dissipation or tidal effects due to external structures (Binney &
Silk, 1979), the conclusion from the calculations is that the evolution of shape of large scale structures is not a promising method for determining the magnitude of the cosmological constant any better than with the classical tests.

2 Theory

2.1 The model

Following Icke (1973) and White & Silk (1979), Newtonian dynamics is used throughout this paper. The Friedman-Lemaitre models are then defined by the following differential equation describing the separation $R_b$ between two fundamental observers in a dusty universe, (the contribution of the cosmological constant is given in parentheses)

$$\frac{d^2 R_b}{dt^2} = -\frac{4\pi G \rho_b R_b}{3} \left( + \frac{\Lambda}{3} R_b \right). \quad (6.1)$$

This equation also describes the radius of a perfectly homogeneous sphere of mass $M_b = 4\pi \rho_b R_b^3/3$ with the addition of the term with the cosmological constant. As is well known (Lyttleton, 1953) this formula can be generalized to the case of an isolated uniform ellipsoid with axes $a_1$, $a_2$ and $a_3$ as follows:

$$\frac{d^2 a_i}{dt^2} = -2\pi G \rho_e a_i \left( + \frac{\Lambda}{3} a_i \right), \quad (6.2)$$

where the $\alpha_i$ are given by

$$\alpha_i = a_1 a_2 a_3 \int_0^\infty \frac{du}{(u + a_i^2) \Delta} \quad (6.3)$$

$$\Delta^2 = (a_1^2 + u)(a_2^2 + u)(a_3^2 + u) \quad (6.4)$$

and are related by $\alpha_1 + \alpha_2 + \alpha_3 = 2$.

To apply this to the case of an ellipsoidal overdensity in an otherwise unperturbed, homogeneous universe White & Silk (1979), following Icke (1973), modified the equations by the introduction of a background term to obtain

$$\frac{d^2 a_i}{dt^2} = -2\pi G \left( (\rho_e - \rho_b) a_i + \frac{\Lambda}{3} \rho_b a_i \right) \left( + \frac{\Lambda}{3} a_i \right). \quad (6.5)$$

The density of the ellipsoid is governed by

$$\rho_e a_1 a_2 a_3 = const \quad (6.6)$$

The contribution of the background term will be governed by Eq. 6.1 together with a constraint equation governing the background density :

$$\rho_b R_b^3 = const. \quad (6.7)$$
We will limit ourselves to oblate spheroids, $a_1 < a_2 = a_3$. This will allow us to calculate the expressions for the $\alpha_i$ explicitly (e.g., Watanabe & Inagaki, 1991) and furthermore decreases the number of free parameters, while preserving the typical behaviour found in non-spherical collapse. The main qualitative difference between spheroidal and the more general tri-axial evolution is that collapse of the major axes is more singular due to the collapse of two axes simultaneously in the spheroidal case; the calculations will therefore be stopped when this happens. The choice between different models is made by varying initial conditions. The values for $H_0, \Omega_0$ and $\lambda_0$ are fixed at the present epoch and the initial axial ratio, $q \equiv a_3/a_1$ and the over-density of the ellipsoid, $\delta \equiv (\rho_e - \rho_b)/\rho_b$ are set at the initial redshift which will be $z_m = 1000$ in all calculations. The expansion is assumed to follow the universal Hubble flow initially: $H_i \equiv \dot{a}_i/a_1 = \dot{H} \equiv \ddot{R}_b/R_b$.

We will concentrate on three different cosmologies. An open universe: $\Omega_m = 0.1, \Lambda = 0$, an Einstein-de Sitter universe: $\Omega_m = 1, \Lambda = 0$ and a flat Lemaître universe: $\Omega_m = 0.1, \lambda_0 = 0.9$, where $\lambda_0 \equiv \Lambda/(3H_0^2)$. Comparison between the open and the Lemaître model will show the pure effect of the cosmological constant, whilst the differences between the Lemaître – and the Einstein-de Sitter model show the varying possibilities one may get when abiding by the inflationary paradigm.

The evolution will be followed through collapse of the minor axis. It will be assumed that this collapse is a dissipationless process and the spheroid is thus allowed to expand afterwards. When many oscillations occur the following integral will be calculated,

$$I_D \equiv \int_{\text{period}} (\dot{a}_1)^2 dt$$

Following Dekel (1983) we may expect this integral to be an adiabatic invariant and thus to be constant over successive oscillations, as long as the periods are short compared to the typical dynamical timescale of the major axis’ evolution. In his approximation of quasi one dimensional collapse near the moment of pancaking, Dekel calculated that a typical length in the oscillating direction, $h$, should scale with the typical size of the perpendicular directions, $R$, as $h \propto R^{1/3}$. The flattening should thus increase as $R/h \propto R^{1/3}$, provided the major axes are still expanding. This will be tested by calculating the average values of $a_1$ and $a_2$ between the successive pancaking events.

