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Economic and Accounting Rates of Return

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Abstract

The rate of return on invested capital is a central concept in financial analysis. The purpose of calculating the rate of return on investment in general is to measure the financial performance, to assess the desirability of a project and to make decisions on the valuation of firms. Financial statement users make regular use of the accounting rate of return (ARR) rather than the economic rate of return (IRR) to assess the performance of corporations and public-sector enterprises, to evaluate capital investment projects, and to price financial claims such as shares. Since ARR measures are based on published accounting statements, there has been a long and sometimes heated debate as to whether such measures have any economic significance. This paper aims to provide a summary of the economic and accounting rates of return discussions in the literature. We analyze the concepts of ARR and IRR and explore possible relationships between them. We extend the previous studies in this line to provide more specific relations of IRR and ARR.

1. Introduction

The rate of return on invested capital is a central concept in financial analysis. The purpose of calculating the rate of return on investment in general is to measure the financial performance, to assess the desirability of a project and to make decisions on the valuation of firms. Rates of return indicators are important for monitoring the economic performance of both public listed corporations and government enterprises.

Economists need measures of business performance for a variety of purposes, including as guides to antitrust policy and in controlling of private sector monopolies. Economic performance can be understood as a *real* rate of return earned on a completed project where all cash outlays and receipts are expressed in monetary units of equivalent purchasing power. Economists' concepts of internal rate of return (IRR) and present value are now widely employed in business for evaluating capital investment projects, pricing shares and assessing managerial efficiency.

Where economists wish to conduct empirical investigations requiring calculations of the internal rate of return (IRR), measurement problems are very common in determining the cash flows which have occurred. Although the concept of IRR is generally associated with ex ante project evaluation, empirical studies must rely on ex post measures for testing models or hypotheses. In the case of either a completed project or a liquidated firm, the IRR can be calculated ex post. But even here there is a problem, particularly where the analyst is limited to externally available information, as the desired cash flow data is unavailable, which is usually the case.

The unavailability of cash flow information has forced researchers to look for other information that is publicly available: a prime source is published accounting data. Financial statements provide the most widely available data on public corporations' economic activities: investors and other stakeholders rely on them to assess the plans and performance of firms. The accounting rate of return, based on accrual concepts and defined as net income divided by book value of equity, is not only a central feature of any basic text on financial statement analysis, but also figures commonly in the evaluation by investment analysts of the financial performance of firms.

Financial statement users such as practicing accountants, information intermediaries, loan officers and government policy advisers make regular use of the accounting rate of return (ARR) rather than the IRR to assess the performance of corporations and public-sector enterprises, to evaluate capital investment projects, and to price financial claims such as shares. Accounting reports constitute the only systematically compiled, publicly available, alternative source of information about the financial affairs of business corporations, and are largely standardized and

audited, too. ARR, based on accrual concepts and defined as a periodic variable, is considered to be a more pragmatic performance indicator. Vatter (1966, p. 682) points out that " accounting measurements are ... the only basic sources of data which establish (however imperfectly) the income for a period, the amount of investment, and the bases of classification and matching which provide the rate of return currently being realized by operations or projects...". ARR is then viewed as a proxy or surrogate for IRR in the various contexts where measures of, and comparisons involving IRR are deemed relevant.

Discretionary choice of accruals, constructed under generally accepted accounting principles (GAAP), include inventory valuation, depreciation, and foreign currency translation (Kelly, 1996a). Hall and Weiss (1967) note that there is a variety of reasons to believe that the profit data will tend to be understated, particularly in large and profitable firms; whilst managers of unprofitable firms, if concerned with retaining control, might well overstate profits. Managers thus possess much discretionary power under GAAP, regarding inventory valuation, depreciation, research and development costs, goodwill, etc. to capture different accounting realities, which yield measurement errors of ARR. Thus ARR in reality is a potentially noisy monitoring device for economic performance appraisals.

Since ARR measures are based on published financial statements, there has been a long and sometimes heated debate as to whether such measures have any economic significance[1]. The debate began with the seminal papers of Harcourt (1965) and Solomon (1966). They discuss the relationship between ARR and IRR and the specific circumstances under which ARR equals to IRR. Fisher and McGowan's paper (1983) has stimulated a most lively discussion on the subject. The findings of analytical work have generally been pessimistic. The early literature investigating the relationship between ARR and IRR models the problem of a firm investing in individual projects and a mix of projects under alternative assumptions about depreciation policy and growth[2]. Researchers find that unless very strong assumptions are made the ARR is not an accurate measure of IRR, that there is no systematic pattern in the errors, and that the biases can be very large.

An important conceptual breakthrough was Kay's (1976) discovery of an exact mathematical relationship between the economist's IRR and ARR. The work that developed from Kay (1976), models the total cash flows of a firm as one project, then transforms the cash flows using the clean surplus accounting profits identity to discover a simple relationship between ARR and IRR. The IRR is found to be a

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[1] See, for example, Harcourt (1965), Solomon (1966), Kay (1976), Fisher and McGowan (1983), Whittington (1988) and Peasnell (1982, 1996).
[2] See also in Livingston and Salamon (1970), Stauffer (1971), Gordon (1974), Fisher and McGowan (1983), and Long and Ravenscraft (1984).

weighted average of ARRs plus an error term that depends on opening and closing valuation differences. Whittington (1979) and Stark (1982) discuss the work of Kay (1976) and Peasnell (1982) extends the analytical model. Edwards, Kay and Mayer (1987) introduced the "value-to-the-owner" concept in this context and developed an economic analysis of accounting profitability. Further Brief and Lawson (1992) presented ARR as a key role to play in the valuation process.

An approach to estimate IRR from the firm's stock price was introduced by Kelly and Tippett (1991). Butler, Holland and Tippett (1994) and Kelly (1996b) employ this method to investigate IRR-ARR relations. They find that ARR in general is not a good surrogate for IRR.

The cash recovery rate (CRR) method was developed by Ijiri (1978) and Salamon (1982) as an alternative way of using accounting information to derive the IRR. Salamon (1988) empirically estimates the conditional IRR and believed that the cash recovery rate (CRR) plays a more reliable role than ARR. Stark (1987), Lee and Stark (1987) and Gordon and Stark (1989) comment on these works.

This paper summarizes the literature about the IRR-ARR relationship, both analytical and empirical. The remainder of the paper is arranged as follows. The next section discusses the definitions of IRR and ARR used in the literature and investigates what kinds of IRR-ARR relations have been examined. Section three provides a summary of analytical and empirical research on the IRR-ARR relationship. It also discusses the cash recovery rate (CRR) to estimate the IRR. Finally, a brief conclusion is formulated in section four.

2. IRR and ARR: definitions and relations

Closely related to the measure of economic return is the measurement of the internal rate of return of a project, a firm or an industry. The economic value of an investment project equals the present value of cash flows expected from that project discounted at a rate given by the appropriate opportunity cost of capital. The economic, or internal rate of return (IRR) is usually defined as that discount rate which equates the present value of its expected cash flows stream to its initial outlay. This concept is central to economic investment theory, and is hence of great interest to economists (Luckett, 1984, p. 213).

Initial researchers usually assumed IRR as a constant rate calculated under certainty. Harcourt (1965, p. 68) asserts that the expected rate of profit in a 'Golden Age' is the internal rate of return – the rate of discount which makes the present value

of the expected quasi-rents equal to the supply price of each machine. He defines (p. 66) "Golden Age" conditions as complete certainty, or total realized expectations. Solomon (1966) measures IRR as the true yield, that is the annual rate of discount at which the present value of investment outlays is just equal to the present value of cash receipts flowing from the investment. According to Solomon (p. 233), "the size and timing of all investment outlays and of all net cash receipts flowing from these outlays are available, or can be estimated either retrospectively or prospectively."

Concerning with the use of annual IRR, Vatter (1966, pp. 695-696) points out that IRR is an average rate of return over the project term, not an annual one; even though it may be expressed as a rate per year. IRR is established and refers to the term of the project as a whole, being a nominal rate derived from the way in which calculations are made.

