Linear differential systems
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Chapter 9
Conclusions

The research presented in this dissertation concerns a number of different directions in system theory: the problem of state construction, $J$-spectral factorization of polynomial matrices, construction of storage functions, and the Nevanlinna interpolation problem. We now review the main results that have been achieved. In Chapter 2, we discussed the calculus of representations and some important technical issues like the choice of the trajectory space and the latent variable elimination. In Chapter 3 the problem of state construction has been investigated; we have shown that a state for a behavior described by higher order linear differential equations with constant coefficients can be computed acting on the trajectories of the behavior with a polynomial differential operator called a state map. Concrete procedures have been illustrated to compute a state map starting from a hybrid or a kernel representation of the behavior.

In Chapter 4 we reviewed some basic notions concerning quadratic differential forms (QDF's) and we described some features of their calculus. In Chapter 5 we applied these concepts to study storage functions for dissipative systems. The main result of the chapter is a characterization of the storage functions corresponding to dissipation rates that enjoy certain spectral properties. We have shown that such storage functions can be computed by means of a state map and a constant matrix obtained from a Pick matrix. In Chapter 6 we studied the problem of balancing general quadratic measures on the external signals of a behavior using the framework of QDF's. Having split the behavior...
in a past and a future subbehavior, we define two QDF's measuring
the past and the future norm of the trajectories. Each past trajectory
is associated with the future trajectory emanating from the same state
and having minimal norm. The main contribution of this chapter is the
notion of balanced state map. In a balanced state space, the external
properties of the behavior are reflected in the relative contribution of
each state component to the behavior of a system.

The problem of J-spectral factorization has been examined in Chap-

ter 7. We solved the problem using the calculus of two-variable polyno-
mial matrices, the notion of dual QDF's, and that of storage function.
The main result of this investigation is an algorithm to perform poly-
nomial J-spectral factorization based on lifting the original one-variable
polynomial problem to a two-variable polynomial one.

In the last chapter of the dissertation we examined a generalization
of the tangential Nevanlinna interpolation problem. We de-
defined an equivalent formulation in behavioral terms as the computation of the
representation of a behavior (the model) fitting a set of polynomial
exponential trajectories (the data). We showed that by dualizing the
original data and considering an enlarged data set, the metric aspect
of the interpolation problem can be neglected. The main result of the
chapter is the formulation of two necessary and sufficient conditions for
the existence of a solution to the Nevanlinna interpolation problem and
a recursive algorithm to compute a solution.

Each of the topics considered in this thesis is quite broad in itself,
and is connected with several others arising in different areas of systems
and control theory. Consequently, the results reported in this disserta-
tion are far from conclusive, and additional work is required in order
to make them truly useful, and to fully clarify their relationship with
known techniques and algorithms. In the following I will sketch what
in my opinion are the limitations of my research work as presented in
this dissertation, and I will suggest some possible directions for future
research.

The main limitation of the results on state construction illustrated
in Chapter 3 and Chapter 6 is that the problem of state construction
is only the first step in the computation of a state space or an in-
put/state/output representation; the second step is the computation
of the state space equations themselves. However, such problem has
not been considered in this dissertation. Consequently, a pressing issue is the design of algorithms to compute a state space or an input/state/output representation starting from a kernel or hybrid representation of a system and a state map. It is reasonable to believe that one-variable polynomial equations will play a prominent role in the solution of this problem. Note that in this way the missing link between first principles models and corresponding state space representations will be provided, and it will be possible to start from a first principles model and proceed to the simulation of the behavior of the system in a direct way.

