Linear differential systems
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Chapter 1

Introduction

For three decades, mathematicians and engineers have been taught to equate “Linear System Theory” with transfer functions and state-space techniques. The reasons for this identification are several. On the one hand, there is the ability of the input-output paradigm to accommodate the engineering point of view of a system as performing certain reactions (the outputs) as a consequence of certain actions (the inputs). On the other hand, there is the insight provided by the notion of state in the internal structure of a system, and the computational advantages of state-space equations over other modes of representation in many areas of application, as exemplified by Kalman filtering, LQ- and LQG control, $H_\infty$-control.

Undeniably, the transfer function and the state space approach are extremely useful ways of looking at systems, and their adoption has given a great momentum to the growth of system theory, transforming it from an assortment of engineering techniques for the analysis and the control of systems into a scientific discipline in its own right. However, it also cannot be denied that the input-output and the state-space paradigms have many important shortcomings. The input-output paradigm fails in accommodating the very often encountered situation in which the variables describing the system cannot be classified in inputs and outputs, i.e. in which no physical causality structure can be assumed. The state-space paradigm is restrictive because in real life linear systems are hardly ever described in terms of first-order differential equations, and therefore the state space itself and a fortiori a state
representation must be deduced from the equations describing a system and is not given as a starting point.

The most important shortcoming of these paradigms, the one that lies at the heart of their inability to accommodate the study of any but a narrow range of physical systems, is that they have been developed without taking into account the most natural approach to devise a mathematical model for a physical system, that of tearing and zooming. Consider, in fact, the usual procedure in modeling: a system is viewed as an interconnection of subsystems, and modeling consists of describing the individual subsystems and their interconnection laws. This procedure is often executed hierarchically, with the subsystems in turn viewed as an interconnection. The net result of such a modeling procedure will be a model which involves manifest variables (often called external variables) - these are the variables whose evolution we try to model - and latent variables (often called auxiliary variables) - these are the variables describing the subsystems and their interconnections. The resulting model will typically involve many algebraic relations (for example, interconnection constraints, resistor laws, spring and damper characteristics, kinematic constraints), combined with differential equations. These may be first order (for example, inductors, capacitors, the dynamics of dampers), second order (for example, the dynamics of masses), or higher order (for example, subsystems whose dynamic laws have been obtained from an identification procedure). The following example illustrates how this procedure is applied to a mechanical system.

**Example 1.1** The mechanical system depicted in figure 1.1 is an abstract model of a one-wheel vehicle. The system consists of a mass (the vehicle body) connected by means of a spring and a damper suspension mechanisms to a second mass (the wheel) connected to a surface (the road) through a damped spring (representing the tyre). We are interested in modeling the relationship between the vertical position $w_2$ of the body and the profile $w_1$ of the road. We assume that the vehicle moves forward with a constant velocity. In order to derive an equation involving these two variables, we will tear the system into smaller subsystems, each of which can be modeled simply, and then interconnect these subsystems.
Figure 1.2 illustrates the tearing up of this system into subsystems. Each subsystem is connected to the others via certain terminals. In all there are 11 terminals. Two variables are associated to each terminal, a position $q_i$ and a force $F_i$, $i = 1, \ldots, 11$. All position variables are considered as deviation from their equilibrium point, i.e. the effects of nonlinearity and gravity is neglected. Also, the forces are assumed oriented towards the subsystems.

We now consider the constitutive equations of each of the subsystems. The damped spring subsystem is described by:

$$D_1 \frac{d(q_2 - q_1)}{dt} + K_1 (q_2 - q_1) = -F$$
$$F_1 = -F_2 = F$$

(1.1)

where $D_1$ and $K_1$ are parameters: the damping and spring constants and $F$ is a latent variable used to describe this subsystem.

The wheel subsystem has the following constitutive equations:

$$q_5 = q_4 = q_3 = q$$
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Figure 1.2: Tearing up the one-wheel vehicle model

\[ M_w \frac{d^2 q}{dt^2} = F_5 + F_4 + F_3 \]

where \( M_w \) (the mass of the wheel) is a parameter and \( q \) is a latent variable.

The constitutive equations of the spring subsystem are

\[ K_2 (q_7 - q_6) = -F' \]
\[ F_7 = -F_6 = F' \]

where \( K_2 \) is a parameter, the spring constant, and \( F' \) is a latent variable.
The equations of the damper are
\[
D_2 \frac{d(q_9 - q_8)}{dt} = -F'' \\
F_9 = -F_8 = F'' \tag{1.4}
\]
where the parameter \(D_2\) represents the damping coefficient and \(F''\) is a latent variable.

