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Monsees, G.; Scherpen, J.M.A.

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G. Monsees
G.Monsees@ITS.TUDelft.nl

J.M.A. Scherpen
J.M.A.Scherpen@ITS.TUDelft.nl

Delft University of Technology
Faculty of Information Technology and Systems
Systems and Control Engineering Group
Delft, The Netherlands

Abstract

Sliding Mode Control is a well-known technique capable of making the closed loop system robust with respect to certain kinds of parameter variations and unmodeled dynamics. The sliding mode control law consists of the linear control part which is based on the model knowledge and the discontinuous control part which is based on the model uncertainty. This paper describes two known adaption laws for the switching gain for continuous-time sliding mode controllers. Because these adaption laws have some fundamental problems in discrete-time, we introduce a new adaption law specifically designed for discrete-time sliding mode controllers.

1 Introduction

Sliding mode control is a well known robust control algorithm for linear- as well as nonlinear systems [2],[3],[5],[9]. Continuous-time sliding mode control has been extensively studied and have been applied to various applications. Much less is known of discrete-time sliding mode controllers, in practice it is often assumed that the sampling frequency is sufficiently high to assume that the controller is continuous-time [12]. Another possibility is to design the sliding mode controller in discrete-time, based on a discrete-time model, however stability has not yet been assured [1],[4],[7],[10],[13].

This paper focuses on an adaptive switching gain for a discrete-time sliding mode controller. Section 2 introduces two known adaption laws for the continuous-time sliding mode controller which are discretized in section 3 to work in discrete-time. The first adaption law (Method I) has some severe drawbacks, even in continuous-time, and the second adaption law (Method II) has some problems occurring in discrete-time. To overcome the drawbacks these two adaption laws have in discrete-time, a new adaption law specifically designed for discrete-time sliding mode controllers is introduced (Method III) in section 3.4. Section 4 demonstrates the Method II and Method III adaption laws by a simple simulation example of a pendulum. Finally section 5 presents the conclusions.

2 Continuous-Time Sliding Mode Control

2.1 Introduction

We consider the (single input) system:

$$\dot{\mathbf{x}}(t) = (A + \Delta A)\mathbf{x}(t) + (B + \Delta B)u(t) + d(t) \qquad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state of the system, $u(t) \in \mathbb{R}$ the input, $d(t) \in \mathbb{R}^n$ an unknown disturbance, (A, B) the nominal model and $(\Delta A, \Delta B)$ the modeling error. The disturbances and modeling errors are assumed to be matched, i.e, they are in the subspace spanned by B and thus can be directly cancelled by the input. For simplicity we will omit the time variable t in the rest of this paper. We will now define the switching surface S by:

$$S = \Lambda \mathbf{e} \qquad (2)$$

with $\mathbf{e} = \mathbf{x} - \mathbf{x}_d$ and $\Lambda \in \mathbb{R}^{1\times n}$. After transforming the system to the regular form Λ can be determined with the aid of a classical control method [3].

We now define a Lyapunov function $V = \frac{1}{2}S^2$ which proves asymptotical stability if

$$\dot{V} = S\dot{S} \leq -\eta|S| \qquad (3)$$

($\eta > 0$) is fulfilled. Inequality (3) is called the *Sliding Condition* [8] or *Reaching Law* [4], it guarantees that the system will (asymptotically) reach sliding motion. If we choose a constant $K \geq \eta$ we get $\dot{S} = -K\ \mathrm{sign}(S)$. Substituting equations (1), (2) and $\mathbf{e} = \mathbf{x} - \mathbf{x}_d$ in the previous equation results in:

$$u = -\frac{1}{\Lambda(B + \Delta B)}\left(\Lambda(A + \Delta A)\mathbf{x} + \Lambda d - \Lambda\dot{\mathbf{x}}_d - K\ \mathrm{sign}(S)\right) \qquad (4)$$

where of course $(\Delta A, \Delta B)$ and d are unknown. The sliding mode controller is now defined as:

$$u = \underbrace{-\frac{1}{\Lambda B}\left(\Lambda A\mathbf{x} - \Lambda\dot{\mathbf{x}}_d\right)}_{=u_l}\underbrace{-\frac{1}{\Lambda B}K\ \mathrm{sign}(S)}_{=u_d} \qquad (5)$$

