Chapter 3

Simulation of the Effect of Rent Assistance on Housing Demand

3.1 Introduction
In the previous chapter we have observed that RA-recipients consume on average more housing services than non-recipients, viz. Dfl. 5620 against Dfl. 4340 (table 2.1 on page 21). *Prima facie*, it is not clear that this difference can be attributed to the RA program: the groups differ in income and preferences. In this chapter, we use the structural model of the previous chapter to decompose this difference, and we examine the effect of RA on housing demand. We shall give estimates of housing consumption in the absence of RA, the effect of application costs and the efficiency of RA.

Rents in counterfactual situations refer to utility-maximizing levels of housing consumption. On the assumption that the policies of the Dutch government are aimed at the satisfaction of demand at a fixed unit price of housing services, we can consider the outcomes as long-run equilibria.

3.2 Decomposition of the Difference
In this section we decompose the difference in housing demand between RA-recipients and non-recipients into a number of components. The decomposition is based on the basic model of housing demand introduced in section 2.5 and the extended model introduced in section 2.6.
3.2.1 The Basic Model

Both the basic and the extended model are nonlinear in the explanatory variables. Hence to determine the effect of interventions on the total demand for housing of the population or of some subpopulation we must simulate the model using the joint distribution of the explanatory variables over the population or subpopulation of interest. In this chapter we use a simpler approach that can be seen as an approximation to this simulation. Instead of determining the average demand in some (sub)population we calculate the demand of the average of the explanatory variables in that (sub)population. In that case, we can interpret the effects as the effects of a representative household of that subpopulation. In the sequel we use this simpler approach to decompose the observed difference in average housing demand between recipients and non-recipients, but we examine whether simulation yields different results.

Below the subscript $A$ indicates non-RA recipients and $B$ RA recipients. $\bar{Y}$ and $\bar{Y}_v$ indicate income and virtual income in the regime allocation equation. The corresponding variables in the demand equations are denoted by $Y$ and $Y_v$. We make this distinction to allow for counterfactual independent variation in these incomes between the demand and regime choice equations.

We decompose the difference in average rent between recipients and non-recipients in three broad components, which consist of a number of subcomponents. The first component reflects the incentives of the RA program. It consists of the effect of the lower price paid by RA-recipients and the lower income that these households have due to the implicit entry fee. Moreover, an RA household is restricted in its choice to rents that exceed the norm rent $R_n$. These effects will be referred to as the price effect, entry fee effect, and truncation effect respectively. The second component is the difference in demand due to the lower income of RA-households (see table 2.1). The third component reflects the difference in preferences between RA- and non RA-recipients. The three components (and subcomponents) can be expressed as differences of subpopulation averages. The relevant formulae are given in table 3.1 where subpopulation averages are expressed as conditional expectations (see also Appendix 3.A). The subscripts indicate the subpopulation. For example, $Y = \bar{Y}_{EB}$ means that income in the demand equation is set equal to the average income in the subpopulation of RA-recipients.
Decomposition of the Difference

Price effect
\[
E \left( R_B \mid p = 1 - \delta, Y = \tilde{Y}_{eB}, R_B > R_n, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right) \\
- E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, R_B > R_n, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right)
\]

Entry fee
\[
E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, R_B > R_n, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right) \\
- E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, R_B > R_n, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right)
\]

Truncation effect
\[
E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, R_B > R_n, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right) \\
- E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right)
\]

Income difference
\[
E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right) \\
- E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right)
\]

Preference heterogeneity
\[
E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right) \\
- E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right)
\]

Total difference
\[
E \left( R_B \mid p = 1 - \delta, Y = \tilde{Y}_{eB}, R_B > R_n, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right) \\
- E \left( R_B \mid p = 1, Y = \tilde{Y}_{eB}, I = 1, \tilde{Y} = \tilde{Y}_{eB}, \tilde{Y}_v = \tilde{Y}_{vB} \right)
\]

Table 3.1: Measurement of effects

The results\(^1\) of the decomposition are reported in table 3.2. As in table 2.1, all values are given in thousands of guilders. We see that

\(^1\) For the decomposition presented here, both the estimates of the reduced form parameters of table 2.2 and the estimates of the reduced form parameters implied by the structural parameters can be used. We use the structural parameters because the effect of application costs discussed in section 3.3.2 below can be calculated only using the structural parameters, and because the structural parameters are invariant under policy changes.
the most important effect is the price effect. Other important effects are the preference heterogeneity effect, which is positive implying that RA-recipients have a relative preference for housing, and the truncation effect.

