Chapter 1

Cluedo

Imagine a country mansion with a couple of partying guests. Suddenly the host is discovered, lying in the basement, and murdered. The guests decide to find out among themselves who committed the murder.

The body is discovered by the butler, under suspicious circumstances that indicate that the location is not the actual murder room. In order to solve the murder it is required to find out who the murderer is, what the murder weapon was, and in which room the murder was committed. The butler is exonerated, the six guests are therefore the suspects. The guests are: Colonel Mustard (colour yellow), Professor Plum (colour pink), the Reverend Green, Mrs. Peacock (colour blue), Ms. Scarlett (colour red, i.e. ‘scarlet’), and Mrs. White. There are six possible murder weapons: candlestick, rope, leaden pipe, wrench, gun, knife. The house consists of nine different rooms: hall, kitchen, dining room, study, sitting room, patio, ballroom, library, pool room.

The game consists of a board with a picture of the house, with the nine rooms in it and ‘paths’ leading in a certain number of steps from one room to another. Also there are six suspect cards, six weapon cards, nine room cards. A pair of dice, six pawns for the (six) players, in colours matching the guests’ names, and six weapon tokens complete the picture.

There are six players. The three types of cards are shuffled separately. One suspect card, one weapon card and one room card are blindly drawn and put apart. These ‘murder cards’ represent the actual murderer, the murder weapon and the murder room. All remaining cards are shuffled together. They are then dealt to the players. Every player gets three cards. Some player starts the game, which is determined by throwing dice, or by the general rule that the player with the red pawn starts. That player then makes a move. A move consists of the following:

- throwing the dice

- trying to reach a room by walking your pawn over the game board

- if a room is reached voicing a suspicion about it, i.e. about a suspect, a weapon and that particular room
Figure 1.1: Keychain with miniature Cluedo game board

- gathering responses to that suspicion from the other players

- optional: making an *accusation* about a suspect, a weapon and a room

The number of steps on the board may not exceed the outcome of the throw of dice. As a consequence of the suspicion, the pawn with the same colour as that of the suspected player is moved to the suspected room. The other players are supposed to respond to the suspicion in clockwise fashion: the first player that holds at least one of the three cards mentioned in the suspicion, must show exactly one of those to the requesting player, and to him only. This ends the move. Whoever is next in turn is again determined clockwise. Just like a suspicion, also an *accusation* is the combination of a suspect, a weapon and a room card. Each player can make an accusation only once in the game. It is not voiced but written down. The accusing player then checks the three murder cards, without showing them to others. If the accusation is false, that player has lost and the game continues. The first player who correctly guesses the murder cards, wins the game. Note that, although pawns are identified with guests, you don’t even know whether you have committed the murder ‘yourself’.

The suspicion one makes is naturally supposed to elicit as much information as possible. It is based on knowledge of one’s own cards and on knowledge of other players’ cards. In order to justify preferring one suspicion over another, we have to determine *what* knowledge is gained from the possible answers to a suspicion.
1.1 Actions

We first discuss some examples of moves in Cluedo.

Example 1

Assume that one of the cards of player 3 is the candlestick card and that two of the cards of player 4 are the green card and the ballroom card, see figure 1.2. Assume that player 1 starts the game. In his first move, player 1 reaches the ballroom. Now the following happens, see also figure 1.3:

- **suspect** Player 1 says ‘I think Reverend Green has committed the murder with a candlestick in the ballroom’;

- **noshow** Player 2 says that he does not have any of the requested cards;

- **show** Player 3 shows the candlestick card to player 1;

- **nowin** Player 1 ends his move.

Players 4, 5 and 6 never get to respond to the suspicion by player 1. If 4 had been asked to, he could have chosen between two cards to show to player 1.

What is the effect of these four actions on the players' knowledge? Note that, as this is the first move in the game, the players only know their own cards.

**suspect** Any combination of a room, weapon and guest card can be asked, provided one occupies that room. Also, it is permitted to ask for one or more of one's own cards (so that one knows the suspicion to be false). Therefore, nothing can be deduced from the suspicion.

**noshow** After player 2 has said that he does not have any of the requested cards, this is commonly known to all players: player 1 knows that player 2 doesn’t have
them, but also player 5 knows that player 1 knows that, etc. What can further be deduced from that information depends on the players' own cards. E.g. player 5 now knows that player 2 does not have 6 particular cards from the total of 21 cards: the three cards asked for by 1, and the three (different) cards that 5 holds himself.

**show** Player 3 has the candlestick card and shows this card to player 1. He shows the card to player 1 *only*, by handing the card face down to player 1. Player 1 then looks at the card, and returns the card the same way. The other players therefore only see that a card has been shown, and know that the others have seen that, etc.