with $\Lambda B \neq 0$ which is easily satisfied if $B \neq \mathbf{0}$ since Λ is a design parameter. The controller part u_l is called the *linear control part*, if the system is perfectly known (and with the proper initial conditions) it keeps the system on the desired surface $S = 0$. The controller part u_d is called the *discontinuous control part*, it drives the system towards the switching surface $S = 0$. In case $\Delta B = \mathbf{0}$, the sliding condition (inequality (3)) is met if:

$$K \geq \eta + |\Lambda(\Delta A\mathbf{x} + d)| \tag{6}$$

If the above condition is fulfilled (and the modeling error is matched) then, once the system is in sliding mode, the system's dynamics become invariant to the modeling errors [9]. This rule can be extended to disturbances and uncertainties in B, see for example [8].

One major problem in determining the switching gain in this way is that one has to know the bound on the uncertainty. In practice this information may not always be available so the switching gain has to be determined by experimentation (tuning) or by the use of an adaption law for the gain. Another disadvantage is that the bound on the error is not only a function of the model but also on the demands. A slowly varying and amplitude limited input signal could result in a smaller error than a rapidly varying and high amplitude input signal. Determining the switching gain for the worst case will result in a relatively high switching gain which leads to high controller activity. It would be better to have a switching gain which is automatically adapted to the circumstances, two methods methods of doing this are described in the next sections.

2.2 Adaptive Switching Gain, Method I

The most straightforward adaption mechanism for the switching gain is given by Wang et. al. [11]:

$$\hat{K} = \int |S| dt \tag{7}$$

This adaption law is based on the fact that once the switching gain is sufficiently large, the system will be forced to the switching surface $S = 0$. However, this adaption law has three major drawbacks:

(1) In case of a large initial error, the switching gain $\hat{K}$ will increase fast due to this error and not because of a model-mismatch. This may result in a switching gain which is significantly larger then the desired gain.

(2) Noise on the measurements will prevent S to be exactly zero so the adaptive gain will continue to increase.

(3) The adaption law can only increase the gain but never decrease it. So if the circumstances change such that a smaller switching gain is permitted the adaption law is not able to adapt to these new circumstances.

2.3 Adaptive Switching Gain, Method II

Another way of determining the switching gain is by the adaption law introduced by Lenz et. al. [6]. For this adaption law it is necessary to replace the switching term $-\frac{1}{\Lambda B}K\ \mathrm{sign}(S)$ in the sliding control law (5) by:

$$u_d = -\frac{1}{\Lambda B}\hat{K}\ \mathrm{sat}(\frac{S}{\Phi}) = \begin{cases} \text{if } -\frac{1}{\Lambda B}\hat{K} & S > \Phi \\ \text{if } -\frac{1}{\Lambda B}\hat{K}\frac{s}{\Phi} & |S| < \Phi \\ \text{if } \frac{1}{\Lambda B}\hat{K} & S < -\Phi \end{cases}$$

Now, u_d steers the system within the boundary region $|S| < \Phi$. Once the system enters the boundary region and stays in it, the system is said to be in *pseudo sliding mode* [8].

The switching gain $\hat{K}$ can now be adapted according to:

$$\hat{K} = \int (|S| - \Psi)\, dt \tag{8}$$

where $0 < \Psi < \Phi$ is constant. Intuitively equation (8) is simple to explain: increase the switching gain $\hat{K}$ while you are outside the region $|S| < \Psi$ and decrease $\hat{K}$ if $|S| < \Psi$. If we compare this adaption law with the drawbacks of the first method then we see that:

(1) In case of a large initial error, the switching gain $\hat{K}$ will increase fast due to this error, but once the system has reached the boundary region $|S| < \Psi$ the switching gain will be decreased again.

(2) Noise on the measurements do not disturb the adaption procedure if the boundary region is chosen sufficiently large.

(3) The Method II adaption law seeks the lowest possible switching gain which keeps the system within the boundary region $|S| < \Psi$. So when the circumstances permit a lower switching gain, the adaption law will automatically adjust the switching gain to the new circumstances.

