Chapter 2

A Structural Model of Rent Assistance and Housing Demand

2.1 Introduction

In most developed nations the government intervenes in the housing market, and The Netherlands is no exception (see e.g. Ball, Harloe and Maartens (1988)). Some of the policies pursued by the Dutch government stimulate the supply of (low-cost) housing, e.g. subsidies for the construction of housing for low-income households. Other policies stimulate the demand for housing, e.g. (full) deductibility of interest payments on mortgages for owner-occupiers and direct rent subsidies for low-income renters. In this chapter, we study the effect of direct rent subsidies on housing demand.

The rent subsidy program in The Netherlands is called Individuele Huursubsidie (IHS) which we shall translate as Rent Assistance (RA). In the program year 1985/86 777 thousand households received RA, that is 25% of all renting households. They received Dfl. 1.344 million (approximately US$ 675 million) in RA subsidies, i.e. Dfl. 1,729 per household that is 33% of the average rent paid by an RA recipient. The RA program was introduced in 1970 in order to bring good quality housing within reach of low-income households. It was felt that the consumption of housing services should be subsidized, because housing was considered to be a merit good having external effects on the health and ability to work of household members. Moreover, under the assumption that rents

1. The program year for RA runs from July 1 to June 30. The year 1985/86 started on July 1, 1985 and ended on June 30, 1986. All our data pertain to this year.
2. In 1985/86 56% of all households were renters.
can be controlled—and indeed in The Netherlands price controls on the rental market are pervasive—the RA program increased the real income of eligible households. Although there is little discussion of the goal of the RA program, it seems that recently the merit good argument has lost ground to distributional considerations.

The RA program affects the relative price of housing services for eligible households in a rather complicated way. The resulting budget set, when choice is restricted to housing services and other consumption, is non-convex. In this chapter we propose a utility maximizing model of housing demand, that takes account of the budget constraint as implied by RA. In specifying this model, we can draw on the extensive experience of applied econometricians with demand analysis in the presence of non-linear budget sets (see e.g. Pudney (1989) for an introduction and Hausman and Wise (1980) for an application of these methods to housing demand). An additional complication is that about 40% of the households that are eligible for RA do not apply for the subsidy. For that reason, we shall specify a joint model of RA take-up and housing demand.

By making a distinction between household preferences and constraints, including the perceived costs of application for RA, we hope to isolate the parameters of the preference structure. If we succeed, we can simulate the effect of changes in the RA program. In chapter 3 we discuss various simulations with the model developed in this chapter. A structural model is better suited to policy analysis, because its parameters are invariant under policy changes. In particular, we can investigate whether RA achieves its stated goals.

The chapter is organized as follows. In section 2.2 we discuss the rules of the RA program. Section 2.3 introduces a structural model for housing demand. The data are discussed in section 2.4. The basic model is estimated in section 2.5, and an extended model is presented in section 2.6. Section 2.7 summarizes and concludes.

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3. The policy intentions of the Dutch government are summarized in *Volkshuisvesting in de jaren negentig* (Housing in the Nineties).
2.2 The Rent Assistance Program and Rental Housing Supply

2.2.1 The Rent Assistance Program

The eligibility for RA and the amount of the subsidy are determined by three parameters: household income, household composition and rent. We refer to the relevant measure of rent paid as the RA rent. The RA rent includes some service charges, such as charges for heating and cleaning of communal space in an apartment building (but not of the apartments), but it excludes charges for cleaning windows or the rent of a garage that sometimes are paid with the rent.

A household is eligible for RA if the RA rent exceeds the norm rent, but is lower than the maximum rent. The norm rent is the rent that the household is supposed to be able to pay, given its composition and income. It depends on household taxable income in the calendar year preceding the program year, and on household composition. Household taxable income is the sum of the taxable incomes of the household members. The norm rent increases with household taxable income, but decreases with family size. The only distinction made in household composition is between households having one member and households having two or more members. To be eligible for RA in the program year 1985/86 taxable income in 1984 had to be less than Dfl. 35 000 for households with two or more members or Dfl. 31 000 for households with only one member. The maximum rent in 1985/86 was equal to Dfl. 8 040 per year for households with two or more members and Dfl. 6 360 for households with one member. The household received no RA, if the RA rent exceeded the maximum rent. A household did also not qualify for RA, if its RA rent was less than Dfl. 2 960 per year. This is the lower bound on the norm rent.

The amount of the subsidy is determined by the difference between the RA rent and the norm rent. The computation is illustrated

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4. The administration of the RA program is in the hands of the municipalities (in Dutch: gemeenten).
5. If taxable income is expected to change by more than 25% in the program year, an estimate of taxable income is used to compute the RA entitlement.
6. A household is not eligible for RA, irrespective of its income, if the value of its assets exceeds Dfl. 107 000.
7. To be precise, the lower bound on the norm rent in 1985/86 was Dfl. 2 780, but RA was only paid if the subsidy exceeded Dfl. 180 per year.
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in figure 2.1. The numbers refer to a household with two or more members. The computation is similar for households with one member. The lower boundary of the region in figure 2.1 is the norm rent\(^8\). The lowest norm rent is Dfl. 2780 per year and the highest norm rent is Dfl. 7540 per year. The norm rent is a step function of taxable household income. It is constant on intervals of width Dfl. 500 (taxable household income less than Dfl. 28000) or Dfl. 1000 (taxable household income between Dfl. 28000 and Dfl. 35000). The regions A to E correspond to different subsidy rates. In region A, the subsidy rate is 100%, in region B 90%, and in regions C, D and E it is 80%, 70% and 60% respectively. The subsidy rates are applied to the difference between the RA rent and the norm rent that is in the relevant region. Consider e.g. a household with a taxable income of Dfl. 27250 and an RA rent of Dfl. 7000. The norm rent for this household is Dfl. 4600, so that the RA computation is based

\(^8\) In figure 2.1 the relation between household taxable income and norm rent is somewhat simplified.
on the difference, Dfl. 2400. This gap is covered by the regions B, C and D, Dfl. 760 in B, 1200 in C and 440 in D. Hence, the subsidy is equal to $0.90 \times 760 + 0.80 \times 1200 + 0.70 \times 440 = \text{Dfl. 1952.}$ The subsidy is rounded to a smaller integer multiple of Dfl. 60, so that the subsidy is Dfl. 1920, 27% of the RA rent.

From figure 2.1 it is clear that the marginal price of housing services is not constant. Depending on household taxable income and the RA rent a household pays 0% (if the RA rent is in region A) to 100% (if the RA rent is not in the regions A to E) of an additional guilder spent on housing.

Note that the dependence of the norm rent on taxable income also increases the income tax rate, in particular for low-income families. Hence, the RA program could have an effect on the work effort. We neglect possible simultaneity of the labour supply and housing demand decision. In general, RA is a non-negligible part of disposable income. In the data used in this chapter, the average fraction of household disposable income derived from RA is 10% for RA recipients. For families in the first quartile of the income distribution, this fraction is 13%.

### 2.2.2 Rental Housing Supply

We estimate the effect of RA on rental housing demand. In general, price subsidies raise the price of the subsidized good. Hence, it is important to consider the supply of rental housing to low-income renters. Most of the rental housing stock (71%) in this segment is owned by housing associations, non-profit organizations that are subsidized by the central government. Municipalities and other non-profit organizations own 15% and the remaining 14% is owned by the private sector. There is national rent control in this segment of the market through the 'rent scoring system' (in Dutch: puntensysteem). In this system scores are associated with size, year of construction, building costs and amenities of the dwelling and the rent is determined by multiplying the total score by the rent per point. The central government determines the yearly change in the rents. The rent scoring system has two effects. First, it increases the correlation between the (observed) quality of the dwelling and the rent. In the sequel we assume that the flow of housing services provided by a dwelling is proportional to its rent. Second, it limits the scope for

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9. Some deviation from this rent per point is allowed.
fraudulent deals between renters and landlords, that are unlikely anyway because of the incentives of the owners.

The government subsidizes the construction of housing and the renovation of existing dwellings for low-income households. These subsidies aim at satisfying the demand for housing at the price set by the government\(^{10}\). These policies were first implemented after the Second World War in reaction to an acute housing shortage, but have remained in place to this time in which there is no indication of aggregate excess demand\(^{11}\). The centrist parties that have been in power in this period have consistently supported these policies, and, as one would expect, vested interests that have developed in this period resist changes. Hence, government intervention ensures that the supply of rental housing in the market segment under consideration is infinitely elastic at a given price per unit of housing services, a price that is moreover approximately the same for all households\(^{12}\). These conditions ensure that in estimating the effect of RA on housing demand we can ignore the supply side of the housing market.

2.3 A Model of Housing Demand with Rent Assistance

2.3.1 Household Utility Maximization

In this section we propose a model of housing demand in the presence of RA. We assume that the household is the decision making unit, and that its preferences can be described by a single utility function. The household divides its income between housing services and other consumption. The price of a unit of housing services is the same for all dwellings, and without loss of generality we set it to Dfl. 1\(^{13}\). Hence, the rent equals the quantity of housing services provided by the dwelling.

\(^{10}\) The data for this study were taken from the Housing Needs Survey, the government-sponsored 'market-research' that is used to predict the need for additional housing construction.

\(^{11}\) We do not claim that every household lives in its preferred dwelling. Households may have to settle for a dwelling that is suboptimal. In our model we allow for this type of 'rationing'\(^{14}\).

\(^{12}\) Of course, delays in construction may cause temporary shortages if actual demand exceeds the predicted demand.

\(^{13}\) As pointed out before, the purpose of the rent guidelines of the Dutch government is to reduce dispersion of unit prices. Moreover, if the household faces unit price dispersion, it may use the expected unit price to determine its demand for housing...
We assume that the household maximizes its utility function subject to a budget constraint that is affected by the RA program. We also must take account of the partial take-up of RA benefits. First, we discuss the budget constraint. We then specify household preferences and we consider the household maximization problem. Finally, we propose a model for the take-up of RA.

2.3.2 The Budget Constraint with RA

The budget constraint of the household is

\[ R + X = Y + S, \tag{2.1} \]

where \( R \) denotes the rent, \( X \) the consumption of other goods, \( Y \) is disposable income and \( S \) is the RA subsidy, which may be 0. For \( R \) we shall use the RA rent.

\( S \) is determined by the difference between the RA rent \( R \) and the norm rent \( R_n(Y_T, H) \) that depends on household taxable income \( Y_T \) and household composition \( H \). Although the subsidy rate \( \delta \) depends on \( R \) (and \( Y_T \) and \( H \) ) (see figure 2.1), we apply a constant subsidy rate to the difference. We set \( \delta = 0.823 \), which is the average rate for RA recipients in our sample. Using this simplification, we can compute the RA subsidy by

\[ S = \begin{cases} \delta(R - R_n(Y_T, H)) & \text{if } R_n(Y_T, H) \leq R \leq R_{\text{max}}(H), \\ 0 & \text{if } R < R_n(Y_T, H) \text{ or } R > R_{\text{max}}(H), \end{cases} \tag{2.2} \]

where \( R_{\text{max}}(H) \) is the maximum rent, that depends on the household composition.

Substitution of equation (2.2) in equation (2.1) and some rewriting gives the budget constraint

\[
\begin{align*}
R + X & = Y & \text{if } R_n(Y_T, H) \leq R \leq R_{\text{max}}(H), \\
(1 - \delta)R + X & = Y - \delta R_n(Y_T, H) & \text{if } R_n(Y_T, H) \leq R \leq R_{\text{max}}(H). \\
\end{align*}
\tag{2.3}
\]

services. This price need not reflect the costs of providing these housing services. Building costs are sometimes subsidized by the government.
If we set $R_n = R_{\text{max}}$ for households that do not qualify for RA because their taxable income is too high, then equation (2.3) applies to all households in the population.

