Selective transport phenomena in coastal sands
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Document Version
Publisher's PDF, also known as Version of record

Publication date:
1996

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Chapter 4

Measurements at the beach

4.1 Introduction

In 1982 high concentrations of heavy minerals were found on the Island of Ameland (see fig. 4.1) which thus became an area of research interest. As mentioned in Chapter 1 heavy mineral placers have traditionally been considered to be the result of winnowing of light minerals from beach sediments due to erosion. This concept, however, does not explain the presence of high amounts of radiogenic minerals like garnet, zircon and tourmaline in areas such as ‘Zwanewaterduinen’. The ‘Zwanewaterduinen’ were formed some 160 years ago after a man-made dike was constructed to close off a tidal inlet. It would appear that accumulations of such minerals also occur when sand is deposited; a hypothesis that is supported by the 25.000 m$^3$ of heavy minerals that were found on the beach between beach pole (RSP) 17-19, after a storm surge in the January - February 1992. This amount exceeds the amount of heavy minerals available in the eroded beach and dunes by an order of magnitude [Mei92, Bos92].

To study the mechanisms of beach erosion and accretion, elevation and radiation were measured at two rectangular test sites during a period of more than three years; from August 1991 till December 1994, 29 measurements were carried out.

Elevations of the Dutch coast are regularly measured for coastal studies; the JARKUS profiles, for example, form a data set of the elevation of the Dutch coast at 250 m intervals recorded at a yearly basis since 1967. The measurement of elevation levels with intervals only in the order of months is less common especially for a fine measuring grid. Greenfield et al (1989) [Gre89] measured time evolutions of elevation and natural radiation levels
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at three test sites at the beach of the Island of Texel. The measurements were mainly of exploratory nature and formed a prelude for the present set of measurements. Because of the high frequency of the measurements and the total time period that was covered the data provide information on the short term behaviour of the beach that would not be noticeable in annual recordings. Due to the combination of elevation and radiation measurements knowledge may be obtained on processes that influence sediment transport.

In the pilot study at the Island of Texel it was shown that radiometric measurements are a sensitive tool for studying dynamical processes at the beach [Gre89, Les89]. Wind driven sand transport is characterised by an inverse correlation between changes in elevation and changes in radiation level; the light, low activity, minerals are mainly involved in transport and cover or uncover the heavy, more active minerals in the top layer of the beach. More generally one may say that because the radiation level contains direct information on the heavy mineral content of the sand and light and heavy minerals are transported in a different way the data entail additional information on the history and therefore on the possible future of the beach.

The dimensions of the sites on Texel were limited (600-1000 m²), with a grid size of 2 m. The processes studied were on a micro scale compared to the size of the island. In the present study on Ameland with sites of 240,000 ('Oerd') and 30,000 m² ('Bornrif') and with grid sizes of 25 and 12.5 m, respectively, it was possible to cover a considerably larger field and study processes on a larger scale (see fig. 4.1). The obtained data are processed by an empirical eigenfunction analysis which allows for a study of the spatial and temporal variations of the data, by calculating 'eigenmodes' of the system. In this way a distinction can be made between significant variations and fluctuations in the data. A positive correlation between evolution in time of radiation and elevation was found for the flooded part of the test site. This shows that heavy minerals are transported onto the beach from the sea (see section 4.4).

After the description of used methods and techniques, section 4.2, first some features of the radiation patterns recorded at the smaller test site, 'Bornrif' will be discussed, section 4.3. The measurements on this field lasted only one year, after which the area was fully below the low-water line. The data from test site 'Bornrif' is unfortunately not suited for an Empirical Eigenfunction analysis because the measurements are often incomplete due to the rapidly progressing erosion. However, the recordings reveal some typical features of the correlation between elevation and radiation which will be discussed.
4.2 Measuring techniques and methods

A more detailed discussion of $\gamma$-ray detection is given in Chapter 1, here only some aspects important for the beach measurements will be given. In section 4.2.3 the chosen method of analysis, empirical eigenfunction analysis, is discussed. A complete listing of all measurements and whether conditions is given in appendix C.

4.2.1 Radiation Measurements

A portable solid-state gamma-ray spectrometer SCINTREX GIS-5, as described in Chapter 1, was used for the radiation measurements in the field. All recordings were ‘total-counts’ measurements with an integration time of 10 seconds. Enhancements in radioactivity due to higher heavy mineral concentrations on the beach occur in the form of lamina, extending over areas that are an order of magnitude larger than the action radius of the detector. Because the ‘action radius’ of the detector is several meters any inaccuracies in the measuring locations are considered to be negligible.

Small variations in the distance of the detector to the ground will not influence the measurements because the absorption of radiation in air is very low. However, two centimeters of over-lying water on the field means a reduction of approximately 20% in the measured intensity. Because it is practical impossible to correct for this absorption, no measurements were taken when the field was inundated.

4.2.2 Elevation measurements

The elevation was measured by the personnel of ‘Rijkswaterstaat’ (RWS) using laser-based and optical level equipment. All values are relative to reference
poles (RSP) at the beach. These poles were calibrated in 1990 according to the Dutch Ordinance Datum (NAP) based on an average sea level. Uncertainties are due to the irregular surface of the beach and the positioning of the measuring rod; they are estimated to be of the order of 2 cm.

4.2.3 Empirical eigenfunction analysis

With an empirical eigenfunctions analysis it is possible to separate spatial and temporal variations of a measured parameter. A data set consisting of measurements at a large number of locations and moments in time can be reduced considerably with this method by calculating ‘eigenmodes’ or ‘eigenfunctions’ of the system. It allows for discrimination between significant variations and ‘noise’.