### 2.2 Assumptions and expectations

The behaviour of the spheroidal model in the Einstein-de Sitter – and the open universe is well known (Icke, 1973; White & Silk, 1979; Watanabe & Inagaki, 1991). Just as in the case of the isolated ellipsoid investigated by Lin et al., the axial ratio $q \equiv a_3/a_1$ grows with time. In the Einstein-de Sitter universe $q$ diverges after a finite time at the moment of pancaking, $a_1 = 0$. This need not happen in an open universe. For a range of initial conditions an effect equivalent to the ‘freezing in’ of perturbations in linear theory of the open universe occurs: when at $z_c \sim \Omega^{-1} - 1$, one of the axes is expanding close to the expansion rate of the background, it will keep expanding indefinitely. This may clearly lead to highly flattened structures, especially when the minor axis does collapse.

In the Lemaître universe we expect behaviour qualitatively similar to that in an open universe. The cosmological constant starts to dominate the expansion of the background
at $z_A = (2\lambda_0/\Omega_m)^{1/3} - 1$ when $\dot{R}_b = 0$. From Eq. 6.6 we see that if $a_i/\delta \ll 1$ at $z_A$, $\Lambda$ will start to dominate the dynamics of the corresponding axis as well, and the spheroid will not collapse along that direction. This may happen to the major axis while the minor axis is at or past turn-around at $z_A$. The minor axis may thus collapse, causing very strong flattening which appears to be stable. The important quantitative difference between the Lemaître – and the open universes is that in the former case the critical epoch occurs later in the evolution of the universe; for $\lambda_0 = 0.9$ and $\Omega_m = 0.1$, $z_A \approx 1.6$, compared to $z_e \approx 9$ for an open universe with $\Omega = 0.1$. This implies that overdensities will have had a longer period of uninhibited growth and thus initial fluctuations of smaller amplitude are needed to obtain a certain over density at present.

The approximation made in modeling the reaction of the background to the collapse of the ellipsoid requires more discussion. The constraint equations, 6.6 and 6.7 and the fact that the axes $a_i$ will grow at a slower rate than $R_b$, physically imply a flux of background matter through all surfaces surrounding the ellipsoid. This inflow of matter ‘fills the gap’ caused by the contraction of the over dense region and is due to the assumption of persistent homogeneity of the background. White & Silk (1979) argue that this assumption is correct in the linear regime, and, whilst this will no longer be true when the over-density becomes non-linear, the self-gravity of the structure then dominates the background.

Following the evolution through and beyond pancaking within this approximation is harder to defend. Expansion in the $a_1$ direction together with the constraint equations would now imply an outflow of background matter where shortly before there was inflow, clearly an unphysical situation. The magnitude of this effect can be estimated in the following way: The total amount of background matter which has entered a spherical shell of radius $R \gg a_1$ expanding with the background comprises a mass nearly equal to the mass of the ellipsoid itself. As long as $a_2$ is in its linear regime this is true for $R = a_2$ as well. The mass which would flow out again is $M_e \rho_b/\rho_e$, where the densities are determined at the point of maximum expansion of $a_1$. This factor will clearly be much smaller than $M_e$ itself and the magnitude of this effect is not the main problem. The problem is that, instead of flowing out, matter actually will continue to accrete onto the perturbation. One would expect that the longer the oscillation continues the more matter will fall in making the approximation worse and worse. There is however a competing effect which will limit the the total amount of matter that might flow in. In the real universe there will be more peaks in the neighbourhood which will compete for the background material. Arguing along the lines of Press & Schechter (1974) we may expect that the maximum amount of background matter accreting on our ellipsoid will only have a mass equal to the original mass of the ellipsoid. We may thus conclude that, while the model will not describe the physics correctly in detail, it will be a good approximation for the early part of the evolution, before and shortly after first pancaking.

3 Results and discussion

3.1 The growth of anisotropy

In Fig. 1a we show for three cosmologies the evolution of the axes of the spheroid
as function of the background scale-factor \( \dot{R}_b \) for some representative choices of the initial conditions. The values of the variables are normalized using their initial values, \( \dot{a}_i = a_i / a_{i,\text{in}} \) etc.. This implies that \( \partial \dot{a}_i / \partial \dot{R}_b \equiv 1 \) initially due to our choice of comoving perturbations. In Fig. 1b we show the corresponding dependence of the axial ratio, \( q \), and the relative overdensity, \( \delta \), on the redshift. The evolution was continued until \( R/R_{\text{in}} = 2000 \), corresponding to \( z = -1/2 \), or it was stopped when the major axes collapsed if this happened sooner. As promised, the minor axis is followed through collapse, which is shown by giving negative values for its size after pancaking.

