Transport at low electron density in the two-dimensional electron gas of silicon MOSFETs

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
1998

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Chapter 5

Scaling analysis of the metal-insulator transition

5.1 Introduction

Recently, evidence is beginning to emerge that the low temperature resistance of a two-dimensional electron gas does, in contrast to previously held convictions, not go to infinity for all electron densities. It is found[1] that for electron densities \(n_s\) below a critical value \(n_c\) the resistivity tends to infinity, but for \(n_s > n_c\) the system tends to a true metallic state. This observation contrasts sharply with the scaling concepts [2] developed in 1979 which state that any two-dimensional system will for low temperatures become insulating. Although much remains unclear about the nature of the observed phase transition and the states on both sides it is assumed to be related to the complicated interplay of disorder and electron-electron interactions.

In this chapter we will confirm the temperature and electric field scaling form applied by Kravchenko et al.[3]. Furthermore the limitations of these scaling forms will be shown by using an analysis based on the differential resistivity near the metal-insulator transition. The symmetry on both sides of \(n_c\) will be discussed in Sec.5.6.

5.2 Present state

One of the first reports about experiments at low electron density in high mobility material was by Wilson et al.[4]. They measured the conductance at low electron density in Si MOSFETs with a mobility between 1 and 2 \(m^2/Vs\).
Cyclotron resonance measurements showed an anomalous behavior. The conductance measurements were not analyzed in detail.

Later a metal-insulator transition at low electron density and high magnetic fields in a high mobility GaAs-heterostructure was reported by Tsu et al. Near the Fractional Quantum Hall state (FQH-state) with filling factor $\nu = 1/3$, an increase in the resistance was observed. GaAs-heterostructures have been studied extensively at the end of the eighties. A metal-insulator transition near filling factor 1/5 is observed, current-voltage measurements were used to characterize the metallic and the insulating phase. These results were brought in relation to a possible Wigner crystallization, because of the strong electron-electron interaction with respect to the Fermi-energy and the scattering potential of the impurities (high mobilities $10^2 - 10^3 \, m^2/V\,s$) and the observation of the FQH-states. In other 2-dimensional systems and in other materials with relative strong electron-electron interactions also metal-insulator transitions were observed. In a 2-dimensional hole gas in GaAs-heterostructure by Santos et al. and in Si MOSFETs by D'Iorio et al. and Kravchenko et al. In Si MOSFETs the metal-insulator transition occurs between the IQH-states instead of the FQH-states, while the mobility is two orders smaller than in GaAs-heterostructures. Jiang et al. showed that a metal-insulator transition also exists between IQH-states in GaAs-heterostructure at low electron density.

In a theoretical paper of Kivelson et al. it was argued that the observed metal-insulator transition in 2-dimensional systems at low electron density can be described as a transition from a metallic QH-state to an Anderson insulator. The various QH-states are related to each other, and can be depicted on each other in a phase diagram reflecting a charge-flux duality. For large disorder an insulating phase is expected.

In 1993 the first paper about a metal-insulator transition in zero magnetic field was reported by Pudalov et al. Si MOSFETs were used in which in comparison to GaAs-heterostructure the electron-electron interaction is stronger due to a lower dielectric constant and the Fermi-energy is lower for a given $k$-value due to the higher effective mass of the electrons. At first this metal-insulator transition was not expected to be a similar metal-insulator transition in the extreme quantum limit. Scepticism about the relevance of the electron-electron interaction exists because of the absence of clear FQH-states. Moreover a metal-insulator transition is in contrast with the widely accepted scaling theory of Abrahams et al., which states that any 2-dimensional system will become insulating at zero temperature.
Of course the existence of the metallic QH-states are an exception attributed to the influence of high magnetic fields as can be easily understood in the picture of Büttiker\cite{18}. Extended states exist at high magnetic fields in a 2-dimensional system, because of a reduction of the backscattering.

