Proton-proton bremsstrahlung in a relativistic covariant model
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2. PROTON-PROTON BREMSSTRAHLUNG
AND THE NUCLEAR INTERACTION

In this Chapter the important ingredients for the calculation of proton-proton bremsstrahlung observables are described. First, we define the kinematics of the bremsstrahlung process, in particular for the case of a real photon. This is followed by the definition of the principal observables, the cross section and the analyzing power. The observables can be expressed in terms of phase-space factors and the dynamical variables, the matrix elements. The matrix elements contain the information of the interaction between the participating particles such as the nucleon-nucleon interaction, discussed in Sec. 2.3, and the nucleon-photon vertex. The precise way in which the NN interaction enters the matrix element is discussed in Sec. 2.4. In the last section we discuss briefly the effects of the Lorentz boosts between the nucleon center-of-mass frames. These effects are important for our particular NN interaction which is determined only correctly in this specific Lorentz frame.

2.1 Kinematics of bremsstrahlung

The kinematics of real bremsstrahlung can be expressed in six independent variables. Although in some particular calculations it is more convenient to choose a reference frame in which the photon defines the \( \hat{z} \) axis, the physical observables are usually shown in the laboratory system. In this frame the \( \hat{z} \)-axis is defined by the momentum of the incoming proton, \( p_{1\mu} = (E_p, 0, 0, p) \) and \( p_{2\mu} = (m, 0, 0, 0) \). This gives one independent variable, since the incoming particles are on-shell, so \( E_p = \sqrt{p^2 + m^2} \), with \( m \) the mass of the proton. In this reference frame the outgoing momenta are defined by

\[
\begin{align*}
    p'_{1\mu} &= (E'_1, p'_1 \sin \theta_1 \cos \phi_1, p'_1 \sin \theta_1 \sin \phi_1, p'_1 \cos \theta_1) \\
    p'_{2\mu} &= (E'_2, p'_2 \sin \theta_2 \cos \phi_2, p'_2 \sin \theta_2 \sin \phi_2, p'_2 \cos \theta_2) \\
    q_\mu &= (\omega, q \sin \theta_\gamma \cos \phi_\gamma, q \sin \theta_\gamma \sin \phi_\gamma, q \cos \theta_\gamma).
\end{align*}
\]  

(2.1)

These momenta must satisfy conservation of total momentum,

\[
p_{1\mu} + p_{2\mu} = p'_{1\mu} + p'_{2\mu} + q_\mu.
\]  

(2.2)

With the definition of the \( \hat{z} \)-axis by the momentum of the beam, the frame is not fully specified. A possible choice is to set the photon \( \phi \)-angle equal to
zero. This is a convenient definition when some of the calculations are done in the reference frame where the photon defines the $\hat{z}$-axis, since the boosts simplify somewhat. The resulting frame is shown in Fig. 2.1. With this choice of reference frame eight variables remain, of which four are independent. The observables are usually expressed in terms of the polar angles $\theta_1, \theta_2$ and $\phi_2$, and for example the azimuthal angle $\phi_1$, solving the equations from momentum conservation for the other four variables. This gives

$$
\begin{align*}
p_1' &= p \sin \theta_2 \sin \theta_\gamma \sin (\phi_\gamma - \phi_2)/\mathcal{N} \\
p_2' &= p \sin \theta_1 \sin \theta_\gamma \sin (\phi_1 - \phi_\gamma)/\mathcal{N} \\
q &= p \sin \theta_1 \sin \theta_2 \sin (\phi_2 - \phi_1)/\mathcal{N}, \tag{2.3}
\end{align*}
$$

where the common denominator is

$$
\mathcal{N} = \cos \theta_1 \sin \theta_2 \sin \theta_\gamma \sin (\phi_\gamma - \phi_2) + \sin \theta_1 \cos \theta_2 \sin \theta_\gamma \sin (\phi_1 - \phi_\gamma) \\
+ \sin \theta_1 \sin \theta_2 \cos \theta_\gamma \sin (\phi_1 - \phi_1), \tag{2.4}
$$

for the case of out-of-plane scattering. The remaining dependent variable (for example $\phi_2$) can be found numerically by solving the equation from energy conservation. In the coplanar case momentum conservation in the $\hat{y}$-direction yields a trivial relation, which in practice dictates all $\phi$-angles to be zero or $\pi$. In that case $p_1'$ and $p_2'$ can be solved using the remaining equations from momentum conservation. Choosing particle 1 as that which is on the same side as the photon with respect to the beam ($\phi_1 = \phi_\gamma = 0, \phi_2 = \pi$), the momenta in terms of $q$ and the independent variables are

$$
\begin{align*}
p_1' &= \frac{p \sin \theta_2 - q \sin (\theta_1 + \theta_2)}{\sin (\theta_1 + \theta_2)} , \quad p_2' = \frac{p \sin \theta_1 - q \sin (\theta_1 - \theta_\gamma)}{\sin (\theta_1 + \theta_\gamma)}. \tag{2.5}
\end{align*}
$$