Fisher and McGowan (1983, p. 82) define IRR as the discount rate that equates the present value of its expected net revenue stream to its initial outlay, and state (p. 82) that IRR is the only correct measure of the profit rate for purposes of economic analysis. Formally the IRR is defined as r such that

$$C_0 = \sum_{i=1}^{n} \frac{C_i}{(1+r)^i} \qquad (1)$$

where C_0 is the initial investment outlays, C_i is net cash flow in period i.

Likewise, Gordon (1974), Kay (1976), Whittington (1979), and Edwards et al. (1987) derive IRR based on the initial cost of a firm's assets.

It is widely accepted that IRR is unobservable for a firm's remainder of life in a world of uncertainty. The IRR is assumed as the appropriate rate discounting by the firm's cash flows which follow a stochastic function of time. The empirical work by Kelly and Tippett (1991), Butler et al. (1994) and Kelly (1996b) consider an ex ante IRR under uncertainty.

Kelly and Tippett (1991, p. 325), Butler et al. (1994, pp. 307-312) and Kelly (1996b, p. 353) note that IRR can be determined as the appropriate discount rate which equates the expected net present value of the firm's future cash flow stream with its time zero security price. Kelly (1996b, p. 347 footnote 2) defines an interval prospective IRR utilizing discounted cash flows which equal net cash flows divided by the average number of ordinary stock.

According to IRR calculations relying on cash flow information, Butler, Holland and Tippett (1994) denoted cash flows given three kinds of definitions. The three cash flow definitions are (p. 307):

> CF1 = net profit after tax, minority interests and preference dividend
> + extraordinary items after tax for the period
> + depreciation + amortization
> + other non-cash adjustments + deferred tax charges;

> CF2 = CF1 − change in inventory and work in progress; and

> CF3 = CF2 − change in debtors + change in short-term provisions + change in creditors.

Butler et al. utilize the Kelly-Tippett method and applied the three definitions of cash flows to estimate IRR.

Peasnell (1996, p. 294) points out that these definitions of IRR differ from the normal view of the concept in two respects. That is, first, it is based on the current market value of the firm rather than the initial cost of the firm's assets; and second, it refers to a (future) time interval that does not include the (past) period(s) from which the accounting data are drawn.

The accounting rate of return (ARR), calculated from the financial statements, is a periodic and an ex post indicator. Vatter (1966, p. 696) asserts that ARR is a figure based only on the data related to a given year, and has no reference to any other part of the project than that year to which it applies. ARR is usually defined as the ratio of accounting profit earned in a particular period to the book value of the capital employed in the period. According to the different numerators and denominators applied to calculate ARR, there are several kinds of definitions used in analysis. For the numerator of ARR, it is usually financial annual accounting profit or income, while the denominator is often determined by book value of assets or book value of equity.

Employing the 'clean surplus' concept, Peasnell (1982, p. 367) defines ARR as the ratio of the accounting profit to the book value of assets at the beginning of the period. The accounting profit was defined in 'clean surplus' terms, where accounting profit equals net dividends paid plus the change in the net book value of the firm's assets during the period. He then shows (p. 369) that the constant ARR equals to IRR when there are no opening and closing valuation errors.

Average ARR calculation was developed by Kay (1976). Kay uses average ARR to overcome 'creative' accounting practices that may distort reported profits in the 'short run'. The average ARR is the weighted average of the lifetime series of ARRs, with the weights obtained by discounting book values at IRR.

According to the use of average ARR, Peasnell (1982, p. 371) mentions that "the IRR is defined to be constant throughout the investment holding period whereas ARRs can and do vary through time. Perhaps the most obvious way of utilizing a time-series of ARRs in practice is to take a simple arithmetic average of them and to treat the result as a proxy for the (constant) IRR."

Using the cash recovery rate (CRR) to estimate the internal rate of return of a firm was considered by Ijiri (1978 and 1979) and Salamon (1982 and 1985) and others. A major benefit claimed for the cash recovery rate (CRR) is that it is not vulnerable to the choice of accounting methods, while ARR is. Thus, the cash recovery rate (CRR) is seen as a useful alternative to the use of the accounting rate of return (ARR) to estimate the economic performance.

Ijiri (1978, p. 347) defines the cash recovery rate (CRR) as a ratio of the cash recoveries with the gross investment for the year. Cash recoveries are the sum of funds from operations, interest, proceeds from the disposal of long-term assets and the net decrease in total net current assets. Gross investment for each year was calculated as the average of beginning and ending total assets. Salamon (1982, 1985) extends Ijiri's work, and believed that it enhances the possibility of using reported accounting data to derive meaningful discounted cash flow rates of return.

As Stark (1987, p. 10) points out, the key element of the cash recovery rate approach is the modeling of the process by which firms' cash flows are generated, given a past sequence of investments. The approach assumes that all investments produce the same sequence of subsequent cash flows, adjusted for scale.

We will survey the scenarios of the relationship between IRR and ARR as follows[3]:

1) annual ARR versus ex post IRR,

2) average ARR versus ex post IRR,

3) average ARR versus ex ante IRR, and

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[3] Kelly's paper (1996a) discussed the debate of prospective IRR versus realized ARR. We extend his work by classifying the IRR-ARR relations in more detail.

4) cash recovery rate (CRR) versus IRR.

The next section will summarize the analytical and empirical research for IRR-ARR relations following these four scenarios.

3. Theoretical and Empirical Studies on IRR-ARR Relations

3.1 Annual ARR versus ex post IRR

When comparing the annual ARR with ex post measure of IRR, researchers usually address the issue whether an ex post and periodic ARR gives a 'right answer' to the constant IRR. Initial researchers examine the machine lives or length of project life, cash flow pattern, growth rate, and accounting policy with respect to the capitalization and depreciation of investment outlays which would affect the ARR as a 'right answer' to IRR.

Harcourt (1965, p. 67) considers four main cases relating to accounting and economic rate of return under Golden Age conditions. The first two cases are that the firm invested solely in physical assets that have a balanced stock, or a constant growth rate each year. The second two cases are that the firm invested both physical and financial assets that under a balanced case, or a constant growth case[4]. For each of the four main cases, four special cases are considered, i.e., variations of cash flow pattern that yielded four alternatives of expected abnormal returns: equal each year, falling, rising, and combination of falling and rising. For each of these cases, he examines the simultaneous effects for ARR and IRR relationships of variations in cash flow patterns, machine lives, growth rates and IRR. He also reports separate ARR's in terms of IRR expressions for straight-line depreciation and declining balance depreciation methods.

Harcourt discusses (pp. 69-70) that if the accountant valued capital as the sum of the discounted values of the expected quasi-rents (abnormal return), the value of capital for the year as a whole can be shown as follows:

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[4] Kelly (1996a, p. 78) thus summarized Harcourt's study into two main cases, first, where firms hold physical assets only and second, where firms hold both physical and financial assets.

$$K = \frac{L}{2r} [(Q + Q^*) - (S + S^*)] \qquad (2)$$

where K denotes the capital value for the year; L denotes the number of machines in any age group and purchased in any year; r denotes the expected rate of profit (IRR); $Q = \sum_{i=1}^{n} q_i$, in which q_i represents the expected abnormal return in year i; $Q^* = \sum_{i=2}^{n} q_i$; S denotes the supply price of machine: $S = \sum_{i=1}^{n} \frac{q_i}{(1+r)^i}$ and $S^* = \sum_{i=2}^{n} \frac{q_i}{(1+r)^{i-1}}$.