A breakthrough in the characterization of the metal-insulator transition was the report of Kravchenko et al.\cite{1,3} in which temperature scaling and electric field scaling analogous to other continuous quantum phase transitions\cite{19} is used. The temperature scaling and electric field scaling is used to prove that the metal-insulator transition is a true metal-insulator transition. The argument in favour of the approach based on continuous quantum phase transitions is the strong electron-electron interaction. It is assumed that the electron-electron interaction invokes a metallic phase in a high mobility 2-dimensional electron system at intermediate electron density \footnote{Intermediate electron density is defined in Chap. 4 as those electron densities at which the resistance increases with temperature.}. This studied Shepelyanski\cite{20} and Efros et al.\cite{21}. Their theoretical studies showed that electron-electron interactions can cause delocalization of the electrons. The scaling theory of Abrahams has been reanalyzed by Dobrosavljevic et al.\cite{22} in response to the work of Kravchenko and co-workers.

In parametrizing the resistive behavior in a system, one is interested in analyzing how the conductance changes when the length of the conductor is changed. In the scaling analysis of Abrahams et al. therefore a "beta function" is defined:

$$\beta(g) = \frac{\partial \log(g)}{\partial \log(L)}$$

(5.1)

which a function of the conductance \(g\) normalized to \(e^2/h\) itself, but not an explicit function of \(L\). The function \(\beta(g)\) is known in the two limits of very large and very small disorder. From Ohm’s law, we know that \(\beta(g) = d - 2\) for \(g \to \infty\), while for \(g\) small one expects exponentially localized states, so that \(\beta(g) \to -\infty\). In between the function \(\beta(g)\) is assumed to be continuous. For noninteracting electrons it was assumed that:

$$\beta(g) = (d - 2) - 1/g + ...$$

(5.2)

which indeed fulfills the two limits. However, Dobrosavljevic et al argue that for interacting electrons one might postulate that

$$\beta(g) = (d - 2) + A/g^k + ...$$

(5.3)
with $A$ positive, which means that for $d = 2$, $\beta(g)$ has to change from a positive quantity to a negative quantity at some critical value $g_c$.

In the experiments[3] it was found that the resistivity scales with temperature as:

$$\rho(T, n_s) = f(|\delta_n|/T^b) \quad \text{with} \quad b = 1/z\nu \quad (5.4)$$

Here $\delta_n$ is defined as $(n_s - n_c)/n_c$ with $n_s$ the electron density and $n_c$ the critical electron density. The exponent of the temperature dependence, $b$, is determined by two scaling parameters, $z$ and $\nu$, the dynamical exponent and the correlation length exponent, respectively[19]. The electric field scaling is found to be given by:

$$\rho(E, n_s) = f(|\delta_n|/E^a) \quad \text{with} \quad a = 1/(z + 1)\nu \quad (5.5)$$

The electric field scaling exponent, $a$, is determined by the same scaling parameters. When both the electric field and the temperature scaling is determined, the two scaling parameters can be calculated.

The temperature dependence of the resistivity in the insulating phase has been examined by Mason et al.[23] and compared to the temperature dependence in the variable-range hopping regime[2]. The temperature dependence of the resistivity derived by Mott[24] is based on a single-particle picture which ignores electron-electron interaction. Under the condition that the density of states is constant at the Fermi-energy the resistivity will be given by:

$$\rho(T) = \rho_0 \exp \left(\frac{T_0}{T}\right)^{\frac{1}{2}} \quad (5.6)$$

Here $\rho(T)$ is the resistivity at temperature $T$, and $T_0$ is some characteristic temperature at which the resistivity is $\rho_0$. Efros et al.[25] argued that the electron-electron interaction between localized electrons creates a Coulomb gap in the density of states near the Fermi-energy. This will manifest itself in the change of the temperature dependence of the resistivity by replacing the exponent in Eq.5.6 by $1/2$. Mason et al. found a temperature dependence in agreement with the theory of Efros et al., which supports that the electron-electron interaction is influencing the electron transport.

Another remarkable observation about the metal-insulator transition reported by Shahar et al.[26] is the symmetry in the current-voltage response (IV-curves) near the transition. When the current or voltage is scaled with

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2The transport in the variable-range hopping regime occurs by phonon assisted tunneling to electron states nearby in energy and in distance.
the resistivity at the critical electron density, then the IV-curves are symmetric near the transition. This symmetry between the IV-curves of the metallic and insulating phase is also verified by Simonian et al. [27] in zero-magnetic field for Si MOSFETs.