In this case $q$ is most readily found from solving the equation from energy conservation numerically.
2.2 Physical observables

In this section we discuss the definition of the physical observables of interest. As mentioned above these quantities are usually measured, and therefore also given in the literature, in the laboratory frame where the \(z\)-axis is defined by the beam (initial nucleon 1) and the target (initial nucleon 2) is at rest. With the choice of dependent and independent variables as given above the 5-fold cross section in the lab. system is given by

\[
\frac{d^5 \sigma}{d\Omega_1 d\Omega_2 d\phi_\gamma} = \frac{m^3 p_1^2 p_2^2 q}{p E_1 E_2 2\omega(2\pi)^5 N_a} \sum_{\lambda_i \lambda_f} |M_{\lambda i \lambda f}|^2, \tag{2.6}
\]

where

\[
N_a = \frac{p_1^1}{E_1} (\sin \theta_2 \cos \phi_2 - \cos \theta_2 \sin \phi_2) + \frac{p_2^1}{E_2} (\sin \theta_1 \cos \phi_1 - \cos \theta_1 \sin \phi_1) + \frac{q}{\omega} (\sin \theta_1 \cos \phi_2 - \cos \theta_1 \sin \phi_2), \tag{2.7}
\]

and in the sum over initial spins \(\lambda_i\) and final spins \(\lambda_f\) averaging over initial spins results in the factor \(\frac{1}{2}\). For real bremsstrahlung \(\omega = q\) and all factors \(q/\omega\) are equal to 1 in Eqs. (2.6-2.8). In most experiments only in-plane scattering has been considered in which all azimuthal angles are zero.

The other observable of interest is the analyzing power. It is related to the spin structure of the \(NN\) interaction, and is defined by

\[
A_\nu = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}. \tag{2.8}
\]

Here \(d\sigma^\uparrow\) is the differential cross section for incoming proton 1 with spin in the \(+\hat{y}\)-direction and \(d\sigma^\downarrow\) is the differential cross section for incoming proton 1 with spin in the \(-\hat{y}\)-direction.

2.3 \(NN\) interaction

In this section we will briefly summarize the framework of the field-theoretical Bethe-Salpeter (BS) equation for the interacting two-proton system and outline the quasi-potential approximation. The \(NN\) interaction is based on a one-boson exchange (OBE) model with only nucleonic and mesonic degrees of freedom.

Within the relativistic field theory two particle scattering is described by the scattering T-matrix. This T-matrix \(T(p, p'; P)\) is a solution of the inhomogeneous BS equation,

\[
T(p, p'; P) = V(p, p') - i \int \frac{d^4 k}{(2\pi)^4} V(p, k) S_2(k, P) T(k, p', P), \tag{2.9}
\]
where \( S_2(p, P) = S^{(1)}(p, P)S^{(2)}(p, P) \) and \( S^{(i)} \) are the free one-particle propagators of the two nucleons with relative momentum \( p \) and total momentum \( P \). In our case the NN interaction \( V(p, p') \) is assumed to be given by the one-boson exchange model of Fleischer and Tjon \([16, 21]\). In this OBE model the interaction is described by the exchange of \( \pi, \eta, \rho, \omega, \delta \) and \( \epsilon \) (or \( \sigma \)) mesons.

The contributions from the isovector mesons \( \pi, \rho \) and \( \sigma \) to the interaction are given by

\[
V_\pi(k, p) = -i\frac{g^2}{4m^2} (\gamma_5 (\not{p} - \not{y}))^{(1)} \Delta_\pi(k - p) (\gamma_5 (\not{y} - \not{q}))^{(2)} \tau_1 \cdot \tau_2
\]

\[
V_\rho(k, p) = -ig_\rho \gamma_\mu \left( \gamma_\mu^{(1)} - \frac{i g_\rho \tau_1^{(1)}}{2M} \sigma_{\mu \nu}(k - p)^\nu \right) \Delta_\rho(k - p)
\]

\[
\times \left( \gamma_\gamma^{(2)} + \frac{ig_\rho}{2M} \sigma_{\gamma \delta}(k - p)^\delta \right) \tau_1 \cdot \tau_2
\]

\[
V_\sigma(k, p) = -ig_\sigma^2 \Delta_\sigma(k - p) \tau_1 \cdot \tau_2,
\]

which are of pseudovector, vector and scalar type, respectively. The \( \omega, \epsilon \) and \( \eta \) mesons give the isoscalar contributions to the interaction, which are of the form

\[
V_\eta(k, p) = -i\frac{g_\eta}{4m^2} (\gamma_5 (\not{p} - \not{y}))^{(1)} \Delta_\eta(k - p) (\gamma_5 (\not{y} - \not{q}))^{(2)}
\]

\[
V_\omega(k, p) = -ig_\omega \gamma_\alpha \Delta_\omega^{(1)} \gamma_\beta(k - p) \gamma_\beta^{(2)}
\]

