1. Introduction

In recent years a considerable advance has been made in the quest for an accurate description of the nucleon-nucleon (NN) interaction. High-precision experiments warranted the development of a multi-energy partial-wave analysis of the nucleon-nucleon scattering data, such as those of the Virginia Tech [1] and the Nijmegen [2, 3] groups. As a result 'Reid-like' NN potentials have been developed which describe the world data accurately, with $\chi^2$/data $\simeq 1$. At the same time, potentials based on a meson-exchange model show reasonable agreement with the data, with $\chi^2$/data $\simeq 1.5 - 2$.

The elastic data which activated the development of modern NN potentials have one major drawback. Inside nuclei the nucleons are bound, and thus the energy $E$ and momentum $p$ do not obey the famous equation

$$E^2 = M^2 c^4 + p^2 c^2,$$

where $M$ is the mass of the nucleon. Nucleons satisfying Eq. (1.1) are called on-shell, whereas nucleons violating (1.1) are off-shell, or off the mass-shell. Due to the binding energy, nucleons inside the nucleus are off-shell. Hence, the NN interaction has to be evaluated at off-shell points and although the modern potentials are essentially on-shell equivalent, i.e. they give the same NN phase shifts, they may lead to different predictions for off-shell kinematics.

One of the most simple tools to investigate the NN interaction at off-shell kinematics is provided by the scattering of protons in which a photon is emitted, proton-proton ($pp$) bremsstrahlung. Since the proton emitting the photon is on-shell in the asymptotic region, it is clearly off-shell between emission of the photon and the interaction with the other proton. Furthermore, since both protons are charged particles this process has the advantage that the leading-order dipole radiation is suppressed. Because this leading-order term can be described by classical Maxwell theory, it is uninteresting from our point of view. Due to this suppression, higher-order contributions play an important role. This is in contrast with proton-neutron ($pn$) bremsstrahlung, where the leading-order dipole term dominates. The $pn$ case has additional disadvantages, in particular the lack of a free neutron target and the difficulties in fixing the energy of the neutrons when used as projectiles.

Prior to the early 1990's, data from proton-proton bremsstrahlung experiments was reasonably well described by soft-photon calculations with non-relativistic potential models. This was for a large part the result of the poor statistical accuracy due to the small cross sections. However, the advance in experimental techniques facilitated the relatively high precision of the TRIUMF [4]
experiment. As a result, the interest in the bremsstrahlung process was renewed. Soon it was seen that the different potential model calculations [3, 6, 7] all give rise to a similar discrepancy between theory and experiment. Moreover, there is an uncertainty in the absolute normalization of the cross sections of the TRIUMF experiment. In the original publication [4] a normalization factor of 2/3 was introduced to facilitate the comparison with theory, and an argument was given for the introduction of such a factor. Thus, the need for more involved theoretical calculations, as well as more high precision experimental data was clearly felt. Recent and ongoing experiments [8, 9, 10] will yield such high-precision data.

In response to the high-precision data, advances in the theoretical description of the process have been made. Although the fundamental theory for the strong interaction is quantum chromodynamics (QCD), at low and intermediate energies QCD is a non-perturbative theory, and the quarks are confined. Thus, calculations in a QCD framework are not practical, and only a description in terms of effective degrees of freedom, such as nucleons and mesons, is feasible. Starting from a potential model for the NN interaction, the emission of the photon is accounted for by including a non-relativistic reduction of the nucleon-nucleon-photon vertex. In such a reduction the current contains a leading-order term, which arises from the large components of the Dirac spinors, and so-called ‘relativistic corrections’, which come from the small components of the Dirac spinors and are proportional to the momenta of the interacting particles. Due to the large mass of the nucleons, the latter are usually small corrections to the leading order term. However, since the leading-order term is suppressed, the relativistic corrections play an important role. The inclusion of these corrections tends to suppress the cross section [6]. A competing mechanism is provided by the rescattering of the two protons. This contribution enhances the cross section by approximately 20%, in particular for the forward angles [6, 11, 7].

Including both the relativistic corrections and the rescattering contribution, the resulting nuclear current contains all possible conventional one-body contributions. The description obtained for the bremsstrahlung process by including these one-body contributions generally leads to predictions for the cross sections that are too low as compared to the data as found in the TRIUMF experiment. Thus it is clear that some contributions to the current are missing. The inclusion of negative-energy states (pair currents) provides a possible extension of the one-body current, along with explicit two-body contributions such as the $\omega \pi \gamma$ and $\rho \pi \gamma$ decay graphs and the $\Delta$-isobar currents. In recent studies [12, 13, 14] some or more of these effects were included by adding the Born-contributions to the one-body current, whereas de Jong et al. [15] refitted the NN interaction explicitly including the $\Delta$ degrees of freedom in the NN interaction.

An alternative way to include the contribution of negative-energy states is to start from the fully relativistic framework of the Bethe-Salpeter (BS) equation. Fleischer and Tjon [16] have developed a one-boson exchange model based on this equation. Although the full BS equation thus in principle can be
solved in ladder approximation, it is very complicated due to the integration over the relative energy of the two intermediate nucleons. A more practical approach is to assume that a 3-dimensional reduction of this integration can be made, such that the two-nucleon unitarity is retained. Examples of such reduction schemes are the Blankenbecler-Sugar approximation [17, 18], the Gross equation [19], and the equal time approximation [20]. The major advantage of these descriptions is that the negative-energy states are included dynamically. Earlier studies of the deuteron [21, 20] have shown that the effects of negative-energy states are relatively small in contrast to the results of non-relativistic calculations in which the negative-energy state contributions are included in a perturbative way. A careful analysis [21] showed that the corrections are canceled by dynamical corrections. This clearly indicates that it is important to have a consistent description of both the electromagnetic and dynamical properties of the interacting nucleon pair.