The empirical eigenfunctions method has been applied to analyse data sets consisting of elevation profiles measured at several moments in time [Win75, Aub79, Aub80, Wij95]. The application as described in this work is different in the sense that a grid of data points is considered instead of one dimensional profiles. Moreover, two quantities are measured (radiation and elevation). The aim of the analysis is not merely to extract eigenmodes of these two quantities and their variation in time but also to investigate possible correlations between elevation and radiation.

The goal of the procedure is to represent the data as a linear combination of two sets of functions with one depending only on position and the other only on time. If the data are denoted by $D(p, t)$, where $p$ is the position $p(x, y)$ and $t$ is the time of the measurements, this separation of variables may be written as:

$$D(p, t) = \sum_i l_i S_i(p)T_i(t),$$

where $S_i(p)$ are spatial functions depending only on position and $T_i(t)$ temporal functions depending only on time. The coefficients $l_i$ give the ‘weight’ of the contribution of the respective functions. Of course there are an endless number of functions that could be chosen but the power of an empirical eigenfunction analysis is that the data are represented by a linear combination of a set of orthogonal functions, ‘generated’ by the data itself [Win75].

The method is based on the Eckart-Young theorem stating that for any real matrix $X$ two orthogonal matrices $U$ and $V$ can be found for which the product $V^T X U$ is a real diagonal matrix $A$ with no negative elements [Dav73]. If $V$ and $U$ are matrices whose columns are orthonormal this theorem can be written as:

$$X = V A U^T.$$
4.2 Measuring techniques and methods

The non-zero eigenvalues of the minor and major product matrices $R = X^TX$ and $Q = XX^T$, respectively, are the same and equal to the square of the singular matrix $\Lambda$ and hence\(^1\):

$$\Lambda^2 = \text{diag}(\lambda),$$

where $\lambda$ is a vector containing the non-zero eigenvalues. Furthermore the columns of matrix $U$ and $V$ contain the eigenvectors of product matrices $R$ and $Q$, respectively. The relation between the $U$ and $V$ follows from equation 4.2

$$V = XU\Lambda^{-1},$$

$$U = X^TV\Lambda^{-1}.\quad (4.4)$$

The Eckart-Young theorem expresses the relation between the data matrix $X$ and the eigenvalues and eigenvectors of its two cross-product matrices.

If the present data are written in the form of a data matrix $D$, where every column represents one recording of the 429 measuring points, the spatial functions $S_i$ are eigenfunctions of the major product of $D$ or correlation matrix $A$

$$A = \frac{1}{n_p n_t} DD^T,\quad (4.5)$$

with

$$a_{ij} = \frac{1}{n_p n_t} \sum_{k=1}^m d_k(p_i, t)d_k(p_j, t),$$

where $n_p$ is the number of positions (429), $n_t$ the number of measurements (27 or 28) and $d_k(p_i, t)$ the radiation or elevation level as measured at position $p_i$ at time $t$. The diagonal elements $a_{ii}$ of this matrix are

$$a_{ii} = \frac{1}{n_p n_t} \sum_{k=1}^m d_k(p_i, t)^2,\quad (4.7)$$

so that the trace $\text{Tr} A$ of the matrix is given by

$$\text{Tr} A = \sum_{p=1}^{n_p} a_{ii} = \frac{1}{n_p n_t} \sum_{i=1}^{n_t} \sum_{k=1}^{n_p} d_k^2(p_i, t),\quad (4.8)$$

which is the mean of the squared values of all data.

The set of eigenvalues $\lambda_i$ belonging to $A$ with corresponding eigenfunctions $S_i$ are defined by

$$AS_i = \lambda_i S_i.\quad (4.9)$$

\(^1R = X^TX = U\Lambda^2 V^T V U^T = U\Lambda^2 U^T \Rightarrow RU = \Lambda^2 U, \text{ and thus } U \text{ contains the eigenvectors of } R \text{ with eigenvalues } \Lambda^2\)
Because the trace of the matrix $A$ is equal to the sum of the eigenvalues one may think of each eigenvalue being representative for a percentage $\frac{\lambda_i}{\sum \lambda_i}$ of the mean of the squared values (MSV) of the data [Win75].

The spatial eigenfunctions form a new set of basis vectors for the data; the temporal eigenfunctions $T_i(t)$ times the coefficients $l_i$ are projections of the data on this new basis. They can be determined by

$$l_i T_i(t) = \sum_{p=1}^{n_p} d_k(p,t) S_i(p).$$

(4.10)

From equations 4.3 and 4.4 it follows that

$$l_i = (\lambda_i n_p n_t)^{\frac{1}{2}}.$$ 

(4.11)

The coefficients $l_i$ express the relative importance of the respective eigenfunctions since they comprise the square root of the eigenvalues. The temporal eigenfunctions give the weight at time $t$ for the spatial eigenfunctions and thus describe the time evolution of the spatial eigenfunctions.

The functions $T_i(t)$ are also eigenfunctions of the minor product matrix $B$:

$$B = \frac{1}{n_p n_t} D^T D,$$

(4.12)

with the elements

$$b_{ij} = \frac{1}{n_p n_t} \sum_{p=1}^{n_p} d_k(p,t_i) d_k(p,t_j).$$

(4.13)

The word ‘function’ suggests a mathematical form and therefore the term eigenprofile (EP) will be adapted for the spatial eigenfunctions; the temporal eigenfunctions will be referred to as weightings.