\[
V_\epsilon(k, p) = -ig_\epsilon^2 \Delta_\epsilon(k - p),
\]

which are of the vector, scalar and pseudovector type, respectively. The bracketed numbers denote the nucleon on which the matrices \( \gamma_\mu \) and \( \sigma_{\mu \nu} \) act. The propagator \( \Delta \) for the pseudoscalar (\( \pi \) and \( \eta \)) and scalar (\( \delta \) and \( \epsilon \)) mesons is

\[
\Delta(p) = \frac{1}{m^2 - p^2},
\]

whereas the propagator \( \Delta^{\mu \nu} \) for the vector mesons \( \rho \) and \( \omega \) is given by

\[
\Delta^{\mu \nu}(p) = \left( -g^{\mu \nu} + \frac{p^\mu p^\nu}{m^2} \right) \frac{1}{m^2 - p^2},
\]

with \( m \) the mass of the meson.

With the OBE exchange as defined in Eqs. (2.10) and (2.11) the integrations in the BS equation do not converge. To ensure the correct behavior for high momenta a phenomenological cutoff is introduced of the monopole form,

\[
F(p^2) = \frac{\Lambda^2}{\Lambda^2 - p^2},
\]

at each meson-nucleon vertex, with \( \Lambda \) the cutoff mass. In this OBE model the cutoff masses are taken to be the same for all mesons.
In principle the full field-theoretical Bethe-Salpeter equation can be solved \cite{28}. However, the calculations are highly non-trivial due to the 4-dimensional integration and the complex pole structure of the propagators. Hence, in practice usually a quasi-potential approximation is made. In the quasi-potential framework the two-particle propagator is replaced by one where the relative energy variable is restricted in such a way that properties like two-particle unitarity and relativistic covariance are maintained. Several approximations have been studied in the literature (for a review see \cite{29}). Here we choose the approximation in which the two nucleons are treated in a symmetrical way, the Blankenbecler-Sugar-Logunov-Tavkhelidze (BSLT) approximation \cite{17,18}. The scalar part of the two-nucleon propagator,

\[ G_0 = \frac{1}{(\frac{1}{2}P + p)^2 - M^2 + i\epsilon} \frac{1}{(\frac{1}{2}P - p)^2 - M^2 + i\epsilon}, \]  

(2.15)

is replaced by the dispersion relation

\[ G_{2}^{\text{BSLT}} = \int_{4M^2}^{\infty} ds' \frac{f(\sqrt{s'}, \sqrt{s})}{s' - s} \text{Disc}(G_0), \]  

(2.16)

where \( s = P^2 \) is the total invariant energy squared and the discontinuity of \( G_0 \) is taken to be

\[ \text{Disc}(G_0) = i\pi\delta^+ \left( \left( \sqrt{\frac{s'}{s}} P + p \right)^2 - M^2 \right) \delta^+ \left( \left( \sqrt{\frac{s'}{s}} P - p \right)^2 - M^2 \right). \]  

(2.17)

The function \( f \) can be arbitrary, except that it has to be free of singularities in the physical region and is constrained by \( f(\sqrt{s'}, \sqrt{s}) = 1 \). The definition in the form of a dispersion relation guarantees that \( G_2^{\text{BSLT}} \) has the same discontinuity as \( G_0 \). Consequently two-particle unitarity is preserved. Thus we assume implicitly that inelastic processes, which in principle included in the full BS equation for energies beyond the pion-production threshold, are not important and can be neglected.

We may now use the freedom in \( f \) to regulate the two-nucleon propagator for large momenta. Choosing

\[ f(\sqrt{s'}, \sqrt{s}) = \frac{2\sqrt{s'}}{\sqrt{s'} + \sqrt{s}}, \]  

(2.18)

we get in the center-of-mass frame of the two-particle system

\[ G_{2}^{\text{BSLT}} = i\pi \frac{1}{E_p - E} \frac{1}{(E_p + E)^2} \delta(p_0), \]  

(2.19)

where \( E = \frac{1}{2}P_0 \) and \( E_p = \sqrt{p^2 + M^2} \). Then the full two-nucleon propagator, including the spinor structure, is given by
The coupling parameters for the various fits. Fit A corresponds to the parameters used in Ref. [16]. In the calculations including negative-energy states the parameters of fit B are used. In all fits we have $g_\pi^2/4\pi = 14.2$, $g_K^2/4\pi = 0.43$, $g_\rho^2/g_\pi^2 = 6.8$, $g_\omega^2/4\pi = 11.0$ and $g_\rho^2/4\pi = 3.09$. While the cutoff mass is $\Lambda^b = 1.5M^2$.