Accounting profit was defined (p. 70) as:

$$A = L(Q - S) \qquad (3)$$

In this situation, ARR can be written as:

$$ARR = \frac{2r(Q - S)}{\left[(Q + Q^*) - (S + S^*)\right]} \qquad (4)$$

Harcourt shows that ARR in equation (4) is approximately equal to r (IRR). He demonstrates (p. 71) that if the accountant valued capital as the accounting's average book value of capital for the year, then:

$$k = \frac{LnS}{2} \qquad (5)$$

where k is the accounting's average book valuation of capital and n is the machine life duration. In this instance,

$$ARR = \frac{2(Q - S)}{nS} \qquad (6)$$

Harcourt observes (p. 71) that ARR in equation (6) is not generally equal to r. In the case of "one-hoss shay" (a constant cash flow stream), he finds (p. 71):

$$ARR = \frac{2}{n} \left[\frac{nr}{1 - (1/(1+r))^n} - 1 \right] \qquad (7)$$

Given a balanced stock of identical machines, he points out (p. 71) that ARR will give different answers for two businesses that are alike in every respect except that the machines of one are longer-lived than those of the other. He also reports (pp. 72-80) large differences between IRR and ARR because depreciation methods utilized by accountants rarely match the economic depreciation implicit in IRR calculations. He characterizes (p. 67) ARR (the ratio of annual accounting profit to average as well as the annual book values of capital) as "...extremely misleading, even under 'Golden Age' conditions." He concludes (p. 80) that anyone "...who compares rates of profits of different industries, or of the same industry in different countries, and draws inferences from their magnitudes as to the relative profitability of investments in different uses or countries, does so at his own peril."

The results by Harcourt (1966) were hardly surprising when only considering the differences between accountant's and economist's views on asset valuations and depreciation calculations for income determination (Luckett, 1984, pp. 215-216). However, Harcourt asserts (p. 67) that ARR is influenced by the pattern of the quasi-rents associated with individual machines in a stock of capital, the method of accounting depreciation used, whether or not the stock of capital is growing, and by the type of assets included in the stock of capital.

Further studies have been made to investigate the relationship between IRR and ARR under specific limiting conditions. Solomon (1966) studies the relationship between the book-yield (ARR) on investment and the true yield (IRR) on investment for a firm that consists solely of projects with the same life and IRR. Solomon (p. 234) divides his analysis into two basic models: the zero-growth situation and growth situation. This looks as same as what Harcourt divided his first two cases. Solomon examines the effect on book-yield and true yield relationships of variation in capitalization policy, depreciation methods, revenue patterns and growth rates.

In the zero-growth case, Solomon (p. 240) concludes that the book-yield (ARR) is not an accurate measure of true yield (IRR) and that the measurement error in the book-yield is neither constant nor consistent. Specifically, he indicates four basic factors that affect the degree to which ARR deviates from IRR:

1) Length of project life: the longer the project life, the greater the overstatement.

2) Capitalization policy: the smaller the fraction of total investment capitalized in the balance sheet, the greater will be the overstatement.

3) The choice of depreciation methods: depreciation procedures faster than a straight line basis will result in higher book-yields.

4) The greater the lag between investment outlays and the recoupment of these outlays from cash inflows, the greater the degree of overstatement.

Under growth situations, Solomon (p. 240) finds that "... the rate at which a division or a company or an industry acquires new investments is a major variable affecting the size of the error contained in the observable book-yield." He observes (p. 241) that if the observable book-yield is higher than true yield for a non-growth situation, a positive growth will tend to lower ARR relative to IRR, and the faster the growth, the more will ARR decline relative to IRR. Furthermore, he distinguishes between two kinds of growth: real growth and inflationary growth. For the real growth, he observes (p. 242) that if ARR is higher than IRR in the zero-growth case, then as growth rate g increases ARR falls continuously toward IRR. Specially, when g is equal to IRR, ARR is also equal to IRR. Inflation, he presents (pp. 242-243) money true yield $m = i + ri + r$, where i is the inflation rate and r is IRR. Meanwhile, "...actual investment and hence, depreciation expense and net book value are themselves affected by the inflationary process." In this case, ARR is totally different from money true yield (IRR).

Like Harcourt (1965), Solomon basically concludes (p. 243) that ARR is "not a reliable measure" of IRR. Hepworth (1966, p. 247) comments that Solomon (1966) provides an excellent case against the indiscriminate use of the straight-line depreciation method in accounting, and also against capitalization policies, as these represent divergence from what would be expected under the compound-interest model.

Another study by Solomon (1970) indicates the relationship between the IRR and ARR measures and shows the impact of some variations in depreciation and expensing procedure, growth rate, etc. In the steady state, the company's observable book rate[5] ARR is a function of the following variables,

$$ARR = f (r, x, d, n, w, l, c) \qquad (8)$$

where r = the discount cash flow rate it is achieving (IRR),

     x = the average expensing policy,

     d = the depreciation policy,

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[5] Solomon (1970, p. 75) denotes b as the accounting rate of return.

n = the average productive life of assets,

w = the fraction of working capital to total capital,

l = the average time lag between the outlay for each asset and the commencement of net cash flows from its use,

c = the time pattern of cash inflows.

He finds (pp. 71-72) that the effects of accounting expensing policies on ARR are clearly powerful. He explains the reason for the higher-than-normal ARR for companies or industries is that they are either riskier, more efficient, or having monopoly powers. The fact that many high-book-rate companies or industries also follow high "expensing" policies suggests strongly that the observable ARR significantly overstates the underlying IRR actually being earned. Solomon (1970, p. 74) then considers the influence of the depreciation method and concludes that in the steady state the longer the duration of each asset, the greater the discrepancy in the ARR measure relative to the IRR measure. In the growth state (p. 77), the ARR is an unbiased and accurate measure of the IRR for a company that is growing steadily at a rate equal to IRR or ARR, if all the net cash flow is reinvested.

Similar to Solomon (1966), Solomon (1970) believes that ARR and IRR measures are not different estimates of the same thing but rather estimates of different things.

Following Harcourt (1965) and Solomon (1966), Livingstone and Salamon (1970) derive a relationship between ARR and IRR which depends on the length of the project (n), the project's IRR (r), the pattern of cash flows associated with the projects (b), and the proportion of annual cash flows that are reinvested (c). Like the other studies cited, they also assume a constant IRR. Employing from Solomon (1966) as

$$ARR_t = \frac{F_t - D_t}{K_t} \tag{9}$$

where F_t denotes fund flows from operations before taxes in period t, D_t is depreciation charges in period t, and K_t is net book value of assets at the beginning of period t.

They denote I_0 as the cost of an infinitely divisible asset which generates cash benefits of Q_1, Q_2,..., Q_n and represents the exogenous investment which is made

by the firm each year for n years. Thus, for years t such that $t \geq 2n$, $F_t =$
$\dfrac{c}{I_0} \sum\limits_{i=1}^{n} F_{t-1} Q_i$; $D_t = \dfrac{c}{n} \sum\limits_{i=1}^{n} F_{t-i}$; and $K_t = \dfrac{c}{n} \sum\limits_{i=1}^{n} (n+1-i) F_{t-i}$. They undertake a
deterministic simulation analysis by substituting various values for the parameters in the model such that,

$$ARR_t = \frac{\sum\limits_{i=1}^{n} (n(1+r-b)(1+r)^n b^{i-1} - (1+r)^n + b^n) F_{t-i}}{((1+r)^n - b^n) \sum\limits_{i=1}^{n} (n+1-i) F_{t-i}} \tag{10}$$

One of the results shows (pp. 206-211) that ARR cycles symmetrically around a constant. The constant around which ARR cycles and the amplitude of the ARR cycle is affected by the model parameters n, r, b, and c. Particularly, the amplitude of the ARR cycle: (1) decreases as t increases for all values of n and IRR; (2) increases as n increases for any given IRR; (3) decreases as IRR increases for any given n.

Another result indicates (p. 212) that when the reinvestment rate equals one (c = 1, i.e., 100% reinvestment) then IRR $\approx$ ARR, regardless of the values of the remaining parameters. This result was shown by Solomon (1966) for the case of level project cash flows. Livingstone and Salamon apply it more generally for other project cash flows. They noted that there is a class of cases for which ARR is a good proxy for IRR.