Although a microscopic view is still missing, it seems to be possible to describe the metal-insulator transition phenomenologically with an electric field and temperature scaling form.

5.3 Electric field scaling

The electric field scaling is assumed according to Sondhi et al. [19] to be:

$$\rho(E, n_s) = \rho_c F\left(\frac{\delta_n}{E^a}\right)$$

with $\rho_c$ the resistivity at the critical density and $|\delta_n| = (n_s - n_c)/n_c$ the relative electron density. The electric field scaling exponent $a = \frac{1}{2z+1}$, can be determined from the data.

The scaling of the electric field dominates for high electric fields, while for low electric fields the temperature scaling dominates. The potential energy of the electric field should be larger than the thermal energy to measure the influence of the electric field scaling. Therefore not all data of the IV-curves are useful for the analysis of the electric field scaling. Especially for values of the relative electron density $\delta_n > 0$ the number of datapoints are limited because of the lower resistivity.

For several sets of IV-curves the electric field is determined. In Fig. 5.1.a the resistivity versus the electric field of the IV-curves shown in Fig. 4.3 is plotted. The data in Fig. 5.1 is used to obtain the electric field scaling exponent. For several exponents the parameter $\delta_n/E^a$ is calculated and used to find the best fit to let all the curves of Fig. 5.1 to collapse onto two branches. In Fig. 5.2.a the best fit is shown for sample #3 of size $10 \times 120 \ \mu m$ at a temperature $T = 200 mK$. The critical electron density is $n_c = 7.45 \times 10^{14} \ m^{-2}$. All values for the resistivity collapse onto two curves, the upper curve for the insulating side and the lower curve for the metal side. The electric field scaling exponent, $a = 0.26$ is a representative value for a temperature $T \geq 0.2 \ K$. In this temperature range the electric field

The value is determined with an accuracy of $10^{12} \ m^{-2}$ at this temperature, but cannot be determined with the same accuracy for all temperatures. At higher temperature the transition is more smooth.
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Figure 5.1: From the IV-curves the electric field and the resistivity can be determined. In a. the resistivity of sample #3 at $T = 200 \text{ mK}$ is shown versus the electric field. The lowest curve corresponds to an electron density of $9.85 \times 10^{14} \text{ m}^{-2}$, the upper curve corresponds to $6.40 \times 10^{14} \text{ m}^{-2}$. b. A similar plot for sample #2 at a temperature of 150 mK.
5.3. Electric field scaling

Figure 5.2: a. The electric field scaling of the resistivity for sample #3 at $T = 200$ mK. The optimal scaling exponent $a = 0.26$. b. Similar for sample #2 at a temperature of 150 mK. The optimal scaling exponent $a = 0.48$ which diverges from values at temperatures $T \geq 200$ mK.
field scaling exponent is on average $0.25 \pm 0.01$. This value is common for all four samples which are checked for electric field scaling in this temperature range. On the other hand the exponent determined by Kravchenko et al. at a temperature of $T = 0.22 \, K$ is $0.37$. This is a striking difference from the values found in the samples studied in this thesis. At a temperature $T < 0.2 \, K$ a larger value for the electric field scaling exponent is found. In Fig. 5.2.b for sample #2 of $2 \times 8 \, \mu m$ at a temperature, $T = 150 \, mK$ an electric field scaling exponent of $a = 0.48$ is found. This value has also been observed in sample #5 at the same temperature. Higher values than the common value are found in our samples for $T < 0.2 \, K$.

In Fig. 5.2.a the data at extreme high electric fields are not included. In contrast Fig. 5.2.b also data are used which have been measured with a current, $I_{DC} > 1 \, nA$. These current values are about ten times larger than used in the data of Fig. 5.2.a. For high electric field it is expected from
5.3. Electric field scaling

Figure 5.4: The offset voltage in sample #2 is shown as a function of $\delta_n = (n - n_c)/n_c$ at a temperature of 150mK. For relatively small positive $\delta_n$ the nonlinear behavior disappear as shown before in Chap. 4.