The equations of the body are
\[
q_{10} = q_{11} = q' \\
M_b \frac{d^2 q'}{dt^2} = F_{10} + F_{11} \tag{1.5}
\]
where the parameter \(M_b\) represents the mass of the body and \(q'\) is a latent variable.

The assignment of the manifest variables yields \(w_1 = q_1\) and \(w_2 = q_{10}\).

In order to derive a model for the system depicted in Figure 1.1, we proceed to interconnect the subsystems. This procedure amounts to imposing constraints on the variables describing the subsystems, namely that the positions of the terminals connected together are equal, and that the corresponding forces are the opposite of each other.

The interconnection of the subsystems yields the following interconnection laws:
\[
q_2 = q_3 \quad F_3 = -F_2 \\
q_{11} = q_7 \quad q_{10} = q_9 \\
F_{11} = -F_7 \quad F_{10} = -F_9 \\
q_6 = q_5 \quad q_4 = q_8 \\
F_6 = -F_5 \quad F_4 = -F_8 \tag{1.6}
\]
The equations (1.1)-(1.6), qualify, in our opinion, for being called a perfectly well-defined mathematical model. It has been obtained in a very systematic way, namely writing down the equations of each of the subsystems according to first principles and imposing interconnection constraints on the variables. The potential of such a procedure for computer assisted modeling is evident. However, the model derived in this way contains a large number of equations and of variables and it is not easy to obtain insight in the relation between \(w_1\) and \(w_2\). In order
to come up with a model that relates directly \( w_1 \) and \( w_2 \), it is necessary to eliminate the auxiliary variables. For example, it is evident that since the interconnection constraints (1.6) hold, the variables \( q_2, q_3, F_2 \) and \( F_3 \) can be eliminated. The model obtained in this way is

\[
\begin{align*}
D_1 \frac{d(q - q_1)}{dt} + K_1(q - q_1) &= -F \\
F_1 &= F \\
q_5 &= q_4 = q \\
M_w \frac{d^2q}{dt^2} &= F_5 + F_4 + F \quad (1.7)
\end{align*}
\]

together with the equations (1.3)-(1.6). Proceeding with the elimination of the remaining auxiliary variables yields the equation

\[
\begin{align*}
K_1 K_2 w_1 - (D_2 K_1 - D_1 K_2) \frac{dw_1}{dt} - D_1 D_2 \frac{d^2w_1}{dt^2} - \\
K_1 K_2 w_2 - (D_2 K_1 + D_1 K_2 - 2D_2 K_2) \frac{dw_2}{dt} \\
-(D_1 D_2 - 2D_2^2 - K_1 M_b + K_2 M_b + K_2 M_w) \frac{d^2w_2}{dt^2} + \\
(D_1 M_b - D_2 M_b - D_2 M_w) \frac{d^3w_2}{dt^3} + M_b M_w \frac{d^4w_1}{dt^4} &= 0 \quad (1.8)
\end{align*}
\]

Equation (1.8) is what we were after. As a description of the relation between \( w_1 \) and \( w_2 \) it is equivalent to equations (1.1)-(1.6), but of course it is much more compact and clear than the original model.

Example 1.1 demonstrates two things. The first one is that modeling a physical system from first principles does not yield as end result a transfer function or a state space representation. In fact, modeling from first principles will result in general in a number of higher order differential equations including many algebraic equations and involving many latent variables; in order to come up with a model involving only the manifest variables (the ones we are interested in), the latent variables must be eliminated; and finally, if needed, a state space or a transfer function representation can be computed.
Consequently, in order to obtain first order or transfer function models, some manipulation has to be performed or physical insight into the structure of the system must be used. Granted, for simple systems such as the one considered in the example, such manipulations can be done relatively easily, and experience can dictate which variables can be chosen as state variables, but for more complicated systems such approaches are bound to become awkward. Furthermore, despite the claims to the contrary of Artificial Intelligence scientists, experience and physical insight cannot yet be efficiently represented in a computer program, and the available techniques for deriving standard representations from first principles models, say graph theoretical algorithms for circuit analysis, are ad hoc methods whose applicability is limited to very specific domains. Therefore, the applicability of the traditional approaches to the representation of physical systems cannot take full advantage of the increasing computing power made available by the advancement of technology.