3 Discrete-Time Sliding Mode Control

3.1 Introduction

The switching part in a sliding mode control brings the system to the switching surface and keeps the system on the surface despite any modeling errors and disturbances with known bound. The underlying motivation is given by the fact that the switching part can instantaneously react to an error such that it is cancelled out directly, which is in discrete-time no longer possible. The switching function can only change its value at specific time-instants dictated by the sampling frequency. Because of this limitation to the switching time, the system will no longer stay on the switching surface and the closed loop system is no longer invariant against matched disturbances. According to Gao [4], a discrete-time sliding mode controller should have the following properties:

I Starting from any initial state, the trajectory will move monotonically toward the switching plane and cross it in finite time.

II Once the trajectory has crossed the switching plane the first time, it will cross the plane again in every successive sampling period, resulting in a zigzag motion about the switching plane.

III The size of each successive zigzagging step is nonincreasing and the trajectory stays within a specified band.

Once the systems' (error) trajectory fulfills the last two conditions the system is said to be in *Quasi Sliding Mode* [4] which we will call in this paper *Discrete-time Sliding Mode*. The band within which quasi sliding mode takes place is called the *Quasi Sliding Mode Band* [4]. A Sliding Mode Controller is said to satisfy a *reaching condition* if the resulting closed-loop system possesses all three attributes (I, II and III) [4].

We will now define a sliding mode controller for the discrete-time system defined by:

$$\mathbf{x}[k+1] = (A_d + \Delta A_d)\mathbf{x}[k] + (B_d + \Delta B_d)u[k] + d[k] \tag{9}$$

where the disturbance $d[k]$ and the model uncertainties ΔA_d and ΔB_d are assumed to be matched again. The switching function is defined by $S[k] = \Lambda\mathbf{e}[k]$ ($\mathbf{e}[k] = \mathbf{x}[k] - \mathbf{x}_d[k]$) where

Λ can be found in the same way as for the continuous-time case [4]. The discrete-time version of the continuous-time reaching condition (3) can be defined by [4]

$$S[k+1] - S[k] = -QS[k] - K\ \mathrm{sign}(S[k])$$

$(0 < Q < 1)$ from which, together with equation (9), we can determine the required input to be $u[k] = u_l[k] + u_d[k]$ where:

$$\begin{aligned} u_l[k] &= \frac{1}{\Lambda B_d}\left(\Lambda \mathbf{x}_d[k+1] - \Lambda A_d \mathbf{x}[k] + (1-Q)S[k]\right) \\ u_d[k] &= -\frac{1}{\Lambda B} K\ \mathrm{sign}(S[k]) \end{aligned}$$

3.2 Adaptive Switching Gain, Method I

The Method I adaption law defined for the continuous-time sliding mode controller (section 2.2) cannot be applied directly in discrete-time. The adaption law expects the system to reach sliding mode which is for a discrete-time system no longer possible. However, we can use the definition of discrete-time sliding mode in a similar way to come to an adaption law. Rule II for discrete-time sliding mode demands that the switching surface is crossed within each sampling interval so we should increase the switching gain until this happens by:

$$\hat{K}[k] = \hat{K}[k-1] + \begin{cases} \gamma & \text{if} \quad \mathrm{sign}(S[k])\ \mathrm{sign}(S[k-1]) = 1 \\ 0 & \text{if} \quad \mathrm{sign}(S[k])\ \mathrm{sign}(S[k-1]) = -1 \end{cases} \tag{10}$$

This adaption law works in a similar way as the continuous-time version of Method I, it increases the switching gain until the system is in sliding mode. Unfortunately, this discrete-time implementation of Method I still has the same disadvantages as the continuous-time version.

3.3 Adaptive Switching Gain, Method II

The Method II adaption law introduced in section 2.3 can be discretized by:

$$\hat{K}[k] = \hat{K}[k-1] + T_s\left(|S[k]| - \Psi\right)$$

where T_s is the sampling time. The above adaption law still increases the gain $\hat{K}$ until the system remains within in the boundary $|S| < \Psi$. However, since in discrete-time "true" sliding mode is no longer achievable, the boundary region $|S| < \Psi$ can be chosen so small that it cannot be reached regardless the value of $\hat{K}$. In that case the adaption scheme will not stop and $\hat{K}$ will grow unbounded. The simulation example in section 4 demonstrates this.