Eq. 5.7 that for all electron densities in the range of the scaling theory the resistivity will lead to the value $\rho_c$. Fig. 5.2 shows that for high electric fields the data deviate from the upper and lower scaling curves. At the insulating side the resistivity tends towards a higher resistivity than $\rho_c$, and likewise for the metal side the resistivity tends towards a lower resistivity.

This deviation is expected from the IV-curves shown in Fig. 5.3. For large current values the IV-curves form a fan, reflecting that the slope at high electric field gradually increases. This tendency can also be seen in the data of Krawchenko et al [3]. Extrapolation of the linear part of the IV-curves to zero bias current shows that none of the nonlinear IV-curves passes through zero. For the insulating side a positive offset voltage is found, whereas for the metallic side a negative offset voltage is found. At the critical electron density the resistivity is linear. In Fig. 5.4 it is shown how the offset voltage
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Figure 5.5: a. The differential resistivity of sample #2 derived from Fig. 5.3 at $T = 150$ mK. b. For high electron density the nonlinearity slowly disappears, the difference between $\rho_0$ and $\rho_E$ becomes smaller. The differential resistivity is taken from sample #3 at $T = 200$ mK, and the electron density is in the range of $6.42 \times 10^{14} \text{ m}^{-2}$ to $9.87 \times 10^{14} \text{ m}^{-2}$. The lowest curve corresponds to the highest electron density.
5.3. Electric field scaling

Figure 5.6: A comparison among the differential resistivity $\rho_0$, $\rho_E$ and the resistivity at $T = 4.2$ K. shows that the dependence on electron density of the differential resistivity $\rho_E$ resembles to the resistivity at $T = 4.2$ K. The data is from sample #2 and both differential resistivities are measured at $T = 200$ mK. 

evolves with the relative electron density. The offset voltage is determined by extrapolating the linear part in Fig. 5.3 to zero current.

From Fig. 5.3 it appears that the differential resistivity is nearly constant at high electric field. In Fig. 5.5a it is shown that the differential resistivity is indeed only weakly dependent on the electric field for high electric fields. The nonlinear behavior extends over a larger current range for high electron density than for low electron density. The difference between the resistivity at zero bias, $\rho_0$, and the differential resistivity at high electric field, $\rho_E$ increases with the absolute value of the relative electron density, $|\delta_n|$. However, for $\delta_n \sim 0.15$ and larger the difference between the resistivity $\rho_0$ and the differential resistivity $\rho_E$ slowly disappears. In other words for high
enough densities the nonlinearity weakens. This can be seen in Fig. 5.5.b where the differential resistivity is shown of sample #3 at $T = 200 \text{ mK}$.

It appears instructive to make a comparison between the resistivity $\rho_0$, $\rho_E$ and $\rho_{T=4.2K}$. The resistivity $\rho_{T=4.2K}$ is the linear resistivity at the temperature, $T = 4.2 \text{ K}$. The comparison is shown in Fig. 5.6 for sample #2. The resistivity $\rho_E$ at high electric fields almost coincides with the resistivity at high temperature. At $T = 4.2 \text{ K}$ the thermal energy $0.36 \text{ meV}$ is much larger than the Fermi-energy at the critical electron density ($0.03 \text{ meV}$), while at these values the Coulomb energy equals $9 \text{ meV}$. At the temperature of $4.2 \text{ K}$ no metal-insulator transition is observed, but the temperature dependence at low and intermediate electron densities shows metallic behaviour. The agreement between both resistivities $\rho_E$ and $\rho_{T=4.2K}$ is an indication that for high electric fields the 2-dimensional system is pushed in a state similar to the one at $4.2 \text{ K}$. Nevertheless for both temperatures around $200 \text{ mK}$ the value of $\rho_E$ is always smaller than $\rho_{T=4.2K}$, which may result from the positive temperature coefficient of $\rho$ at high temperature. One might argue that the electric field heats the electron gas to temperatures close to $4.2 \text{ K}$. This ignores, however, that $\rho_E$ is the slope of the curve and ignores also the offset voltage.