Often the arithmetic mean is subtracted from the data (principal component analysis) so that the trace of the product matrices is equal to the total variance and all off-diagonal elements represent the covariance between the gridpoints. The advantage of using the raw data without subtracting the mean, is that the mean beach profile (shape) is represented by eigenprofile associated with the largest eigenvalue; this profile will be called mean radiation or elevation eigenprofile (MREP and MEEP). The higher-order profiles account for the variance from this mean beach profile, where the second and the third EP are most important. The data may be described accurately with only three or four profiles. An Empirical Eigenfunction analysis was used successfully by Wijnberg et al. (1995) to describe the morphological behaviour.
of the multiple bar system for the Dutch coast over a period of decades using the JARKUS-data set [Wij95].

Eigen vectors, eigen values and all matrix manipulations have been calculated with routines from the NAG (Numerical Algorithms Group) Fortran Library (1975).

4.3 Test site ‘Bornrif’

4.3.1 Description of the site

This site is located near the west point of the island with RSP 3.0 in its centre; the area is known as ‘Bornrif’. Measurements were started in March 1992 because of the presence of high concentrations of heavy minerals were spotted there by the personnel of ‘Rijkswaterstaat’. The field measured 200 \( \times \) 150 m\(^2\) and was located between RSP 2.9 - 3.1, \( y = 525 - 675 \). The whole area was inundated regularly and eroded fast. After a year the measurements were stopped because the total site had disappeared below the low-water line. In total 10 recordings were made of the site.

The field is not marked by any particular features like dunes; the area is flat. It is an interesting area because of the high erosion rate and the relatively high count rates. After two recordings it was decided to reduce the grid size to 12.5 m; 25 m was too large to cover the interesting phenomena.

4.3.2 Results and discussion

Especially in the first four recordings of the survey the radiation pattern as measured at the test site Bornrif could be called spectacular. The average level was relatively high (compared to the site at ‘Oerd’) and the variation large; areas with count rates approximately twice as high as in the rest of the field occurred. Spots of dark coloured heavy mineral grains could be noticed at the surface of the beach. In time the differences within the field decreased but the average level remained relatively high.

The Bornrif area was eroding fast during the period of the survey, especially in the winter period; the net sediment transport must have been mainly seaward. The logical question arises: is the occurrence of high radiation levels caused by high concentrations of heavy minerals, supporting the lag-deposit hypothesis? It will be shown that this is not necessarily true.

**Radiation swash bar.** In figure 4.2 two characteristic recordings of the elevation and count rate of the site at ‘Bornrif’ are shown. The recordings
Measurements at the beach

were made at a) 22 April 1992 and b) 21 October 1992. Both elevation plots show a (local) maximum as a result of sand deposited by the swash run-up of the waves. It will be called a ‘swash bar’ (SB). Notice that the orientation of the SB is different at the two dates. Both radiation plots also show well defined peaks in the measured count rates. Such a peak area will be called a ‘radiation swash bar’ (RSB). It was observed that an RSB is caused by an accumulation of heavy mineral grains at the surface close to the peak of the SB. Another example of an occurrence of an RSB is given in figure 4.3 where

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.2}
\caption{Two recordings of the count rates (cp10s) and elevation (cm) at ‘Bornriif’: a) at 22 April 1992 and b) at 21 October 1992.}
\end{figure}

two recordings, one of 10 March 1993 and one of 7 April 1993, are shown. Contrary to April no RSB is present in March. At first sight the elevation plots do not seem to differ significantly but a more thorough examination reveals that in March the crest of the SB has migrated outside the test site, as is clear from figure 4.4, which shows cross sections (in the y direction) of the beach and the measured count rates at the four mentioned dates. From these transects one notices that the occurrence of an RSB coincides with the presence of a swash bar. A (local) maximum in the elevation seems to be a necessary condition for the development of an RSB. There are two possible reasons for the occurrence of an accumulation of heavy minerals: 1. removal of light material due to erosion or 2. deposition of heavy minerals. Because a swash bar is formed by the wave run-up it implies that heavy minerals are
Figure 4.3: Two recordings of the radiation (cp10s) and elevation (cm) levels at ‘Bornrif’: a) at 10 March 1993 and b) at 7 April 1993. (The white part of the field was already below the low water line when the recording was made.)

Figure 4.4: Measurements on single lines in the seaward direction (y) from four recordings of the elevation (solid lines) and count rates (dashed lines) at ‘Bornrif’: 22 April 1992, 21 October 1992, 10 March 1993 and at 7 April 1993.
Measurements at the beach

deposited by a landward directed (bed load) transport and are not (solely) the result of winnowing of light minerals (lag deposit). However, the local wave climate which causes the erosion might be the reason why the large amounts of heavy minerals, which are a prerequisite to form an RSB, are present in the area. Erosion would then correspond to selective removal of predominantly light minerals.

That deposition of heavy minerals is not restricted to areas suffering erosion becomes clear in the next section where the measurements at the second test site ‘Oerd’ will be discussed.

4.4 Test-site ‘Oerd’

4.4.1 Description of the site

This major site is located at the north-east side of the island, between RSP 23.2 and 24.0. It will be referred to as site ‘Oerd’, named after the nearby natural reserve. The location was chosen because at this location the beach is broader than at the rest of the island and a significant variation in the count rate is present within the field. During the period of the recordings (August 1991 until December 1994) the beach was stable; no obvious erosion or accretion of the beach was observed.

The cross-shore positioning, indicated by $y$, is between pole $y = 0$ and $y = 300$. The shore parallel direction, $\approx$ west-east, will be denoted with $x$. Position $(x = 0, y = 0)$ denotes the southwest corner of the site and $(x = 800, y = 300)$ the northeast one. Hence the total site measures $800 \times 300$ m$^2$. Every 25 m in both directions a small pole marks a measuring location. The total field is covered with a grid of 429 points where every position $p$ can be denoted by $p(x, y)$ with $x = m \times 25$ and $y = n \times 25$ where $m = 0, .., 32$ and $n = 0, .., 12$.