\[ S_2^{\text{BSLT}}(p, P) = \frac{1}{2\pi i} \left( \frac{1}{2}P + \gamma + M \right)^{(1)} \left( \frac{1}{2}P + \gamma + M \right)^{(2)} G_2^{\text{BSLT}} \]
\[ = \frac{1}{2\pi i} \left( (E + E_p)\Lambda_+^{(1)} + (E - E_p)\Lambda_-^{(1)} \right) \times \left( (E + E_p)\Lambda_+^{(2)} + (E - E_p)\Lambda_-^{(2)} \right) G_2^{\text{BSLT}} \]
\[ = \frac{1}{2} (E_p - E)\delta(p_0)S^{(1)}(p, P)S^{(2)}(p, P), \]

where the projection operators $\Lambda_{\pm}^{(i)}$ are defined in Appendix A. In particular, with this choice we get for the two-nucleon propagator in the positive energy spinor states

\[ S_{++} = \frac{1}{2} \frac{1}{E_p - E} \]

Using the two-nucleon propagator from Eq. (2.20), the integration over the relative energy $p_0$ in the inhomogeneous BS equation can be performed, and the BSLT equation is obtained,

\[ T(\hat{p}, \hat{p}'; P) = V(\hat{p}, \hat{p}') + \frac{1}{(2\pi)^3} \int d^3k V(\hat{p}, \hat{k}) S_2^{\text{BSLT}}(\hat{k}, P)T(\hat{k}, \hat{p}', P). \]

Here $\hat{p}, \hat{p}'$ and $\hat{k}$ are the relative 4-momenta $p, p'$ and $k$ under the restriction that in the c.m. system of the nucleons the energy component is zero, $p_0 = 0, p'_0 = 0$, and $k_0 = 0$.

The BSLT equation can be solved in a partial-wave basis [20], yielding a number of coupled-channel equations, that involve essentially a coupled set of one-dimensional integral equations due to the quasi-potential approximation. Aside from the physical $(+, +)$ positive-energy states, also combinations involving negative-energy states occur $(-, -)$, and even and odd combinations
of \((+,-)\) and \((-,+)\). The T-matrix has been fitted to the experimental phase shifts of Arndt et al. [1] by varying the meson-nucleon coupling constants. Fits have been made both without (set A) and with (set B) intermediate negative-energy states for energies up to 200 MeV. Starting from the fit found with the full Dirac structure included, we simply have varied only the coupling constants \(g_t\) and \(g_b\) to obtain a fit for the case of the absence of intermediate negative-energy states (set C). The sets of the coupling parameters for the different fits are given in Table 2.1 and the corresponding caption. The resulting phase shifts for both fits can be considered almost phase equivalent and are up to 300 MeV in reasonable agreement with the Arndt phases.

### 2.4 The bremsstrahlung amplitude

The dynamics of the bremsstrahlung process is contained in the invariant matrix \(M_{JF} = \varepsilon^\mu (i J_\mu |F\rangle\langle J_\mu|)\) with \(\varepsilon^\mu\) the photon polarization vector. If the T-matrix is properly antisymmetrized, the nuclear current \(J_\mu\) is given by

\[
\langle J_\mu |F\rangle = \langle p', P'|T(p', \bar{p}; P')S^{(1)}(\bar{p}, P'), \mu(q)|p, P) \\
+ \langle p', P'| \mu(q)S^{(1)}(\bar{p}, P)T(\bar{p}, p; P)|p, P) + (1 \leftrightarrow 2) \\
- i \int \frac{d^4 k}{(2\pi)^4} \langle p', P'|(T(p', k'; P')S^{(1)}(k', P') \\
\times \mu(q)S_2(k, P)T(k, p; P)|p, P). \quad (2.23)
\]

The sum of the first two terms will be referred to as the Impulse Approximation (IA). The corresponding diagrams are shown in Figs. 2.2(a) and 2.2(b). The last term is referred to as the rescattering contribution, corresponding to the diagram in Fig. 2.2(c). Using the anti-symmetry of the protons, the diagram where the photon is emitted by intermediate particle 2 can be rewritten as a diagram where the photon is emitted by particle 1. If the T-matrix is antisymmetrized, it can be shown that the antisymmetrized sum of all rescattering diagrams is given by the diagram with antisymmetrized T-matrices with emission from particle 1 only. The momenta in Eq. (2.23) are defined through conservation of 4-momentum at the \(NN\gamma\) vertex and of total momentum, so that \(p' = p - \frac{1}{2}q\) and \(P' = P - \frac{1}{2}q\).