5.4 Temperature scaling

The scaling function for the temperature is assumed to be:

$$\rho(T, n_s) = \rho_0 F \left( \frac{c_b |\delta n|}{T^b} \right)$$

with the scaling exponent, $b = \frac{1}{z \nu}$. The temperature scaling exponent differs from the electric field scaling exponent, but both depend on the dynamic scaling parameter $z$ and the correlation length exponent $\nu$. When both temperature and electric field scaling is applied, then both parameters $z$ and $\nu$ can be determined.

The resistivity at zero bias is used for the temperature scaling. In Fig. 5.7 the resistivity is shown as a function of temperature for several electron densities in the range from $6.05 \times 10^{14} \text{ m}^{-2}$ to $9.85 \times 10^{14} \text{ m}^{-2}$ of sample #3. Around zero bias the influence of the electric field scaling is minimal. The data at a temperature of $150 \text{ mK}$, as well the data above $500 \text{ mK}$ are left out, because the $\rho$ fall outside the expected range of validity of the scaling analysis. In Fig. 5.7.b the resistivity of sample #3 is plotted versus
5.4. Temperature scaling

Figure 5.7: a. The differential resistivity around zero bias, $\rho_0$, as a function of temperature for sample #3. b. Temperature scaling of the differential resistivity $\rho_0$ is applied to sample #3 in the temperature range of 200 mK to 500 mK. The temperature scaling exponent $b = 0.48$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{figure5_7.png}
\caption{a. The differential resistivity around zero bias, $\rho_0$, as a function of temperature for sample #3. b. Temperature scaling of the differential resistivity $\rho_0$ is applied to sample #3 in the temperature range of 200 mK to 500 mK. The temperature scaling exponent $b = 0.48$.}
\end{figure}
the temperature scaling parameter, $|\delta_n|/T^b$. The data collapse onto two curves when the temperature scaling exponent $b$ is 0.48. The upper curve corresponds to the insulating state, whereas the lower curve corresponds to the metallic state. This value for $b$ is also found in sample #4, $b = 0.46$. The accuracy is worse than for the electric field scaling. This is due to the small temperature range that we had data for. The average value is $0.47 \pm 0.04$. The value found by Popovic et al. [28] is 0.62. Kravchenko et al. [3] found an even higher value of $b = 0.83$.

Similar to the electric field scaling Eq.5.8 suggests that for high temperature the resistivity will go to a single value $\rho_c$. It is apparent from Fig. 5.6 that the resistivity at high temperature ($T = 4.2 K$) deviates from this single value. The temperature scaling is further analyzed with the differential resistivity. In Fig. 5.8 the differential resistivity of sample #4 at an electron density of $7.0 \times 10^{14} m^{-2}$ (insulating side) at several temperatures in the range of the temperature scaling.
density \( n_s = 7.0 \times 10^{14} \text{ m}^{-2} \) is shown as a function of current for several temperatures. The differential resistivity at zero bias decreases with temperature as mentioned in Chap. 4 for the insulating state. For high electric fields the temperature dependence is different, the differential resistivity increases with temperature. A larger differential resistivity at high electric field, \( \rho_E \), implies that for high electric field the resistivity will eventually be larger at high temperature than for low temperature. Fig. 5.9.a shows that indeed the curves cross each other when the temperature is varied, indicating different behavior for low voltages compared to high voltages. This is not expected for simple heating. The temperature dependence of \( \rho_E \) is further investigated by extracting a temperature coefficient \( \alpha_E \) defined as in Eq.4.1, for \( \rho_E \) instead of \( \rho_0 \). The temperature coefficient \( \alpha_E \) is compared with the temperature coefficient \( \alpha \) in Fig. 5.9.b. The temperature coefficient \( \alpha_E \) valid at high electric fields seems to be the extrapolation of \( \alpha \) at zero field on the metallic side. This suggests the earlier indication that the state at high electric field is similar to the metallic state due to interacting electrons. The temperature coefficient seems to increase monotonously with decreasing electron density.

5.5 Offset voltage

The offset voltage is derived from the nonlinear IV-curves by extrapolation of the resistivity \( \rho_E \) towards zero bias current as illustrated in Sec. 5.3. This offset voltage is determined as a function of electron density around the critical electron density at several temperatures. The result is shown in Fig. 5.10. The offset voltage is positive for low electron density and increases with decreasing electron density. It is negative on the metallic side and decreases with increasing electron density until approximately the turning point at which also the temperature coefficient changes drastically. Both the offset voltage and the temperature coefficient are independent of temperature in the temperature range \( T < 0.5K \).