The northwest part $(x = 0 - 400, y = 175 - 300)$ of the field was flooded during almost each tide while during northerly storm events the total site gets inundated usually with the exception of the dune area. In figure 4.5 an elevation plot is shown that is typical for the site. In the central part of the field $(x = 300 - 575, y = 50 - 150)$ small dunes are present which are characteristic of the area.

The south side of the field is surrounded by dunes of a few meters high at $y \approx -50$m. When going eastwards they bend a little to the south. During the measuring period the sea managed once to break through this dunes in the winter 1992-1993 leaving an ‘inlet’ behind. Behind the dunes a stretched flat area is located, the nature reserve ‘Oerd’.
A characteristic radiation pattern of the site is shown in the left part of figure 4.5. Two parallel ridges ($y \approx 50$ and $y \approx 200$ m) in the west part of the field with an enhanced radioactivity level show up. From a possible third ridge, in the east part directed from $(x = 700, y = 50)$ to $(x = 800, y = 150)$ only the tail is visible. The rest of the site has a lower activity. The presence of dunes does not seem to influence the radiation pattern; no sudden decrease or increase occurs.

The elevation and radiation level (count rate) of the test site ‘Oerd’ are recorded for a period of three and a half year. In this period the radiation was measured 28 times and the elevation 27 times\(^2\). The frequency of the recordings was irregular; the winter period is the most dynamic and therefore interesting period and it was tried to do as many measurements as possible within this season. Unfortunately weather conditions and high tide did not always permit measurements. During summer months changes were hardly observed and in 1993 it was decided to drop measurements during this period also in view of limited support by RWS in this holiday period.

\subsection*{4.4.2 Results}

The data set from the major test site ‘Oerd’ contains 27 elevation and 28 radiation measurements at 429 grid points each. To extract information about the development in time from the data, one has to separate space and time

\footnote{\text{In total 29 measurements were undertaken: at one occasion only part of the field could be measured due to inundation and once it was not possible to measure elevations because of dense fog.}}
dependence of both quantities. This is done by means of the Empirical Eigen-
function Analysis described in section 4.2.3.

The result of the analysis procedure is a set of eigenprofiles that serves
as a new basis for the measured data. Every recording of the site can be
'constructed' by a combination of these eigenprofiles multiplied with the
associated weightings. The time evolution of the weightings are representa-
tive for the dynamic behaviour of the site. Investigating correlations between
the weightings on the elevation and radiation eigenprofiles, EEP's and REP's,
respectively, gives information about accretion and erosion of the field and
the type of sediment involved in these processes.

First some general features of the calculated eigenprofiles will be discussed
after which the time evolution of the weightings are evaluated. The last part
of this section shall be devoted to the correlation between the weightings on
elevation and radiation eigenprofiles.

Eigenprofiles In figure 4.6 the first three radiation and elevation eigen-
profiles $S_i$ multiplied with $\sqrt{\lambda_i}$ ($i = 1,..,3$) are shown. In table 4.1 the
percentage of the mean of the squared values (MSV) of the data is shown.
The numbers in the parentheses give the percentage of the variance from the
mean profile. The fourth eigenprofile accounts for only 7 % of the variance of
the mean elevation eigenprofile (MEEP). It represents extreme values in the
elevation measurements and is therefore not shown in figure 4.6 but will be
discussed separately later on in this section. In the following discussion the
letters refer to the marked areas in the figures.

Table 4.1: The percentage of the MSV of all data (left) and of all data
except for the recordings in week 78 and 83 (right), that is described by the
eigenprofiles associated with the four largest eigenvalues. The numbers in
parentheses give the percentage of the variance that is represented by the
eigenprofiles. The columns with 'R' are for the radiation and 'E' for elevation
data.

<table>
<thead>
<tr>
<th></th>
<th>EP</th>
<th>R</th>
<th>E</th>
<th>R</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>98.95</td>
<td>99.73</td>
<td>99.16</td>
<td>99.73</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.33 (32)</td>
<td>0.12 (45)</td>
<td>0.25 (31)</td>
<td>0.13 (49)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.15 (15)</td>
<td>0.04 (15)</td>
<td>0.11 (13)</td>
<td>0.03 (10)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.11 (10)</td>
<td>0.02 (7)</td>
<td>0.07 (8)</td>
<td>0.02 (7)</td>
<td></td>
</tr>
</tbody>
</table>

The mean elevation eigenprofile (MEEP) accounts for 99.73 % of the MSV
of the data. The main features, the dunes (D) and the decreasing height in the seaward direction, are visible. The letter 'H' marks roughly the position of the high-water line. The second and third elevation eigenprofiles (EEPs), which together represent 60% of the variance of the MEEP, account for the variation in elevation as present in, for example, the dune area.\(^3\)

![Radiation and Elevation plots](image)

**Figure 4.6:** First three eigenprofiles multiplied with \((\lambda_{np})^{1/2}\) (i = 1, 2, 3) associated with the three largest eigenvalues of the radiation (left) and elevation data from site ‘Oerd’. The higher-order profiles can have a positive or a negative weighting. The letters refer to the text.

The mean radiation eigenprofile (MREP) accounts for almost 99% of the MSV of the data. It clearly shows two ridges of enhanced radiation in the west (left) part of the field with the peaks located at \(y = 250\) m (R1) and

\(^3\)The weightings of the higher eigenprofiles can either be positive or negative; they should be superimposed or subtracted from the mean eigenprofile, respectively.
y = 25 m (R2). It seems that the ‘tail’ of R2 extends until the dune area. A possible third area of enhanced radiation is starting at (x = 700, y = 100) and is mainly located outside the area.