McHugh (1976) examines Livingstone and Salamon's model in a more analytic fashion and finds it to be not universally valid. He provides a mathematical proof of the results derived by Livingstone and Salamon and a questioning of some of their conclusions. McHugh (p. 183) analyzes the asymptotic form for the cash flow F_t and shows the derivation relied on some results of matrix theory. He concludes that the asymptotic limit for ARR_t has been given an analytic form. He also finds that ARR asymptotically approaches IRR in the long run.

Livingstone and Van Breda (1976) reply McHugh (1976) comments on Livingstone and Salamon's model. Livingstone and Van Breda (1976) then discuss McHugh's criticisms by using standard difference equation methodology. They show (p. 188) that the solution to the cyclical damping effect only "...permits an analytical relationship to be established when n $\rightarrow$ ∞". They argue that McHugh's criticisms were unwarranted, "since Livingstone and Salamon specifically avoided any claim of generality for the findings in question".

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Further work by Stauffer (1971) is concerned with a more realistic model deriving general conditions under which ARR deviates from IRR. Stauffer analyzes the problem considering varying cash flows and income taxes, and extends Solomon (1970) by including non-depreciable capital in his model. He assumes that the firm can be represented by a convolution-type investment process, whereby the cash flow in any year t is the sum of the investment outlays made in all prior years, each investment outlay being weighted by the cash flow associated with a unit investment of age T. Stauffer proposes (pp. 436-437) the model as,

$$Y(t) = K(t) + \int_0^t \pi(t-T)Y(t-T)K(t,T)dT \qquad (11)$$

where $Y(t)$ is the cash flow for the firm, $K(t, T)$ idem cash flow generated in year $t + T$, from a unit investment in year t, and $\pi(t)$ idem fraction of the firm's cash flow which is reinvested in any year. The cash flow $K(t, T)$ depends both upon asset age and the time of investment. He further assumes:

(1) The process is stationary, i.e., $K(t, T)$ equals $K(t - T)$. In other words, the cash flow pattern produced by a unit investment is independent of the time at which the investment is made and is also independent of all prior or subsequent investments made by the firm.

(2) $K(t)$ is a bounded, non-negative function of t for $1 \leq t \leq N$ and vanishes for all $t \succ N$. This ensures that there exists one and only one real internal rate of return for $K(t)$, i.e., a unique, positive discount rate, r, for which the present value of the cash flow stream $K(t)$ equals unity.

(3) Expectations are always realized; if the firm invests one dollar now, the resulting cash flow will be precisely that given by $K(t)$.

(4) $\pi(t) = \pi_0$; the reinvestment fraction is taken to be constant.

Stauffer notes (pp. 440-441, 468-469) that the ARR will exactly equal the IRR for an arbitrary growth path, if and only if the cash flow profile, $K(t)$, and the accounting depreciation schedule, $D(t)$, jointly satisfy the integral equation

$$D(t) = K(t) - r\, e^{rt}(1 - \int_0^t K(T)e^{-rT}\, dT \qquad (12)$$

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where r represents IRR which is defined analytically as the values of r for which $\int_0^N K(t)e^{-rt}dt = 1$ or $\sum_{i=1}^N K(i)(i+r)^{-i} = 1$. Thus, if K(t) is given, D(t) is uniquely specified. Otherwise, if K(t) and D(t) are not related to each other as stipulated by the equation, the ARR will in general diverge from the IRR.

Stauffer achieves a general synthesis of IRR and ARR relationships as described in the bulk of the literature analyzed. In particular, he makes clear the direct and indirect roles which growth plays in generating measurement error in ARR. He (p. 467) examines that "if the firm's financial structure involves working capital, using the exact depreciation schedule will not yield an exact measure of the economic rate of return." Like Harcourt (1966) and Solomon (1966), Stauffer concludes that realized ARR is generally a very poor proxy for ex ante IRR.

Bhaskar (1972) regresses 1,000 IRR observations against those for ARR to ascertain functional relationships between the two indicators. He proposes the following regression model:

$$ARR = \alpha + \beta\,IRR + u \tag{13}$$

where α is the intercept term, β is the slope coefficient, and u is the error term. He states his results (p. 49) that as the degree of riskiness increased from experiment one to three, the intercepts of the regression equations approached zero, the slopes approached one, the correlation coefficients were higher, and the standard error of estimate decreased. He concludes (p. 51) that ARR asymptotically approaches IRR in more uncertain environments, and that the results of the straight-line method of depreciation were always dominated by the annuity method.

Gordon (1974) investigates some special cases for analytical differences between realized ARR and IRR measures. Gordon believes (p. 345) that the components of the traditional ARR can be useful in approximating the true IRR. He notes (p. 352) that although ARR in general is not a good measure of the IRR, supplementary information can be generated by an enlightened accounting profession that will greatly enhance the usefulness of the components of the ARR for approximating the IRR.

Consistent with Solomon (1970), Livingstone and Salamon (1970) and Stauffer (1971), Gordon confirms (pp. 348-349) that where the growth rate equals the internal rate of return and there is 100% reinvestment of cash flows, ARR will be on average approximately equal to IRR over the long-run. He shows that Livingstone and Salamon's (1970) case where the reinvestment rate equals 100% is a special case of

Stauffer's (1971) case where the growth rate equals the IRR. He observes (p. 353) that income determination and valuations of assets are the heart of the disparities between ARR and IRR. He concludes (p. 349) that "...the major underlying conditions, for the ARR of any particular period to be a meaningful approximate of the IRR are that the accountant's income and asset valuations must approximate the economic income and economic asset values."

It is a sufficient justification that IRR be a desired measure, in which case it would be useful to have a conversion formula from the (usually available) ARR to the (usually not available) IRR (Livingstone and Salamon, 1970, p. 214).

The work by Livingstone and Salamon (1970), Stauffer (1971) and Gordon (1974) extends the earlier work of Harcourt (1965) and Solomon (1966). The important generalizations for ARR that best approximate IRR are two situations: first, for a firm in steady state growth at a rate g which equals IRR and second, ARR be measured net of depreciation. Otherwise, the findings of initial analytical work on IRR-ARR relation have generally been pessimistic. However, Gordon (1974) shows that the discrepancies between ARR and IRR are minimized if the accountant chooses a depreciation method that approximates the economic depreciation implicit in IRR.

Fisher and McGowan (1983, p. 84) reassert that the depreciation schedules affecting ARR differ from year to year and make the ARR does not equal IRR in general. They comment (p. 82) that the accountants view on depreciations may differ from economically acceptable definitions so that ARR provide almost no information about IRR. Fisher and McGowan develop an analytic model and showed that ARR is influenced by accounting methods, investment growth rates, and project cash flow profiles. They find (p. 84 and footnote 11) that if ARR is higher than the growth rate, then IRR is also higher than the growth rate; if ARR is lower than the growth rate, then IRR is lower than the growth rate; if ARR equals the growth rate, IRR equals to ARR (while ARR calculated on beginning-of-year total assets). They conclude (p. 90) that there is no way in which one can look at accounting rates of return and infer anything about relative economic profitability. This result is criticized by further studies, such as Long and Ravenscraft (1984).

Long and Ravenscraft (1984) have explicitly criticized the work of Fisher and MacGowan and, thus, implicitly criticized all of the analytic studies about the ARR-IRR relationship. Long and Ravenscraft (1984, p. 497) believe that it is inappropriate to draw conclusions about the relationship between the ARR and the IRR in empirical settings based upon the nature of the relationship in the highly simplified, hypothetical, and, perhaps, unrepresentative "examples" which are presented in the analytical studies.

Gordon and Stark (1989, p. 425) point out that it is not solely the accountant's inability to correctly measure economic depreciation that causes inaccuracies in the ARR as an indicator of the IRR. "Other inaccuracies are caused by the fundamental difference between cash flows and accrual accounting profit flows. ... Most importantly, these additional differences can cause a situation where the use of economic depreciation, as conventionally defined, could actually *decrease*, rather than increase, the accuracy of the accountant's rate of return". They model (pp. 427-429) the effect of a net difference between accrual accounting profit flows and the underlying cash flow pattern for a typical firm project, apart from those caused by accounting depreciation, on the relationship between ARR-IRR. They also find that under some conditions ARR equals IRR.