In the temperature range of \( 0.5 - 1 \text{ K} \) the offset voltage decays. At first the offset voltage around the critical electron density disappears, while at low electron density the offset voltage is still present. The lower the electron density the higher the temperature should be to allow the offset voltage to disappear.

As a consequence of the increasing temperature the IV-curves become linear around the critical electron density, while at low electron density the
Figure 5.9: a. Part of the IV-curves are shown to illustrate that IV-curves at higher temperature cross the IV-curves of lower temperature. The IV-curves are from sample #4 at an electron density of $7.0 \times 10^{14} \text{m}^{-2}$. b. A comparison between the temperature coefficient of the differential resistivity at high electric field with the temperature coefficient at zero bias as a function of electron density.
Figure 5.10: The offset voltage as a function of electron density for several temperatures of sample #4. The offset voltage is temperature independent in the temperature range of 200 to 500 mK. For higher temperature the offset voltage decreases and becomes zero.
Figure 5.11: A comparison of the IV-curves of sample #4 between $T = 200 \text{ mK}$ and $T = 500 \text{ mK}$ shows that at higher temperature the IV-curves becomes linear around the critical electron density. The electron density is in the range $8.38 \times 10^{14} \text{ m}^{-2}$ to $7.00 \times 10^{14} \text{ m}^{-2}$. 
5.5. Offset voltage

Figure 5.12: The nonlinear IV-curves for low electron density at $T = 500 \text{ mK}$ compared with those at $T = 200 \text{ mK}$. At high temperature the IV-curves are more smooth and the differential resistivity (slope) is for large current larger. The electron densities are 5.09, 4.92 and $4.75 \times 10^{14} \text{ m}^{-2}$ respectively.

IV-curves are nonlinear. This is illustrated in Fig. 5.11, which show the IV-curves of sample #4 at a temperature of 200 mK and 500 mK. The IV-curves are linear around $n_c = 7.4 \times 10^{14} \text{ m}^{-2}$ at 500 mK, while at 200 mK they are nonlinear. At lower electron density the IV-curves are nonlinear for $T = 500 \text{ mK}$ as well, although the nonlinearity is not as abrupt as for low temperature (Fig. 5.12). The result at $T = 500 \text{ mK}$ is consistent with the earlier result of Pudalov et al. [17]. They report in this article only nonlinear behavior for low electron density. It is expected that also in their samples the nonlinear behavior for electron densities above the critical electron density can be observed at lower temperatures, such as reported by Krawchenko et al. in similar samples.
Figure 5.13: The symmetry of the IV-curves is demonstrated by scaling the voltage-axes with the critical resistivity. The symmetry holds only for small $\delta_n$. The IV-curves for positive $\delta_n$, 0.0222, 0.0447, 0.0897 respectively 0.1349 are mirrored.

5.6 Symmetry

The symmetry of the IV-curves is investigated by Shahar et al.[26]. They found that the IV-curves on the metallic side can be mapped on the IV-curves at the insulating side, when they are scaled with $\rho_c$. The symmetry is attributed to the charge-flux duality near disordered dominated transitions in quantum Hall systems. The mapping follows the law of corresponding states as derived by Kivelson et al.[16].

The symmetry is also observed in zero-magnetic field by Simonian et al.[27]. The symmetry follows in zero-magnetic field[22], when the conductivity, $\sigma^+ = \sigma/\sigma_c$, is scaled:

$$\sigma^+(\delta_n, T) = \frac{1}{\sigma^+(-\delta_n, T)}$$  \hspace{1cm} (5.9)
The symmetry should hold only for $T > T_0(\delta_n)$ in the quantum critical region.