The second and third radiation eigenprofiles (REPs) are less dominant than in the case of the elevation and the fourth profile is needed to describe more than half (57%) of the remaining variance. The areas within the test site where the variance in the radiation levels is most significant are well defined:

- The area near the high waterline (x = 0, y = 150–200 until x = 600, y = 300) is prominently present in the two higher profiles (2B and 3B).
- The radiation ridges R1 and R2 visible in the MREP, are also present in the third (3A) and second REP (2A and C), respectively.

The fourth REP is shown in figure 4.7-A. It contains a ridge where the variance is most significant. This is caused by two recordings from February and March 1993 where a ‘radiation swash bar’ (RSB), as was so frequently observed at the test site ‘Bornrif’, occurred. The purpose of this analysis is to extract trends (eigenmodes) in the data. Because in the 28 measurements this phenomena is observed only at the two mentioned occasions, it seems justified to exclude them in the analysis. The procedure as explained in section 4.2.3 is therefore repeated without the two recordings. The result of this second analysis for

The fourth REP is shown in figure 4.7-B; it now represents the variance at the locations of the two radiation ridges R1 and R2. It may be concluded that just as in the case of the elevation eigenprofiles, the fourth REP is representing extreme values.
The results for the highest three eigenprofiles when the data from week 78 and 83 are left out are shown in figure C.1 in appendix C. The main features have not changed except that within in the second order REP the places of maximal variance have become less pronounced (2A,B). In the right hand side of table 4.1 the percentage of the MSV of the data that is represented by the profiles generated by the ‘peakless data’ is shown. As expected the MREP represents a higher fraction than in the case where all the data is included. The remaining variance is divided over the other profiles in a similar way as before.

From this comparison one may conclude that the method produces stable results: exclusion of two recordings does not yield significant different results.

**Weightings** Figure 4.9 b and c, show the weightings, multiplied with $\sqrt{\lambda_{ij}}$, for the first two radiation and elevation eigenprofiles$^4$. Appendix C contains figure C.2 where the weightings of the first three radiation and elevation eigenprofiles are shown as a function of time for both analyses, with and without the RSB of week 78 and 83.

The weightings of the mean profiles are positive during the entire period. Any increase (decrease) means an overall higher (lower) elevation or radiation level and that the differences as present in the mean EP will become more (less) pronounced. The increasing trend of the elevation weightings points at a beach that is becoming higher but also steeper.

The weightings of the higher-order eigenprofiles can either be positive or negative and the associated EP has to be superimposed or subtracted, respectively, from the mean eigenprofile.

**An example** To give an impression of the results of the method figure 4.8 shows the measured and the reconstructed elevation profile from the western part of the site, at $x = 125 m$. The solid line is the measured profile, the dashed line the MEEP and the dash-dotted line is the (weighted) sum of the first three eigenprofiles. Two recordings are shown: a) from 16 October 1991 (week 10) and b) from 11 March 1994 (week 133). The weighting of the MEEP at 11 March 1994 is larger than the weighting of 16 October 1991 (see fig. 4.9); the MEEP in March lies lower than the measured profile which is steeper than in October. The second-order eigenprofile in March is scaled by a relative large positive weighting to raise the middle part ($y = 100 - 225 m$) and lower the sea and duneside. The third eigenprofile is used to somewhat decrease this effect by a positive weighting (see figure C.2 at page 167).

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$^4$Figure 4.9 is printed on page 131
In October 1991 the opposite is the case; the mean eigenprofile is too steep and needs to be flattened by the second and third order EEP: the middle part is lowered by a negative weighting on the second order EEP which also means that the sea and duneside are somewhat elevated; the latter effect is increased by a positive weighting on the third EEP.

Figure 4.8: Measured and the reconstructed elevation for transects from the western part of the site Oerd, at $x = 125m$: a) 16 October 1991 (week 3) and b) 11 March 1994 (week 133). The solid line is the measured profile, the dashed line the first eigenprofile and the dash-dotted line is the (weighted) sum of the first three eigenprofiles.

4.4.3 Time evolution of the EPs

In figure 4.9 the weightings of the mean and second-order EEPs and REP (including all data) multiplied with $\sqrt{\lambda_1}$ are plotted. The average value of the mean elevation and radiation eigenprofile is approximately 1 and the weightings times $\sqrt{\lambda_1}$ are therefore equal to the averaged height, in centimeters, and average count rate, in counts per 10 seconds, respectively. Summer (1 April - 31 September) and winter (1 October - 31 March) periods are denoted by dashed lines. The upper part of the figure shows the average (dashed) and highest (solid) water levels as measured at the west side of the neighbouring Island of Terschelling. It shows that the winter periods are marked by higher water levels were a peak value was reached in the winter of 1993 (≈ week 70 - 80). Within the summer of 1993 (week 90 - 110) when no measurements were taken the weather was calm and the measured water levels were normal.

\(^5\)Values from a measuring station located more near to Ameland were not available for the full period. However, the recordings from both stations are similar.
Figure 4.9: a. average (dashed) and highest (solid) water levels measured at the west side of Terschelling. b. Time evolution \( (T_1(t) \sqrt{\lambda_1 n_1}) \) of the mean and c. of the second order eigenprofile \( (T_2(t) \sqrt{\lambda_2 n_1}) \) for elevation (squares) and radiation (circles). The summer, ‘S’ (1 April - 31 September), and winter, ‘W’ (1 October- 31 March), periods are indicated by dashed lines. The beach nourishment that took place in July - September 1992, is indicated in figure b. between week 50 - 60 with ‘bn’.