Equation (2.3) makes clear that RA has two effects on the budget constraint. First, it reduces the (marginal) price of housing services from 1 to $1 - \delta$. Second, it has a negative effect on disposable income. To be eligible for RA the household must consume an amount of housing services that exceeds the norm rent $R_n$. A fraction $1 - \delta$ of $R_n$ has to be paid anyway, but the amount $\delta R_n$ is the household contribution to the ‘entry fee’ for RA. In other words, $\delta R_n$ can be considered as a fixed cost, which has to be incurred in order to be eligible for RA. Following e.g. Blomquist (1983) we define virtual income $Y_v$ by

$$Y_v = Y - \delta R_n(Y_T, H).$$

Hence we can rewrite the second line in equation (2.3) as

$$(1 - \delta) R + X = Y_v \quad R_n(Y_T, H) \leq R \leq R_{\text{max}}(H).$$

The fixed cost is on average Dfl. 2,735 per year which equals 14% of average disposable household income.

The budget constraint of an RA recipient is drawn in figure 2.2. Income of the household is given by $OY$ (i.e. $OY'$) and virtual income is $OY_v$. It is evident that the budget set of an RA recipient is non-convex. The slope of the segments $YA$ and $A''Y'$ is 1, while the slope of the segment $AA'$ is $1 - \delta$, reflecting the lower marginal price of housing services under RA.

From figure 2.2 we see that households that would choose an $(R, X)$ combination on the segment $AA'$ in the absence of RA move to $AA'$ after introduction of RA. Moreover, some households that give housing low priority move from $YA$ to $AA'$ and some households with strong relative preferences for housing services move from $A''Y'$ to $AA'$.

Without knowledge of the preferences of the household we can not make more precise predictions.

### 2.3.3 Preferences and Utility Maximization

We assume that household preferences can be represented by the utility function

$$u(R, X) = \left( \frac{R}{\beta_1 + \beta_2} \right) \exp \left( \frac{\beta_1 X - \beta_2 R + \beta_3 \beta_4}{\beta_3 + \beta_4 R} \right).$$

(2.4)
If we maximize (2.4) subject to a linear budget constraint
\[ pR + X = Y, \] (2.5)
we obtain the indirect utility function
\[ \nu(p, Y) = \left( Y + \frac{\beta_2}{\beta_1} p + \frac{\beta_3}{\beta_1} + \frac{\beta_0}{\beta_1} \right) \exp(-\beta_1 p), \] (2.6)
and the demand for housing services
\[ R = \beta_0 + \beta_1 Y + \beta_2 p. \] (2.7)

We are somewhat restricted in our choice of preference structure, because we need an explicit expression for either the direct or the indirect utility function. In section 2.5 we shall test whether this restrictive specification of preferences biases our results.

According to the Slutsky condition, the parameters of demand equation (2.7) have to satisfy the following restriction:
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\[
\frac{\partial R}{\partial Y} \cdot R + \frac{\partial R}{\partial p} = \beta_1 R + \beta_2 \leq 0.
\]  

(2.8)

If the parameters do not satisfy this restriction, then the solution does not satisfy the second-order conditions for the maximization of utility function (2.4) subject to budget constraint (2.5).

The budget set in Figure 2.2 is non-convex. In Figure 2.3 we decompose this budget set in two convex sets whose union is the original non-convex budget set. We consider utility maximization subject to the budget constraints A and B separately. The optimal choice with budget constraint A, which is the constraint faced by households that are not eligible for RA, is denoted by \((R_A, X_A)\). The optimal choice with budget set B is \((R_B, X_B)\). The utility maximizing \((R, X)\) is found by comparing \(u(R_A, X_A)\) and \(u(R_B, X_B)\).

Note that this solution method requires knowledge of the direct utility function \(u(R, X)\). A solution method that only requires the indirect utility function is preferable, because by Roy’s identity we can obtain the demand for housing services directly from the indirect utility function. Hence, expressing the decision to apply for RA in terms of the indirect utility function gives us additional flexibility in the selection of functional forms, because an explicit solution for the direct utility function is not required. If we ignore the constraint \(R < R_{\text{max}}(H)\) in

\begin{align*}
\text{Figure 2.3: Decomposition of the non-convex budget set in two convex budget sets}
\end{align*}
A Model of Housing Demand with Rent Assistance

If we assume that the preferences are such that optimal choice under RA is always on the interior of $Y^e$, then an eligible household will choose a dwelling with RA if and only if

$$\nu(1 - \delta, Y^e) > \nu(1, Y).$$  

(2.9)

and the indirect utility function in (2.6) leads to the following demand equations (here and in the sequel $R_A$ and $R_B$ refer to unrestricted choices):

$$R = \begin{cases} R_A = \beta_0 + \beta_2 + \beta_1 Y & \text{if not RA}, \\ R_B = \beta_0 + \beta_2 (1 - \delta) + \beta_1 Y^e & \text{if RA}, \end{cases}$$

(2.10)

with according to equation (2.9)

$$RA \Leftrightarrow I^* = \nu(1 - \delta, Y^e) - \nu(1, Y) > 0,$$  

(2.11)

where

$$I^* = \left( \frac{\beta_2}{\beta_1} (1 - \delta) + \frac{\beta_2}{\beta_1} \right) \exp(-\beta_1 (1 - \delta)) +$$

$$- \left( \frac{\beta_2}{\beta_1} + \frac{\beta_2}{\beta_1} \right) \exp(-\beta_1) +$$

$$+ Y^e \exp(-\beta_1 (1 - \delta)) - Y \exp(-\beta_1).$$  

(2.12)

We can easily acknowledge the constraint $R < R_{\text{max}}$ in the RA regime. Using the results in Neary and Roberts (1980) we define $\tilde{p}$ and $\tilde{Y}$ as the price and income that support the choice $R_{\text{max}}$, and we obtain the demand equation

$$R = \begin{cases} R_A = \beta_0 + \beta_2 + \beta_1 Y, \\ \nu(1, Y) > D(R_B < R_{\text{max}}) \nu(1 - \delta, Y^e) + \\ + D(R_B \geq R_{\text{max}}) \nu(\tilde{p}, \tilde{Y}), \\ R_B = \beta_0 + \beta_2 (1 - \delta) + \beta_1 Y^e, \\ \nu(1 - \delta, Y^e) \geq \nu(1, Y), R_B < R_{\text{max}}, \\ R_{\text{max}}, \\ \nu(\tilde{p}, \tilde{Y}) > \nu(1, Y), R_B \geq R_{\text{max}}, \end{cases}$$

(2.13)

14. These satisfy the equations

$$Y^e - (1 - \delta) R_{\text{max}} = \tilde{Y} - \tilde{p} R_{\text{max}}$$

$$R_{\text{max}} = \beta_0 + \beta_2 \tilde{p} + \beta_1 \tilde{Y}. $$
where $D(\cdot)$ is the indicator function of the event in parentheses. Households that face a binding constraint in the RA regime either choose the corner solution $R_{\text{max}}$ or leave the RA regime and choose a dwelling on $A^\rho Y'$. Note that only households with a binding constraint in the RA regime choose rents that exceed $R_{\text{max}}$. Comparison of (2.10) and (2.13) shows that the latter is less suited for an empirical model, in particular if we want to allow for population variation in the parameters. Equation (2.10) results in a reduced form that is linear in parameters, but such a simplification is not obtained in (2.13), because $\rho$ and $Y$ are nonlinear functions of the parameters. Although these problems are not insurmountable, we shall see in section 2.4 that the fraction of households that choose a corner solution is negligible, and that very few households pay a rent larger than $R_{\text{max}}$. Hence, in practice the constraint is not binding, and we use equation (2.10) as the basis for the empirical analysis.

### 2.3.4 Modelling the Take-up of RA

It is well known, that the take-up of income-support programs is in general less than 100%, see for instance Blundell, Fry and Walker (1988) and Moffitt (1983). For the RA program, this fact has also been documented. Estimates of the take-up rate for RA vary from 44% to 76% (Konings and Van Oorschot (1990)). We shall incorporate a take-up decision in model (2.10)–(2.12).

One can think of at least two reasons why households do not apply for RA, even though they are entitled to benefits. First of all, the household may be unaware of its entitlement. As seen in section 2.2, the program is rather complex, and it is not immediately clear if a household is entitled to an RA subsidy, given its income and rent. Schep (1991) summarises research on the take-up of social benefit programs in The Netherlands, one of the programs being the RA program. In a test, application forms were given to a group of low-income households and a group of households headed by someone with a university degree. It turned out that no household of the first group was able to fill in the form correctly, and only 12% of the second group was able to do so. The second reason for not using the program is the existence of application costs. These costs can be monetary (one has to make xeroxes, fill in forms, read information, etc.) and non-monetary (stigma associated with using a government income-support program, cf. Moffitt (1983)).
Our empirical results show that the take-up is strongly related to the amount of benefit that one would obtain under RA. This is consistent with the presence of application costs, and hence we model the take-up by introducing such costs.

Let the costs be denoted by $C'$. Household income under RA is now $Y_v - C'$, with indirect utility $\nu(1 - \delta, Y_v - C')$. Hence, a household will choose a dwelling with RA, if $\nu(1 - \delta, Y_v - C') > \nu(1, Y)$. In this approach we can also take account of non-monetary indirect utility costs. Suppose these non-monetary costs are $\bar{c}$ (measured in utils). Then, the household will choose a rent with RA, if

$$\nu(1 - \delta, Y_v - C') - \nu(1, Y) > \bar{c},$$

which with specification (2.6) can be rewritten as

$$\nu(1 - \delta, Y_v) - \nu(1, Y) > C' \exp(-\beta_1 (1 - \delta)) + \bar{c}.$$  \hfill (2.14)

If we redefine the costs incurred as $C = C' + \bar{c} \exp(-\beta_1 (1 - \delta))$, one sees that a household will choose a rent with RA if

Figure 2.4: The consequences of application costs
\[ \nu(1 - \delta, Y_v - C) - \nu(1, Y) > 0. \]  

The non-monetary costs \( \tilde{C} \) are valued at the marginal utility of income. In the present model, monetary and non-monetary application costs reduce virtual income \( Y_v \) under RA. We can not distinguish between monetary application costs (\( C' \)) and non-monetary application costs (\( \tilde{C} \)) in this functional specification.

The effect of application costs on the budget constraint is illustrated in figure 2.4. The budget constraint with application costs is \( YAB' A'' Y' \). The effect of RA on households with rents on \( AA'' \) is different with and without application costs. In both cases households on \( B'' A'' \) will apply for RA. However, if \( C = 0 \) all households on \( AB' \) will apply, but whether a household on \( B'' A'' \) will apply if \( C > 0 \) depends on its relative preference for housing services. Households with low relative preferences will choose not to apply. Hence, if there are application costs then application for RA is positively related to the amount of the entitlement.

2.4 The Data

2.4.1 Description of the Data and Descriptive Statistics

For the empirical analysis we have used data from the Woningbehoefstenonderzoek 1985/86 (Housing Needs Survey 1985/86, to be abbreviated as HNS 1985/86). This survey is based on a large sample from the Dutch population (54342 responding households, with the sample size being 70816). The sample and the sample design are described in detail in CBS (1990). For our purposes, we can consider the sample as a random sample of households. The survey contains detailed information on the dwelling of the households, as well as on their socio-economic characteristics.

We do not use all sample households in the analysis. We restrict ourselves to renters who satisfy certain criteria. These criteria are listed in Appendix 2.A. Most selections are made to ensure that the utility-maximizing model is a reasonable description of household behaviour. We retain only households of which either the head of the household or his/her partner are interviewed. Moreover, we only consider households with a taxable income that entitles it to RA. Whether a potential RA recipient actually receives RA is another matter.
There are three reasons why a potential RA recipient does not receive RA: the rent paid is smaller than the norm rent, the rent paid is higher than the maximum rent or the household is eligible for RA, but it does not apply for the subsidy. In the sample we find that very few households do not receive RA because their rent exceeds the maximum rent. Moreover, there is no indication that households in the RA regime are constrained by the restriction that the rent should not exceed the maximum rent. If this were the case, we would observe a clustering of observed rents at and slightly below the maximum rent. We do not observe such a clustering as is evident from the density of observed rents in figure 2.5. For these reasons, we select only those households whose rent is below the maximum rent and for these households we neglect the constraint that the rent should not exceed the maximum rent. This selection facilitates the empirical analysis.