The example shows also that there is not a unique way of representing a physical system. This remark is less a truism than what seems at first reading, and one has only to think about the ubiquity of state space representations in textbooks and scientific literature to convince oneself. Indeed, different representations can and will be produced of the same system, each suitable for different purposes; the equations of a model can be manipulated and rearranged, and variables can be eliminated. Consider example 1.1: the sets of equations (1.1)-(1.6), (1.7) together with (1.3)-(1.6), and (1.8), all represent models for the relation between \( w_1 \) and \( w_2 \). However, if elimination of other latent variables than \( q_2, q_3, F_2 \) and \( F_3 \) had been performed, a different model from the one consisting of equations (1.7) and (1.3)-(1.6) would have been obtained. And of course a state space model or a transfer function model would also be a description of the relation between \( w_1 \) and \( w_2 \). All such representations reflect in some way or another the reality of the physical system. None of them is a priori better than the others, but each has certain advantages over the others in terms of suitability for simulation, for analysis, for control, and so on. How are they related with each other? What common features do they share? Is it possible to derive one from the other? How? To these basic questions, classical system theory gives no answer, dominated as it is by the input-output
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and the state-space paradigm.

The narrowness of the transfer function and of the state space paradigm becomes all the more evident when one considers three areas in which system and control theory has achieved important results in terms of practical applications as well as theoretical advances: feedback control, optimal control, and Lyapunov theory.

In classical feedback control the control input is chosen as a function of an observed output of the system to be controlled, and the problem of synthesis amounts to devise a controller that implements this function. The controller accepts as input the observed output of the plant, and produces as output the control signal. In many cases the interconnection of the plant and the controller in the feedback loop induces constraints on the variables of the system, so that certain variables that are inputs or outputs in the plant or the controller can no longer be classified as such in the closed loop system. There are also many situations in which it is unnatural to view the action of the controller as feedback, and to think of it in terms of signal flow graph. This is the case for the model of Example 1.1.

**Example 1.1 revisited** The spring and the damper connecting the mass of the body and the mass of the wheel in Figure 1.1 can be viewed as implementing a control law aimed at the achievement of certain control objectives. However, the spring and the damper cannot be classified as “controllers” in the usual sense, since it is impossible to regard one variable (the position or the position and the velocity of the body and the wheel masses) as being measured and used in order to decide what value another variable (the force exerted on the body mass) should take on.

In the context of optimal control and Lyapunov theory, the attention of classical system and control theory has been directed exclusively to first order models. For example, classical optimal control theory assumes that the control objectives are specified through the minimization of the integral of a quadratic functional of the state and of the input and disturbance variable. But as is shown in Example 1.1, a model de-
derived from first principles is in general not a first order model, nor does it allow a clear classification of the variables into inputs and outputs. The question arises as whether it is possible to develop a theory of optimal control and Lyapunov stability that proceeds on the basis of a first principles model. Such a theory would enable the modeler and the control engineer to describe the control objectives and to study the stability properties entirely in terms of the variables describing the system. The following example illustrates this point.

**Example 1.2** Many visco-elastic mechanical systems (for example the system considered in Example 1.1) are described by equations of the form

\[ Kw + D \frac{dw}{dt} + M \frac{d^2w}{dt^2} = 0 \]  

(1.9)

where \( K, M, D \in \mathbb{R}^{q \times q} \), \( K = K^T \geq 0 \), \( M = M^T \geq 0 \), and \( D + D^T \geq 0 \). Here \( M \) is the matrix associated with the inertial effects, and \( D \) and \( K \) are associated with the viscous damping and the elasticity effects.

The system of equations (1.9) is not of first order and therefore a state space representation should be computed before being able to apply classical Lyapunov theory. However, the equations describing the system will in general include static relations among the variables, so that state space equations of the kind used in classical Lyapunov theory would be non-trivial to compute. Moreover, the need for such a transformation of the model is questionable by itself: it is evident that a stability theory for higher-order differential equations would allow to study stability in terms of the original parameters \( K, D, \) and \( M \). It can be shown (see [58], Example 4.5) that the system (1.9) is asymptotically stable if and only if the matrix

\[ \begin{pmatrix} K + D\lambda + M\lambda^2 \\ \sqrt{(D + D^T)}\lambda \end{pmatrix} \]

has full column rank for all \( \lambda \in \mathbb{C} \). This is true, for example, in the case if \( K \) is nonsingular and \( \ker((D + D^T)) \subset \ker(M) \).