It should be noted the study of Beaver (1998) of measurement error on accounting net income and accounting rate of return[6]. The nature of the measurement error in general will be a function of (a) the cash flow pattern of the assets (including useful life), (b) the acquisition cost of the assets, (c) the accounting alternative chosen, (d) the growth rate, and (e) the internal rate of return. The accounting rate of return will not only depend upon the accounting method used but also the growth rate.

It seems clear that because of the different choice of accounting methods ARR can be viewed as a surrogate measure of IRR that will contain a measurement error in most settings. In general, there are opposing positions on whether or not the measurement error will affect ARR as a good approximate for IRR. Apparently, Long and Ravenscraft (1984) believe that the measurement error in the ARR is so small that it does not contaminate the results of empirical research on firm profitability, whereas Fisher and McGowan (1983) believe that the measurement error is potentially so severe that profitability research that relies on the ARR is as likely to be misleading as it is to be useful. Furthermore, the analytical research on the IRR-ARR relationship demonstrates that only under limited circumstances the periodic ARR can be expected to be a useful surrogate for ex post IRR.

3.2 Average ARR versus ex post IRR

Average ARR calculation was developed by Kay (1976). Gordon (1974) once points out that under some conditions ARR is on average approximately equal to IRR over the long-run. In addition, Long and Ravenscraft (1984, p. 494) believe that the use of

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[6] For more detail, see Beaver (1998, pp. 51-57).

accounting profits does on average, yield important insights into economic performance.

Kay (1976) develops a continuous time mathematical model to focus on the relationship between the internal and accounting rates of return over a finite time. He models (p. 449) under the condition of balanced growth a t-year-old machine that generates cash flows at a rate f(t) and requires expenditures at a rate g(t). Accountants depreciate it at a rate d(t), so that its book value at t will be,

$$v(t) = \int_0^t (g(x) - d(x))dx \qquad (14)$$

The accountant's rate of profit a(t) is defined as,

$$a(t) = \frac{f(t) - d(t)}{v(t)} \qquad (15)$$

The internal rate of return r is defined by the equation:

$$\int_0^\infty f(t)e^{-rt}dt = \int_0^\infty g(t)e^{-rt}dt \qquad (16)$$

If a(t) is constant and equal to a, then f(t) = av(t) + d(t) and so

$$\int_0^\infty f(t)e^{-at}\,dt = \int_0^\infty av(t)e^{-at}dt + \int_0^\infty d(t)e^{-at}dt \qquad (17)$$

where v(0) = 0, since t = 0 is the instant before the first expenditure on the machine is incurred. All expenditures are written off sooner or later, so that $\int_0^\infty g(t)dt = \int_0^\infty d(t)dt$ and v(t) $\to$ 0 as t $\to$ ∞; and since $\dot{v}(t)$ = g(t) - d(t) integration by parts gives

$$\int_0^\infty f(t)e^{-at}\,dt = \int_0^\infty g(t)e^{-at}\,dt \qquad (18)$$

comparing equation (18) with equation (16), we get a = r, and thus ARR = IRR. Kay noted (p. 449) that this result makes no assumptions about the shape of the f, g, and d schedules. In this case the ARR is not a misleading indicator.

Kay further states (pp. 451-452) that if a is the average ARR and if the value of capital employed is a or r, the weighted average ARR is equal to the IRR. More formally, if

$$a = \frac{\int_0^z a(t)v(t)e^{-at}dt}{\int_0^z v(t)e^{-at}dt} \qquad (19)$$

with integration and substitution of $\overset{\bullet}{v}(t) = g(t) - d(t)$, then $\int_0^z f(t)e^{-at}dt = \int_0^z g(t)e^{-at}dt$, and hence a = r.

Kay also models (pp. 454-456) a firm in steady state growth for which the ARR is constant and the book value of capital grows at rate n. If W is the economist's valuation of the firm, and V is that of the accountant, he develops the model which yields the relations as

$$(n - r)W = (n - a)V \qquad (20)$$

so that in special cases where n = r or W = V, then a = r. He concludes (p. 459) that if ARR is measured over a number of years, it will be an acceptable indicator of the true rate of return; if it is measured over a single year than it may prove seriously misleading.

Wright (1978) doubts about some technical points and statements on Kay (1976), while Kay (1978) gives him a reply afterwards. Wright agrees (p. 466) that over the entire life of an investment, the accounting rates of return must average out (with suitable weights) to the internal rate of return. However, Wright believes that the distortions need not be equal and opposite, i.e., the weighting of the average permits a few years of low ARR early in the life of the investment to offset an indefinitely large number of years of high ARR later, and vice versa. He thus concludes (pp. 466-468) that ARR can be over- or understated for an indefinitely long period that is still at the user's peril.

Kay (1978) replies that Wright emphasizes the difficulties of the intermediate case. Kay states (p. 469) that while IRR is equal to the weighted average of ARRs when there is a complete sequence of accounting information, substantial difficulties arise in attempting to estimate IRR when there is incomplete data in the absence of a plausible belief that ARR is constant. He thus confirms (p. 470) that if the value of

capital employed is a or r, the weighted average ARR, corrected for errors in opening and closing asset valuations, is equal to the IRR.

Whittington (1979) further investigates the deficiencies of ARR as a proxy for IRR. He aims (pp. 201-202) to "...define those uses in which the deficiencies of ARR are relatively unimportant and to identify the specific sources of deficiencies in ARR, so that they can be corrected or allowed for in uses in which they are potentially important." Moreover, he pointed out (p. 202) that although the user of ARR in Harcourt's words 'does so at his own peril', it seems likely that the absence of better information will force him to continue to use ARR. It is better to define the nature of the peril and draw up safety rules, rather than to forbid the use of ARR.

Based on his study, Whittington (p. 207) summarizes Kay's results as follows:

1) There is a general analytical relationship between ARR and IRR. IRR can be derived as an appropriately weighted average of ARRs.

2) For an individual project, this weighted average may be calculated exactly, but for a continuing firm, errors may remain because of the discrepancy between accounting and economic values of assets at the beginning and the end of the period.

3) If a simple unweighted average of a project's ARR is taken, this will be a good proxy for IRR when there is no time trend in ARR, and a perfect one when ARR is constant. When ARR declines through time, the simple average will underestimate IRR; when ARR rises, the simple average will overestimate IRR.

4) In the case of a firm in balanced growth, ARR is equal to IRR when the growth rate is equal to ARR (and therefore = IRR). In cases where the rate of growth is less than IRR, it is reasonable to assume that ARR $\geq$ IRR, because of the accountant's conservative tendency to undervalue assets.

5) One might reasonably expect that for a firm as opposed to a project, the process of aggregating a number of projects of different ages, length of life, etc., would lead to relative stability of ARR and thus to relatively small divergences between average ARR and IRR. One might also expect that the process of averaging over a longer period of years will diminish the effect of the discrepancies between the economist's and the accountant's valuations of opening and closing assets. Unless these discrepancies grow proportionately with time, their importance will be reduced because they will be quantitatively small relative to the flows, as the period for measuring the flows increases.

Whittington recognizes (p. 207) that there can be considerable divergences between ARR and IRR and that any correspondence between them in practice is likely to be a statistical average relationship rather than an exact one. He believes that ARR and IRR measures "...do have an analytical relationship to one another and that, in certain circumstances, there can be an exact correspondence".

However, Stark (1982) opposes this view. Stark notes that Kay's conclusions are drawn from a simple model of the firm that does not explicitly include expenditure on working capital requirements, loan financing, interest expenses and taxation expense. He thus demonstrates an extension of Kay's model to the more complicated world of working capital requirements, loan financing and taxation. His main conclusion (p. 525) is that ARR, even if measured over a number of years, is not necessarily an acceptable indicator of IRR.