Player 1 now knows that player 3 holds the candlestick card. Player 1 doesn’t know whether player 3 holds one, two or all three of the requested cards. That he just holds candlestick, is only known by player 4, by deduction. Curiously enough, player 1 doesn’t know that player 4 knows that. Nor does player 3. Common knowledge among the 6 players is only, that player 3 holds at least one of the three requested cards. From this, e.g., everybody can deduce that 3 does not hold the cards white, scarlett, and conservatory.

**nowin** Player 1 ends his move. This is an implicit action, only inferred because 1 does not make an *accusation*. A successful accusation corresponds to publicly announcing knowledge of the murder cards. Not accusing therefore corresponds to a public announcement that you are ignorant of the murder cards. In this example, it is unclear how that announcement changes the knowledge of other players. We therefore present a different example in which it is obvious:

**Example 2**
Suppose that player 1 had moved to the kitchen instead of to the ballroom, and that the murder cards are ‘kitchen, scarlett, knife’. Player 1, by mere incredible luck, chooses to voice the suspicion ‘I think that Scarlett has committed the
murder in the kitchen with a knife'. The player playing red moves his pawn to the kitchen. None of the other players can show a card. Player 1 writes down the accusation ‘kitchen, scarlett, knife’, checks it, and announces that he has won.

Example 3
Now compare the previous example with the state of the game where, instead, the cards ‘kitchen, scarlett, knife’ are not on the table but are held by player 1. Again, 1 voices the suspicion ‘I think that Scarlett has committed the murder in the kitchen with a knife’. Obviously, again none of the other players can show a card. However: player 1 now ends his move. The other players now deduce that player 1 holds at least one of the requested cards! Observe that they do not know that he holds all three of them: it could have been the case that, instead, player 1 only holds ‘kitchen’ and that the murder cards are ‘conservatory, scarlett, knife’. Once more, nobody would have shown a card to 1.

Apart from the actions apparent in example 1, three other sorts of action may occur in Cluedo: **accuse**, **check** and **announce**.

**accuse** At any moment during his move, a player may **guess** what the murder cards are, i.e. he may write down a final accusation, that will then be checked by him. **Any** combination of a room, weapon and guest card is permitted. One does not have to occupy the room of the accusation, as was required for a suspicion. Note that the content of the accusation is hidden to other players. An accusation does not effect the knowledge of the players.

**check** After a player has checked his accusation and has told the other players that he has indeed won, it is public knowledge (common knowledge to all) that he knows the cards on the table. However, as the game is over, it serves no purpose to describe these changes. If he told them that he lost, the game continues. Now the other players haven’t learnt anything at all, because the content of the accusation was hidden to them.

**announce** What makes a real play of the Cluedo game even more interesting, is that players allow themselves slips of the tongue such as: ‘Ha! I now know who the murderer is’, or: ‘Arrrgh, I still don’t know the murder room!’ Such announcements often result in interesting updates.

**On perfect logicians** We assume that the players are perfectly logical. In actual plays of Cluedo, this assumption is dangerous. You may have all the knowledge required to deduce the cards on the table, but not make the deduction. Also, you may have forgotten earlier moves. Therefore, ending your own move does not imply that you cannot win. The next player to move, who is reasoning from the incorrect assumption that you do not know the murder cards, may now incorrectly deduce what these cards are, make that accusation, and lose.
Although it is illegal not to show a card if you hold it, it is perfectly legal not to win even though you can.

1.2 Strategy

Some actions affect the knowledge of the players, such as show. Other actions don’t, such as accuse. However, actions also may affect the beliefs of the players. Such beliefs determine preferences among their strategies: what suspicion to make, which card to show, make an accusation now or later? E.g., one can ask for a combination of three cards from which one holds either none, one, two or three oneself. From what type of suspicion can we expect to gain the most information? We discuss this topic by way of examples.

Example 4
It is your turn. You may go to either the billiard room or to the conservatory. You know that player 2, the first to answer your suspicion, has the billiard room card or the scarlett card or the rope card. You don’t know whether 2 has the conservatory card. Should you prefer a suspicion about the card that he is more likely to have, billiard room? Or about the card that he is less likely to have, conservatory? If you ask for the conservatory card, and if 2 does not hold that card, the next player after 2 will still have to answer your request, so you can gather yet more information in your move. On the other hand, now everybody will know that that 2 does not hold the conservatory card, which may not be to your advantage.

Example 5
You are being asked to show one of the cards scarlett, gun, and kitchen. You hold the cards scarlett, gun and conservatory. Is it better to show scarlett or gun? If you have previously shown scarlett to the requesting player, it is better to show scarlett again.