In table 3.2 we also report the standard errors of the components (see also Appendix 3.B). These reflect the sampling variation of the parameter estimates. Note that all components are significantly different from 0.

The same decomposition obtained by simulating over the sample is presented in table 3.3. By comparing the entries of table 3.3 with those of table 3.2, we see that it hardly matters whether the effects are calculated by simulating over the sample or by using representative households.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Difference</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price effect</td>
<td>5.57 - 4.73</td>
<td>0.84 (0.18)</td>
</tr>
<tr>
<td>Entry fee effect</td>
<td>4.73 - 4.86</td>
<td>-0.13 (0.0088)</td>
</tr>
<tr>
<td>Truncation effect</td>
<td>4.86 - 4.43</td>
<td>0.43 (0.077)</td>
</tr>
<tr>
<td>Income Difference</td>
<td>4.43 - 4.82</td>
<td>-0.39 (0.045)</td>
</tr>
<tr>
<td>Preference heterogeneity</td>
<td>4.82 - 4.15</td>
<td>0.67 (0.29)</td>
</tr>
<tr>
<td>Total difference</td>
<td>5.57 - 4.15</td>
<td>1.42 (0.11)</td>
</tr>
</tbody>
</table>

Table 3.2: Decomposition of the difference, representative households
(standard errors in parentheses)

<table>
<thead>
<tr>
<th>Effect</th>
<th>Difference</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price effect</td>
<td>5.62 - 4.78</td>
<td>0.84 (0.11)</td>
</tr>
<tr>
<td>Entry fee effect</td>
<td>4.78 - 4.90</td>
<td>-0.12 (0.0084)</td>
</tr>
<tr>
<td>Truncation effect</td>
<td>4.90 - 4.44</td>
<td>0.46 (0.079)</td>
</tr>
<tr>
<td>Income Difference</td>
<td>4.44 - 4.83</td>
<td>-0.40 (0.048)</td>
</tr>
<tr>
<td>Preference heterogeneity</td>
<td>4.83 - 4.14</td>
<td>0.69 (0.30)</td>
</tr>
<tr>
<td>Total difference</td>
<td>5.62 - 4.14</td>
<td>1.48 (0.11)</td>
</tr>
</tbody>
</table>

Table 3.3: Decomposition of the difference, simulation over sample (standard errors in parentheses)
Decomposition of the Difference  

<table>
<thead>
<tr>
<th></th>
<th>Model E1</th>
<th>Model E2</th>
<th>Model E3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price effect</td>
<td>0.72</td>
<td>0.57</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.25)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>Entry fee effect</td>
<td>−0.12</td>
<td>−0.12</td>
<td>−0.064</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.015)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Truncation effect</td>
<td>0.34</td>
<td>0.26</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.073)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>Income Difference</td>
<td>−0.40</td>
<td>−0.43</td>
<td>−0.35</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.041)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>Observed preference heterogeneity</td>
<td>0.00</td>
<td>−0.03</td>
<td>−0.03</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.040)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>Unobserved preference heterogeneity</td>
<td>0.94</td>
<td>1.27</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(0.34)</td>
<td>(0.53)</td>
</tr>
<tr>
<td>Total difference</td>
<td>1.49</td>
<td>1.53</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.12)</td>
<td>(0.12)</td>
</tr>
</tbody>
</table>

Table 3.4: Decomposition of the difference, extended models (standard errors in parentheses)

3.2.2 The Extended Model

We repeat the decomposition for the extended model of section 2.6. In the extended model, a fraction of the preference heterogeneity is explained by household characteristics. As a consequence, we can decompose the difference due to preference heterogeneity in subcomponents due to observed and unobserved preference heterogeneity. The difference due to observed heterogeneity can be expressed as

\[
E \left( R_B \mid p = 1, Y = \bar{Y}_A, I = 1, \bar{Y} = \bar{Y}_A, \bar{Y}_v = \bar{Y}_{v,A}, x = \bar{x}_B \right)
\]

\[
- E \left( R_B \mid p = 1, Y = \bar{Y}_A, I = 1, \bar{Y} = \bar{Y}_A, \bar{Y}_v = \bar{Y}_{v,A}, x = \bar{x}_A \right)
\]  (3.1)

where \( x = \bar{x}_B \) denotes the average of the explanatory variables in the subpopulation of recipients.