During the past decade, Willems in [51, 52, 53, 56, 54, 55, 31] has proposed a paradigm for the description of systems (not necessarily
linear and time-invariant), that encompasses the input-output and the state-space one and overcomes their inadequacies. This approach centers around the idea of a system as a set of time-trajectories, taking on values in a space of signals that includes besides the variables whose evolution we try to model some latent variables introduced in the modeling process. The set of all possible trajectories of the system variables is called the *behavior* of the system. This is called the *behavioral approach*. It treats the variables on an equal footing, thus not assuming any decomposition into inputs and outputs. Moreover, it makes a sharp distinction between the *system*, i.e. the behavior, and its *representation*, the set of equations that describe it. The same system can be described by different systems of higher order differential equations, maybe by a set of differential equations of first order, or maybe by input-state-output equations, each suitable for different purposes: analysis, control, simulation, etc. We now go back to Example 1.1 to illustrate the behavioral point of view.

**Example 1.1 revisited** From the behavioral point of view, the mechanical system of Example 1.1 is characterized by two manifest variables, \( w_1 \) and \( w_2 \). To describe the interaction between the two, i.e. to specify the set of time trajectories \((w_1, w_2)\) that the system can produce, various choices of representations can be made. By introducing the latent variables \( q_i, F_i, i = 1, \ldots, 11, q, F, F', F'', q' \) and writing down the equations (1.1)-(1.6), one such representation is obtained. We call such a representation *hybrid*, because it involves both the manifest and the latent variables.

However, the relation between \( w_1 \) and \( w_2 \) is not very evident from this model. Therefore, the need arises to devise a model that involves only \( w_1 \) and \( w_2 \). Equation (1.8) is such a model. We will call such a representation a *kernel representation* of the behavior of the system. It can be derived in a systematic way from (1.1)-(1.6) with a procedure that we will outline later on in this thesis, the *latent variable elimination*.

Consequently, the set of all trajectories \((w_1, w_2)\) that the system can

---

1In a different context, another famous Belgian put it in this way: "Ceci n'est pas une pipe".

produce is described by
\[
\{(w_1, w_2) \quad \text{s.t.} \quad \exists q_i, F_i, i = 1, \ldots, 11, q, F, F', F'', q' \\
\text{s.t. equations (1.1)-(1.6) are satisfied}\}
\]
or equivalently by
\[
\{(w_1, w_2) \text{ s.t. equation (1.8) is satisfied } \}
\]

The behavioral approach to control centers around the idea of interconnection. Instead of considering a controller as a signal processor, in behavioral control it is viewed as a dynamical system that is interconnected with the plant through certain control variables. In the non controlled plant, the control variables of the plant are constrained by the dynamical laws describing the behavior of the plant; after interconnection, they must satisfy also the laws imposed by the controller. In this way the dynamic behavior of all the plant variables is modified, since the interconnection of plant and controller affects not only the interconnection variables but, through them, also the other variables describing the plant. Thus, this interconnection can be viewed as imposing additional laws on the variables in order to achieve the modification of the behavior of the plant to a desired set of possible trajectories. This point of view has the advantage that whenever the input-output structure of the plant is not specified, or the combination of plant and controller does not exhibit a natural signal flow, the physical structure of the system is faithfully reflected in the description. In order to illustrate the behavioral approach to control, we now consider a model for a feedback amplifier.

**Example 1.3** Quite surprisingly, one of the success stories in the history of control, the invention of the feedback amplifier by Black (see [6]), is an example of a real situation in which the feedback paradigm, centered around the signal flow structure of the plant and the controller, is not applicable.

The circuit in figure 1.3 represents a voltage amplifier. It consists of two capacitors $C_1$ and $C_2$, a resistor $R_1$ and a voltage controlled ideal
amplifier $Kv_u$. We describe it as a behavioral system in the following way. The variables at the external terminals are the voltages $v_1$, $v_2$, $v_3$, and $v_4$. It is possible to derive in a systematic way (which we will not illustrate here) a model describing the relation between these variables. This is represented by the following differential equation:

$$
R_1(C_1 + C_2) \frac{d}{dt}v_3 + v_3 - R_1(C_1 + C_2) \frac{d}{dt}v_4 - v_4 -
$$