3.4 Adaptive Switching Gain, Method III

3.4.1 Adaption Law Definition: The Method II adaption law works in continuous-time (for matched uncertainties) rather well but as described in the previous section it is not always suitable in the discrete-time case. To overcome this problem we introduce a new adaption law in this section.

The optimal gain K_o is defined as the smallest possible switching gain which drives and keeps the system in discrete-time sliding mode. We propose the following adaption law:

$$\hat{K}[k] = \left|\hat{K}[k-1] + \gamma\ \mathrm{sign}(S[k])\ \mathrm{sign}(S[k-1])\right| \tag{11}$$

where $\gamma > 0$ is the adaption constant which determines the speed of adaption. The term $\mathrm{sign}(S[k])\ \mathrm{sign}(S[k-1])$ is $+1$ if the system did not cross the switching surface (gain should be larger) and -1 if the system did cross the switching surface (gain should be smaller) so the adaptive gain $\hat{K}$ is altered in the appropriate direction. In the next section it will be shown that the adaptive gain $\hat{K}$ will converge to the region:

$$K_o - \gamma < \hat{K} < K_o + \gamma \tag{12}$$

where K_o is the optimal gain which is the smallest switching gain which results in:

$$0 = \sum_{k=1}^{\infty} \mathrm{sign}(S[k])\ \mathrm{sign}(S[k-1]) \tag{13}$$

Important to notice is that the previous equation does not satisfy condition II for discrete-time sliding mode. However, the smallest switching gain which fulfills the above equation will keep the system close to the switching surface and thus can be considered to be a valid (discrete-time) sliding mode definition. In the remainder K_o is considered to be constant.

3.4.2 Proof of Convergence: Any switching gain which leads to

$$0 = \sum_{k=1}^{\infty} \mathrm{sign}(S[k])\ \mathrm{sign}(S[k-1]) \tag{14}$$

(with proper initial conditions) is denoted by $K_{II'}$. The lowest possible gain which fulfills this equation is called the optimal switching gain K_o (which is by assumption constant). Once again we study the adaption law (11). First it will be shown that $\hat{K}$ will increase when $\hat{K} < K_o$ and successively that $\hat{K}$ will decrease when $\hat{K} > K_o$:

1. $\underline{\hat{K}[k-1] < K_o}$: If we will assume that $\hat{\rho}$ will reach or pass ρ_o in N_1 time-steps, we have to show:

$$\frac{1}{N_1}\sum_{k=1}^{N_1}\left\{|\hat{K}[k] - K_o| - |\hat{K}[k-1] - K_o|\right\} < -\epsilon \tag{15}$$

with $\epsilon = \frac{K_o - \hat{K}[1]}{N_1}$. This leads to:

$$\frac{1}{N_1}\sum_{k=1}^{N_1}\left\{\hat{K}[k-1] - \hat{K}[k]\right\} = -\epsilon \tag{16}$$

With equation (11) this results in:

$$\frac{1}{N_1}\sum_{k=1}^{N_1}\{\gamma\ \mathrm{sign}(S[k])\ \mathrm{sign}(S[k-1])\} = \epsilon \tag{17}$$

As long as the system is not in discrete-time sliding mode, $S[k]$ and $S[k-1]$ will most of the time have equal signs. The above equation implies $\gamma = \epsilon$ which can easily be satisfied since N_1 can be chosen arbitrarily large.

2. $\underline{\hat{K}[k-1] > K_o}$: Again we assume that $\hat{K}$ will reach or pass K_o in N_2 time-steps. We have to show:

$$\frac{1}{N_2}\sum_{k=1}^{N_2}\left\{|\hat{K}[k] - K_o| - |\hat{K}[k-1] - K_o|\right\} < -\epsilon \tag{18}$$

with $\epsilon = \frac{\hat{K}[1]-K_o}{N_2}$. Rewriting (18) gives:

$$\frac{1}{N_2}\sum_{k=1}^{N_2}\left\{\hat{K}[k]-\hat{K}[k-1]\right\} = -\epsilon \qquad (19)$$

With equation (11) this results in:

$$\frac{1}{N_2}\sum_{k=1}^{N_2}\{\gamma \operatorname{sign}(S[k]) \operatorname{sign}(S[k-1])\} = \epsilon \qquad (20)$$

Since the system is in discrete-time sliding mode, $\sigma[k]$ and $\sigma[k-1]$ will have opposite signs. The above equation then implies $\gamma = \epsilon$ which again can be easily satisfied since N_2 can be chosen arbitrarily large.