This symmetry is checked in our samples for small $T_0$, thus for electron densities near the critical electron density. The relative electron density $\delta_n = 0.0222$ is mapped on the relative electron density $\delta_n = -0.0228$ as can be seen in Fig. 5.13. There is no symmetry for higher relative electron density. This is observed in all three samples checked for symmetry. This result is consistent with the previous observations. It was observed that the scaling theory is limited to small positive $\delta_n$. Based on the scaling theory the conductivity is expected to be an odd function around $\delta_n = 0$. In Fig. 5.7.b the resistivity at zero bias, $\rho_0$, is shown. The resistivity is linear around $\delta_n$ on a logarithmic scale, which implies an odd function. For $\delta_n > 0$ the resistivity deviates at small $\delta_n$ from linear on the logarithmic scale, therefore only a small range in $\delta_n$ satisfies the symmetry.

Besides the small range in which the symmetry is valid, it is also observed that the range to which the electric field scaling can be applied is also small. Especially for samples with a large length to width ratio. In Fig. 5.5 the data of sample #2 (2 $\times$ 8 $\mu$m) can be compared with sample #3 (10 $\times$ 120 $\mu$m). The nonlinearity in the IV-curves of sample #3 stretches over a smaller range than in sample #2. While the nonlinearity in the IV-curves of sample #5 (not shown here) stretches over an even larger range than in sample #2. It can be concluded that the geometry influences the range of electric field scaling.

### 5.7 Conclusions

The electron transport is studied in a 2-dimensional electron gas at low temperature, $T < 1$ K. The electron density of the system has been varied from high electron density ($10^{16}$ m$^{-2}$) to low electron density ($10^{14}$ m$^{-2}$). The resistivity of the system is measured, and the data show a transition from metallic behavior to insulating behavior at decreasing electron density. This is in contrast with the predictions of Abrahams et al.[2], which state that a 2-dimensional system will never be metallic at zero temperature.

The experimental system studied here was a Si MOSFET for two reasons. First high quality Si MOSFETs were available. Secondly, in Si MOSFETs the electron-electron interaction is relatively high with respect to the Fermi-energy, because of the material properties. The effective mass is large and the dielectric constant at the interface is small. The ratio between the Coulomb
Low $n_s$ | Intermediate $n_s$ | High $n_s$
---|---|---
$\alpha < 0$ | $\alpha > 0$ | $\alpha \approx 0$
nonlinear | (non)linear | linear
$E_p < E_{\text{Coul}}$ | $E_p < E_{\text{Coul}}$ | $E_{\text{Coul}} < E_p$
Insulating | Metallic | Weak Localized

Figure 5.14: There are three regimes at low temperatures. By varying the electron density the three different regimes can be entered. The transition between low and intermediate electron density is the critical one. The disorder should be weak enough to maintain $\rho < 3\hbar/e^2$ in the regime of intermediate $n_s$.

At low temperature three different regimes can be distinguished high, intermediate and low electron density. In Fig. 5.14 the three regimes are shown with their characteristics.

At high electron density, $n_s > 2 \times 10^{16} \text{ m}^{-2}$ the resistivity is linear and almost temperature independent. The Fermi-energy is larger than the Coulomb energy. The 2-dimensional system is weakly localized. When the electron density decreases, then at approximately $2 \times 10^{16} \text{ m}^{-2}$ the temperature dependence slowly increases, becoming more metallic.

At intermediate electron density, $n_c < n_s < 2 \times 10^{16} \text{ m}^{-2}$ the temperature coefficient is positive and increases with decreasing electron density. The Fermi-energy is smaller than the Coulomb energy, and the ratio $E_{\text{Coul}}/E_F$ increases with decreasing electron density. The IV-response is initially still
linear, except near the critical electron density. The resistivity increases with increasing electric field. Also the temperature dependence is different near the critical electron density, the temperature coefficient changes sign with decreasing electron density. At the critical density the temperature coefficient is zero and only there the resistivity is linear.

At low electron density the temperature coefficient is negative and the resistivity non-linear. The non-linear resistivity is opposite to the non-linear resistivity above the critical electron density, also the temperature dependence is reversed. For high electric field and high temperature it is observed that the behavior becomes similar to the phase at intermediate electron density. The resistivity is nearly constant. Secondly the temperature coefficient is positive, and its dependence on electron density is extended.