The first 70 to 80 weeks of the survey are characterised by rather regular patterns. In the mean elevation profiles one notes an increase in the winter
periods (week 10 - 40 and 60 - 80) and slight decrease during the summer. The mean radiation eigenprofiles show the opposite behaviour: a decrease during the winter season and an increase in the summer. The second-order eigenprofiles show a slight oscillatory behaviour in phase with the mean elevation eigenprofile and with an increasing amplitude in time. These observations correspond to a beach with an elevation profile that hardly changes in shape during the first 70 - 80 weeks but gets steeper later on. For the radiation the mean count rates are high in the summer and low in the winter and secondary features, represented by higher-order profiles, only play a minor role in the first part of the surveys. After week 80 the average level of the countrate as well as the variations in count rate within the site increase.

In the period between week 80 to 130 the storm event around week 80 produces a swash bar with a strong RSB in addition to an overall increase in elevation of about 5 cm. No measurements were taken during the summer period, but when the measurements are resumed in week 110 the overall elevation of the beach is still higher than expected and its mean radiation level is comparable to the situation at the end of the previous summer. It takes until week 120 before the high water levels of the winter period manifest themselves again and the situation in the profiles changes. Around week 130 the second-order elevation profile has become quite strong meaning that the dunes (D) are considerably reduced in height and that in the northwest corner of the site the beach has obtained a well developed swash bar as indicated by 2B in figure 4.6. Simultaneously the overall radiation levels dropped but, due to the strong increase in weighting of the second-order radiation eigenprofile, increase on the seaward slope (A) of the elevation swash bar. The effect on the physical appearance of the site is that the dry part of the beach becomes more flat especially in the dune area. The seaward side becomes more steep. This is somewhat reduced by the larger third-order weightings (see figure C.2).

For the rest of the period the elevation profile of the site maintains this shape due to the remaining high weighting on the second-order elevation profile. The radiation profiles seem to resume their seasonal behaviour of the first 80 weeks.

After this overall description of the evolution of the elevation and radiation properties of the site, the time evolution of individual profiles will be discussed.

**Time evolution of the MEEP** (mean elevation eigenprofile) During the first 70 - 80 weeks the MEEP is (slightly) higher in the winter than it is in the summer, contrary to the behaviour of the MREP. Remarkable is the fact
that storm events that did occur during the period of the survey did not cause erosion but accretion. In for example October 1991 recordings were made just before and right after a storm surge (16 and 23 October 1991, week 10 and 11); in figure 4.9 one may see that this caused a sudden increase of the mean beach profile.

Over the total period of 174 weeks there is an increase in the elevation. In figure 4.10 a linear fit through the weightings of the MEEP gives a yearly (relative) increase of the beach of 2.5 cm. However, the first 60 - 70 weeks of the survey and in the last 80 weeks no real increase can be observed. Therefore, the data are suggestive for a stepwise increase in the period of week 60 to 90. One should realise that this increase is relative to beach poles, which have subsided due to the extraction of natural gas by the NAM since 1986. The rise of the beach relative to the subsiding beach poles is such that the elevation relatively to the average sea level (NAP) is practically unchanged. In the period of 1986 to 1994 the beach is lowered by 12 - 14 cm according to measurements of the NAM [NAM95]. The total measured relative lift of the test site, in the period from August 1991 and October 1994 is approximately 6 - 8 cm which is similar to the measured subsidence of the area.

![Figure 4.10: Time evolution ($T_{1}(t)(\lambda_{1}n_{1})^{1/2}$) for the mean elevation profile. The solid line is a linear fit with a slope of approximately 2 cm per year. The dashed line is 'to guide the eye' and shows the 'jump' between week 60 - 90. The total subsidence during the 3.5 year of the survey is approximately 6 - 8 cm.](image)

From these observations one may conclude that natural processes tend to keep the beach height at a more or less fixed position, relative to sea
Measurements at the beach

level. Under normal conditions the sand required for such supply has to come from the active zone of the island, including the foreshore. A beach nourishment is a man-made effect that distorts the quasi-equilibrium coastal profile. From July until September of 1992 (week 46 to 58) an amount of 1.7 million m$^3$ of sand, coming from the NAP -20 line, was brought on the beach of Ameland between RSP 12 and 20 west of the site. The period is indicated in figures 4.9-b. and 4.10 by ‘bn’. The increase in accretion velocity of the beach is starting roughly 15 weeks later (during the winter period). This could indicate that the extra sand needed to compensate the subsidence of the beach comes from the beach renourishment location. Moreover, it was observed that a large part of the beach renourishment eroded within a few months time [RWS92]. Changes in the behaviour of the mean radiation eigenprofile in the same period (figure 4.9), indicate that sand with different radiometric properties is deposited on the beach than before. The weightings on the second order eigenprofiles suggests that indeed sand with a higher concentration of heavy minerals, is deposited within the flooded part of the field where in the earlier part of the survey sand eroded. Samples from the beach nourishment area did not contain higher amounts of heavy minerals. However, the higher energetic conditions required to raise this site may very well be the reason for deposition of heavy minerals.

**Second-order eigenprofiles** The weightings of the second-order radiation and elevation eigenprofiles show a seasonal behaviour during the full 3.5 years. One notices that in the first 70 weeks the overall sign of the second-order weightings is negative, meaning a flat beach. In the winter period the weightings increase, or become less negative, meaning a deposition of sand containing heavy minerals in the area around the high water line (2B), whereas the dunes (2D) will decrease in height. The dry parts of the site (2C and E) increase slightly in height causing a small decrease in radiation.

During the summer the weightings decrease again; the dunes grow with light, low-active minerals while sand is eroding from the rest of the site. Because of the overall increase in count rate, the area around the high water line becomes less pronounced as an active area.