$$KR_1C_1 \frac{d}{dt}v_1 + KR_1C_1 \frac{d}{dt}v_2 = 0 \quad (1.10)
$$

Typically, the gain $K$ of the amplifier (for example a transistor) is very sensitive to temperature, aging, the load, etc., and the need arises to make the amplification insensitive to changes in these operating conditions. Black’s intuition was that this objective (which in modern terms we would call one concerned with robustness, with sensitivity reduction) could be achieved in a simple way by connecting the amplifier with the voltage divider depicted in figure 1.4. We now proceed to show how this connection cannot be construed easily as “feedback” in the ordinary, signal flow oriented, sense of the word. The voltage divider is a system whose evolution is described by four variables, the voltages $v_5$, $v_6$, $v_7$ and $v_8$, that obey the laws

$$
(R_2 + R_3)v_7 - (R_2 + R_3)v_8 - R_3v_5 + R_3v_6 = 0
$$

$$v_6 = v_8 \quad (1.11)
$$

Figure 1.3: A voltage amplifier
The very powerful idea of Black was to connect the amplifier with the voltage divider as depicted in Figure 1.5. The laws describing the evolution of the interconnected system are (1.10) and (1.11) together with the interconnection constraints

\[
\begin{align*}
    v_5 &= v_3 \\
    v_6 &= v_4 \\
    v_8 &= v_1
\end{align*}
\]

(1.12)

It can be shown that if \( K \gg 1 \) and \( R_1 \gg R_2 \) and \( R_1 \gg R_3 \), then the gain between \( v_2 - v_7 \) and \( v_3 - v_4 \) essentially equals

\[
\frac{R_2 + R_3}{R_3}
\]

Consequently, this gain does not depend on time and on the amplifier characteristics, and the interconnection of the amplifier with the voltage divider achieves the desired robustness aim.

The interconnection of Figure 1.5 does not allow an easy interpretation as the result of “closing the loop” around a plant with a controller that processes observed outputs in order to produce control inputs. In the plant represented in Figure 1.3 one can interpret \( v_1 - v_2 \) as the input and \( v_3 - v_4 \) as the output. However, it is quite awkward to interpret the voltage divider as a processing any of these two signals. Indeed, the action of the voltage divider on the amplifier is of imposing additional

Figure 1.4: A voltage divider
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Figure 1.5: A feedback amplifier

laws on the control variables $v_1$, $v_3$ and $v_4$ in order to achieve the desired control objective. Many other examples involving low distortion circuits could be made in which terms like “restriction of trajectories” or “imposition of additional laws” are more fitting descriptions of the control problem at hand (see [38], Chapter 5).

Starting from the point of view of a system as a set of time trajectories, the disadvantages and contradictions of the state space and the input-output paradigms can be overcome, and many of the questions posed above can be solved. We are far from suggesting here that the behavioral approach is opposed to the transfer function and the state space paradigms; in fact, some of the results presented in this thesis originate from research on the concept of state in a behavioral framework. We see behavioral system theory as providing a natural way of looking at systems, in which no unjustified assumptions on the structure of a system and on the nature of its variables are made. This point of view is more general than the input-output and the state space one; however, both approaches are accommodated in the behavioral framework. We will
attempt to support the validity of this claim in two ways throughout this thesis. The first one is the translation of behavioral definitions in the language of the state space and of the transfer function approach. The second one consists in working out some of the original results of this thesis in the case of systems described by state space equations. We must however admit that we did not try to draw a complete picture of the connections between the results available in classical system theory and those derived in this dissertation in the behavioral framework. We believe that further investigation in this direction is needed, and that such an effort would not be entirely trivial.

We now proceed to give a review of the contents of this thesis. In Chapter 2 the aspects of the behavioral approach to system theory that are more relevant for the topics discussed in this dissertation are illustrated. Besides providing an overview of some basic notions, we discuss several important issues like the choice of the trajectory space and the elimination of the latent variable.

In Chapter 3 we consider the problem of state construction. In the behavioral approach, the input, output or state nature of a variable is not assumed as given \textit{a priori}, but as a feature that must be \textit{deduced} from the description of the system. In particular, the state variable is not given from first principles, but it is a latent variable that must be \textit{constructed} on the basis of the description of the system. In the case of systems described by linear differential equations with constant coefficients, a state variable can be computed as the image of a polynomial differential operator acting on the trajectories of the behavior. Such a polynomial differential operator is called a \textit{state map}, and concrete procedures can be given to compute a state map from the equations describing the system.