The transition at the critical electron density is a true metal-insulator transition. Such a transition is described as a continuous quantum phase transition by Sondhi et al.\[19\]. The dependence on the electric field and the temperature of the resistivity can be described with a scaling factor, $\delta_n/E^a$ respectively $\delta_n/T^b$. Both the exponent $a$ and $b$ are determined from the data in several samples. For the electric field exponent a common value of $0.25 \pm 0.01$ for temperatures $T \geq 200\text{mK}$ is determined. The exponent of the temperature dependence is determined as $0.47 \pm 0.04$. The dynamical scaling parameter and the correlation length exponent can be derived from the exponents $a$ and $b$. The dynamical scaling parameter is $1.14 \pm 0.18$ and the correlation length exponent is $1.87 \pm 0.24$. These values are higher than the earlier ones observed by Krawchenko et al.\[3\] and Popovic et al.\[28\]. The dynamical scaling parameter is close to $z = 1$, which points theoretically to a strongly interacting system\[19\].

The exponent of the temperature dependence, $b$ is compared with the result of Mason et al.\[23\]. They observed a temperature dependence of the resistivity near the critical electron density that is in agreement with:

$$\rho = \rho_0 \exp \left(\frac{T_0}{T}\right)^{0.5}$$

(5.10)

The exponent 0.5 corresponds theoretically to a system of localized electrons in which a Coulomb gap is created due to the interaction between the electrons. Our data is also consistent with this dependence. The data confirm that electron-electron interactions are dominant in high mobility Si MOSFETs at low and intermediate electron densities.

At high temperature ($T > 1 \text{K}$) or for high electric field a deviation is observed from the scaling as proposed by Dobrosavljevic et al.\[22\]. The
deviation is also shown partly in the data of Kravchenko et al. The resistivity approaches for high electric field and high temperature a constant value for the differential resistivity which depends on the electron density and weakly on temperature and electric field. While theory expects that the resistivity will asymptotically approach the resistivity at the critical electron density, \( \rho_c \). The deviation suggests that the scaling might be described by:

\[
\rho(n_s, E, T) = \rho_E(n_s) F(\delta_n, E, T)
\]

with \( \rho_E \) the resistivity at high electric field. Although it is suggested that the resistivity will reach asymptotically the differential resistivity for high electric fields, it contains no offset potential.

For low electron densities the differential resistivity for high electric field or at high temperatures has similar properties as the differential resistivity for intermediate electron density. The differential resistivity hardly depends on electric field, and also the temperature dependence at high electric field is an extension of the temperature dependence at intermediate electron density.

At intermediate electron density the electrons are supposed to be delocalized by the relative strong electron-electron interaction, analogous to the delocalization of the electrons as described by Shepelyanski[20] and by Efros et al.[21]. At low electron density the electrons are supposed to be localized by the influence of the disorder potential for three reasons. At first in Si MOSFET's with a mobility of about a factor 4 lower than the mobility in our Si MOSFET's there is no metal-insulator transition observed[28], and the electrons are strongly localized by the disorder potential upon decreasing the electron density. At an approximately four times higher mobility it can be expected that the disorder potential will be more of influence when the electron density is lowered. Although the electron-electron interaction is stronger in a Si MOSFET compared to \( E_F \), it decreases with decreasing electron density. When the electron density is low enough the disorder potential can dominate over the Coulomb energy associated with the electron-electron interaction.

Secondly, there is a tendency that for samples with a lower mobility the critical electron density is at higher electron density, when the results described in this thesis and those of Kravchenko et al. and Popovic et al. are compared.

Thirdly, at high temperature there is no metal-insulator transition observed, but a continuation of the phase with metallic behavior. This phase is supposed to be metallic due to the relatively strong electron-electron interaction. The influence of the disorder potential can be reduced by increasing
the temperature or by increasing the electric field. This is consistent with the observation at low electron density for high electric fields, that the metallic phase due to electron-electron interactions reappears.

Hence, it can be concluded that the metal-insulator transition is a complicated interplay of localization due to disorder together with the electron-electron interactions. A microscopic picture describing the metallic phase and the metal-insulator transition is needed. Experimentally we have been able to confirm the phenomenologically description of the metal-insulator transition as proposed by Kravchenko et al. We have also shown some of the limitations of their scaling analysis.

References


5.7. Conclusions


Chapter 5. Scaling analysis of the metal-insulator transition