In Equation (2.23) the on-shell form of the electromagnetic (e.m.) vertex \(\mu\) is taken

\[
\gamma^{(1)}(q) = e \left( F_1^{(1)}(q^2)\gamma^\mu - \frac{i}{2M} F_2^{(1)}(q^2)\sigma^{(1)}_{\mu\nu}q^\nu \right), \quad (2.24)
\]

thus ignoring a possible dependence of the form factors on the off-shell mass of the intermediate proton. The on-shell form factors are given by

\[
F_j^{(1)}(q^2) = F_j^{(1)}(q^2) + F_j^{(1)}(q^2)\tau_3^{(1)} = F_j^{(1)}(q) + F_j^{(1)}(q). \quad (2.25)
\]
For proton-proton bremsstrahlung with real photons $q^2 = 0$, and the NN $\gamma$ vertex reduces to

$$\Gamma^{(i)}(q) = e \left( \gamma^{(i)}_{\mu} - \frac{i\kappa}{2M} \sigma^{(i)}_{\mu\nu} q^\nu \right), \quad (2.26)$$

where $e$ is the charge and $\kappa = 1.79$ is the anomalous magnetic moment of the proton. We have used the fact that all intermediate nucleons are protons, hence $\tau_3$ in Eq. (2.25) always gives +1 when sandwiched between isospin states. With this choice of the vertex, the nuclear current as defined in Eq. (2.23) is conserved in the Bethe-Salpeter formalism, provided that the kernel is local ($V(k, p) = V(k - p)$). Details of the proof are given in Sec. 2.6.

Due to the presence of the photon in the final state, in general the NN $T$-matrix is needed in different Lorentz frames. Usually the NN interaction is determined in the center-of-mass (c.m.) system of the nucleon pair. If we choose the c.m. system of the initial protons to calculate the amplitude, the $T$-matrix for the diagrams involving the NN interaction after emission of the photon is obtained through the Lorentz structure of the $T$-matrix

$$T(p', p; P) = \Lambda(\mathcal{L}) T^{\text{c.m.}}(\mathcal{L}^{-1} p', \mathcal{L}^{-1} p; \mathcal{L}^{-1} P) \Lambda^{-1}(\mathcal{L}), \quad (2.27)$$

Here $\Lambda = \Lambda^{(1)} \Lambda^{(2)}$ is the spinor transformation for the boost $\mathcal{L}$ from the calcu-


\[ L_{\mu}^\nu = \begin{bmatrix} \sqrt{1 + \eta} & 0 & 0 & -\sqrt{\eta} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sqrt{\eta} & 0 & 0 & \sqrt{1 + \eta} \end{bmatrix}, \] (2.28)

where \( \sqrt{\eta} = q/M_{pp} \) and \( \sqrt{1 + \eta} = (2E - q)/M_{pp} \). \( E \) is the total energy of the initial protons in their c.m. frame and we have defined an effective “proton-proton mass” \( M_{pp} = 2\sqrt{E(E - q)} \). The corresponding one-particle spinor transformation operator is

\[ \Lambda(\mathcal{L}) = \sqrt{\frac{2E - q + M_{pp}}{2M_{pp}}} \left( 1 - \gamma^0 \gamma^3 \frac{q}{2E - q + M_{pp}} \right), \] (2.29)

with \( \mathbb{1} \) the identity matrix. In all calculations presented below the boosts are taken into account.

### 2.5 Quasi-potential approximation in the rescattering contribution

For the rescattering contribution an integration over the internal 4-momentum \( k \) has to be performed. Again we would like to make a quasi-potential reduction to simplify the integration. Such a reduction must be consistent with the approximation made in solving the T-matrix. We have used the equal-time framework, where it is assumed that the NN interaction does not depend on the relative energy of the two nucleons in its center-of-mass system. Then the integration over the relative energy \( k_0 \) can be done analytically, since the integrand is of the form

\[ I_0^{(i)} = \int \frac{dk_0}{2\pi} S^{(i)}(k_0, k - \mathbf{q}; E'), (i) (q) S_2(k_0, k; E), \] (2.30)

where \( E' = E - \omega \) with \( \omega \) the energy of the photon. A contour integration over \( k_0 \) can be carried out, resulting in three terms, which are given by the residues at the poles where one of the intermediate nucleons is on-shell. The poles of the two-nucleon propagator \( S_2(k, P) \) are at

\[
\begin{align*}
k_0^a &= E + E_k - i\varepsilon & k_0^c &= E - E_k + i\varepsilon \\
k_0^b &= -E + E_k - i\varepsilon & k_0^d &= -E - E_k + i\varepsilon,
\end{align*}
\] (2.31)

and the poles of the one-particle propagator \( S^{(i)}(k', P') \) are at

\[
\begin{align*}
k_0^c &= (\omega - E) + E_{k - \mathbf{q}} - i\varepsilon, & k_0^d &= (\omega - E) - E_{k - \mathbf{q}} + i\varepsilon.
\end{align*}
\] (2.32)

If we choose to close the contour in the upper half-plane, we get contributions from poles \( k_0^a, k_0^b \) and \( k_0^c \). The first of these corresponds to the spectator model,
where particle 2 is on its mass shell. The remaining 3-dimensional integration has to be done numerically. An analysis of the pole structure of the remaining integrand shows that there are two poles in the spatial momentum $k$, both arising from the spectator term. These poles correspond to the situation where either before or after emission of the photon the two protons are on-shell. In Appendix B a detailed discussion of these poles is given.