The models behave as expected. For \( \delta_{\text{in}} \) large enough, the Einstein-de Sitter and the Lemaître models correspond closely to one another: all axes collapse early when turn around has occurred before \( z_\Lambda \). At some critical initial density, the value of which depends on the initial axial ratio, the minor axis in the Lemaître universe still collapses, but the major axes keep expanding, in fact followed by the minor axis after it has oscillated once or twice. At a still lower density even the minor axis does not collapse. The open model has qualitatively the same behaviour as the Lemaître model, but needs still higher initial

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{(a)Evolutionary tracks showing \( \dot{a}_i = a_i / (a_{i,\text{in}}) \) as function of \( \dot{R}_b \), for the three cosmologies. All figures keep the following identification: Einstein-de Sitter universe: solid line; open universe: dash-dotted line; Lemaître universe: dashed line. Initial parameters are \( \delta_{\text{in}} = 0.003 \), topframe: \( q_{\text{in}} = 2 \), bottomframe: \( q_{\text{in}} = 7 \).
(b)Evolutionary tracks showing \( q = a_2/a_1 \) and \( \delta = \rho_c/\rho_b - 1 \) as function of \( z \) for the three cosmologies. The lower sets of lines show \( \delta \), the higher sets show \( q \). Characteristics as in (a).}
\end{figure}
densities for the minor axis to collapse.

If the anisotropy of large scale structure can be used to determine cosmological parameters it will be by using statistics of many objects which will all have started their evolution from slightly different initial conditions. I have therefore shown in Fig. 2 the results of evolution from initial conditions which were distributed along an ellipse in the $(\delta_{in}, q_{in})$-plane. In a simple model for the initial conditions this might correspond to an average value plus $n\sigma$ deviations. The Einstein-de Sitter and Lemaître models coincide at $z \gtrsim 1.5$ after which the Lemaître model lags behind. The open model has separated from the other two before $z = 5$ and has effectively frozen in. Pancaking is only encountered in the Einstein-de Sitter model. At the highest $q_{in}$-values the density and the axial ratio first diverge and reach more moderate values after crossing the plane $a_1 = 0$. This effect spreads to lower values of the

![Figure 2: Density contrast $\delta$ versus axial ratio $q$ at indicated redshifts, for elliptically distributed initial values $(\delta_{in}, q_{in})$. Centers and axes of initial ellipses were $(\delta \pm \delta, q \pm r_q) = (0.002 \pm 0.0001, 4 \pm 3)$. The center of the ellipses is indicated by the dot.](image)
initial axial ratio at later times.

The results of the calculations for the three different models do show qualitative differences. At the same time it is clear that any final state can be obtained by a suitable choice of initial conditions; these must be different for the three world models. To be able to predict possible observable consequences of living in a certain cosmology, we must put constraints on the calculations. One possibility would be to restrict the allowed range of initial conditions, but we will instead compare the evolution of models which lead to the same final state, i.e. $\delta$ and $q$, in all three world models. By following the evolution towards this final state we may then look for observable differences at higher redshifts. This may be compared to the calculations by Lahav et al. (1991) on their non-linear spherical infall models where they fix the amplitude of the initial fluctuation spectrum by observed fluctuations at the present epoch.

For the final state we choose parameters corresponding to a typical structure at present. For the cosmological constant to have maximum effect the over-density at $z_\Lambda$ should not be too high, but to be able to observe the structure unambiguously against the background it should be past the linear regime. Since the mildly non-linear structures observed at present are indeed highly anisotropic one may hope that possibly discriminating results might be obtained in this regime. The evolution of the axes for the corresponding initial parameters is shown in Fig. 3a. In Fig. 3b are also plotted the peculiar velocities, represented by the

![Graphs showing evolutionary histories](image1)

![Graphs showing dynamical histories](image2)