The relativistic NN interaction is the starting point for the description of pp bremsstrahlung in this thesis. A review of the field-theoretical formalism and its implications for the NN interaction is given in Chapter 2. After a brief discussion of the observables of the bremsstrahlung process, the framework of the relativistic description of pp bremsstrahlung is given, and it is shown that the nuclear current is conserved if the full BS T-matrix is used. The assumptions used in solving the BS equation are discussed, and the implications of these approximations in the case of pp bremsstrahlung are investigated. In particular, in the equal-time approximation the NN interaction is used at the point where the relative energy in the center-of-mass (c.m.) frame of the interacting protons is zero. It is shown that for this approximation the effects are at most of the order of 5% just below the pion-production threshold ($T_{lab} = 280$ MeV).

The effects of including contributions from negative-energy states, or pair contributions, are addressed in Chapter 3. The NN interaction couples dynamically to these states. In a non-relativistic calculation these contributions can be included in a perturbative way. Such a perturbative inclusion corresponds to including the negative-energy states in the single-scattering contributions only. It is shown that in this case the cross section at large photon momenta is enhanced up to over 40%, which is roughly of the order one would expect on the basis of a naive estimate. Inclusion of the intermediate negative-energy states also in the rescattering contribution gives a large suppression, and the remaining effects at the same kinematics is of the order of 10-15%.

The cancellation between contributions from the single-scattering and rescattering contributions seems surprising, since the lowest order of the interaction, $V$, for the former is $V$, whereas for the latter it is $V^2$. Any cancellation between contributions from negative-energy states can only occur order-by-order in the strong coupling constant. The solution to this apparent paradox is found in the soft-photon theorem [22] for proton-proton bremsstrahlung. It can be proven that for low photon momenta the negative-energy states do not contribute to the current. Using the kernel of the BS equation, it is seen both numerically and analytically that the Born contributions indeed cancel. Using the theorem more generally, it can be shown that due to the symmetry
of the $pp$ system the same conclusion holds for the full $NN$ interaction. We may conclude that negative-energy states give an appreciable contribution at intermediate energies, but that this contribution is considerably overestimated if the negative-energy states are included perturbatively. At the same time the clear discrepancy between theory and experiment remains, even if these relativistic contributions are accounted for.

Another possible contribution is the two-body current. In contrast to, for example, proton-neutron processes, the longitudinal two-body currents are suppressed since the current is conserved at the level of the one-body current. Furthermore, since the protons are equally charged particles, within the framework of a one-boson exchange model the exchange of charged mesons, and therefore also the coupling of the photon to the exchanged mesons, is suppressed. Thus for $pp$ bremsstrahlung only the transverse two-body currents contribute. Contributions to these currents come from the meson-decay graphs ($\rho\pi\gamma$ and $\omega\pi\gamma$) and the $\Delta$-isobar. It has been suggested in the literature [11, 12, 14, 15] that these contributions can account for much of the discrepancy between experiment and theory. In Chapter 4 the inclusion of these currents in the relativistic framework of the previous chapters is discussed. It is found that the static limit for the meson-decay graphs give similar results as the fully relativistic current up to the pion-production threshold. Such a conclusion does not hold for the $\Delta$-isobar current. In this case, it is essential to account for the difference in the energy-dependence of the $\Delta$ propagator in the different diagrams.

In a recent experiment at KVI [8, 23] was possible to measure not only real photons, but also lepton pairs originating from virtual photons [24]. Thus, for the first time, data on virtual bremsstrahlung will be available, and the energy-momentum range is extended to regions not accessible before. A drawback is that the magnitude of the cross section is reduced, roughly by a factor $\alpha$ (the fine-structure constant), compared to real bremsstrahlung. Therefore, at present only inclusive cross section data is available. In Chapter 5 the fully relativistic description of the preceding chapters is applied to virtual bremsstrahlung. The calculation of the longitudinal structure function $W_L$ is problematic due to large cancellations in the matrix elements of the charge operator $\langle J_0 \rangle$. We argue that in virtual bremsstrahlung it is preferable to eliminate the charge in favor of the longitudinal current $J_3$. Hence, $W_L$ is given entirely in terms of $J_3$.

Using the current with $W_L$ defined in terms of $J_3$, predictions for the structure functions are given, and these predictions are compared to soft-photon calculations of Ref. [25], using two different realizations of the soft-photon theorem [22, 26]. It is seen that even at the intermediate energies of the KVI experiment [27] there are large deviations between our predictions and those of the soft-photon calculations, as well as between the predictions of the two different soft-photon amplitudes. A comparison of Monte Carlo simulations using the two soft-photon amplitudes as theoretical input shows that one of these calculations tends to agree better with experiment than the other, but the dependence on the invariant mass of the photon is not well reproduced. Due to the integration over the available phase space, it is not straightforward to
draw conclusions concerning our calculations. Exclusive data is clearly needed to obtain a better understanding of the virtual bremsstrahlung.

The structure functions that play a role in virtual bremsstrahlung are shown to be even more sensitive to the higher-order effects than is the case in real bremsstrahlung. In particular, the transversal interference structure function $W_{TT}$ has a large contribution from the two-body current whereas these hardly contribute to the structure functions involving the longitudinal components of the current ($W_L$ and $W_{LT}$). This behavior is readily seen from the non-relativistic limit to these currents. The negative-energy states give a substantial contribution to $W_L$ and $W_{LT}$. 
Ch. 1 Introduction