We want to include only households that are utility maximizers. A standard approach in the literature is to select households that have moved recently (see, e.g. Ball and Kirwan (1977)). Households that moved a long time ago may no longer be in equilibrium, because adjustment costs may prevent them from moving to another dwelling.
By retaining only those households that have moved recently, we hope that the observed consumption of housing services is close to the utility maximizing level of consumption. In section 2.5 we test whether this restriction biases our results.

We have used some additional information to identify utility-maximizing households. In the IHS 1985/86, households were asked if they intended to move within two years and whether they were satisfied with their dwelling and neighbourhood. We select those households which claimed to have no intentions of moving within two years and which were reasonably happy with their dwelling and neighbourhood. Even though this selection is based on intentions and not on observed behaviour, we think that it improves the correspondence between the data and the model.

A problem in analyzing housing demand is that we only observe housing expenditures. Housing expenditures are the product of the unit price of housing services and the quantity of housing services. However, price and quantity are not observed separately. For that reason we assume that the unit price of housing services is the same for all rental dwellings. In other words, differences in rents reflect differences in the quantity of housing services rather than differences in the price of housing services. We normalize the price component to 1. Every other normalization would do, because it merely changes the units of measurement of the quantity of housing services. Hence, the only price variation we allow for is the price variation due to the RA program.

For each household in the sample we computed its RA entitlement using information on household taxable income and family composition. The income measure needed for the calculation of the RA benefit in the year July 1 1985 – June 30 1986 is taxable household income in 1984. However, taxable income in the IHS 1985/86 is measured over the year 1985. We assumed that taxable wage income increased by 2% from 1984 to 1985\textsuperscript{15}, and we assumed that social security benefits remained constant. This enabled us to estimate taxable household income in 1984.

We present some summary statistics in table 2.1.

All variables have been introduced before, except SIZE and AGE. SIZE is the size of the household and AGE is the age of the head of the household. All monetary variables are measured in thousands of guilders.

\textsuperscript{15} See Central Planning Bureau (1986), table IV.8.
The Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full sample</th>
<th>RA-recipients</th>
<th>Non-recipients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income ($Y$)</td>
<td>22.93</td>
<td>20.83</td>
<td>24.28</td>
</tr>
<tr>
<td></td>
<td>(6.00)</td>
<td>(5.32)</td>
<td>(6.03)</td>
</tr>
<tr>
<td>Virtual Income ($Y_v$)</td>
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<td>18.02</td>
<td>20.42</td>
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<tr>
<td></td>
<td>(5.07)</td>
<td>(4.78)</td>
<td>(5.03)</td>
</tr>
<tr>
<td>Rent ($R$)</td>
<td>4.84</td>
<td>5.62</td>
<td>4.34</td>
</tr>
<tr>
<td></td>
<td>(1.52)</td>
<td>(1.23)</td>
<td>(1.48)</td>
</tr>
<tr>
<td>Norm rent ($R_n$)</td>
<td>4.12</td>
<td>3.36</td>
<td>4.61</td>
</tr>
<tr>
<td></td>
<td>(1.47)</td>
<td>(0.90)</td>
<td>(1.56)</td>
</tr>
<tr>
<td>Rent assistance ($S$)</td>
<td>1.08</td>
<td>2.01</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>(1.20)</td>
<td>(1.05)</td>
<td>(0.86)</td>
</tr>
<tr>
<td>Entitled to RA</td>
<td>61.4%</td>
<td>100%</td>
<td>36.5%</td>
</tr>
<tr>
<td>Price ($p$)</td>
<td>0.68</td>
<td>0.18</td>
<td>1.00</td>
</tr>
<tr>
<td>Size</td>
<td>2.35</td>
<td>2.40</td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td>(1.31)</td>
<td>(1.28)</td>
<td>(1.32)</td>
</tr>
<tr>
<td>Age</td>
<td>43.72</td>
<td>45.73</td>
<td>42.42</td>
</tr>
<tr>
<td></td>
<td>(18.75)</td>
<td>(19.50)</td>
<td>(18.14)</td>
</tr>
<tr>
<td>Observations</td>
<td>1809</td>
<td>710</td>
<td>1099</td>
</tr>
</tbody>
</table>

Table 2.1: Means of variables, standard deviations in parentheses

(per year). The variable Rent assistance in table 2.1 is the computed RA subsidy, i.e. the outcome of our computation of the RA benefits. The household may or may not take up these benefits.

Note that RA recipients spend, on average, more on housing than non-recipients. This may be due to the lower price of housing in the RA regime, but it may also be a consequence of the threshold, i.e. the norm rent, in the RA program. We also see that the fixed cost of entering the program (the difference between $Y$ and $Y_v$, see section 2.3.2) is higher for non-recipients than for recipients. The differences in household size and age between the two groups are small.

Note that the average computed RA subsidy is not zero for households that do not receive RA. This means that there are households that are entitled to an RA benefit, but that do not receive the benefit. In fact, in our sample the take-up is 63.9%. The partial take-up of RA benefits will receive explicit attention in our empirical model.
2.4.2 A Preliminary Analysis

From table 2.1 we can obtain crude estimates of the price and income elasticity of housing demand. We estimate the price elasticity by

$$\hat{\eta}_p = \frac{(\bar{R}_B - \bar{R}_A)}{(\bar{p}_B - \bar{p}_A) / \bar{p}} = -0.22,$$

where $\bar{R}_A$ is the average rent paid by non RA-recipients, $\bar{R}_B$ the average rent paid by RA-recipients, $\bar{R}$ the average rent paid in the sample, etc. If the income elasticity of housing demand is positive, this is an underestimate because the average income of RA-recipients is lower than that of non-recipients. However, if we use a similar procedure to estimate the income elasticity of housing demand, we obtain $\hat{\eta}_Y = -1.76$. This counterintuitive result is a direct consequence of the stronger incentives of the RA program for lower income households.

We can avoid the use of between-regime income variation by a slightly more sophisticated analysis in which we regress the rent on price and income. The resulting price and income elasticities are $\hat{\eta}_p = -0.27$ and $\hat{\eta}_Y = 0.43$.

It must be stressed that these estimates may still be biased. First, the norm rent may have an upward effect on the rents paid in the RA regime, resulting in an upward bias in the absolute value of the price elasticity. Moreover, its dependence on income may induce an upward bias in the estimate of the income elasticity. Second, the price may be endogenous, e.g. because RA-recipients may have a relatively strong preference for housing services causing an upward bias in the absolute value of the price elasticity. Third, we have not distinguished between non-recipients with and without entitlement to RA. Fourth, for RA-recipients the appropriate income measure is virtual income $Y_v$ that includes the fixed costs of RA. The structural model of the next section will deal with these potential biases.

One implication of our theoretical model is that there is a positive relationship between the take-up of the RA-benefits and the amount of the benefit (see subsection 2.3.4). We examine this by estimating a probit model for households entitled to RA, with the dependent variable being 1 if the households exercises its entitlement to RA and 0 otherwise, and with independent variables the amount of RA $(S)$ and income $(Y)$. The estimation results and standard errors are:
\[-0.30 + 0.40S - 0.00076Y\]
\[(0.19)\quad (0.040)\quad (0.0075)\]

Only the coefficient of $S$ is significant at a 5% level. We conclude that there is a strong positive relation between take-up and the amount of the benefit, as is predicted by the model in section 2.3.4.

### 2.4.3 Measurement Error and the Take-up of RA

We encounter two problems when we determine the take-up status of a household. First, the assessment of eligibility for RA is based on predicted household taxable income in 1984. Hence, prediction errors may lead to erroneous inclusion or exclusion of some households in our sample, i.e. those households whose taxable income is erroneously predicted to be below or above the maximum eligible income. Second, some households that receive RA may not report this. The reason is that RA is paid either to the landlord or the household. In the first case RA is subtracted from the rent. Some households that receive RA through their landlord may report that they do not receive the subsidy. Using information on the population fraction of households that receive RA directly, we can re-estimate the take-up rate.

Let $I$ be an indicator of whether a household receives RA ($I = 1$) or not ($I = 0$), and $D$ an indicator of whether the RA is paid to the household ($D = 1$) or to the landlord ($D = 0$). $D$ is defined only if $I = 1$. The households report $\tilde{I}$ and $\tilde{D}$. We want to estimate $\Pr(I = 1)$, the population fraction that receives RA. We assume that there are no households that report incorrectly that they receive RA, i.e. $\Pr(I = 0, \tilde{I} = 1) = 0$ so that we have

$$\Pr(I = 1, \tilde{I} = 1) = \Pr(\tilde{I} = 1).$$

We have

$$\Pr(D = 0 \mid I = 1) = \Pr(D = 0 \mid I = 1, \tilde{I} = 1) \Pr(\tilde{I} = 1 \mid I = 1) + \Pr(D = 0 \mid I = 1, \tilde{I} = 0) \Pr(\tilde{I} = 0 \mid I = 1). \quad (2.16)$$

Hence,

$$\Pr(\tilde{I} = 1 \mid I = 1)$$

---

16. As discussed in section 2.4.1, we only consider those households, that are eligible for RA.
A Structural Model of Rent Assistance and Housing Demand

\[
\Pr(I = 1) = \Pr(\tilde{I} = 1) \times \\
\times \frac{\Pr(D = 0 \mid I = 1, \tilde{I} = 1) - \Pr(D = 0 \mid I = 1, \tilde{I} = 0)}{\Pr(D = 0 \mid I = 1) - \Pr(D = 0 \mid I = 1, \tilde{I} = 0)} (2.18)
\]

which is rewritten as

\[
\Pr(I = 1) = \Pr(\tilde{I} = 1) \times \\
\times \frac{1 - \Pr(D = 0 \mid I = 1, \tilde{I} = 1)}{1 - \Pr(D = 0 \mid I = 1)} (2.19)
\]

In our case, the factor on the right-hand side of equation (2.19) is 1.08, so that the estimate for the take-up rate, which uses the additional information about the way RA is received, is 69%.

2.5 An Empirical Model of Rental Housing Demand

In this section, we first discuss our estimation strategy, which we then use to obtain estimates of the parameters of the model. The estimation strategy consists of three steps. First, we choose a stochastic specification for the structural model of rental housing demand. Next, we note that the structural model can be obtained by restricting the parameters of a reduced form model. We derive the likelihood function of this reduced form model. Finally, we obtain the structural parameters from the reduced form parameters by the minimum distance method. This estimation procedure is computationally simpler than and asymptotically equivalent to maximum likelihood estimation of the structural model. In section 2.5.2 we present our empirical results. The results of some specification tests are discussed in section 2.5.3.
2.5.1 Stochastic Specification and Estimation Strategy

The model of the previous section is not suitable for estimation because it assumes that every household has the same preference structure, i.e., the same $\beta$. It is unreasonable to impose this restriction. One can model variation in preferences by making $\beta$ dependent on demographic characteristics, but not all variation can be explained. Moreover, we have seen in the last section that with respect to demographic variables as household size and age of the head, households that receive RA do not differ much from households that do not receive RA. Therefore, it is unlikely that the difference in average housing demand between RA recipients and other households can be attributed to differences in demographic characteristics. We model heterogeneity of preferences by making $\beta_0$ a random variable that varies over the population. The marginal rate of substitution between housing services and other goods is $-\frac{\beta_0 X - R + \beta_0}{\beta_0^2 + \beta_1^2} = \frac{u_X}{u_X}$. If the Slutsky condition is satisfied, the denominator is negative and the marginal rate of substitution increases linearly with $\beta_0$. Households with a large $\beta_0$ strongly prefer housing over other goods.