Example 6
You have just been shown a card, and have to decide whether to pass your move to the next player. You know the murder room and the murder weapon, and you know that the murderer is scarlett or green. Should you make an accusation or should you wait another round? This may never come to pass, as one of your opponents may win before. Now imagine that you also know that one of your opponents must already have gained full knowledge of the murder cards because of your move. Now, clearly, you must guess the murder cards and make an accusation.

Example 7
It seems not smart to ask for three cards that you all hold yourself, because you will not gain any information from the other players that way. However, you may
successfully mislead your opponents that way. By tricking an opponent into making a false accusation, based on the incorrect assumption that you have not asked for your own cards, you may get another turn and a chance to win. Otherwise, that opponent may have won instead, by preferring a different accusation (from the false one that he actually made), that turns out to be correct.

**Example 8**

Apart from the issue what suspicion should be preferred, actual Cluedo players do prefer some suspicions over others. It is claimed to be a good strategy to try to prevent opponents from reaching a specific room. Suppose you want to prevent player 2, who plays white, to reach the kitchen. You can try to prevent that by suspecting white to have committed the murder in the ballroom, in your turn. Also, actual Cluedo players often follow the tactic of asking for a combination of cards from which they hold one or two cards themselves. If we take (a computational model of) limited processing capability and limited short-term memory into account, this behaviour may be quite justified, although not rational. See, for a different application, [Taa99].

**1.3 Simplifications**

Before we model Cluedo game states and describe Cluedo game actions, we make some simplifications: We disregard the role of the board, dice and pawns. We only model actions that affect the knowledge of the players. We ignore that there are different types of cards. As we are only interested in the dynamics of knowledge, the second simplification is obvious. As concerns the first and third, we will argue that the difference with the real game is less than one may think.

**Board, dice and pawns** Board, dice, and pawns determine which suspicions players can make. The outcome of the throw of dice determines which rooms you can reach with your pawn, and therefore about which rooms you can utter a suspicion. Also, when voicing a suspicion, the pawn for that guest is moved to the room of the suspicion. Therefore the player with that pawn has to start his next move from that room. Again, that determines what room that player can reach later.

Disregarding board, dice and pawns is less of a simplification than one might think, because one can generally reach a room, and because it is totally unclear why some suspicions should be preferred over others.

For each player, the first move in the game starts from his initial pawn position. For Peacock, the closest room is reached from that position in 7 steps, for the other pawns this is 8 steps. Therefore, the player playing with the pawn Peacock will reach a room with probability $\frac{21}{36}$. For the other players the probability is $\frac{15}{36}$.

Other moves typically start from a room. From any room on the board, the number of steps needed to reach the closest room is at most 4. (From a corner
room one can reach the opposite corner with any throw.) The average outcome of a throw of – two – dice is 7. Therefore, the probability to reach some other room in one’s move is at least \( \frac{33}{36} \). Also, one may ask a different question about the room one already occupies.

**Actions that do not change knowledge** Some sorts of action do not effect the knowledge of players: suspect, and accuse. Although a suspect action has no epistemic effects, it raises the issue to which noshow and show actions are the answer. We will model the combination of a suspicion, i.e. a question, with an answer to it as a game action in the more technical meaning of the word. Similarly, an accuse action raises an issue, although not publicly. If an accusation is confirmed by a check action, that combination has epistemic effects. However, as the game is over, we are not interested in those effects. If the accusation is falsified by the check action, the players are just being told that it is unsuccessful. In that case there are no epistemic effects. An announce action is not strictly a move according to the rules of Cluedo, also it is always to your disadvantage. Therefore, we will only model the actions noshow, show and nowin.

**Types of cards** Not just any suspicion can be made but only a suspicion consisting of a card of each type. This restricts the strategies for gathering information. Also, not just any three cards lie on the table but one of each type. This restricts the number of different deals of cards that players have to consider. In the initial state of the game there are \( 6 \times 6 \times 9 = 324 \) possible combinations of cards on the table. Without this restriction, there would have been \( \binom{27}{3} = 1330 \) to consider.

The epistemic consequences of a show action are independent of the type of card that is shown. Also, the issue of strategic preference seems complicated enough when all cards are of the same type. Therefore, we have abstracted from that information too.

### 1.4 Historical note

Cluedo was invented by Anthony E. Pratt, a solicitor’s clerk, in 1944. He said to have invented it when he was temporarily laid off because of World War II and instead doing a, mostly boring, fire brigade duty. Cluedo was first marketed by Waddington’s Games, England, in 1949. In the USA, the game is called Clue instead of Cluedo. Anthony Pratt died in 1994, in obscurity. His death only became generally known in 1996, after a public appeal by Waddington’s (he had already sold his rights to the game in the fifties). His tombstone reads ‘inventor of Cluedo’.