The results of the decomposition are given in table 3.4. The labelling of the columns corresponds to those in tables 2.8 and 2.9. Model E1 is the model with household size as an additional explanatory variable, and in model E2 preferences vary with household size and
age of the head. Finally, model E3 is the model with the more flexible preference specification of page 38. The results of the first column are very similar to those of the basic model in table 3.2. It is clear that the fraction of the difference due to preference heterogeneity that can be attributed to differences in average household size is small. The decomposition in the last two columns differ from the one in the first. Note that the estimates for both the price effect and income difference are of the same order of magnitude in all three models.

Similar estimates can be calculated by simulating over the sample. As in the previous section, these results are not markedly different from the ones obtained by using representative households and are not given here.

3.3 The Effect of RA on Housing Demand

In this section we analyse the effect of RA on housing consumption. All calculations are based on the basic model of section 2.5.

3.3.1 Housing Consumption in the Absence of RA

The effect of the elimination of RA on a representative RA household is equal to the sum of the three program effects reported in table 3.2. Housing consumption would be reduced by Dfl. 1140 (20%), from Dfl. 5570 to Dfl. 4430. For RA-recipients, the fraction of income spent on housing would decrease from 26.7% to 21.3%. It is clear that the RA program has a large positive effect on housing demand.

One reason cited to justify housing subsidies is that housing consumption should exceed some minimal standards (e.g. Rosen (1985), and Smith, Rosen, and Fallis (1988)). Apparently, the government does not consider the amount of housing consumption by non-recipients as much too low. If this were the case, the government could change the parameters of the RA program such that these households received RA as well. Our results show that the expected housing demand by RA-recipients in the absence of RA is larger than that by non-recipients. Hence, this argument does not provide a strong basis for the existence of the RA program.
The Effect of RA on Housing Demand

<table>
<thead>
<tr>
<th>Probability of taking up RA</th>
<th>$\Pr(I = 1 \mid C, \bar{Y}, \bar{Y}_w)$</th>
<th>0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected rent</td>
<td></td>
<td>4.66</td>
</tr>
<tr>
<td>Probability of RA, no application costs ($C = 0$)</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>Expected rent</td>
<td></td>
<td>5.02</td>
</tr>
<tr>
<td>Probability of RA, no application costs ($\sigma^2_3 = 0$)</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>Expected rent</td>
<td></td>
<td>5.18</td>
</tr>
</tbody>
</table>

Table 3.5: Probability of taking up RA for an average household in the sample and the expected rent of that household

### 3.3.2 The Effect of Application Costs

In section 2.3 we discussed the effects of application costs on housing demand. Using the estimates of the structural model we are able to quantify the effect of application costs. In table 3.5 we consider the average household in our sample, i.e., the household with average values of $Y$ and $Y_w$. We obtain the effect of application costs by setting $C = 0$. If we interpret $v_3$ as unobserved heterogeneity in $C$, then elimination of application costs would also imply $v_3 \equiv 0$. Hence, in table 3.5 we also compute the probability of taking up RA if $\sigma^2_3 = 0$. We also give the expected rents under these hypotheses, computed by

$$E(R) = E(R_A \mid I = 0) \Pr(I = 0)$$

$$+ E(R_B \mid R_B > R_n, I = 1) \Pr(I = 1).$$

In table 3.6 we concentrate on households with rents that entitle them to RA. If application costs were eliminated, almost all these households would use their entitlement.

### 3.3.3 The Efficiency of RA

The principal goal of the RA program is to make good quality housing affordable for low-income households (see Ministerie van Volkshuisvesting, Ruimtelijke Ordening en Milieubeheer (1989b)). This goal has not been made more precise by the government so that it is difficult to assess whether the program is successful in reaching this goal. Because
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<table>
<thead>
<tr>
<th>Probability of RA</th>
<th>Probability of RA, no transaction costs</th>
<th>Probability of RA, no transaction costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pr(I = 1 \mid C, \bar{Y}, \bar{Y}, R_A &gt; R_n) )</td>
<td>( 0.44 )</td>
<td>( 0.74 )</td>
</tr>
<tr>
<td>( \Pr(I = 1 \mid C = 0) )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( \Pr(I = 1 \mid \sigma^2 = 0) )</td>
<td>( 0.97 )</td>
<td>( 0.79 )</td>
</tr>
</tbody>
</table>

Table 3.6: Probability of taking up RA for a household with income equal to the average income of households which do not take up their benefit

of the lack of quantitative objectives of the program we define two efficiency measures ourselves. These measures correspond to different goals of the program. The first measure is the increase of housing consumption per guilder of subsidy, which is appropriate if the goal of the program is to increase housing consumption. The second efficiency measure is the equivalent income allowance\(^2\) per guilder of subsidy. This measure is appropriate if the goal of RA is income redistribution.