Let $\beta_0$ be normally distributed with mean $\delta_0$ and variance $\sigma_0^2$: $\beta_0 \sim \mathcal{N}(\delta_0, \sigma_0^2)$. The deviation of $\beta_0$ from its mean is denoted by $\zeta$. If all variation in housing demand is due to preference and income variation, the stochastic version of our demand model becomes:

\[
R = \begin{cases} 
\delta_0 + \beta_0 + \beta_1 Y + \zeta & I = 0 \\
\delta_0 + \beta_0(1 - \delta) + \beta_1 Y_0 + \zeta > R_n & I = 1 
\end{cases} 
\]

\[
I^* = \left( \frac{\beta_2}{\beta_1} (1 - \delta) + \frac{\beta_2}{\beta_1^2} + \frac{\delta_0}{\beta_1} \right) \exp(-\beta_1(1 - \delta)) + 
- \left( \frac{\beta_2}{\beta_1} + \frac{\beta_2}{\beta_1^2} + \frac{\delta_0}{\beta_1} \right) \exp(-\beta_1) + \exp(-\beta_1(1 - \delta))Y_0 + 
- \exp(-\beta_1)Y - \exp(-\beta_1(1 - \delta))C + 
+ \left( \frac{\exp(-\beta_1(1 - \delta)) - \exp(-\beta_1)}{\beta_1} \right) \zeta, 
\]

\[
I = \begin{cases} 
0 & I^* < 0 \\
1 & I^* \geq 0 
\end{cases} 
\]

Since rents in the RA-regime ($I = 1$) necessarily exceed the norm rent $R_n$, the distribution of rents in this regime is truncated from below.
In equation (2.20), and later on, this is indicated by \( I > R_{n} \) after the demand equation. Note that if \( \beta_{1} > 0 \), then \( I^* \) is increasing in \( \zeta \), i.e. households with a relatively strong preference for housing are more likely to receive RA.

Of course, it is overly restrictive to allow only for preference heterogeneity. Another source of variation in the demand equation is the difference between the realized consumption of housing services and the desired consumption of these services. At the moment of the decision the desired type of dwelling may not be available, and the household must settle for a dwelling that provides either a larger or smaller amount of housing services. We assume that on average households realize their desired level of consumption. This assumption will be tested in section 2.5.3. The assumption is in line with the fact that aggregate demand and supply are approximately equal (see section 2.2.2). Because it may be easier to find a dwelling with the desired level of housing services in either the RA- or non-RA regime, the variance of this optimization-failure disturbance term need not be equal in both regimes. Households that prefer the RA regime face a restriction when choosing a particular dwelling. Even if the actual level of housing services provided by the dwelling is not equal to the desired level, it must exceed the level corresponding to the norm rent. We assume that households in the RA regime are aware of this restriction, so that the rents in the RA regime are truncated at the norm rent. Households that prefer the non-RA regime do not face a similar restriction, because there is no obligation to take up the RA benefits. Note that the truncation in the RA regime only is needed if we allow for optimization errors. In (2.20) the rent in the RA regime necessarily exceeds \( R_{n} \). We also allow for additional variation in the regime allocation equation that reflects among other things unobserved heterogeneity in \( C \).

The complete stochastic specification of our model is now:

\[
R = \begin{cases} 
\delta_{0} + \beta_{2} Y + \zeta + v_{1} & I = 0 \\
\delta_{0} + \beta_{2}(1 - \delta) + \beta_{4} Y_{v} + \zeta + v_{2} > R_{n} & I = 1 
\end{cases}
\]

\[
I^* = \frac{\beta_{2}}{\beta_{1}} (1 - \delta) + \frac{\beta_{2}}{\beta_{1}} + \frac{\delta_{0}}{\beta_{1}} \exp(-\beta_{1}(1 - \delta)) + \\
- \left( \frac{\beta_{2}}{\beta_{1}} + \frac{\beta_{2}}{\beta_{1}} + \frac{\delta_{0}}{\beta_{1}} \right) \exp(-\beta_{1}) + \exp(-\beta_{1}(1 - \delta))Y_{v} + 
\]
\[-\exp(-\beta_1 Y) - \exp(-\beta_1 (1 - \delta)) C + \]
\[\left( \frac{\exp(-\beta_1 (1 - \delta)) - \exp(-\beta_1)}{\beta_1} \right) \zeta + v_3, \tag{2.23} \]

\[I = \begin{cases} 
0 & I^* < 0 \\
1 & I^* \geq 0
\end{cases} \]

We assume that the preference heterogeneity \( \zeta \) is independent of the optimization errors \( v_1, v_2 \) and \( v_3 \). The variances of these terms will be denoted by \( \sigma_\zeta^2, \sigma_v^2, \sigma_v^2 \) and \( \sigma_v^3 \) respectively.

If we ignore the parameter restrictions on (2.22) and (2.23), the corresponding reduced form model is:

\[ R = \begin{cases} 
\alpha_0 + \alpha_Y Y + \varepsilon_1 & I = 0 \\
\alpha_1 + \alpha_{Y_1} Y_1 + \varepsilon_2 > R_n & I = 1
\end{cases} \tag{2.24} \]

\[ I^* = \gamma_0 + \gamma_{Y_1} Y_1 + \gamma_Y Y + \eta \tag{2.25} \]

\[ I = \begin{cases} 
0 & I^* < 0 \\
1 & I^* \geq 0
\end{cases} \]

For future reference, we define \( \tilde{R}_A \) to be the systematic part of the first equation, i.e. \( \tilde{R}_A = \alpha_0 + \alpha_Y Y \). \( \tilde{R}_B \) and \( \tilde{I} \) are defined analogously as the systematic parts of the second demand equation and the regime allocation equation.

The distribution of the disturbances is

\[ \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \eta \end{pmatrix} \sim N_3 \begin{pmatrix} 0, \begin{pmatrix} \sigma_{\varepsilon_1}^2 & \sigma_{\varepsilon_1 \varepsilon_2} & \sigma_{\varepsilon_1 \eta} \\ \sigma_{\varepsilon_2}^2 & \sigma_{\varepsilon_2 \eta} & \sigma_{\varepsilon_2 \eta} \\ \sigma_{\eta}^2 & \sigma_{\eta} & 1 \end{pmatrix} \end{pmatrix} \].

We impose the conventional normalization \( \sigma_{\eta}^2 = 1 \).

The identification of the structural parameters from the reduced form parameters proceeds as follows. First, \( \beta_1 \) is equal to \( \alpha_Y \) or \( \alpha_{Y_1} \). The equality of \( \alpha_Y \) and \( \alpha_{Y_1} \) is an overidentifying restriction on the demand equations. Secondly, \( \alpha_0 - \alpha_1 = \beta_2 \delta \) and hence this difference identifies \( \beta_2 \).

Because we have identified \( \beta_1 \), we can identify \( \sigma_\zeta^2 \) from either \( \text{cov}(\varepsilon_1, \eta) \) or \( \text{cov}(\varepsilon_2, \eta) \). The equality of these two covariances is a second overidentifying restriction. In the regime allocation equation the ratio of \( \gamma_{Y_1} \) and \( \gamma_Y \) identifies \( \beta_1 \). This is a third overidentifying restriction:

\[ \alpha_Y (= \alpha_{Y_1}) = \frac{1}{\delta} \log \left( -\frac{\gamma_{Y_1}}{\gamma_Y} \right). \tag{2.26} \]
Because $\delta_0$, $\beta_1$, and $\beta_2$ are identified from the demand equations the constant of the regime allocation equation just identifies $C$. Hence, there are three overidentifying restrictions. If we set the application costs $C$ to zero, then there is an additional overidentifying restriction. Since all parameters in the regime allocation equation are identified from the parameters of the demand equations, the variance of $\eta$ is identified as well. Since $\sigma^2_v$ is identified, this in turn identifies the variance of $v_2$.

The model in (2.24) and (2.25) is a switching regression model. The only regressors that appear are $Y$ and $Y_u$ and they appear in both the demand and the regime choice equations. Switching regression models that have the same regressors in the regression and allocation equations are identified if the selection effect in the regression equations can be expressed as a nonlinear function of the regressors. This is a weak basis for identification, because the nonlinearity is due to arbitrary assumptions on the joint distribution of the disturbances. We can only avoid such arbitrary identifying restrictions, if there are regressors that enter the regime allocation but not the demand equations. Candidates are variables that affect the take-up of RA, but not housing demand. However, even if such variables are not available, identification of the reduced form can be secured.

To see this, we rewrite the reduced form using the definition of $Y_u$ to obtain

\[
R = \begin{cases} 
\alpha_0 + \alpha_Y Y + \varepsilon_1, & I = 0, \\
\alpha_1 + \alpha_{Y_u} Y - \alpha_{Y_u} \delta R_n + \varepsilon_2 > R_n, & I = 1 
\end{cases}
\]

\[
I^* = \gamma_0 + (\gamma_{Y_u} + \gamma_Y) Y - \gamma_{Y_u} \delta R_n + \eta
\]

Now note that the entry fee $\delta R_n$ enters in the allocation equation, but not in the demand equation of the non-RA households. Hence, the demand equation in the non-RA regime is not just identified from any arbitrary nonlinearity. The demand equation in the RA regime depends both on $Y$ and $\delta R_n$, and hence we need a restriction on $\alpha_{Y_u}$ to identify this equation. The obvious restriction is $\alpha_Y = \alpha_{Y_u}$. Rewriting the allocation equation as a function of $Y_u$ and $\delta R_n$ gives the same result. An obvious objection is that $Y$ and $\delta R_n$ may be strongly correlated. However, it should be remembered that the entry fee depends on taxable income and deductibles and progressive taxation reduce the correlation between disposable income and taxable income. The relation between taxable income and the entry fee is nonlinear and has been determined.
by the central government. If identification results from the fact that \( \delta R_i \) is just a nonlinear function of \( Y \) then estimates of the reduced form parameters should be sensitive to the inclusion of powers of \( Y \) and \( Y_v \) in the demand equations. We test this in section 2.5.3. Hence the implicit entry fee secures identification if we maintain the hypothesis \( \alpha_Y = \alpha_{Y_v} \). In the estimation we did not impose this restriction and tests of this restriction should be considered with some reservation.