### 2.6 Current conservation

The nuclear current as defined in Eq. (2.23) is conserved, provided that the propagator structure in the $NN$ interaction is consistent with the propagator structure used in the rescattering contribution. This is most readily seen if a continuum two-nucleon scattering state is introduced,

$$\Psi(k, p; P) = [(2\pi)^4 \delta^4(p - k) - iS_2(k, P)T(k, p; P)] [p, P]. \quad (2.33)$$

The free nucleon pair $[p, P]$ with relative momentum $p$ and total momentum $P$ is given by the anti-symmetric combination of two free Dirac spinors. From this scattering state an ‘amputated’ scattering state can be defined,

$$\phi(k, p; P) = [S_2^{-1}(k, P)(2\pi)^4 \delta^4(p - k) - iT(k, p; P)] [p, P]. \quad (2.34)$$

Substitution of this amputated scattering state in the Bethe-Salpeter equation gives

$$T(p, p'; P)[p'; P] = i (\phi(p, p', P) - S_2^{-1}(p, P)(2\pi)^4 \delta^4(p' - p)[p', P])$$

$$= V(p, p')[p', P] - i \left( \frac{i}{(2\pi)^4} \int d^4k V(p, k)S_2(k, P) \times (\phi(p, p', P) - S_2^{-1}(k, P)(2\pi)^4 \delta^4(p' - p)[p', P]) \right)$$

$$= \int \frac{d^4k}{(2\pi)^4} V(p, k)S_2(k, P)\phi(k, p'; P). \quad (2.35)$$

The initial and final states are on the mass shell, and consequently in the first line in Eq. (2.35) the second term can be eliminated using $S_2^{-1}(p', P)[p', P] = 0$. Thus the scattering state satisfies a homogeneous Bethe-Salpeter equation,

$$\phi(p, p'; P) = \frac{-i}{(2\pi)^4} \int d^4k V(p, k)S_2(k, P)\phi(k, p; P). \quad (2.36)$$

The current for emission from particle $i$ is found from substituting the scattering state, Eq. (2.34), in Eq. (2.23), which gives

$$J_{\mu}^{(i)} = \int \frac{d^4k}{(2\pi)^4} \bar{\phi}(p', k'; P')S^{(i)}(k', P') \cdot \langle (i) | S_2(k, P) \phi(k, p; P), \quad (2.37)$$

where $P^e = P - \frac{1}{2}q$, $k' = k - \frac{1}{2}q$ for emission from particle 1 and $k' = k + \frac{1}{2}q$ for emission from particle 2. The contribution with emission of the photon
without strong interaction is absent, since the term that would arise from
the sandwiching of the photon vertex with the in- and outgoing states vanishes,
and Eqs. (2.37) and (2.23) are indeed equivalent. Using the photon vertex of
Eq. (2.24), the Ward identity for the vertex is

\[ q \cdot J^{(i)}(q) = e F_1^{(i)} \left( S^{(i)}(k', P') - S^{(i)}(k, P) \right). \]  \hspace{1cm} (2.38)

Here we have used that for real photons \( q^2 = 0 \) and written \( F_1^{(i)}(q^2 = 0) = F_1^{(i)} \).
In the following we will concentrate on the contributions due to emission from
particle 1; the derivation for emission from particle 2 is identical, except for
a relative minus sign. The contraction of the nuclear current with the photon
momentum is given by

\[ q \cdot J^{(1)} = -ie \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}(p', k', P) S^{(1)}(k', P') F_1^{(1)} \]
\[ \times \left( S^{(1)}(k', P') - S^{(1)}(k, P) \right) S_2(k, P) \phi(k, p; P). \]  \hspace{1cm} (2.39)

The homogeneous BS equation (2.36) can be substituted for \( \phi \) in the first term
of the previous equation, which gives

\[ \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}(p', k', P) S_2(k, P) \phi(k, p; P) = \]  \hspace{1cm} (2.40)
\[ = \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 l}{(2\pi)^4} \tilde{\phi}(p', l; P') S_2(l, P') V(l, k') S_2(k, P) \phi(k, p; P), \]

whereas substitution for \( \phi \) in the second term gives

\[ \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}(p', k'; P) S_2(k', P') \phi(k, p) = \]  \hspace{1cm} (2.41)
\[ = \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 l}{(2\pi)^4} \tilde{\phi}(p', k'; P) S_2(k', P') V(k, l) \phi(l, p; P). \]

Applying the transformation \( k \rightarrow k + \frac{1}{2} q \) in the second term and renaming \( l \) to
\( k' \) in the first term and \( l \) to \( k \) in the second term, the result can be written as

\[ q \cdot J^{(1)} = \frac{-ie}{(2\pi)^8} \int d^4 k \int d^4 k' \tilde{\phi}(p', k'; P') S_2(k', P') \]
\[ \times \left( V(k', k - \frac{q}{2}) F_1^{(1)} - F_1^{(1)} V(k' + \frac{q}{2}, k) \right) S_2(k, P) \phi(k, p; P). \]  \hspace{1cm} (2.42)