Meanwhile, Peasnell (1982) extends the work of Kay (1976) on the ARR-IRR relationship. He supports the results of earlier work by using an arithmetic mean of ARR which is compared with IRR.

The objective of Peasnell's (1982) paper aims to examine both a firm's economic value and its economic yield derived from accounting data. He presents a common analytical framework concerning the mathematical connection between the conventional economic concepts of value and yield and 'clean surplus' accounting models of profit and return. He defines (pp. 362-371) ARR as

$$a_t = \frac{P_t}{A_{t-1}} \tag{21}$$

where A_{t-1} is the book value of assets at the beginning of the period, P_t is accounting profit which is equal to net dividends (C_t) paid plus the change in the net book value of the firm's assets ($A_t - A_{t-1}$) during the period, i.e., $P_t = C_t + (A_t - A_{t-1})$. Peasnell shows that ARRs are the direct accounting analogues of market rates of interest. He points out (p. 368) that "...the ARRs influence and determine the economic actions of subordinates in 'accounting markets' because the ARRs are principals of direct interest to the actors rather than surrogates of 'true yields' or unobserved market rates".

The economic approach to asset value is

$$V_0 = \sum_{t=1}^{N} v_t C_t + v_N R_N \tag{22}$$

where $v_t = \dfrac{1}{(1+i_1)(1+i_2)...(1+i_t)}$, and $i_1, i_2,...,$ are the one-period opportunity cost rates specified at $t = 0$; R_N is either the liquidating receipt or a valuation at horizon date N of the capital stock, depending on the circumstances. The excess net present value (NPV) is measured by subtracting the sacrifice value (C_0) from the economic value (V_0):

$$NPV = V_0 - C_0 = \sum_{t=1}^{N} v_t C_t + v_N R_N - C_0 \qquad (23)$$

Further, one can substitute C_t by $P_t - (A_t - A_{t-1})$, and define the discounted accounting valuation error at time N, which is $v_N(R_N - A_N)$ and thus $E_0 = C_0 - A_0$.

As $v_t - v_{t-1} = - i_t v_t$, equation (25) can be rewritten as:

$$NPV = \sum_{t=1}^{N} v_t(P_t - i_t A_{t-1}) + (E_N - E_0) \qquad (24)$$

In other words, NPV is equal to the sum of the discounted excess income plus the difference of accounting capital valuation errors. If $v_t = v^t$ and $i_t = r$ for all t and NPV = 0:

$$NPV = \sum_{t=1}^{N} v^t(P_t - rA_{t-1}) + (E_N - E_0) = 0 \qquad (25)$$

Rearranging terms, gives:

$$r = \frac{\sum_{t=1}^{N} v^t P_t}{\sum_{t=1}^{N} v^t A_{t-1}} + \frac{E_N - E_0}{\sum_{t=1}^{N} v^t A_{t-1}} \qquad (26)$$

and noting that $P_t = a_t A_{t-1}$, one obtains

$$r = \frac{\sum_{t=1}^{N} a_t (v^t A_{t-1})}{\sum_{0}^{N} v^t A_{t-1}} + \frac{E_N - E_0}{\sum_{t=1}^{N} v^t A_{t-1}} \qquad (27)$$

If the accounting valuation errors are offsetting, then $E_N = E_0$, and IRR is a straight forward linear weighted sum of ARRs:

$$r = \sum_{t=1}^{N} w_t a_t \qquad (28)$$

where $\sum w_t = 1$. Thus if a is constant, IRR equals a. Peasnell (p. 371) states that:

> "The IRR is defined to be constant throughout the investment holding period whereas ARRs can and do vary through time. Perhaps the most obvious way of utilizing a time-series of ARRs in practice is to take a simple arithmetic average of them and to treat the result as a proxy for the (constant) IRR."

In conclusion, Peasnell asserts (p. 379) that the mathematical relationships between accounting and economic yields "...hold regardless of the profit construct employed. Whether or not accounting yields have any economic significance outside of this mathematical relationship with the IRR depends on the accounting model involved."

It seems clear that the analytic work on the ARR-IRR relationship demonstrates that the average ARR can be viewed as a measure of the firm's IRR[7], while in some settings ARR will contain measurement errors (Stark, 1982). From this viewpoint, the controversy can be interpreted as a difference of opinion over the magnitude and extent of this measurement error in actual settings.

The behavior of this error term has recently been investigated by Steele (1995). Steele starts with Peasnell's (1982) model that IRR is found to be a weighted average of ARRs plus an error term which depends on opening and closing valuation differences. The discovery of Steele (p. 923) is that the error decreases as the length of time of measurement increases, reaching a minimum.

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[7] See particularly in Kay (1976), Whittington (1979), and Peasnell (1982).

It is worth noting the work of Edwards, Kay and Mayer (1987) who tried to justify the use of ARR in the assessment of the performance of activities. They illustrate (p. 63) that ARR computed over a segment of an activity's life time, using value-to-the-owner rules for book value of capital employed, can provide economically relevant information.

Using the book value of the firm's capital as a measure of capital input and output at the beginning and end of the segment respectively, Edwards et al. (p. 37) define ARR over a segment as the discount rate which makes the discounted value of the net cash flows over the segment plus the discounted book value of capital employed at the end of the segment equal to the book value of capital employed at the beginning of the segment. Formally, the ARR over the segment from the end of period 0 to the end of period T is given by $\alpha_{0,T}$ such that

$$V_0 = \sum_{t=1}^{T} \frac{(F_t - K_t)}{(1 + \alpha_{0,T})^t} + \frac{V_T}{(1 + \alpha_{0,T})^T} \qquad (29)$$

where V_0 is the book value of capital employed at the end of period 0; V_T is the book value of capital employed at the end of period T; F_t is revenue generated in t; K_t is new capital required in t; $\alpha_{0,T}$ is the ARR over (0, T) and all cash flows are assumed to occur at the end of the period.

The accounting profit of the activity in period t, Y_t, is defined as

$$Y_t = F_t - K_t + V_t - V_{t-1} \qquad (30)$$

which implies that depreciation in period t, D_t, equals (-(V_t - K_t) - V_{t-1}). Further, the accounting rate of profit in period t is

$$a_t = \frac{Y_t}{V_{t-1}} \qquad (31)$$

Substituting equations (30) and (31) in (29) and rearranging terms, one finds that

$$\alpha_{0,T} = \frac{\sum_{t=1}^{T} \dfrac{a_t V_{t-1}}{(1 + \alpha_{0,T})^t}}{\sum_{t=1}^{T} \dfrac{V_{t-1}}{(1 + \alpha_{0,T})^t}}$$ (32)

This means that the ARR over a segment of an activity's life ($\alpha_{0,T}$) segments' can be calculated as a weighted average of the ARRs of the activity in the individual periods of the segment, where the weights are the book values of capital employed in each period discounted at the ARR.

Edwards et al. (pp. 50-63) demonstrate that an expected accounting rate of return calculated on this basis can, when compared with the cost of capital, provide an appropriate signal to the investor as to whether the firm justifies further investment, maintenance of the current level of investment, or divestment. When calculated on an ex post basis, it can provide a signal as to whether there are barriers to entry in the industry which give rise to monopoly profits, or, in the case of a competitive firm, of the success of management during the past period.

Whittington (1988, p. 264) comments on the work of Edwards et al. (1987) providing a comprehensive case for a system of real-terms accounting, with the value-to-the-owner rule as the valuation basis and using a financial capital maintenance concept. Whittington explains (p. 264) that "the reason for this lies in the nature of the value-to-the-owner rules, which switch the valuation between replacement cost, net realizable value and discounted net present value of future receipts, depending upon the economic status of the relevant asset." He believes (p. 265) that current cost accounting has a sound justification in economic theory and the ARR plays an important role in the interpretation of such data and can be used as a proxy for the economist's internal rate of return.