Between week 70 to 90 and from week 120 on the sign of the weightings changes from negative to positive and a SB is formed with an RSB, indicating the deposition of heavy minerals. The storm event around week 80 caused even a strong RSB. Due to the formation of the SB the site has become steeper as can be seen in the right hand side of figure 4.8. The dunes decrease further.

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$^6$Note the sign of the eigenprofiles in each region.
in height and lose their 'low' activity feature.

4.4 Test-site 'Oerd'

4.4.4 Correlations

One of the goals of this study was to investigate correlations between elevation and radiation. However, first inter correlations have to be considered between eigenprofiles of different order but obtained from the same quantity (elevation or radiation). This is necessary because the mean is not subtracted and consequently the mean eigenprofile passes through the arithmetic mean of the data. Because higher-order eigenprofiles are in the direction of the maximum variance, in sequential order, this means that weightings of the mean eigenprofiles are not necessarily uncorrelated to the weightings of the higher eigenprofiles. For the correct interpretation of any cross correlations between elevation and radiation EPs one therefore has to start by considering the inter correlations.

**Inter correlations** To test the significance of correlations the linear correlation coefficient \( r \) is calculated. It is defined as the ratio of the covariance of two variables and the product of their standard deviations

\[
    r_{xy} = \frac{\text{cov}_{xy}}{s_x s_y},
\]

where \( \text{cov}_{xy} = \frac{1}{n-1} \sum_{i,j} (x_i - \bar{x})(y_j - \bar{y}) \) and \( s^2_x = \frac{1}{n-1} \sum_{i} (x_i - \bar{x})^2 \). Since the covariance can never exceed the product of the two standard deviations the correlation \( r \) will be a number between -1 and +1 where zero means no linear correlation and +1 (-1) a perfect positive (negative) linear correlation.

In table 4.2 the values of \( r \) are given between the weightings of the mean and second to fourth-order radiation and elevation eigenprofiles. To be significant on a 1% level the absolute values of the correlations should exceed 0.45 when all data are included and 0.47 for the data without the RSB.

Table 4.2 shows that for the elevation the significance of a correlation is not influenced when the recordings of February and March 1993 are excluded (see table 4.2) and that the second-order EEP is strongly correlated to the shape of the MEEP.

---

7 Orthogonality does not per definition imply that profiles are uncorrelated. A counter example is a first and second-order sinus function.

8 These values are according to the \( t \)-test. For this test to be valid it must be assumed that the variables are normally distributed and that they are randomly chosen from the population. Although the number of recordings is too low to tell if this requirement is satisfied, there is no reason to assume otherwise.
**Table 4.2**: Correlations $r$ between the weighting on the first and higher-order eigenprofiles. The left side of the table gives the linear correlation coefficient for the weightings of elevation and the right side on the radiation eigenprofiles. Correlation coefficients are calculated for the data with (a) and without (w) the recordings containing the RSB. Correlations are significant on a 1% level according to the $t$-test if $|r| > 0.45$ and $|r| > 0.47$, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Elevation</th>
<th>Radiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>w</td>
<td>a</td>
</tr>
<tr>
<td>2</td>
<td>0.76</td>
<td>0.07</td>
</tr>
<tr>
<td>3</td>
<td>-0.39</td>
<td>-0.28</td>
</tr>
<tr>
<td>4</td>
<td>-0.23</td>
<td>-0.45</td>
</tr>
</tbody>
</table>

Figure 4.8 on page 130 illustrates the correlation between the first two elevation eigenprofiles. A low mean elevation eigenprofile is apparently equivalent to a flat beach; when the area is raised the beach gets more pronounced features represented by a larger weight on the second EEP.

The negative correlation between the first and both the third and fourth-order elevation profile, which is not significant on a 1% confidence level, is due to the fact that large part of the site lies at the dry beach. A large weighting on these higher-order profiles means an increase in height at the seaward side and therefore a shift of the high water line in the seaward direction. Because it has been shown that an increase of the whole site occurs when the field is flooded (see figure 4.9), this has a negative effect on the first-order weightings.

For the radiation a significant correlation exists solely between the mean and fourth REP and only without the two recordings of 1993. The change from no correlation to a relatively strong one between the two eigenprofiles, as illustrated in figure 4.11, can be understood from figure 4.7 at page 128. In case of the ‘all-in’ analysis the fourth radiation eigenprofile is dominated by the presence of the RSB; when this is omitted from the analysis it represents (among others) variations in radiation level at the location of the two radiation ridges R1 and R2.

The absence of a correlation between the first three radiation eigenprofiles emphasizes the independent behaviour of the steady and the dynamical part of the radiation pattern represented by the mean and higher REP, respectively. Since the elevation eigenprofiles are correlated and radiation eigenprofiles are not it may be concluded that any correlation between the mean and higher-order eigenprofiles is not due to the method of analysis.
4.4 Test-site ‘Oerd’

Figure 4.11: Correlations between the weightings of the mean and the fourth REP: A, when all data is included and B, without the two recordings of week 78 and 83 (RSB).

Table 4.3: Linear correlation coefficients between the weightings of the elevation (horizontal) and radiation (vertical) eigenprofiles. The left half of the table includes all data (A) on the right half RSB is excluded (W).

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>EP</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-0.12</td>
<td>-0.11</td>
</tr>
<tr>
<td>2</td>
<td>0.72</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>-0.06</td>
<td>-0.36</td>
</tr>
<tr>
<td>4</td>
<td>0.58</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Cross correlations As can already be seen in figures 4.9 and C.2 the weightings of the radiation and elevation eigenprofiles are correlated. In figure 4.12 the correlation between elevation and radiation weightings of the same order are illustrated and in table 4.3 the corresponding linear correlation coefficients are given. The correlations can be understood by looking in detail at figure 4.6.