In general the commutator of \( V \) and \( F \) does not vanish, even if the interaction
\( V \) is local, \( V(p, k) = V(p - k) \), due to a possible isospin dependence. In
that case an extra contribution has to be included which is proportional to
the commutator of \( V \) with \( F^{V, 2} \). For the present NN interaction with the
one-boson exchange kernel that contains the isovector mesons \( \pi, \rho \) and \( \delta \) with
isospin structure \( \tau^{(1)} \cdot \tau^{(2)} \), the additional contribution is proportional to

\[ \left[ \tau^{(1)} \cdot \tau^{(2)} , \tau^{(1)} \right]_3 = 2i(\tau^{(1)} \times \tau^{(2)})_3 \]
\[ \left[ \tau^{(1)} \cdot \tau^{(2)} , \tau^{(2)} \right]_3 = -2i(\tau^{(1)} \times \tau^{(2)})_3. \]  \hspace{1cm} (2.43)
If the final and initial states, as well as all intermediate states, consist only of protons, the commutators vanish and thus the additional terms needed for current conservation in general are absent.

In this proof we have made use of the fact that in the expression for the current and in the Bethe-Salpeter equations the same propagator structure is used. In practical calculations the BSLT approximation was used for the T-matrix, while in the integration over the relative energy \( k_0 \) in Eq. (2.37) the equal-time approximation was made. Thus the T-matrices were assumed not to depend on the relative energy in the c.m. system of the protons. Ignoring boost effects this implies that in Eqs. (2.40) and (2.41) the propagators that arise from substituting the homogeneous BS equation are to be replaced by BSLT propagators, and instead of Eq. (2.42) we get

\[
q \cdot J^{(1)} = e \int \int \frac{d^3 q'}{(2\pi)^3} \frac{d^3 p}{(2\pi)^3} \delta_{\text{BSLT}}(p', k'; P') S_2^{\text{BSLT}}(k', P') \times V^{\text{BSLT}}(k' - k + \frac{1}{2}q) S_2(k, P) \delta_{\text{BSLT}}(k, p; P)
- e \int \int \frac{d^3 q'}{(2\pi)^3} \frac{d^3 p}{(2\pi)^3} \delta_{\text{BSLT}}(p', k'; P') S_2(k', P') \times V^{\text{BSLT}}(k' - k + \frac{1}{2}q) S_2^{\text{BSLT}}(k, P) \delta_{\text{BSLT}}(k, p; P).
\]  

(2.44)

Here \( \delta_{\text{BSLT}}, S^{\text{BSLT}} \) and \( V^{\text{BSLT}} \) are the scattering state, two-particle propagator and one-boson kernel with 4-momenta restricted according to the BSLT prescription that the relative energy is zero in the c.m. system of the nucleons.

In the equal-time approximation the integration over the relative energy remains. The dependence on this variable only enters in the full propagator \( S_2 \). Writing the propagators in terms of the projection operators (compare Eq. (A.7)) the integration over the contour closed in the upper half-plane gives for the first term in Eq. (2.44)

\[
\int \frac{dk_0}{i\pi} S_2(k, P) = \int \frac{dk_0}{i\pi} \left( \frac{\Lambda_+^{(1)}(k)}{E + k_0 - E_k + i\epsilon} + \frac{\Lambda_-^{(1)}(k)}{E + k_0 + E_k - i\epsilon} \right)
\times \left( \frac{\Lambda_+^{(2)}(k)}{E - k_0 - E_k + i\epsilon} + \frac{\Lambda_-^{(2)}(k)}{E - k_0 + E_k - i\epsilon} \right)
= \frac{\Lambda_+^{(1)}(k) \Lambda_-^{(2)}(k)}{E_k - E} + \frac{\Lambda_-^{(1)}(k) \Lambda_+^{(2)}(k)}{E_k + E}.
\]  

(2.45)

since contributions are picked up from the term with \( \Lambda_+^{(2)} \) at \( k_0 = E - E_k \) and from the term with \( \Lambda_-^{(1)} \) at \( k_0 = -E - E_k \). The state where particle 1 has \( \rho \)-spin + and particle 2 has − (the +− state) is absent since this has no poles in the upper half-plane, whereas the −+ state is absent due to a cancellation between the contributions from the first and second pole. A similar expression is found for the second term in Eq. (2.44). The BSLT propagator is given by
Therefore, if we retain only the positive-energy contributions in both the expression for the current and in the evaluation of the T-matrix, the current is conserved.