On the assumption that the regime choice precedes the choice of a dwelling, the loglikelihood of this model is given by

\[
\ell(\theta) = \sum_{I,=0} \log f(R_i, I) + \sum_{I,=1} \log f(R_i \mid I, R_i \geq R_{ni}) f(I)
\]

\[
= \sum_{I,=0} \log \int_{-\infty}^{\bar{R}_{A_i}} f_{\varepsilon, \eta}(R_i - \bar{R}_{A_i}, \eta) d\eta + \sum_{I,=1} \log \frac{\int_{\bar{R}_{B_i}}^{\infty} f_{\varepsilon, \eta}(R_i - \bar{R}_{B_i}, \eta) d\eta}{\Pr(R_{B_i} \geq R_{ni}, I_i \geq 0)} \int_{\bar{R}_{B_i}}^{\infty} f_{\eta}(\eta) d\eta
\]

\[
= \sum_{I,=0} \left( \log f_{\varepsilon_1}(R_i - \bar{R}_{A_i}) + \log \Pr(\eta < -\bar{I}_i \mid \varepsilon_1 = R_i - \bar{R}_{A_i}) \right) + \sum_{I,=1} \left( \log f_{\varepsilon_2}(R_i - \bar{R}_{B_i}) + \log \Pr(\eta \geq -\bar{I}_i \mid \varepsilon_2 = R_i - \bar{R}_{B_i}) \right)
\]

\[
\quad + \log \Pr(\eta \geq -\bar{I}_i, \varepsilon_2 \geq R_{ni} - \bar{R}_{B_i}) + \log \Pr(\eta \geq -\bar{I}_i)
\]  \quad (2.27)

Here, \( f_{\varepsilon, \eta} \) denotes the bivariate density of \((\varepsilon_1, \eta)\), \( f_{\varepsilon_1} \) the marginal density of \( \varepsilon_1 \), etc., and \( \theta \) is the vector of identified parameters:

\[
\theta = (\alpha_0, \alpha_Y, \alpha_1, \alpha_{Y_v}, \gamma_0, \gamma_Y, \sigma_{\varepsilon_1}, \sigma_{\varepsilon_2}, \sigma_{\varepsilon_1_\eta}, \sigma_{\varepsilon_2_\eta}, \sigma_{\varepsilon_1}^2, \sigma_{\varepsilon_2}^2)^t
\]

The loglikelihood function does not depend on \( \text{cov}(\varepsilon_1, \varepsilon_2) \). Since we only observe housing demand in one of the two possible regimes, it is hardly surprising that this parameter is not identified.
The structural model in equation (2.22) follows from the reduced form model by imposing parametric restrictions. Let these restrictions be given by:

$$\theta = \pi(\psi).$$

The exact form of the restrictions is given in Appendix 2.B. We estimate the structural parameters $\psi$ by the minimum distance method (see, for instance, Chamberlain (1984)). An estimate of $\psi$ is obtained by minimizing the quadratic form

$$S_N = (\hat{\theta} - \pi(\psi))^T A_N (\hat{\theta} - \pi(\psi)),$$  \hspace{1cm} (2.28)

with $A_N$ a possibly stochastic, symmetric weighting matrix and $\hat{\theta}$ the maximum likelihood estimator of $\theta$. Under certain regularity conditions, the asymptotic distribution of $\hat{\psi}$ is

$$\sqrt{N}(\hat{\psi} - \psi) \sim N \left( 0, (F'AF)^{-1} F' A \left( \text{var} \hat{\theta} \right) A F (F' AF)^{-1} \right)$$

where $A = \text{plim} A_N$ and $F = \frac{\partial \pi(\psi)}{\partial \psi}$. It is easily seen that choosing the weighting matrix $A_N = \left( \text{var} \hat{\theta} \right)^{-1}$ yields the estimator for $\psi$ with the smallest variance. However, the minimizer of (2.28) is a consistent estimator for $\psi$, regardless of the choice of $A_N$. If the restrictions are true, then minimum distance estimation with weighting matrix $\left( \text{var} \hat{\theta} \right)^{-1}$ yields an estimator which has the same asymptotic distribution as the maximum likelihood estimator.

If the structural model is just identified, $\psi(\cdot)$ will be one-to-one and the minimum of the quadratic form (2.28) is 0. On the other hand, if the structural model is overidentified, then $S_N$ can be used to test these restrictions. To be precise, under the null hypothesis that the restrictions are satisfied, we have that $S_N \overset{\text{a.s.}}{\sim} \chi^2(p)$, with $p$ the number of overidentifying restrictions.

We shall estimate our model by maximum likelihood. A disadvantage of this method is that the resulting estimates are inconsistent if the distribution of the disturbances is misspecified. The standard test of distributional assumptions in this kind of models is the LM-test based

17. In Appendix 2.B we neglect the restriction $\sigma_{x_1 z} = \sigma^2$ because $\sigma_{x_1 z}$ is not identified in the reduced form model.
An Empirical Model of Rental Housing Demand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter</th>
<th>Parameter</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>2.26 (0.27)</td>
<td>$\sigma_{z_1 \eta}$</td>
<td>0.16 (0.15)</td>
</tr>
<tr>
<td>$\alpha_Y$</td>
<td>0.089 (0.0088)</td>
<td>$\sigma_{z_2 \eta}$</td>
<td>0.37 (0.25)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>4.11 (0.26)</td>
<td>$\sigma_{z_1}$</td>
<td>1.37 (0.030)</td>
</tr>
<tr>
<td>$\alpha_Y$</td>
<td>0.058 (0.011)</td>
<td>$\sigma_{z_2}$</td>
<td>1.28 (0.051)</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>1.12 (0.13)</td>
<td>$\gamma_{Y_y}$</td>
<td>0.75 (0.054)</td>
</tr>
<tr>
<td>$\gamma_Y$</td>
<td>$-0.71$ (0.047)</td>
<td>$\gamma_Y$</td>
<td>$-3973.52$</td>
</tr>
</tbody>
</table>

Table 2.2: Estimation results, reduced form model (standard errors in parentheses)

on a parametric family of distributions that has the normal distribution as a special case (see Bera, Jarque, and Lee (1984)). This test can not be used here because, to our knowledge, a convenient parametric generalization of the trivariate normal distribution is not available. Distributional assumptions can be avoided altogether by estimating the model semi-parametrically (for instance using the Gallant and Nychka (1987) approach). This is left for future work.

2.5.2 Empirical Results

The estimation results for the reduced form model in equations (2.24) and (2.25) are given in table 2.2. All calculations were performed using the MAXLIK- and OPTMUM-routines of GAUSS386VM on a 486-personal computer.

The empirical results are in accordance with our expectations: the income effect is positive and significantly so in both demand equations. The price effect is negative, as can be seen from the difference between the intercepts. The estimates of $\gamma_{Y_y}$ and $\gamma_Y$ have an opposite sign, as in the regime allocation equation (2.23). Moreover, $\gamma_{Y_y}$ is slightly larger in absolute value than $\gamma_Y$, which was expected from the

18. This general family of distributions is the so-called Pearson family.

19. A trivariate normal distribution is a member of the family of so-called 'elliptical distributions' (see Muirhead(1982)). In principle, one could test whether the distribution is multivariate normal or another member of this family. As far as we know, no such tests have been developed as yet.
A Structural Model of Rent Assistance and Housing Demand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Application costs</th>
<th>No application costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>4.08 (0.13)</td>
<td>3.89 (0.13)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.079 (0.0053)</td>
<td>0.065 (0.0069)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-1.50 (0.23)</td>
<td>-0.83 (0.23)</td>
</tr>
<tr>
<td>$C$</td>
<td>1.03 (0.050)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>0.59 (0.12)</td>
<td>0.88 (0.085)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>1.24 (0.061)</td>
<td>1.09 (0.070)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>1.11 (0.068)</td>
<td>1.03 (0.075)</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>1.23 (0.087)</td>
<td>4.15 (0.55)</td>
</tr>
<tr>
<td>$S_N$</td>
<td>4.74</td>
<td>265.95</td>
</tr>
<tr>
<td>Overidentifying restrictions</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2.3: Estimation results, structural model (standard errors in parentheses)

Theoretical model as well. The covariances between the disturbance of the regime allocation equation and those of the demand equations are small and positive, though not significantly different from 0. The implied correlations are $\hat{\rho}_{\varepsilon_1\eta} = 0.14$ and $\hat{\rho}_{\varepsilon_2\eta} = 0.33$. Two restrictions implied by the structural model can be imposed on the reduced form directly, viz. $\alpha_\varepsilon = \alpha_\eta$ and $\text{cov}(\varepsilon_1, \eta) = \text{cov}(\varepsilon_2, \eta)$. The resulting reduced form estimates are very similar to the ones reported in table 2.2 and the restrictions are not rejected as is seen from the likelihood-ratio test statistic ($LR = 4.70, \chi^2_{0.05}(2) = 5.99$).

The parameter estimates for the structural model, obtained by minimum distance estimation, are given in table 2.3. The weighting matrix used is $A_N = (\text{var} \hat{\theta})^{-1}$.

We give estimates of the structural model both with and without application costs. If we estimate the structural model with $C = 0$ then the restrictions are rejected ($S_N = 265.95, \chi^2_{0.05}(4) = 9.47$). Allowing for application costs yields larger estimates for the price and income effects and the remaining restrictions on the reduced form are not rejected ($S_N = 4.74, \chi^2_{0.05}(3) = 7.81$). The reason that the restrictions for the structural model without application costs are rejected is that the over-identifying restriction on the intercept of the regime allocation equation is rejected. As indicated above, no problems arise from the restrictions
An Empirical Model of Rental Housing Demand

\(\alpha_Y = \alpha_{\gamma_Y}\) and \(\text{cov}(\varepsilon_1, \eta) = \text{cov}(\varepsilon_2, \eta)\). Moreover, note that the estimate for the income effect based on \(\gamma_{\gamma_Y}\) and \(\gamma_Y\) (see equation (2.26)) is 0.067, which is neatly between \(\hat{\alpha}_{\gamma_Y}\) and \(\hat{\alpha}_{\gamma_{\gamma_Y}}\). Hence, the rejection of the restrictions is caused by rejection of the restriction on \(\gamma_0\) and hence, by the restriction that there are no application costs \((C = 0)\).

The implied price elasticity evaluated at the average rent and price \((R = 4.84\) and \(p = 0.68)\) equals \(-0.21^{20}\) and the income elasticity evaluated at \(R = 4.84\) and \(Y = 22.93\) is 0.37. These estimates are both somewhat smaller in absolute value than the crude estimates obtained in section 2.4.2 but the differences are remarkably small. This can partly be explained by the small estimates of \(\sigma_{\varepsilon_{\gamma}}\) and \(\sigma_{\varepsilon_{\eta}}\), since these imply that the biases due to self-selection are small.

The application costs \(C\) are significantly positive, as we had expected. The estimated costs are Dfl. 1.031, which is 18% of the average rent paid by RA-recipients and 51% of the average RA subsidy received\(^{21}\). The estimates imply that 18% of the residual variance in the non-RA regime and 22% in the RA regime is explained by preference variation.

The Slutsky-condition (equation (2.8)) is satisfied for all observations with \(R \leq 19.0\) which is much higher than the maximum rent in our sample.

### 2.5.3 Tests of the Reduced Form

The restrictions on the reduced form parameters implied by the structural model are not rejected. However, we can only derive confidence from that, if the reduced form parameters are not sensitive to changes in the specification of the reduced form model. We consider three extensions of the reduced form model.

---

20. In section 2.3.4 we introduced application costs to explain the partial take-up of RA-benefits. Strictly speaking application costs affect the demand for housing services in the RA regime, because they reduce the virtual income of the household. As a consequence the constant of the demand equation in the RA regime is \(\beta_0 = \beta_1 C + \beta_2 (1 - \delta)\). Hence, the estimate of the price effect reported in table 2.3 may be too small in absolute value. Because \(\beta_1 C\) is very small, the potential bias is negligible.

21. The application costs can be considered as a lump-sum ‘payment’ and to appreciate their size we should take account of the tenure of the household in a dwelling e.g. by dividing the numbers in the text by the number of years the household stays there. However, the application costs can also be interpreted as a yearly ‘payment’ the household has to make because of the stigma associated with receiving RA. As we discussed in section 2.3.4 we can not distinguish between both interpretations.
First, we allow the demand equations to be nonlinear in \( Y \) and \( Y_v \). We included \( Y^2 \) in the non-RA and \( Y_v^2 \) in the RA demand equation. The resulting parameter estimates are in table 2.4.

The coefficients of \( Y^2 \) and \( Y_v^2 \) are jointly not significantly different from 0 (\( LR = 2.10 \) and \( \chi^2_{0.95}(2) = 5.99 \)) and the other parameter estimates are largely unaffected. The only change is in the coefficient of \( Y_v \), which after the discussion in section 2.5.1 should not surprise us. We conclude that the preference structure is not too restrictive. Moreover, the insensitivity to the inclusion of the powers of \( Y \) and \( Y_v \) shows that identification from the implicit entry fee is possible and not the result of the (arbitrary) assumed functional form of the demand equation.