In Chapter 5 of this thesis we have characterized the storage functions corresponding to a dissipation rate of the form $F^T(\zeta)F(\eta)$ with $F$ square, nonsingular, and such that the roots of $\det(F)$ form a $\Lambda$-set for $F^\sim F$ (cfr. Theorem 5.3.1). However, it is interesting to classify all storage functions corresponding to dissipation rates of the form $F^T(\zeta)F(\eta)$ with $F$ square and nonsingular; observe that such a characterization is essentially already known, since in the context of spectral factorization problems it has been studied with both state space and polynomial techniques (see [16, 33]). A second direction of future research stems from the consideration that the results of Theorem 5.4.1 and Theorem 5.4.2 are probably related to known algorithms (for example the Popov function approach of [18] and the numerical procedures reviewed in Chapters 5 and 7 of [43]) for solving the linear matrix inequality and the algebraic Riccati equation arising in many optimal control and filtering problems. It would be interesting to investigate this relationship and to design an algorithm for the computation of storage functions having the classical algorithms for spectral factorization and for the solution of the algebraic Riccati equation as special cases.

As for what regards the solution to the subspace Nevanlinna interpolation problem illustrated in Chapter 8, we believe that a further investigation on the role of duality will provide more insight in the structure of the problem. Another direction for future research is the study of metric interpolation problems in which the metric is induced by a non constant QDF.

Of course, this thesis only addresses a limited class of problems in the area of behavioral linear systems and control theory: a great deal of work remains to be done in this field, both from a practical
and from a theoretical point of view. From the practical side, it must
be realized that the solution of any control or model approximation
problem consists in the end in developing algorithms that start with
a first principles model specification and end up, with minimal model
transformation, with the specification of a controller, reduced order
model, etc.. Up to now, few of such procedures have been developed
in the behavioral framework. From the theoretical point of view, the
most pressing areas in which work is needed are the following:

Model approximation Given a system, the problem of model ap-
proximation consists in finding a lower complexity system that
approximates the original one. Several methods are known in
the state space and the transfer function approach to perform
model approximation, ranging from model reduction by balanc-
ing to $H_\infty$-approximation. These techniques are representation-
oriented: starting from a representation of the system in state
space or transfer function form, a simplified model in the same
form is computed. It would be interesting to see whether an ap-
proach more consistent with the basic tenets of behavioral system
theory, i.e. based on the system itself rather than its represen-
tation, could be developed. The calculus of QDF’s could be a useful
tool in formulating and solving approximation problems.

Infinite dimensional systems Infinite dimensional systems have been
studied from a behavioral point of view both in discrete and in
continuous time. Issues as controllability and observability, the
specification of canonical computational forms, etc., have been
successfully addressed in the behavioral framework; however, sev-
eral topics have not yet been touched upon. The most prominent
of these is the control of infinite dimensional systems. It is our
belief that the idea of control as imposing additional laws on
the behavior of the plant is better suited to deal with classical
infinite-dimensional control problems as heat conduction, beam
vibrations damping, etc., than the current approaches, which are
based on generalizing finite-dimensional control concepts to the
infinite-dimensional case.

Nonlinear systems This is by and large an open area of research.
With the exception of some unpublished work by J. Michalik, fundamental issues like the choice of the trajectory space, a calculus of representations, the concept of state, the notions of linearization, of controllability and observability, etc., have not yet been satisfactorily addressed in the behavioral setting. The complexity of the problems and the amount of technical difficulties involved in such a research effort should not be underestimated; However, we believe that a clean conceptual framework for discussing non-linear dynamical systems is a direction in which the behavioral ideas can be successfully extended.

CAMD software In Chapter 1 of this dissertation we have discussed by means of an example a modeling procedure based on tearing and zooming, in which a system is viewed as the interconnection of subsystems and modeling consists of describing the subsystems and their interconnection laws. In Chapter 7 we illustrated a point of view on control that puts forward the interconnection of a controller to a plant as the central paradigm of control theory. An ambitious research project is to combine together the tearing and zooming modeling procedure, the control-as-interconnection paradigm, the concept of state map, and concrete procedures to solve specific control, analysis, and approximation problems, in order to come up with a computer-assisted modeling and design software package based on behavioral ideas. The implementation of such a package is a long-term research objective, since the theoretical and practical problems to be faced are complex. In particular, many important issues should first be addressed concerning the representation of physical subsystems in abstract terms, and the design of efficient and reliable numerical procedures to solve concrete algebraic problems arising in behavioral system theory.
CHAPTER 9. CONCLUSIONS