2.7 Retardation effects in the NN interaction

In this section the effect of retardation in the NN interaction is discussed. Bremsstrahlung, with its inherent transfer of energy in the NN interaction, provides a useful tool to study the importance of this effect. From a simple one-pion exchange (OPE) model, which is the longest-range contribution to the interaction between nucleons, it is concluded that retardation effects can be important. The discussion is generalized to the Born approximation to the NN interaction, that is the exchange of a single boson (OBE), which is studied using the kernel of the BS equation. From this it is clear that in particular the vector and pseudovector mesons can give rise to effects of the order of 15%. To see whether in a full T-matrix calculation the importance of retardation is similar, we modify the NN interaction by iterating one full OBE, and conclude that in a full calculation the relevance of retardation is suppressed with respect to the result in the OPE and OBE case.

2.7.1 Two simple models: one-pion and one-boson exchange

In the calculation of the bremsstrahlung amplitude we use the equal-time or instantaneous framework. In this framework it is assumed that the T-matrices can be evaluated at the point where the relative energy of the protons \( k_0 \) is zero in the center-of-mass system of the interaction. To investigate the effects of retardation in the NN interaction we may first look at a simple one-pion exchange (OPE) model without form factors, where the NN \( \pi \) vertex was either assumed to be given by a pseudovector \((\not{\ell}\gamma_5)\) or pseudoscalar \((\gamma_5)\) coupling. The nuclear current for emission from particle \( i \) in the equal-time approximation is given by

\[
S_{\text{OPE}}^{\text{em}}(p',P') = \frac{1}{2}(E_p - E_0)\delta(p_0)S^{(1)}(p,P)S^{(2)}(p,P)
\]

\[
= \frac{\Lambda_+^{(1)}(k)}{2(E_k - E)} \frac{\Lambda_+^{(2)}(k)}{2(E_k + E)} \quad \frac{\Lambda_-^{(1)}(k)}{2(E_k + E)} + \frac{\Lambda_-^{(2)}(k)}{2(E_k - E)} \frac{E_k - E}{E_k + E} \quad (2.46)
\]

Therefore, if we retain only the positive-energy contributions in both the expression for the current and in the evaluation of the T-matrix, the current is conserved.
the pion in the c.m. frame. In Fig. 2.3 we show the cross section calculated with
the equal-time approximation divided by the result of the calculation using the
exact one-pion exchange, just below the pion production threshold, $E_{\text{lab}} = 280$
MeV, and small proton angles $\theta_1 = 12^\circ, \theta_2 = 12.4^\circ$. For the pseudoscalar
coupling the result is close to 1 at all photon angles, so that we can conclude that
retardation effects in the meson propagator are small. For the pseudovector
coupling we see larger deviations, up to 15%. Thus it is clear that retardation
effects in the nucleon-nucleon meson coupling can be considerable.

If we use the one-boson exchange kernel instead of the OPE interaction,
the effects are of the same order as those found for the pseudovector coupling.
This can also be seen in Fig. 2.3, where the dashed line is the fraction of the
cross section calculated with the OBE kernel in equal-time approximation to
the cross section calculated with the exact OBE-kernel. The effects are mainly
due to the pion and the vector $\omega$ meson.

### 2.7.2 Modified equal-time approximation

To see whether these retardation effects also can be substantial in the T-matrix
calculations, we have constructed a modified equal-time approximation by iterating one full one-boson exchange,

$$T^{\text{mel}}(p', p; P) = V^{\text{OBE}}(p', P) - i \int \frac{d^4k}{(2\pi)^4} \frac{1}{V^{\text{OBE}}(p', \hat{k})} S_\beta^{\text{BSL}}(\hat{k}, P) T^{\text{ret}}(\hat{k}, p; P),$$

(2.48)
in the c.m. system of the incoming nucleon pair, where the relative energy of the incoming nucleons \( p_0 = 0 \), and \( \hat{k} \) is restricted through the BSLT-approximation. \( T^{\text{met}} \) is the modified equal-time T-matrix, whereas \( T^{\text{et}} \) is the T-matrix calculated with the equal-time approximation. Note that the two T-matrices are on-shell equivalent. The relative energy of the final nucleon pair is calculated by assuming that particle 2 is on-shell, so that \( p'_0 = E_{p'} - E_p \). In Fig. 2.4 we show a calculation of the cross section in Impulse Approximation at the same kinematics as in the OPE calculation. The full line is the result using the modified equal-time T-matrix, the dotted line is the result when we use \( T^{\text{et}} \). The retardation effects are somewhat lower than in the case of the OPE, but at these energies and angles still appreciable, of the order of 10%.

In the construction of the \( k_0 \) dependence it is assumed that only one of the particles is on the mass shell. Furthermore the full \( k_0 \) dependence in the meson propagator is retained. Since this dependence will give rise to spurious poles [19], this formalism can not be applied without modification to the rescattering contribution. However, the main interest is the effect of retardation in the meson propagator, which may be expected to cancel out in the integration over the 3-momentum in the rescattering loop. Furthermore we have seen that the effect is at most in the order of 10%, and thus the calculation restricting the use of the modified equal-time approximation to the Impulse Approximation
may be assumed to give a reasonable estimate of the size of these effects.