In a second specification test we extended the dataset. For the estimation results of table 2.2 we used only households that moved in the four years before the survey. Restricting the sample to recent movers may have biased our estimates, but because we compare two groups of renters, the size and the direction of the bias are hard to predict. To investigate this we included the households that satisfied all other selection criteria in Appendix 2A irrespective of their tenure. This increased the number of observations to 6468. The reduced form and structural parameters are given in table 2.5.

Note that the structural restrictions are not rejected. The structural parameter estimates are similar to those in table 2.3. The price and income effect are somewhat smaller and the application costs are

![Table 2.4: Estimation results, reduced form model with \( Y^2 \) and \( Y_v^2 \) (standard errors in parentheses)](image-url)
Reduced form parameters & Structural parameters \\
\(\alpha_0\) & 2.69 (0.11) & \(\delta_0\) & 3.86 (0.084) \\
\(\alpha_Y\) & 0.060 (0.0038) & \(\beta_1\) & 0.057 (0.0033) \\
\(\alpha_t\) & 3.67 (0.20) & \(\beta_2\) & -1.09 (0.13) \\
\(\alpha_{Y_t}\) & 0.052 (0.0086) & \(C\) & 1.30 (0.039) \\
\(\gamma_0\) & 0.93 (0.070) & \(\sigma_\xi\) & 0.74 (0.053) \\
\(\gamma_{Y_t}\) & 0.65 (0.032) & \(\sigma_1\) & 1.08 (0.030) \\
\(\gamma_{Y_t}\) & -0.63 (0.028) & \(\sigma_2\) & 1.17 (0.040) \\
\(\sigma_{\varepsilon_{Y_t}}\) & 0.40 (0.069) & \(\sigma_3\) & 1.34 (0.058) \\
\(\sigma_{\varepsilon_t}\) & 0.52 (0.19) \\
\(\sigma_{\varepsilon_{Y_t}}\) & 1.31 (0.016) \\
\(\sigma_{\varepsilon_{Y_t}}\) & 1.40 (0.049) \\
\[\ln(\ell)\] & -13761.12 \\
Observations & 6468 \\
\(S_N\) & 2.40 \\
Overidentifying restrictions & 3 \\

Table 2.5: Estimation results, reduced form and structural model in extended sample (standard errors in parentheses)

larger. The fact that the structural restrictions are not rejected in this much larger sample is encouraging.

In a third test, we included regional effects. We have assumed that the ‘disequilibrium variables’ \(v_1\) and \(v_2\) have mean 0, i.e. on average households are on their demand curve. Although the HNS does not contain precise information on the location of the households, we can distinguish between four regions: the north (mainly rural), the east (mainly rural), the south (more urban) and the west (urban). In particular, in urban areas (the west) households may have difficulties finding the desired dwelling. The reduced form estimates are given in table 2.6.

The regional dummies are jointly insignificant \((LR = 7.44, \chi^2_{0.95}(6) = 12.59)\). Note that the other parameter estimates are almost identical to those in table 2.2.

The specification checks show that the reduced form model is quite robust.
An Extension of the Basic Model

In this section we extend the empirical model of section 2.5 by adding two additional regressors, viz. the age of the head of the household \( (AGE) \) and the size of the household \( (SIZE) \). Demographic variables are often used to model preference variation between households for housing consumption, see inter alia Mayo (1981) and Rosen (1979).

The demographic variables were included by expressing \( \delta_0 \) as a linear combination of an intercept, \( SIZE \) and \( AGE \). This specification leads to a reduced form where \( SIZE \) and \( AGE \) appear linearly. There is no fundamental reason why only \( \delta_0 \) should be parametrized, apart from the fact that this parametrization yields a reduced form which is linear in the explanatory variables.

The reduced form estimation results are given in table 2.7. Broadly the results are as before: a positive income effect, a negative price effect, and \( \gamma_{Y_v} \), \( \gamma_Y \) with opposite signs. Larger and older households tend to consume more housing services (and hence, are more likely to receive RA). The null hypothesis that the coefficients of the added explanatory variables are jointly zero, is rejected for both models at the 5%-level (the \( LR \)-statistics are 9.94 and 46.78 with 3 and 6 degrees of freedom respectively).

\[
\begin{array}{cccc}
\text{Parameter} & \text{Parameter} & \text{Parameter} \\
\alpha_0 & 2.32 & (0.28) & \gamma_0 & 1.12 & (0.13) \\
\alpha_Y & 0.088 & (0.0088) & \gamma_{Y_v} & 0.75 & (0.65) \\
\alpha_{Anorth} & -0.058 & (0.15) & \gamma_Y & -0.71 & (0.047) \\
\alpha_{Aeast} & 0.10 & (0.13) & \sigma_{\varepsilon_1\eta} & 0.17 & (0.14) \\
\alpha_{Awest} & -0.11 & (0.11) & \sigma_{\varepsilon_2\eta} & 0.37 & (0.24) \\
\alpha_1 & 3.99 & (0.28) & \sigma_{\varepsilon_1} & 1.37 & (0.030) \\
\alpha_{Bnorth} & 0.085 & (0.17) & \sigma_{\varepsilon_2} & 1.28 & (0.051) \\
\alpha_{Beast} & -0.064 & (0.16) & \alpha_{Bwest} & 0.18 & (0.14) \\
\ln(\ell) & -3969.80 \\
\text{Observations} & 1809 \\
\end{array}
\]

Table 2.6: Estimation results, reduced form model with regional dummies (standard errors in parentheses)
An Extension of the Basic Model

Parameter | Model E1 | Model E2
---|---|---
$\alpha_0$ | 2.34 (0.37) | 1.62 (0.40)
$\alpha_Y$ | 0.084 (0.016) | 0.094 (0.015)
$\alpha_{1,SIZE}$ | 0.027 (0.056) | 0.013 (0.0024)
$\alpha_{1,AGE}$ | 0.017 (0.056) | 
$\alpha_1$ | 4.21 (0.25) | 3.71 (0.031)
$\alpha_2$ | 0.021 (0.021) | 0.019 (0.021)
$\alpha_{2,SIZE}$ | 0.16 (0.077) | 0.22 (0.077)
$\alpha_{2,AGE}$ | 0.0076 (0.0029) | 
$\gamma_0$ | 1.21 (0.13) | 1.28 (0.18)
$\gamma_Y$ | 0.67 (0.068) | 0.67 (0.067)
$\gamma_{SIZE}$ | -0.64 (0.055) | -0.65 (0.055)
$\gamma_{AGE}$ | 0.092 (0.044) | 0.089 (0.044)
$\sigma_{\epsilon_1}$ | 0.20 (0.19) | 0.26 (0.19)
$\sigma_{\epsilon_2}$ | 0.59 (0.26) | 0.66 (0.24)
$\sigma_{\epsilon_3}$ | 1.37 (0.032) | 1.36 (0.033)
$\sigma_{\epsilon_4}$ | 1.33 (0.072) | 1.34 (0.074)

ln(ε) | -3968.55 | -3950.13
Observations | 1809 | 1809

Table 2.7: Estimation results of the extended reduced form model (standard errors in parentheses)

As before, we imposed the parameter restrictions implied by the structural model on the reduced form. The restrictions are given in equations (2.B-12)-(2.B-17) in Appendix 2.B and the estimation results are in Table 2.8. The results are similar to the ones in section 2.5.2. The signs of the additional explanatory variables are as expected: larger households and households with older heads consume more housing services. The structural restrictions are not rejected for the model with only SIZE ($S_N = 8.77$ with 5 degrees of freedom), but they are rejected when both SIZE and AGE are included. In the latter case, the value of the criterion function is 23.95, which should be compared with $\chi^2_{0.95}(7) = 14.07$. In both specifications, we have allowed for application costs. The structural model without application costs is firmly rejected and we do not present those results here.
The rejection of the structural restrictions on the parameters might be due to a restrictive specification of the preferences. According to the second column of Table 2.7, the income effects differ markedly between both demand equations. Hence, we assume more flexible preferences that allow for different income effects between both demand equations. Consider for example the following specification of the indirect utility function, which generalizes the specification in equation (2.4):

\[ \nu(p, Y) = Y \exp \left(-\beta_1 p - 1/2\beta_3 p^2\right) - Q(p) \]  \hspace{1cm} (2.29)

with

\[ Q(p) = \int_{-\infty}^{p} (\beta_2 t + \beta_0) \exp \left(-\beta_1 t - 1/2\beta_3 t^2\right) dt. \]

Using Roy’s identity, one obtains the demand function:

\[ R = \beta_0 + \beta_1 Y + \beta_2 p + \beta_3 pY. \]  \hspace{1cm} (2.30)

---

Table 2.8: Estimation results of the extended structural models (standard errors in parentheses)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model E1</th>
<th>Model E2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_0 )</td>
<td>4.06 (0.13)</td>
<td>3.46 (0.17)</td>
</tr>
<tr>
<td>( \delta_{SIZE} )</td>
<td>0.036 (0.021)</td>
<td>0.11 (0.041)</td>
</tr>
<tr>
<td>( \delta_{AGE} )</td>
<td>0.008 (0.0017)</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.067 (0.0092)</td>
<td>0.062 (0.011)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-1.23 (0.27)</td>
<td>-0.93 (0.30)</td>
</tr>
<tr>
<td>( C )</td>
<td>1.18 (0.087)</td>
<td>1.05 (0.052)</td>
</tr>
<tr>
<td>( \sigma_c )</td>
<td>0.46 (0.12)</td>
<td>0.82 (0.11)</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>1.31 (0.045)</td>
<td>1.12 (0.071)</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>1.18 (0.044)</td>
<td>1.00 (0.063)</td>
</tr>
<tr>
<td>( \sigma_3 )</td>
<td>1.09 (0.11)</td>
<td>1.33 (0.10)</td>
</tr>
<tr>
<td>( S_{N} )</td>
<td>8.77</td>
<td>23.95</td>
</tr>
<tr>
<td>Overidentifying</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

---

22. A test shows that \( \alpha_Y \) and \( \alpha_{SIZ} \) are significantly different. The difference between \( \alpha_{SIZ} \) and \( \alpha_{SIZE} \) seems large as well, but it is less significant than the difference between the income effects.
Summary and Conclusions

Table 2.9: Estimation results of the extended structural model (standard errors in parentheses)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model E3</th>
<th>Parameter</th>
<th>Model E3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>3.94 (0.31)</td>
<td>$C$</td>
<td>1.04 (0.053)</td>
</tr>
<tr>
<td>$\delta_{\text{SIZE}}$</td>
<td>0.11 (0.042)</td>
<td>$\sigma_\zeta$</td>
<td>0.77 (0.12)</td>
</tr>
<tr>
<td>$\delta_{\text{AGE}}$</td>
<td>0.008 (0.0017)</td>
<td>$\sigma_1$</td>
<td>1.15 (0.070)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.039 (0.017)</td>
<td>$\sigma_2$</td>
<td>1.03 (0.063)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-1.63 (0.48)</td>
<td>$\sigma_3$</td>
<td>1.37 (0.10)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.031 (0.017)</td>
<td>$S_N$</td>
<td>20.44</td>
</tr>
<tr>
<td>Overidentifying restrictions</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the price is constant within each regime, this specification allows for different income effects between both regimes. The estimation results for the structural model are now given in table 2.9.

The more flexible preference structure is rejected as well, when tested at a 5%-level of significance ($\chi_{0.05}^2(6) = 12.59$). It would be possible to parametrize the other taste parameters (for example, $\beta_3$), but this leads to a decrease in degrees of freedom. Other ways of modelling the age-effect on housing consumption might be preferable, for example stratification with respect to this variable, but we do not pursue this matter any further.

From the estimation results in this section, we can draw the following conclusions. First, incorporating demographic variables is possible, but the resulting parametric restrictions were rejected, even when we allowed for more flexible preferences. However, the results found in the previous section are rather robust against adding extra explanatory variables.