Thus it is shown that $\hat{K}$ will always move in the direction of K_o but because of the step sizes γ the optimal gain K_o cannot be reached exactly. If we assume that $\hat{K}[k-1] = K_o + \delta[k-1]$ and $\delta[k-1]$ approximates zero then $\hat{K}[k]$ will be:

$$\begin{aligned}\lim_{\delta[k]\to 0^+}\{\hat{K}[k]\} &= K_o - \gamma \\ \lim_{\delta[k]\to 0^-}\{\hat{K}[k]\} &= K_o + \gamma\end{aligned}$$

So we may conclude that the best $\hat{K}$ can do is to stay within the region:

$$K_o - \gamma < \hat{K} < K_o + \gamma \qquad (21)$$

3.4.3 Improved Convergence: The adaption constant γ is a trade-off between switching gain accuracy and adaption speed. In the case that the circumstances under which the system operates do not change this trade-off could be by-passed by the introduction of a non-constant adaption constant. If we would start with a relatively high adaption constant and then decrease it, then the optimal gain K_o could be reached fast and accurate. Two ways of doing this are:

- **Time-dependent adaption constant:** A straightforward method of decreasing the adaption constant is make it a function of time, for example by:

$$\gamma[k] = \gamma_0 \exp^{-\frac{kT_s}{\tau_a}} \qquad (22)$$

  where τ_a should be sufficiently large to be able to reach the region $|\hat{K} - K_o| < \gamma[k]$ before $\gamma[k]$ had decayed to much. Despite the fact that this method is easy to implement it has the drawback that it is possible that the adaption constant is decreased too fast which may prevent the switching gain $\hat{K}$ to reach the optimal value K_o as close as possible.

- **Goal-dependent adaption constant:** Another way of adjusting the adaption constant is to monitor the gain $\hat{K}$. It is known (see section 3.4.2) that once $\hat{K}$ in the region $|\hat{K}[\cdot] - K_o| < \gamma$ it will not leave that region anymore. This is based on the assumption that the optimal gain K_o is constant which will generally not be the case. One could however monitor whether $\hat{K}$ stays within a certain region $|\hat{K}[\cdot] - K_o| < \kappa$, where $\kappa > \gamma$ (for example $\kappa = 10\gamma$), for a certain time span. Once $\hat{K}$ stays within the specified region for a sufficient time, the adaption constant could be decreased slowly.

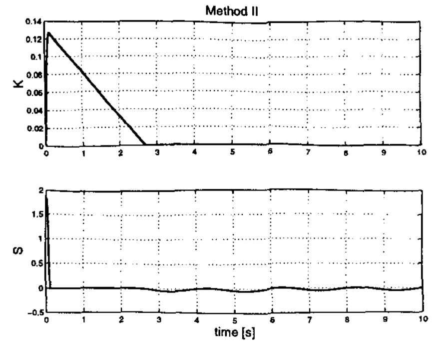

Figure 1: Simulation results for the Method II adaption law with $\Psi = 0.5\Phi$ and $\Phi = 0.1$.

4 Applied to the Pendulum

In order to demonstrate the developed adaption law we will apply it to a driven pendulum [3],[6]. The discretized Method II adaption law (see section 3.3) will be applied as well as the Method III adaption law (see section 3.4). The dynamics of a pendulum with friction can be described by:

$$\ddot{\theta} = -\frac{1}{g}\sin(\theta) - c_f\dot{\theta} + \frac{1}{ml^2}u$$

where θ is the angle, g is the gravity constant, m the mass and l the length of the pendulum. By appropriate scaling this equation can be reduced to the normalized pendulum given by:

$$\ddot{y} + a_2\dot{y} + a_1 sin(y) = u$$

which we can write in state-space form by:

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -a_1\sin(x_1) - a_2x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ u \end{pmatrix}$$

where $x_1 = y$ and $x_2 = \dot{y}$. In this example a_1 and a_2 are taken to be 0.25 and 0.1 respectively.