Furthermore, Brief and Lawson (1992) present ARR as a key role to play in the valuation process and developed an accounting-based DCF formula to value a project. They extended the work of Peasnell (1982) on the economic significance of ARR. Assuming V_0 is the accounting book value at the beginning of the period, a is the constant accounting rate of return, and C_t is the net cash flows in period t, we get

$$V_0 = \frac{C_1}{(1+a)} + \frac{C_2}{(1+a)^2} + ... + \frac{C_n}{(1+a)^n} + \frac{V_n}{(1+a)^n}$$ (33)

where a is defined for any time segment in the life of a project as the weighted average of actual single-period rates of return, a_t, a in equation (33) is the algebraic equivalent of the value of a that solves:

$$a = \sum_{t=1}^{n} w_t a_t \qquad (34)$$

where the weights, w_t, are:

$$w_t = \frac{\dfrac{V_{t-1}}{(1+a)^t}}{\displaystyle\sum_{j=1}^{n} \dfrac{V_{j-1}}{(1+a)^j}} \qquad (35)$$

Brief and Lawson assert (p. 420) that "the definition of a as a weighted average of ARRs is important because it provides insight into the nature of the bias in using more direct ways to transform a sequence of ARRs into an estimate of a".

Brief and Lawson also examine the relationship between a simple arithmetic average of the single-period ARRs and the pseudo IRR for different assumptions about the time series of a_t. They show (pp. 420-421) that when the time series of a_t is reasonable stable, the bias in estimates of a based on a simple arithmetic mean of ARRs will not be large. They conclude that understanding how discounted cash flow analysis can be based directly on accounting data leads to a greater appreciation of the general nature of accounting and also provides a compelling reason to give ARR a more prominent place in financial statement analysis to value a firm.

3.3 Average ARR versus ex ante IRR

Although the discounted cash flow (DCF) rate for a single project is a well known and widely used measure in capital analysis, the corresponding DCF measure for an ongoing company is not generally available. Recent literature concerning ARR and IRR relationships has focused on developing more realistic economic frameworks and generating empirical evidence. Kelly and Tippett (1991), Butler et al. (1994) and Kelly (1996) focus on our third scenario: average ARR with ex ante IRR estimate relationships.

Kelly and Tippett (1991) develop a continuous time econometric model, and for illustration purposes carried out limited empirical analysis, to assess whether ARR provides a reasonable reflection of the economic returns corporations were expected to earn over its remaining lives. Due to the difficulties in calculating unrealized cash flows, they assume (pp. 323-328) future cash flows generated by a stochastic process and develop a non-linear regression technique to estimate the cash flow parameters, from which it was feasible to derive economic return estimates. Employing a paradigm that assumes either an upward or downward movement in accumulated cash flow at time t, they model (pp. 323-324) cash flows as:

$$dC(t) = [\alpha e^{kt} + \beta C(t)]dt + dW(t) \tag{36}$$

where $C(t)$ denotes the level of accumulated cash flows per security at time t, $dC(t)$ denotes the instantaneous or periodic cash flow per stock over the interval $[t, t + \Delta t]$, α, k and β are parameters to be estimated and $dW(t)$ is a white noise process with variance parameter σ^2.

If $\int_0^\infty e^{-it} dC(t)$ is the net present value of the firm's future cash flows, utilizing equation (36) and taking expectations, they showed (p. 325) that

$$E_0 \left(\int_0^\infty e^{-it} dC(t) \right) = \frac{i\alpha}{(i-\beta)(i-k)} \tag{37}$$

where $E_0(.)$ is the expectations operator at time zero. This expression can then be equated with the firm's time zero security price, and by solving for i, an estimate of IRR is obtained.

Kelly and Tippett apply their model to five large Australian companies for the fifteen-year period from 1973 to 1988. The ARR for each corporation was estimated by averaging the ARR over the five years period ending in 1973 (i.e., from 1969-1973). Two other ARR measures were used: the first was the ARR for the 1973 fiscal year and the second was a weighted average of ARRs from 1969 to 1973 in which the weights were determined using the sum-of-years'-digits formula. However, tests show that there were no significant differences between the results obtained using these alternative ARR approaches (see p. 326 and footnote 12).

Annual cash flow and ARR data for five large listed companies from 1973 to 1988 were collected and measured following the 'clean surplus' method consistent with the examples of Kay (1976). Using a non-linear regression procedure, they estimate the parameters, α, β, k, and σ^2 of equation (36) as $dC(t) = [\alpha e^{kt} + \beta C(t)$

]dt + dW(t). Together with the corporation's share price at the beginning of the fifteen year period, IRR was then estimated using the probability density function implied by the stochastic process, i.e.,

$$\frac{i\alpha}{(i - \beta)(i - k)} = H \tag{38}$$

where H is the corporation's share price at the beginning of the fifteen year period. Standard normal z scores[8] for each firm were used to compare average ARR with estimated IRR. Results obtained show that four of the five z scores are different from zero at any reasonable level of significance. Kelly and Tippett thus conclude that ARR is potentially a poor and misleading surrogate for ex ante IRR (pp. 325-327). Meanwhile, they note that further research should apply a larger sample.

Butler et al. utilize the Kelly-Tippett method and applied the three definitions of cash flows to estimate IRR. They provide an empirical analysis of the time series properties of ARR in order to test for IRR and ARR differences. To analyze the time series properties of ARR, they select a sample consisting of 195 non-financial British companies having a continuous set of financial information for the 23 years ending December 31, 1991. They show (p. 304) that "...over this period the ARR varies cyclically, roughly in line with variations on real economic activity."

To test whether the ex post ARR, averaged over a 'short' period of time, can be regarded as a satisfactory proxy for the economic return a corporation is likely to earn over its remaining life, Butler et al. (pp. 304-306) run time-series regressions and represented that there are no significant differences between long-term average ARR and the (ex post) average ARR. Therefore, they use simple averages of four ARRs ending in 1972/1973, and cash flow figures for 18 years ending in 1990/1991 in which three alternative cash flow definitions (see section 2.1) are applied to reveal the impact of allocation decisions inherent in determining accounting data. The final sample, after eliminating companies where the regression procedures provided an unsatisfactory fit to the cash flow data, reduced to 156 firms for cash flow definition one, 146 firms for cash flow definition two and 156 firms for cash flow definition three (p. 309).

Following the Kelly-Tippett (1991) method, Butler et al. observe (pp. 312-313) that the relationship between ARR and IRR "takes a more sophisticated form than that implied by the statistical methodology used so far". They also find (p. 314)

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[8] $z = \sqrt{\dfrac{2H((i - \beta)(i - k) - i\alpha)}{i(i - k)}} \bigg/ \sigma$

monopolists manipulating their accounting profits in a way that "the higher the IRR, the more the incentive for downward manipulation of the ARR." The obtained results show (p. 315) that (1) ARR follows a mean reversion process, (2) on average, IRR is significantly greater than ARR, (3) on average, ARR is inversely related to IRR, although only weakly so, and (4) for given IRR, managers of large firms report lower ARRs than the managers of smaller firms, although the relation is not strong.

Kelly (1996b) investigates the validity of using ARR as a monitoring proxy for IRR by utilizing randomly selected annual reports of 44 Australian listed firms over the 23 years from 1968 to 1990. He supports (p. 353) the idea employed by Kelly and Tippett (1991) and Butler et al. (1994) that IRR can be estimated by determining the appropriate discount rate which equates the expected net present value of the firm's future cash flow stream with its initial or time zero stock price.

Following the Kelly-Tippett framework, Kelly uses ARR (calculated from 1969 to 1973 data) as the discount rate for estimating the IRR that a corporation will earn over its remaining life where time zero is the firm's balance sheet date in 1973 and infinity represents the entity's longevity or approximates its expected termination date. By modeling uncertainty and imposing assumptions on stochastic cash flows, empirical results obtained confirm that ARR is an unreliable symbol for IRR.