The consequences of the RA program for housing demand are shown in figure 3.1. In the absence of RA, the consumption of housing services would be \( OA' \). However, due to the RA program, the household is able to attain a higher level of utility, and housing demand is \( OC'' \). The household attains the same level of utility if it receives an equivalent income allowance \( \Delta Y \) (\( DD' \)), which can be solved from

\[
\nu(Y_n, 1 - \delta) = \nu(Y + \Delta Y, 1).
\]

Hence, we can decompose the total effect of the RA program on housing demand given by \( A'C'' \) into an income effect \( A'B' \) and a price effect \( B'C'' \).

Our first measure of efficiency of the RA program is the ratio of the additional housing demand due to RA and the RA allowance \( CC' \). If this ratio is low then a large part of the RA-allowance is not used for additional housing consumption. We have calculated this ratio for all RA-recipients in the sample. The average increase of housing consumption due to the RA program is Dfl. 1360 per year and the average

\(\text{2. The equivalent income allowance is also known as the equivalent variation, see Varian (1984).}\)
of the efficiency measure is 73%, with standard deviation 33%. If the sole purpose of the RA program is to increase housing consumption, it appears to be reasonably successful.

In table 3.2 we have seen that households receiving RA have a stronger preference for housing consumption than non-recipients. If we do not correct for this selectivity-bias, the estimated efficiency of the RA program is 86%. Housing consumption without RA (OA' in figure 3.1) is underestimated and hence the increase in housing consumption is overestimated.

3. The standard deviation does not incorporate sampling variability of the estimated parameters.
Our second measure of efficiency of RA is $\Delta Y/CC'$, the equivalent income allowance which makes households as well off as under RA divided by the amount of RA. This measure of efficiency is related to the relative income inefficiency measure of Aaron and Von Furstenberg (Aaron and Von Furstenberg (1971)). The latter inefficiency measure is defined as $1 - \Delta Y/CC'$. The average of this efficiency measure is only 50%, so the RA program is not very efficient in this respect: RA-recipients can be made as well off (in utils) as under the RA program by giving them (on average) half of the subsidy in cash.

3.4 Conclusion
In this chapter we have examined some implications of the estimates of the structural housing demand model of chapter 2. The observed difference in housing demand between RA-recipients and non-recipients has been decomposed into a program effect, an income difference effect and a preference heterogeneity effect. It appears that the program effect is large and that the preference heterogeneity effect is non-negligible. We also found that the take-up rate would be almost 100% if the application costs would be 0. Finally, we have examined the efficiency of the RA program. On average, each guilder of RA leads to an increase of housing consumption by 73 cents.

3.A Conditional Expectations
The conditional expectations in both demand regimes are given by:

$$
E (R_A \mid \eta < -\bar{T}) = \bar{R}_A + E (\varepsilon_1 \mid \eta < -\bar{T}) = \bar{R}_A - \sigma_{\varepsilon_1} \Phi(\bar{T})
$$

$$
E (R_B \mid \eta \geq -\bar{T}, \varepsilon_2 \geq R_n - \bar{R}_B) = 
\bar{R}_B + E \left( \varepsilon_2 \mid \eta \geq -\bar{T}, \varepsilon_2 \geq R_n - \bar{R}_B \right) = 
\bar{R}_B + \sigma_2 \frac{\phi \left( \frac{R_n - \bar{R}_B}{\varepsilon_2} \right)}{\Phi \left( \frac{R_n - \bar{R}_B}{\varepsilon_2} \right)} \times \text{Pr} \left( \eta \geq -\bar{T}, \varepsilon_2 \geq R_n - \bar{R}_B \right)
$$

(3.A-1)
Calculation of the Standard Errors

\[
\times \left( 1 - \phi \left( \frac{-\bar{T} - \rho_{\varepsilon_{[n]} R_{n} - \bar{R}_{B}}}{\sqrt{1 - \rho_{\varepsilon_{[n]}^2}}} \right) \right) + \sigma_{\varepsilon_{[n]}} \Pr(\eta \geq -\bar{T}, \varepsilon_{2} \geq R_{n} - \bar{R}_{B}) \times \left( 1 - \phi \left( \frac{R_{B} - \rho_{\varepsilon_{[n]} \cdot (-\bar{T})}}{\sqrt{1 - \rho_{\varepsilon_{[n]}^2}}} \right) \right)
\]