The sliding mode controller is designed for the discrete-time linearized model (sampling time $T_s = 5e^{-3}$ s) around the origin ($\mathbf{x} = [0 \;\; 0]^T$) represented by equation (9) where A_d and B_d are given by:

$$A_d = \begin{bmatrix} 1.0 & 5.0\, e^{-3} \\ -1.2\, e^{-3} & 1.0 \end{bmatrix} \quad B_d = \begin{bmatrix} 0.0 \\ 5.0\, e^{-3} \end{bmatrix}$$

The unmodeled nonlinearity can now be considered as a matched disturbance. The switching surface is computed by the use of LQR techniques, which leads to:

$$S[k] = \Lambda \mathbf{e}[k] = [-6.23 \quad -0.98]\mathbf{e}[k]$$

The control law is taken as derived in section 3 where the switching gain K is now adaptive (i.e. replaced by $\hat{K}[k]$). The system has to track the following signal:

$$\mathbf{x}_d = \begin{bmatrix} \omega\sin(\omega t) \\ \cos(\omega t) \end{bmatrix}$$

so the angle and the angular velocity of the pendulum should be a perfect sine- and cosine function of the time ($\omega = \frac{2\pi}{20}$).

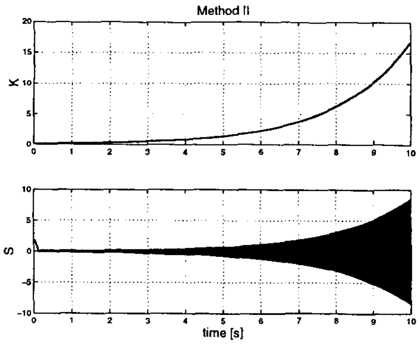

Figure 2: Simulation results for the Method II adaption law with $\Psi = 0.5\Phi$ and $\Phi = 0.01$.

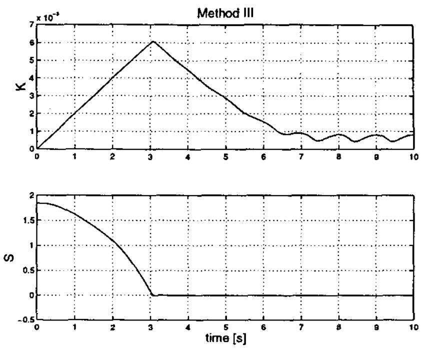

Figure 3: Simulation results for the Method III adaption law with $\gamma = 1e^{-3}$.

The controller is tested on a (simulated) system with $a_{s1} = \frac{1}{2}a_{m1}$ and $a_{s2} = 2a_{m2}$ (where the subscript s stands for the (simulated) system and the subscript m stands for the model which is used for the controller design). The simulation results for the Method II adaption law are given in figures 1 and 2. Figure 1 demonstrates that the Method II adaption law works satisfactory. The gain adapts very rapidly and the sliding variable S stays within the defined quasi sliding mode band $|S| < \Phi$. However figure 2 shows that if the quasi sliding mode band is selected too small, the adaption law will become unstable since it is trying to force the system into the quasi sliding mode band which is no longer achievable.

The simulation results for the Method III adaption law are given in figure 3. Initially the adaptive gain is rather large because of the error in the initial conditions. Once the sliding surface is reached, the adaptive gain is reduced considerably because it only has to compensate for the modeling error. The results are about similar to the Method II results with $\Phi = 0.1$, Method III keeps S slightly closer to zero. However, there is no danger of instability. The adaptive gain can be further smoothened if a smaller adaption constant γ is chosen.

5 Conclusions

In this paper, a new adaption law for for the switching gain was introduced. It has been specifically designed for discrete-time sliding mode controllers. This new method proved to have an important advantage over the discretized version of the adaptive gain introduced in [6], namely that there is no danger of instability of the adaption procedure because of a bad choice of adaption parameters. With the latter method a boundary region (quasi sliding mode band) within which (discrete-time) sliding mode will take place has to be selected. This region can be chosen smaller than achievable in which the adaptive gain can grow unbounded. With the new adaption law this can no longer happen. Simulation results visualize the above statements.

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