Empirical researchers, using IRR and ARR somewhat different to that in earlier work, present that ARR is an unsatisfactory proxy for IRR. Although empirical analysts hope to provide evidence of whether ARR is the best approximate for IRR, they have indeed presented some important properties of ARR which are useful in financial statement analysis.

3.4 Cash Recovery Rate (CRR) versus IRR

Ijiri (1978, 1979, 1980) and Salamon (1982, 1985) have suggested that estimates of economic performance can be obtained by converting an estimate of a firm's cash recovery rate into an estimate of its economic rate of return. This approach depends primarily on the assumption that the firm repeatedly invests in the project (firm is regarded as collection of projects), and the time profile of cash recoveries associated with the project in which the firm repeatedly invests. Together with the (constant) rate of growth of annual investment and the (constant) rate of change in the general price level, researchers developed cash recovery rates related to internal rate of return (Stark, 1987, p. 99).

Ijiri (1978) initially defines the cash recovery rate as the ratio of the sum of funds from operations, interest expense, sale of investments, sale of property, plant, and equipment, and the decrease in total current assets (if it occurs) to the average of beginning and ending total assets.

Under the condition of constant investment growth, Salamon (1982) shows that a firm's CRR converges to a constant given in the following equation when the cash flow profile of the firm's projects can be represented by a single parameter:

$$CRR = \frac{g}{(1+g)^n - 1} \cdot \frac{(1+g)^n - b^n}{1+g-b} \cdot \frac{(1+r)^n(1+r-b)}{(1+r)^n - b^n} \tag{39}$$

where g is the constant growth rate in gross investment, n is the useful life of firm's "representative" composite project, r is the internal rate of return of firm's representative project, and b is the project cash flow pattern parameter. It also assumed that each firm is a collection of projects that have the same useful life, cash flow profile, and IRR (Salamon 1985, p. 498).

Salamon (1985) demonstrates the nature of the measurement error in the ARR. He believes that the conditional IRR estimates are free from some of the sources of measurement error which are known to contaminate the ARR. Salamon (p. 500) examines 197 firms of the five years period 1974-78 from COMPUSTAT. He first calculates CRR for each firm for each year as cash recoveries divided by the gross investment for the year. Then he estimates the useful life of the projects and the investment growth rate of each firm in order to convert (via equation 38) each firm's CRR into conditional estimates of its IRR. Finally, these estimated values along with the values of the parameter b (i.e., b = 0.8, 1.0, 1.1, or a random assignment from (0.8, 1.1)) were substituted into equation (38) and produced four conditional IRR estimates for each firm in the sample. Salamon relied on conditional IRR estimates to show that the ARR contains systematic measurement error in the relationship between firm profitability and firm size. It is an indication of a relative advantage of the CRR approach that the relationship between size and profitability is shown to be different when measures of economic performance derived from CRRs are employed instead of ARRs (Stark 1993, p. 202).

Stark (1987) points out that, if firm projects have multiple investment flows, the cash recovery rate, as defined theoretically, it is impossible to observe the CRR from published financial statements. Lee and Stark (1987) try to test whether the cash recovery rate, as defined in the Ijiri model, is conceptually suitable for conversion into a corporate IRR. Their analysis suggests that it might not, even if an accounting system could be devised suitable for the production of the appropriate cash recovery rates.

Some researchers have expressed criticism on the cash recovery rate approach. Brief (1985) gives the view that it assumes that a firm is a collection of projects that differ only in scale. Every project has the same life of n years, cash flow profile and IRR. The cash flows in any year during the life of a project are assumed to be related to cash flows in the first year by a cash flow profile parameter, b. The CRR method, in effect, assumes that the firm's cash flows is "known". Therefore, as an alternative to the CRR method, the IRR can be determined in the more usual way directly from the firm's cash flows. In comparison to the CRR method, the direct method does not require data about a project's life (n). Nor does it require information about a project's cash flow profile.

Others made more technical criticisms. Lee and Stark (1987) point out that a cash recovery rate based on Ijiri's (1978) system of cash flow accounting might well be unsuitable for use in estimating economic rates of return. Lee and Stark (p. 129) argue that it is not axiomatic in current practice to discount investment flows at a different rate from recovery cash flows, although such a treatment might have its use when net cash flows are negative at dates subsequent to the date of the initial capital investment.

The purpose of Salamon's (1988, p. 269) paper is to provide evidence on whether the properties of the ARR-IRR relationship in analytic models are indicative of the properties of the actual relationship between ARR and IRR in a sample of US manufacturing firms. He discusses the suggestion of Stark (1987) and Lee and Stark (1987) and still believes (p. 277) that the IRR estimates produced by the use of equation (38) and Ijiri's estimates of CRRs "capture enough of the systematic properties of the firm's underlying IRR that the estimates can be useful in providing evidence on whether or not the analytic ARR-IRR literature is or is not just a set of unrealistic examples".

Based on his earlier work, Salamon (p. 275) considers the linkage between the depreciation method adopted for financial reporting purposes and the cash flow profile of the firm's projects. He therefore denotes (p. 276) three conditional IRRs describing straight-line, accelerated depreciation and both methods applied. He then represents the relationship among ARR, IRR and investment growth g. The sample consisted of 965 USA steady-state corporations over the five years 1976 -1980. Only firms that displayed positive growth over the five-year period were included in the sample (p. 277). Given values of average cash recovery rate (CRR), average life of each firm's projects and g for each firm in the sample, the conditional IRR was estimated. Empirical evidence obtained (p. 285) supports that "...it is not investment growth itself that impacts ARR but the difference between investment growth and IRR. Furthermore, whether the impact of g on ARR is in the direction of increasing ARR (relative to IRR) or in the direction of decreasing ARR (relative to IRR)

depends upon the time shape of the cash flow profile of the firm's projects." Salamon concludes (p. 284) that "ARRs of real-world firms are just as systematically influenced by such profit extraneous factors as the rate of growth in gross investment and depreciation method as are the hypothetical firms which were analytically created."

Stark (1993) doubts that the conventional empirical definition of the CRR will not measure accurately the true CRR. He presents (p. 206) the characteristics of error in CRRs when some of total periodic investment is not capitalized in the accounting records, and when components of total periodic investment are retired from the accounting records in advance of the composite project to which they belong ceasing to be active.

Peasnell points out (1996, p. 294):

"The CRR approach now seems less promising than it once did. A difficulty is that once the cash flow profiles of a group of firms have been empirically assessed, the IRR can be derived directly, without going through the roundabout step of computing the conditional IRR (CIRR). The CIRR has been used as a means of obtaining estimates of IRR, which can then be used to test some of the predictions of the analytic literature concerning the errors in ARR (Salamon 1985, 1988). This assumes, of course, that a proxy for IRR obtained from CRR is more likely to approximate the true underlying IRR than is ARR itself."

4. Conclusions

Using the ARR to estimate the IRR is a fundamental application of published financial statements. The early literature on this issue, e.g., Harcourt (1965), Livingston and Salamon (1970), Stauffer (1971) and Fisher and McGowan (1983), draws such dismal conclusions on the perils of this endeavor as to virtually undermine its intellectual credentials. However, due to the insights of Kay (1976), Peasnell (1982), Whittington (1979, 1988), Steele (1995) and Brief and Lawson (1992) scholars are now less pessimistic about the conditions under which accounting measures can be used for valid economic analyses.

Employing the 'clean surplus' concept (Peasnell, 1982) and the 'value-to-the-owner' idea (Edwards et al., 1987), researchers recognize that accounting information has some economic significance. The method of estimating IRR by a firm's stock price (Kelly and Tippett, 1991; Butler et al., 1994; Kelly, 1996b) leads the research to

the uncertainty which assumes that firm cash flows are generated by a stochastic function of time. These studies show ARR as a poor estimator of IRR.

Research on the IRR-ARR relationship has more than a 30-year history and most of it has focused on answering the question of whether or not ARR is an accurate estimator for IRR. The possibility of using ARR into valuation proceses provides a useful way when one conceptualizes how market value relates to accounting data. It also requires empirical work on it.

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