**Figure 3:** (a) Evolutionary histories for initial conditions giving equal results at $z = 0$ in Einstein-de Sitter ($\delta_{in} = 0.0018$), open ($\delta_{in} = 0.0092$) and Lemaitre universe ($\delta_{in} = 0.0031$). In all cases $q_{in} = 2.4$. The vertical line indicates $z = 0$.
(b) Dynamical histories of parameter $\delta_{v,i} \equiv H_i/H - 1$ corresponding to (a).
The most striking aspect is the strong similarity between the open - and the Lemaitre universe throughout most of their evolution, as can be clearly seen from their evolutionary tracks. This is translated in the near equality of their axial ratios at $z \leq 5$. Even the peculiar velocities are remarkably similar, especially so since no constraint was placed on this observable. It is clear that one could from this case never decide between the open and the Lemaitre universe. The Einstein-de Sitter model differs little from these two, the deviation in axial ratio being greatest at $z \sim 0.5$, vanishing at $z = 0$ and increasing strongly at $z < 0$. Only the velocity structure at $z = 0$ is significantly different. While the Lemaitre and open cases have a quasi constant minor axis, the Einstein-de Sitter model is clearly collapsing. This possibility of a long period of quasi static behaviour of the minor axis differentiates between the Einstein-de Sitter universe on the one hand and the open and Lemaitre universe on the other.

As before we will follow the evolution from initial conditions distributed along ellipses in $\delta_{in}$ and $q_{in}$, analogously to the calculations leading to Fig. 3. The initial parameters were now chosen such that the resulting distribution at $z = 0$ would correspond as closely as possible for the three world models. The results in Fig. 4a show that indeed it is possible for such non-trivial distributions to agree completely at $z = 0$. Again the similarity between the open and the Lemaitre universe with respect to their peculiar velocities, shown in Fig. 4b, is remarkable. With respect to this parameter, the Einstein-de Sitter model differs significantly from the other two.

### 3.2 The stability of flattening

Following the evolution through collapse is a new feature of this work. In realistic cases one would expect pancaking to be accompanied by some amount of relaxation (possibly dissipational) creating a thin disk which after a while contracts in the directions of the major axes as well. In this sense a highly flattened structure arises which would remain flat until the major axes collapse. In the present model we consider the other extreme, i.e. no relaxation at all. As was described in the Introduction, the minor axis oscillations that can be observed after its first collapse may give rise to an adiabatic invariant which can explain the persistence of the flattening even in this case (Dekel, 1983). Fig. 5a shows the values for $I_D$ (Eq. 6.8) calculated over periods between successive zero-crossings of the minor axis for an initially highly anisotropic spheroid $\delta_{in} = 0.01$, $q_{in} = 20$) in an Einstein-de Sitter universe. For such initial conditions we can follow the collapse through many oscillations and we see that the corresponding value of the adiabatic invariant is perfectly constant for those periods in which $a_2$ does not change too much. As we see in Fig. 5b the average values for the minor and major axes accurately follow the predicted relation, $a_1 \propto a_2^{2/3}$, for those periods. However, as we saw in the case where the initial asphericity was not so pronounced, the minor axis oscillates only a few times before the major axes collapse. This implies that the central assumption underlying the use of an adiabatic invariant, namely that the oscillation periods should be much shorter than other relevant timescales, is not fulfilled and the scenario will not be important.
This may seem less of a problem for the open and the Lemaître universe since here one can easily find cases where the major axes do not collapse at all. It appears that in those cases after at most a few oscillations the minor will expand indefinitely as well. This of course is due to the continuous expansion of the major axes, which results in an ever decreasing surface density. Although this shows that the adiabatic invariant will be irrelevant in those world models as well, the consequent ‘freezing in’ of the anisotropy at the same time presents a third mechanism for stable flattening.

The question whether relaxation occurs might have more relevance for the evolution of the major axes: a thin structure exerts a greater force along its major axis than does a thick one. To make an estimate on the strength of this effect we have in Fig. 6 calculated the evolution of a structure which remains at zero thickness after pancaking and shown it together with the oscillating case. As we see in the Einstein-de Sitter case the effect on the major axis is small, but on the other hand it may just mean the difference whether it will

Figure 4: (a) : $\delta$ vs. $q$ at indicated redshifts for elliptically distributed $(\delta_{in}, q_{in})$ giving equal results at $z = 0$. Centers and axes of initial ellipses were $(\delta \pm r_{K}, q \pm r_{D})$: Einstein-de Sitter : $(0.0017 \pm 0.0003, 2.4 \pm 1.4)$; open : $(0.0087 \pm 0.0015, 2.4 \pm 1.4)$; Lemaître : $(0.0029 \pm 0.0052, 2.4 \pm 1.4)$. (b) : corresponding relations $\delta$ vs. $\delta_{\nu}$. 
collapse or not as the calculation for the Lemaitre model shows.