### 2.7 Summary and Conclusions

We have developed and estimated a structural model of rental housing demand, and we have used this model to study the impact of a rent subsidy program on the demand for rental housing. Recently, the credibility of structural estimates of program effects has been questioned. Some researchers have taken the position that only (quasi-) experimental
approaches can yield valid estimates of effects. Although this discussion has focused correctly on the weak points of structural methods, it is our opinion that the structural approach, if applied carefully, can yield valuable insights into the working of social programs.

For that reason we have chosen not to impose the restrictions implied by our structural model. Instead, we have tested these restrictions against a reduced form, and we have concluded that the restrictions for the basic model are not rejected by the data. Moreover, the reduced form estimates were not sensitive to changes in the specification and an extension of the sample. These checks confirmed our conjecture that the entry fee effect identifies our model and not the arbitrary distributional assumptions.

2. A Selection of Households

In the empirical analysis, we did not use all cases of the Housing Needs Survey 1985/86, which has 54342 respondents of which 32403 are renters. Of these, 21525 live in a household with a taxable income in the RA range. The cases used satisfied the following criteria, where the number of cases deleted after each selection is given in parentheses:

1. the household is the main occupant of the dwelling (3496);
2. the respondent is either the head of the household or his/her partner (69);
3. the dwelling is not used for business purposes (not a farm, shop, etc.) (1147);
4. the household is a single person household or a couple with or without children (1075);
5. the head of the household is not self-employed (579);
6. the income data of the household are valid as checked by the Central Bureau of Statistics (4338);
7. taxable income of the household in 1984 exceeds Dfl. 10000 (146);
8. rent in 1985 exceeds Dfl. 1200 (379);
9. the household does not receive RA while our simulation program indicates that the household is not entitled to RA (442);
10. rent paid is less than the maximum rent (123);
11. the household has moved in the period 1982-1986 (6839);

23. The identification problem in our model is similar to the identification problem in the Roy model that has been studied by Heckman and Honoré (1990).
12. the respondent is reasonably satisfied with the dwelling and the neighbourhood (858);
13. the household has no intentions of moving within the next two years (225).

2. B  Relation between Structural and Reduced Form Parameters

Let $\theta$ be the vector of parameters of the reduced form model and $\psi$ be the vector of parameters of the structural model:

$$\theta = (\alpha_0, \alpha_Y, \alpha_1, \alpha_{Yv}, \gamma_0, \gamma_{Yv}, \gamma_Y, \sigma_{\varepsilon_0}, \sigma_{\varepsilon_1}, \sigma_{\varepsilon_2})^T$$

$$\psi = (\delta_0, \beta_1, \beta_2, C, \sigma_{\zeta}, \sigma_1, \sigma_2, \sigma_3)^T$$

The relations between the reduced form parameters and the structural parameters are given by:

$$\alpha_0 = \delta_0 + \beta_2$$  \hspace{1cm} (2. B. 1)

$$\alpha_Y = \beta_1$$  \hspace{1cm} (2. B. 2)

$$\alpha_1 = \delta_0 + \beta_2 (1 - \delta)$$  \hspace{1cm} (2. B. 3)

$$\alpha_{Yv} = \beta_1$$  \hspace{1cm} (2. B. 4)

$$\gamma_0 = \left(\frac{\beta_2}{\beta_1} (1 - \delta) + \frac{\beta_0}{\beta_1} + \frac{\delta_0}{\beta_1}\right) \exp(-\beta_1(1 - \delta)) +$$

$$- \left(\frac{\beta_2}{\beta_1} + \frac{\beta_0}{\beta_1} + \frac{\delta_0}{\beta_1}\right) \exp(-\beta_1) - \exp(-\beta_1(1 - \delta)) \frac{\sigma_{\zeta}}{\beta_1} \right) \left( \frac{\exp(-\beta_1(1 - \delta)) - \exp(-\beta_1)}{\beta_1} \right)^2 + \sigma_{\delta}^2$$  \hspace{1cm} (2. B. 5)

$$\gamma_{Yv} = \frac{\exp(-\beta_1(1 - \delta))}{\sqrt{\sigma_{\zeta}^2 \left( \frac{\exp(-\beta_1(1 - \delta)) - \exp(-\beta_1)}{\beta_1} \right)^2 + \sigma_{\delta}^2}}$$  \hspace{1cm} (2. B. 6)

$$\gamma_Y = \frac{-\exp(-\beta_1)}{\sqrt{\sigma_{\zeta}^2 \left( \frac{\exp(-\beta_1(1 - \delta)) - \exp(-\beta_1)}{\beta_1} \right)^2 + \sigma_{\delta}^2}}$$  \hspace{1cm} (2. B. 7)
A Structural Model of Rent Assistance and Housing Demand

\[ \sigma_{e_i}^2 = \sigma_{\zeta}^2 + \sigma_{I_i}^2 \]  
(2.B-8)

\[ \sigma_{e_2}^2 = \sigma_{\zeta}^2 + \sigma_{I_2}^2 \]  
(2.B-9)

\[ \sigma_{e_{1n}} = \sigma_{\zeta}^2 \left( \frac{\exp(-\beta_1(1-\delta)) - \exp(-\beta_1)}{\beta_1} \right) \]  
(2.B-10)

\[ \sigma_{e_{2n}} = \sigma_{\zeta}^2 \left( \frac{\exp(-\beta_1(1-\delta)) - \exp(-\beta_1)}{\beta_1} \right) \]  
(2.B-11)

In the extended model of section 2.6, there are 6 additional reduced form parameters and 2 additional structural parameters due to the additional explanatory variables. The 6 extra relations between these parameters are:

\[ a_{1,SIZE} = \delta_{SIZE} \]  
(2.B-12)

\[ a_{1,AGE} = \delta_{AGE} \]  
(2.B-13)

\[ a_{2,SIZE} = \delta_{SIZE} \]  
(2.B-14)

\[ a_{2,AGE} = \delta_{AGE} \]  
(2.B-15)

\[ \gamma_{SIZE} = \frac{\delta_{SIZE} \left( \exp(-\beta_1(1-\delta)) - \exp(-\beta_1) \right)}{\sqrt{\sigma_{\zeta}^2 \left( \frac{\exp(-\beta_1(1-\delta)) - \exp(-\beta_1)}{\beta_1} \right)^2 + \sigma_3^2}} \]  
(2.B-16)

\[ \gamma_{AGE} = \frac{\delta_{AGE} \left( \exp(-\beta_1(1-\delta)) - \exp(-\beta_1) \right)}{\sqrt{\sigma_{\zeta}^2 \left( \frac{\exp(-\beta_1(1-\delta)) - \exp(-\beta_1)}{\beta_1} \right)^2 + \sigma_3^2}} \]  
(2.B-17)

2.C Details of the Loglikelihood

The demand model in section 2.5 can be written as

\[ R_i = \begin{cases} 0 & I_i = 0 \\ \alpha_1 z_{1i} + \varepsilon_{1i} & I_i = 1 \\ \alpha_2 z_{2i} + \varepsilon_{2i} > R_{mi} & I_i = 1 \\ \alpha_3 z_{3i} + \varepsilon_{3i} & I_i = 1 \\ \end{cases} \]  
(2.C-1)

\[ I_i^* = \gamma_{SIZE} z_{3i} + \eta_i \]  
(2.C-2)

\[ I_i = \begin{cases} 0 & I_i^* < 0 \\ 1 & I_i^* \geq 0 \end{cases} \]
where the vector $\mathbf{z}_i$ contains the regressors of the demand equation for non-recipients, etc.

The vector $(\varepsilon_{1i}, \varepsilon_{2i}, \eta_i)'$ is normally distributed:

$$
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\eta
\end{pmatrix}
\sim 
\mathcal{N}_3
\left(0,
\begin{pmatrix}
\sigma_1^2 & \sigma_{12} & \sigma_{1\eta} \\
\sigma_{12} & \sigma_2^2 & \sigma_{2\eta} \\
\sigma_{1\eta} & \sigma_{2\eta} & \sigma_\eta^2
\end{pmatrix}
\right).
$$

The loglikelihood is now given by:

$$
\ell(\theta) = \sum_{i=1}^{n} \left( \log f_{\varepsilon_i} (R_i - \tilde{R}_{Ai}) +
\frac{1}{\sigma_1} \phi \left( \frac{(R_i - \tilde{R}_{Ai})}{\sigma_1} \right) +
\log \Phi \left( \frac{-\tilde{I}_i - \frac{\sigma_{1\eta}}{\sigma_\eta^2} (R_i - \tilde{R}_{Ai})}{\sqrt{1 - \rho_1^2}} \right) +
\frac{1}{\sigma_2} \phi \left( \frac{(R_i - \tilde{R}_{Bi})}{\sigma_2} \right) +
\log \Phi \left( \frac{-\tilde{I}_i - \frac{\sigma_{2\eta}}{\sigma_\eta^2} (R_i - \tilde{R}_{Bi})}{\sqrt{1 - \rho_2^2}} \right) +
\phi_2 \left( \frac{(R_{ni} - \tilde{R}_{Bi})}{\sigma_2}, -\tilde{I}_i, \rho_{2\eta} \right) +
\log \Phi (\tilde{I}_i) \right)
\right) +
\sum_{i=1}^{n} \left( \log f_{\varepsilon_i} (R_i - \tilde{R}_{Bi}) +
\frac{1}{\sigma_1} \phi \left( \frac{(R_i - \tilde{R}_{Bi})}{\sigma_1} \right) +
\log \Phi \left( \frac{-\tilde{I}_i - \frac{\sigma_{1\eta}}{\sigma_\eta^2} (R_i - \tilde{R}_{Bi})}{\sqrt{1 - \rho_1^2}} \right) +
\frac{1}{\sigma_2} \phi \left( \frac{(R_i - \tilde{R}_{Bi})}{\sigma_2} \right) +
\log \Phi \left( \frac{-\tilde{I}_i - \frac{\sigma_{2\eta}}{\sigma_\eta^2} (R_i - \tilde{R}_{Bi})}{\sqrt{1 - \rho_2^2}} \right) +
\phi_2 \left( \frac{(R_{ni} - \tilde{R}_{Bi})}{\sigma_2}, -\tilde{I}_i, \rho_{2\eta} \right) +
\log \Phi (\tilde{I}_i) \right)
\right).
$$
\[ + \log \Phi \left( \frac{-\gamma' z_{3i} - \frac{\sigma_{1\eta}}{\sigma_1} (R_i - \alpha_1' z_{1i})}{\sqrt{1 - \rho_{1\eta}^2}} \right) + \]
\[ + \sum_{t=1} \left( \log \frac{1}{\sigma_t} \phi \left( \frac{(R_i - \alpha_t' z_{ti})}{\sigma_t} \right) + \right) \]
\[ + \log \Phi \left( \frac{-\gamma' z_{3i} - \frac{\sigma_{2\eta}}{\sigma_2} (R_i - \alpha_2' z_{2i})}{\sqrt{1 - \rho_{2\eta}^2}} \right) + \]
\[ - \log \left[ 1 - \Phi \left( \frac{(R_{ni} - \alpha_1' z_{ni})}{\sigma_1} \right) - \Phi \left( -\gamma' z_{3i} \right) \right] + \]
\[ + \Phi \left( \frac{(R_{ni} - \alpha_2' z_{ni})}{\sigma_2}, -\gamma' z_{3i}; \rho_{2\eta} \right) \]
\[ + \log \Phi \left( \gamma' z_{3i} \right) \right). \]

(2.C.3)

Here, \( \phi(\cdot) \) denotes the standard normal density function, \( \Phi(\cdot) \) the standard normal distribution function, and \( \Phi_2(\cdot; \cdot; \rho) \) is the bivariate standard normal distribution function with correlation parameter \( \rho \). For notational convenience we use both \( \sigma_{1\eta} \) and \( \rho_{1\eta} \) on the one hand and \( \sigma_{2\eta} \) and \( \rho_{2\eta} \) on the other hand even though these are not ‘independent’ parameters. Only \( \sigma_{1\eta} \) and \( \sigma_{2\eta} \) are considered to be parameters of the loglikelihood.