Recently, LQ- and $H_{\infty}$-optimal control problems have been examined from a behavioral perspective. In this setting, optimal control problems are described by certain control objectives that the trajectories of the interconnected system must accomplish; the objectives are specified by quadratic functionals of the system variables and their derivatives. Such functionals are called \textit{quadratic differential forms} and can be effectively represented by two-variable polynomial matrices. In Chapter 4 we give a review of the basic definitions and results concerning quadratic differential forms. The concept of quadratic differential
form is used throughout the rest of the thesis. In Chapter 5, we use it to study dissipative systems from a behavioral point of view. In many cases, a quadratic differential form can be interpreted as the power; consequently, its integral over the real line measures the energy that is being supplied to or flows out from a system. In some cases the system dissipates energy and therefore the net flow of energy going into the system is positive; in the literature, such systems are called dissipative or passive. Dissipative systems store part of the energy that is being exchanged with the external world at a rate measured by a certain *storage function*. In Chapter 5 we examine the computation of storage functions for a special class of dissipative systems. These results are applied also to state space systems, providing a connection between storage functions and solutions of the algebraic Riccati equation.

One of the most important problems in classical system theory, split as it is between the input-output and the state space point of view, is the relationship between the internal (i.e. the state) properties and the external (i.e. the input and output) properties of a system. The first result in the study of this relationship is the Kalman decomposition theorem, that relates controllability and observability of a subsystem with the transfer function of the whole system. Another important contribution in this direction has been the concept of balancing. This amounts to selecting a basis for the state space in such a way that the external properties of the system are reflected in the internal properties, those of the state. This concept has important applications in model reduction, signal processing, and controller design. In Chapter 6 of this thesis we examine the notion of balancing from a behavioral point of view, and we introduce the notion of balanced state map. In the setting we propose, the external properties of the system are measured by quadratic functionals of the manifest variables and their derivatives, and balancing consists in choosing a state map that makes evident the contribution of each component of the state variable to the external behavior of the system.

Chapter 7 of the dissertation deals with the polynomial *J*-spectral factorization problem. In the behavioral approach to optimal control problems, the computation of an optimal controller reduces to the factorization of a polynomial matrix derived from a description of the control objectives and the dynamics of the plant. A very convenient way
of performing the factorization is by associating a two-variable polynomial matrix to the one-variable polynomial matrix to be factored; in Chapter 7 we will illustrate a new algorithm for $J$-spectral factorization that uses this technique. This algorithm should be of interest not only in the behavioral framework, but in all areas of application of $J$-spectral factorization, namely network synthesis, filtering theory, polynomial $H_\infty$-optimal control, to name but a few.

One of the central issues in the mathematical modeling of dynamical systems is how to derive a mathematical model on the basis of observations or experiments. The model is chosen in a model class, a set of candidate models, which reflects the available knowledge about the system and the assumptions the modeler wishes to make on the nature of the system under study. In many cases of interest the given observations can be modeled exactly. One example of such situation is the problem of deriving a state space model from impulse response measurements, the so called realization problem. Another example is the fitting of a finite set of polynomial exponential trajectories to a behavior described by linear constant-coefficient differential equations. In general, many different models explaining the observations can be devised. However, in the behavioral approach it is not the descriptive power of a model that is important, but its prescriptive power: a model is better than another if it precludes more outcomes of the phenomenon to happen. Starting from this point of view it is natural to choose as the “best” model the one that explains the observations and as little else as possible. In the behavioral framework, such model is called the most powerful unfalsified model. The concept of most powerful unfalsified model can be applied also to the problem of rational interpolation with metric constraints, in which a rational function is sought that interpolates certain values and is required to have norm less than a given value. A classical example of such a problem is the Nevanlinna interpolation problem, in which the interpolant is required to be stable and with infinity norm less than one. In Chapter 7 of this thesis a generalization of the Nevanlinna interpolation problem is examined from the point of view of linear exact modeling. It turns out that in this case the representation of the most powerful unfalsified model exhibits a remarkable symmetric structure.

In the last chapter of the thesis we discuss the limitations of the work
presented in this dissertation and give suggestions for further research.

The proofs are collected in Appendix A, and a list of the symbols used in this thesis is given in Appendix B. Appendices C and D contain two summaries of the results presented in this dissertation, one in Dutch and the other in Italian.