\[
= \bar{R}_{B} + \sigma_{3} \frac{\phi \left( \frac{R_{B} - \rho_{\varepsilon_{[n]} \cdot (-\bar{T})}}{\sqrt{1 - \rho_{\varepsilon_{[n]}^2}}} \right)}{\Pr(\eta \geq -\bar{T}, \varepsilon_{2} \geq R_{n} - \bar{R}_{B})} \phi \left( \frac{-\bar{T} + \rho_{\varepsilon_{[n]} R_{B} - \bar{R}_{B}}}{\sqrt{1 - \rho_{\varepsilon_{[n]}^2}}} \right) + \sigma_{\varepsilon_{[n]}} \frac{\phi(-\bar{T})}{\Pr(\eta \geq -\bar{T}, \varepsilon_{2} \geq R_{n} - \bar{R}_{B})} \phi \left( \frac{R_{B} - \rho_{\varepsilon_{[n]} \cdot (-\bar{T})}}{\sqrt{1 - \rho_{\varepsilon_{[n]}^2}}} \right)
\]

(3.A-2)

(see Pudney (1989), Appendix 2, equation (A2.57)). Of course, the last equation contains two correction terms, the first one reflects the truncation \( R_{B} > R_{n} \) and the second one reflects the endogeneity of the regime.

3.B Calculation of the Standard Errors

The effects calculated in section 3.2 were calculated as the difference of conditional expectations. In order to assess whether these estimated effects differ significantly from 0, we have to derive their standard deviations. Each effect can be written as \( m_{1} - m_{2} \) with

\[
m_{1} \equiv f_{1}(\hat{\theta}, \bar{x}_{1}) \quad (3.B-1)
\]

\[
m_{2} \equiv f_{2}(\hat{\theta}, \bar{x}_{2}) \quad (3.B-2)
\]

The functions \( f_{1} \) and \( f_{2} \) are the conditional expectations derived in Appendix 3.A. The variance of \( m_{1} - m_{2} \) is, of course, equal to

\[
\text{var} (m_{1} - m_{2}) = \text{var} f_{1}(\hat{\theta}, \bar{x}_{1}) + \text{var} f_{2}(\hat{\theta}, \bar{x}_{2}) +
\]

\[
-2\text{cov} \left( f_{1}(\hat{\theta}, \bar{x}_{1}), f_{2}(\hat{\theta}, \bar{x}_{2}) \right). \quad (3.B-3)
\]
We define $g_1$ as

$$g_1 = \frac{\partial f_1}{\partial \theta} (\hat{\theta}, \bar{x}_1)$$

and $g_2$ is defined analogously. According to the delta-method, $\text{var} m_1$ is asymptotically equal to

$$\text{var} m_1 \approx g_1^t \Psi g_1$$

where $\Psi \equiv \text{var} \hat{\theta}$. A similar expression holds for $\text{var} m_2$. Furthermore, we have

$$\text{cov} \left(f_1(\hat{\theta}, \bar{x}_1), f_2(\hat{\theta}, \bar{x}_2)\right) \approx \text{cov} \left(g_1^t \left(\hat{\theta} - \theta\right), g_2^t \left(\hat{\theta} - \theta\right)\right) = g_1^t \Psi g_2.$$

Equation (3.B-3) now reduces to

$$\text{var} (m_1 - m_2) \approx g_1^t \Psi g_1 + g_2^t \Psi g_2 - 2g_1^t \Psi g_2$$

$$= (g_1 - g_2)^t \Psi (g_1 - g_2)$$

(3.B-4)

which we used to calculate the standard errors in table 3.2.

The standard errors of table 3.3 were derived similarly. Since we simulate over the sample, equations (3.B-1) and (3.B-2) change to

$$m_1^* \equiv \frac{1}{N_1} \sum_i f_1(\hat{\theta}, x_i)$$

(3.B-5)

$$m_2^* \equiv \frac{1}{N_2} \sum_j f_2(\hat{\theta}, x_j)$$

(3.B-6)

where we assume that the groups consist of $N_1$ and $N_2$ households respectively. Let $G_1$ be an $(N_1 \times k)$ matrix, with $i$-th row

$$G_{1,i} = \frac{\partial f_1(\hat{\theta}, x_i)}{\partial \theta} (\hat{\theta})$$

and let $G_2$ be defined in the same way. Using similar arguments as above, the variance of $m_1^* - m_2^*$ equals

$$\text{var} (m_1^* - m_2^*) \approx \frac{1}{N_1^2} G_1^t \Psi G_1 + \frac{1}{N_2^2} G_2^t \Psi G_2$$

$$- \frac{2}{N_1 N_2} G_1^t \Psi G_2.$$