4 Summary and conclusions

In this article we have, using the model of a homogeneous spheroid, calculated the effect of a cosmological constant on the evolution of shape of gravitationally collapsing structures. It was shown that a non-vanishing cosmological constant may indeed induce strong and lasting flattening. Due to the fact that the gravitational deceleration along the minor and the major axes differs it becomes possible that the cosmological constant dominates the evolution of the major axis while at the same time having negligible effects on the dynamics of the minor axis. The resulting effects on the asphericity of structures are most interesting in those objects which are dynamically in their mildly non-linear regime. That is the epoch where anisotropies first start to develop, and their low density implies that the cosmological constant may have had a relatively important effect on their dynamical evolution.

An open universe without cosmological constant shows the same qualitative behaviour. Here strong and lasting anisotropies are also obtained due to the fact that the initial expansion may be stopped and reversed by the selfgravity in one direction but not in the other directions. The main quantitative difference is that in the open universe initial fluctuations of higher amplitude are required than in the Lemaitre universe.

Figure 5: (a) : adiabatic invariant according to Eq. 6.8 in Einstein-de Sitter universe with initial conditions $\delta_m = 0.01$, $q_m = 20$.
(b) : average values of $a_1$ vs. $a_2$ for each half-period. The line shows the predicted relation $a_1 \propto a_2^{2/3}$. 
In an Einstein-de Sitter universe instances of high flattening occur during collapse, but for moderate initial axial ratios these are short lived as the major axes collapse quickly as

![Graphs](image)

**Figure 6:** Evolutionary tracks for the completely dissipative case when structures stay thin after collapse of the minor axis (solid line), compared to case with no dissipation at all, where oscillations are allowed to proceed without hindrance (dashed line); (a) : Einstein-de Sitter, (b) : Lemaître universe. All calculations had the same initial parameters, $\delta_{\text{in}} = 0.005$, $q_{\text{in}} = 2$. 
well. Structures of quasi stable high flattening in a critical universe are only possible when the fluctuations were initially highly anisotropic. The persistence of the anisotropy can then be understood even without dynamical relaxation, in the context of Dekel’s (1983) adiabatic invariant scenario. Due to the aforementioned collapse of the major axes this mechanism is not relevant for moderate initial axial ratios, \( a_2/a_1 \leq 5 \), unless some other effects, such as tidal forces, could extend the period of expansion for the longer axes. This work therefore casts doubt on the relevance of this scenario for explaining the flattening of large scale structure.

Since we are unable to observe the complete dynamical history of a certain object and do not make any assumptions regarding initial conditions we have normalized the calculations to fluctuations at the present epoch. We choose typical values for the over-density and anisotropy of a present day object and find for the three universes the corresponding initial conditions. One such case was shown in Fig. 4a. There it appears that the open model \((\Omega_m = 0.1, \lambda_0 = 0)\) and the Lemaître model \((\Omega_m = 0.1, \lambda_0 = 0.9)\) are almost identical while the Einstein-de Sitter model differs from these two mainly in the (peculiar) velocity structure at present and in the axial ratio at redshifts \( z \sim 0.5 \), albeit only slightly. This arises from the fact that evolution into non-linearity at recent times is much more extreme in the Einstein-de Sitter model than in the other two models. In those cases a density fluctuation must separate out before some critical redshift \( z_c \) \((z_A, \text{section 2.2})\) or it will evolve with the background thereafter. This will mean that mildly non-linear structures at \( z = 0 \) might have been observable at almost their present day density contrast and shape at \( z = 1 \) in the Lemaître and open universes. However, in the Einstein-de Sitter universe these structures only just reached non-linearity and were still close to their initial shape at \( z = 1 \). A realistic range of initial fluctuations would blur this effect and reduce its potential for discriminating between the various cosmologies.

In the real universe structures with over-densities \( \delta \sim 5 \) are connected to each other and to high density clusters and are in general not very homogeneous (Haynes & Giovannelli, 1988; Geller & Huchra, 1989). These facts undermine the assumptions of the simple model of an isolated homogeneous structure, and it may thus be questioned whether the rather small effects of a possible non-zero cosmological constant can be separated from possible tidal and relaxation effects.

We have confirmed the conclusions of Lahav et al. (1991) and Martel (1991ab), – the dynamical effects of the cosmological constant are too small to be of any use as a test on its value. Discriminating between the open and the Lemaître universe using the shape of large scale structure seems hopeless. In the context of the inflationary scenario the choice is between the Einstein-de Sitter and the Lemaître models. Even here the differences in the dynamical effects on the formation of large scale structure are small and are mainly due to the difference in the value of \( \Omega_m \) rather than the cosmological constant.

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