In order to derive the gradient of this loglikelihood function, we introduce some additional notation:

\[ h_1 = \frac{R_i - \alpha_1' z_{1i}}{\sigma_1} \]
\[ h_2 = \frac{-\gamma' z_{3i} - \frac{\sigma_{1\eta}}{\sigma_1} (R_i - \alpha_1' z_{1i})}{\sqrt{1 - \rho_{1\eta}^2}} \]
\[ h_3 = \frac{R_i - \alpha_2' z_{2i}}{\sigma_2} \]
\[ h_4 = \frac{\gamma' z_{3i} + \frac{\sigma_{2\eta}}{\sigma_2} (R_i - \alpha_2' z_{2i})}{\sqrt{1 - \rho_{2\eta}^2}} \]
\[ h_5 = \frac{R_i - \alpha_2' z_{2i}}{\sigma_2} \]
\[ h_6 = -\gamma' z_3 \]

\[ H_\gamma(h_5, h_6; \rho_{2n}) = 1 - \Phi(h_5) - \Phi(h_6) + \Phi_2(h_5, h_6; \rho_{2n}) \]

where we have omitted the index \( i \). Now the loglikelihood function can be written as:

\[
\ell(\theta) = \sum_{t=0} \left( \log \frac{1}{\sigma_1} \phi(h_{1t}) + \log \Phi(h_{2t}) \right) + \\
\quad + \sum_{t=1} \left( \log \frac{1}{\sigma_2} \phi(h_{3t}) + \log \Phi(h_{4t}) + \\
\quad - \log H_\gamma(h_{5t}, h_{6t}; \rho_{2n}) + \log \Phi(-h_{6t}) \right). \tag{2.C-4}
\]

We differentiate first with respect to the \( h \)-terms. Next these terms are differentiated with respect to the parameters.

\[
\frac{\partial \ell(\theta)}{\partial h_{1i}} = -h_{1i} \\
\frac{\partial \ell(\theta)}{\partial h_{2i}} = \frac{\phi(h_{2i})}{\Phi(h_{2i})} \\
\frac{\partial \ell(\theta)}{\partial h_{3i}} = -h_{3i} \\
\frac{\partial \ell(\theta)}{\partial h_{4i}} = \frac{\phi(h_{4i})}{\Phi(h_{4i})} \\
\frac{\partial \ell(\theta)}{\partial h_{5i}} = \frac{1}{1 - \Phi(h_{5i}) - \Phi(h_{6i}) + \Phi_2(h_{5i}, h_{6i}; \rho_{2n})} \times \\
\quad \times \left( \phi(h_{5i}) - \frac{\partial \Phi_2(h_{5i}, h_{6i}; \rho_{2n})}{\partial h_{5i}} \right) \\
\frac{\partial \ell(\theta)}{\partial h_{6i}} = \frac{1}{1 - \Phi(h_{5i}) - \Phi(h_{6i}) + \Phi_2(h_{5i}, h_{6i}; \rho_{2n})} \times \\
\quad \times \left( \phi(h_{6i}) - \frac{\partial \Phi_2(h_{5i}, h_{6i}; \rho_{2n})}{\partial h_{6i}} \right) - \frac{\phi(h_{6i})}{\Phi(-h_{6i})}
\]
After tedious calculations (and the help of computer algebra package Maple V), the following contributions to the gradient for non-recipients are derived:

\[
\frac{\partial \ell}{\partial \alpha_1} = -h_{11} \frac{\partial h_{11}}{\partial \alpha_1} + \frac{\phi(h_{21})}{\Phi(h_{21})} \frac{\partial h_{21}}{\partial \alpha_1} + \frac{1}{\sigma_1} \frac{\sigma_{1n}}{\sigma_1^2 \sqrt{1 - \rho_1^2}} z_{1i} + \\
+ \frac{\phi(h_{21})}{\Phi(h_{21})} \frac{\sigma_{1n}}{\sigma_1^2 \sqrt{1 - \rho_1^2}} z_{1i} \]

\[
\frac{\partial \ell}{\partial \sigma_1} = -\frac{1}{\sigma_1} h_{11} \frac{\partial h_{11}}{\partial \sigma_1} + \frac{\phi(h_{21})}{\Phi(h_{21})} \frac{\partial h_{21}}{\partial \sigma_1} = -\frac{1}{\sigma_1} h_{11}^2 + \\
+ \frac{\phi(h_{21})}{\Phi(h_{21})} \left( \frac{2 \sigma_{1n} (R_i - \alpha_{1i} z_{1i})}{\sigma_1^2 \sqrt{1 - \rho_1^2}} - \frac{\sigma_{1n}^2}{\sigma_1^2 - \sigma_1^2 \sigma_{1n}^2} h_{2i} \right) \]

\[
\frac{\partial \ell}{\partial \sigma_{1n}} = \frac{\phi(h_{21})}{\Phi(h_{21})} \frac{\partial h_{21}}{\partial \sigma_{1n}} = \frac{\phi(h_{21})}{\Phi(h_{21})} \left( -\frac{R_i - \alpha_{1i} z_{1i}}{\sigma_1^2 \sqrt{1 - \rho_1^2}} + \frac{\sigma_{1n}}{\sigma_1^2 - \sigma_1^2 \sigma_{1n}^2} h_{2i} \right) \]

\[
\frac{\partial \ell}{\partial h_{21}} = \frac{\phi(h_{21})}{\Phi(h_{21})} \frac{\partial h_{21}}{\partial \gamma} = -\frac{\phi(h_{21})}{\Phi(h_{21})} \frac{1}{\sqrt{1 - \rho_1^2}} z_{1i} \]

The contributions to the gradient by the RA-recipients are given by

\[
\frac{\partial \ell}{\partial \alpha_2} = -h_{31} \frac{\partial h_{31}}{\partial \alpha_2} + \frac{\phi(h_{41})}{\Phi(h_{41})} \frac{\partial h_{41}}{\partial \alpha_2} + \frac{1}{H(h_{51}, h_{61}; \rho_{2n})} \times \\
\times \left( \phi(h_{51}) - \Phi \left( h_{61} - \rho_{2n} h_{51} \right) \phi(h_{51}) \right) \frac{\partial h_{51}}{\partial \alpha_2} \]

\[
= h_{31} \frac{1}{\sigma_2} z_{2i} - \frac{\phi(h_{41})}{\Phi(h_{41})} \frac{\sigma_{2n}}{\sigma_2^2 \sqrt{1 - \rho_2^2}} z_{2i} + \\
- \frac{\phi(h_{51})}{H(h_{51}, h_{61}; \rho_{2n})} \left( 1 - \Phi \left( h_{61} - \rho_{2n} h_{51} \right) \sqrt{1 - \rho_2^2} \right) \frac{1}{\sigma_2} z_{2i} \]
\[
\frac{\partial \ell}{\partial \sigma_2} = -\frac{1}{\sigma_2} h_{3i} \frac{\partial h_{3i}}{\partial \sigma_2} + \frac{\phi(h_{4i})}{\Phi(h_{4i})} \frac{\partial h_{4i}}{\partial \sigma_2} + \frac{1}{H(h_{5i}, h_{6i}; \rho_{2n})} \left( \frac{\partial H}{\partial h_{5i}} \frac{\partial h_{5i}}{\partial \sigma_2} + \frac{\partial H}{\partial \rho_{2n}} \frac{\partial \rho_{2n}}{\partial \sigma_2} \right) \\
= -\frac{1}{\sigma_2} + \frac{1}{\sigma_2} h_{3i} - \frac{\phi(h_{4i})}{\Phi(h_{4i})} \left( \frac{2\sigma_{2n} (R_{i} - \alpha_{t} z_{2i})}{\sigma_{2}^{2} \sqrt{1 - \rho_{2n}^{2}}} + \frac{\sigma_{2n}^{2}}{\rho_{2n}^{2} h_{4i}} \right) + \\
+ \frac{\phi(h_{5i})}{H(h_{5i}, h_{6i}; \rho_{2n})} \left( \phi \left( h_{5i} - \rho_{2n} h_{5i} \right) - 1 \right) \frac{1}{\sigma_2} h_{5i} + \\
+ \frac{\phi_{2} (h_{5i}, h_{6i}; \rho_{2n}) \sigma_{2n}}{H(h_{5i}, h_{6i}; \rho_{2n})} \frac{\sigma_{2n}}{\sigma_{2}^{2}} \\
\frac{\partial \ell}{\partial \sigma_{2n}} = \frac{\phi(h_{4i})}{\Phi(h_{4i})} \frac{\partial h_{4i}}{\partial \sigma_{2n}} - \frac{1}{H(h_{5i}, h_{6i}; \rho_{2n})} \frac{\partial H}{\partial \rho_{2n}} \frac{\partial \rho_{2n}}{\partial \sigma_{2n}} \\
= \frac{\phi(h_{4i})}{\Phi(h_{4i})} \left( \frac{R_{i} - \alpha_{t} z_{2i}}{\sigma_{2}^{2} \sqrt{1 - \rho_{2n}^{2}}} + \frac{\sigma_{2n}}{\rho_{2n}^{2} h_{4i}} \right) + \\
- \frac{\phi_{2} (h_{5i}, h_{6i}; \rho_{2n})}{H(h_{5i}, h_{6i}; \rho_{2n})} \frac{1}{\sigma_2} \\
\frac{\partial \ell}{\partial \gamma} = \frac{\phi(h_{4i})}{\Phi(h_{4i})} \frac{\partial h_{4i}}{\partial \gamma} - \frac{1}{H(h_{5i}, h_{6i}; \rho_{2n})} \frac{\partial H}{\partial h_{5i}} \frac{\partial h_{5i}}{\partial \gamma} - \frac{\phi(h_{5i})}{\Phi(h_{5i})} \frac{\partial h_{5i}}{\Phi(h_{5i})} \frac{\partial h_{5i}}{\partial \gamma} \\
= \frac{\phi(h_{4i})}{\Phi(h_{4i})} \sqrt{1 - \rho_{2n}^{2}} z_{2i} - \frac{\phi(h_{5i})}{H(h_{5i}, h_{6i}; \rho_{2n})} \times \\
\times \left( 1 - \phi \left( h_{5i} - \rho_{2n} h_{5i} \right) \right) z_{2i} + \frac{\phi(h_{5i})}{\Phi(h_{5i})} z_{2i} 
\]
Here, $\phi_2(x, y; \rho)$ denotes the standard normal bivariate density function with correlation coefficient $\rho$. In deriving these formulas we have used (Pudney (1989), p. 325):

$$\frac{\partial \Phi_2(x, y; \rho)}{\partial x} = \Phi \left( \frac{y - \rho x}{\sqrt{1 - \rho^2}} \right) \phi(x),$$  \hspace{1cm} (2.C-5)

$$\frac{\partial \Phi_2(x, y; \rho)}{\partial y} = \Phi \left( \frac{x - \rho y}{\sqrt{1 - \rho^2}} \right) \phi(y),$$  \hspace{1cm} (2.C-6)

$$\frac{\partial \Phi_2(x, y; \rho)}{\partial \rho} = \phi_2(x, y; \rho).$$  \hspace{1cm} (2.C-7)

The last equation is derived using the property of the normal distribution:

$$\frac{\partial \phi_2(x, y; \rho)}{\partial \rho} = \frac{\partial^2 \phi_2(x, y; \rho)}{\partial x \partial y}$$

(Stuart and Ord (1987), p. 502). Clearly, Pudney (1989) is mistaken in his equation (A3.29) where he claims

$$\frac{\partial \Phi_2(x, y; \rho)}{\partial \rho} = \frac{\rho}{1 - \rho^2} (\Phi_2(x, y; \rho) - 1).$$