The correlation between the weightings of the mean radiation and elevation profiles is not significant. However, in figure 4.12 one can see that up and till the measurement of week 70 (solid squares) there is a rather strong negative correlation \((r \approx -0.8)\). This negative correlation can be explained by the presence of the radiation ridges (R1) and (R2); the beach is accreted by light, low active sand, the ridges get covered and hence, the radiation level
Measurements at the beach

will decrease. If subsequently due to wind driven transport light minerals are winnowed the radiation level will increase again. After week 70, open squares in figure 4.12, the correlation between the mean eigenprofiles changes and the sign is not evident anymore.

The weightings on the second-order eigenprofiles are strongly correlated. The points lie almost entirely in the first and third quadrant of figure 4.12 and hence both eigenprofiles are scaled with weightings that have the same sign. For areas denoted by C, D and E in figure 4.6 this means that an increase (decrease) in elevation causes a decrease (increase) in radiation. Areas C, D and E are all three located at the dry beach and an inverse correlation between radiation and elevation can be interpreted as wind driven transport of light minerals; they cover (uncover) the heavy minerals present in the beach sand. A similar behaviour was observed at the test sites at Texel [Gre89, Les89].

The part of the field denoted with A in figure 4.6 lies on the border between a positive and a negative correlation while for part B, an area that is located just below the high-water line, a clear positive correlation between elevation and radiation is found; an increase (decrease) in elevation causes an increase (decrease) in radiation. Since in the flooded areas transport by water is dominant, this correlation indicates that heavy minerals are deposited (removed) by the sea. Sand deposited in week 78 in the area contains a high concentration of heavy minerals which has increased the radiation level significantly. This is illustrated by the peak in both the weightings on the first and second-order profiles that is visible in figure 4.9 (the RSB form February and March 1993).

A semi-infinite model, as described in Chapter 3, explains the change in count rate if sand is covered or uncovered by material of different activity. The measured activity level $I_m$ is given by:

$$I_m = I_0 e^{-\mu d} + I_c (1 - e^{-\mu d}),$$

where $I_0$ is the activity of the original sand, $I_c$ of the covering sand, $\mu$ the absorption coefficient and $d$ the thickness of the covering layer [Gre89]. In first-order approximation this gives a linear correlation between changes in radiation and elevation.

A negative correlation is observed between the weightings of the third elevation and radiation EPs. The major part of the points (the bottom plots of figure 4.12) lies in the second and fourth quadrant and the weightings of the radiation and elevation EPs have opposite signs. For areas A and B in

\footnote{Note the sign of the eigenprofiles for the different regions}
Figure 4.12: Relative weightings, $T_n(t)$ ($n = 1, \ldots, 3$), of the first three eigenprofiles for radiation ($T_r^n$) plotted as function of elevation ($T_t^n$) eigenprofiles. The solid squares represent the 15 measurements up and till week 70. The left part of the figure includes all data, for the right part the results from week 78 and 83 are not included.

In figure 4.6, where the eigenprofiles have opposite signs, this is indicative for the deposition of heavy minerals, as was the case in February 1993.

The area denoted with 3B can be regarded as an RSB that is apparently part of the eigenmode of this test site. Although this RSB is less spectacular than the one observed in week 78 and 83 it has the same characteristics; a
band of high radiation level just at the position of the high water line.

Besides the correlation between elevation and radiation eigenprofiles of the same order, correlations exist between EEPs and REPs of different order as may be seen in table 4.3. The strong correlation between the weightings on the second order REPs and the mean elevation eigenprofile is the consequence of the inter correlation between the mean and second order elevation eigenprofile, as discussed in the former paragraph, in combination with the strong cross-correlation between the second-order EPs. The fourth EEP accounts for the variation in height at the locations of the radiation ridges R1 and R2 and is therefore correlated to MREP.

4.5 Conclusions

The conclusions can be summarized as follows:

- The survey at ‘Bornrif’ proves a correlation between the occurrence of a swash bar and a ‘radiation-swash-bar’ (RSB). Similarly at test site ‘Oerd’ such a phenomenon was observed. It indicates deposition of heavy mineral grains by the swash run-up.

- Recordings of the elevation at a monthly basis give valuable information on the ‘short term’ (months, seasons) behaviour of the beach. Seasonal trends were observed in the two highest eigenmodes of the system: in the summer the beach at site ‘Oerd’ is somewhat lower. Measurements before and after a storm period indicate that the beach is raised by such an event.

  The apparent quick response of the beach to the subsidence of the island due to gas extraction is remarkable. There are indications that the increase of the beach elevation during the period of week 70 - 90 can be correlated to the strong erosion of the beach renourishment that took place between RSP 12 - 20.

- The Empirical Eigenfunction Analysis provides a powerful tool to study the time evolution of a beach. The correlation between radiation and elevation data allow for differentiation of transport modes of the sediment.

- The radiation measurements give valuable information on not only the morphology of the beach but also about sediment transport mechanisms. Just as was found in the pilot study in 1989 on Texel, wind-driven
transport is characterised by an inverse relation between changes in radiation and elevation [Gre89].

On both test sites on Ameland deposition of heavy minerals results from water driven transport, resulting in a positive correlation between radiation and elevation. The general theory that heavy mineral placers are solely lag deposits, see p.e. [Eit95, Fri95, Woo75], does not agree with this observation.

The change in correlation between the largest eigenmodes, or mean beach profiles, of radiation and elevation that is observed in the second half of 1992 suggests that the sand that is deposited at the test site has different radiometric characteristics than the original sand.