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Delegation of Authority, Managerial Initiatives, and the Design of Divisional Structure

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Abstract

This paper provides a rationale for a firm to adopt either an integrated or a separated divisional structure, which is based on the interplay between the structure of authority and the costs and benefits of integration vis-à-vis separation. We use the framework of Aghion and Tirole (1997) to explain the structure of authority. This framework captures the notion of managerial initiatives. It shows that monitoring by the head-office decreases divisional managers’ effort levels. We incorporate this framework into the analysis of costs and benefits of integrating or separating divisions. Integration will be beneficial for the head-office if it generates synergy gains. The larger the synergy gains are, the more appealing integration will be. Consequently, the head-office’s incentive to monitor increases. Due to a more intense monitoring, managers exert lower effort levels. For managers, integration entails costs and benefits. If the benefits outweigh the costs, managers will be motivated to exert high effort levels in an integrated divisional structure. The optimality of integrating or separating divisions will then be determined by the trade-off between synergy gains and the managerial effort elicitation.

JEL Classification: D89, L22

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1 Introduction

Firms often reorganize internally in an attempt to create a better functioning of their internal structure. They may decide to integrate divisions under a common manager or to separate divisions and to assign a manager to run each division. Consider, for example, the recent experience of Sony Pictures Entertainment Inc. and Motorola Inc. (Wall Street Journal Europe, 1998). The former decided to integrate its two movie-making subsidiaries, TriStar Pictures and Columbia Pictures, while the latter decided to split its Consumer and Infrastructure Lines.

In some cases such an attempt may work well, but in some others it may instead create inefficiency, as is the case with Andersen Worldwide (Financial Times, 1997). This raises the issue of identifying significant factors behind the success and failure of an internal-reorganization attempt. Business analysts often argue that the existence of synergy benefits is the key factor. It certainly makes sense. However, there is a danger of over-emphasizing the role of synergy benefits. We might tend to overlook the costs of integrating (separating) divisions. If we take Andersen Worldwide case as an example, it is hard to believe that the inefficiency is caused by the lack of synergy gains, given that both Andersen Consulting and Arthur Andersen engage in complementary activities. In principle, having them under the same roof should be beneficial for the parent company. The fact that it is not indicates that integration also entails costs. What are these costs? Will these costs be compensated by the benefits? These are the questions that should be answered if we want to understand better the reason behind the success or failure of an internal-reorganization attempt. Our paper aims to do this.

In the literature, there are some theoretical studies investigating the costs and benefits of integration vis-à-vis separation. Grossman and Hart (1986) argue that integration is beneficial as it solves the hold-up problem. It should be noted, however, that Grossman and Hart’s paper stresses more on the external boundaries of the firm. Their framework

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Footnotes:
1. Internal reorganization may also cover other activities such as expanding (shortening) the hierarchy, changing internal labor policies, changing the design of compensation, etc. However, in this paper we restrict our attention only to the firm’s decision to choose a divisional structure, i.e. integrated or separated divisional structure.
2. There are two divisions under the umbrella of Andersen Worldwide, namely Andersen Consulting (the strategic consulting division) and Arthur Andersen (the accounting division). The corporation was established in 1913, and it initially operated as an accounting firm. Only later that the consulting business was added. Recently, it is reported that there is a conflict between Andersen Consulting and Arthur Andersen (see Financial Times (June, 1997) and the Wall Street Journal Europe (September, 1997)). Andersen Consulting has led a request with an international arbitrator to break away from Andersen Worldwide.
3. See for instance Wall Street Journal Europe (June 1999), which reports that Telefonica SA, a Spanish telecom giant planned to integrate its media units under one roof. Business analysts praised the move, for they argue that it will enable the parent company to generate synergy gains.
explains when a firm should stay independent and when it should be integrated with another firm. Our paper, instead, focuses on the internal boundaries of the firm. We analyze the optimality of integrating or separating internal divisions. Consequently, Grossman and Hart's framework cannot be directly applied to our paper. There is essentially a major difference between the analysis of external and internal boundaries of the firm that hinders such a direct application, namely the structure of authority, i.e. who can decide on what. In an internal organization framework such as ours, the head office can always mandate that divisional integration and interdivisional transaction be carried-out. A division may not have the right to refuse the decision of the head office. On the contrary, in Grossman and Hart's framework a firm ('division') can always refuse to deal whenever an unforeseen contingency that is not governed by the contract occurs, and can always freely decide whether or not to integrate.

Riordan (1990) and Olsen (1996) also analyze the costs and benefits of integration. Similar to Grossman and Hart (1986), their paper focuses on the external boundaries of the firm. Riordan (1990) analyzes the decision of a downstream firm to integrate backwardly with an upstream firm. Such integration enables the downstream firm to obtain better information about the upstream firm. However, it also comes with costs. It lowers ex-ante managerial incentives. Olsen (1996) argues that integration brings complementary gains, however it also creates greater informational rents for managers. Managers will tend to push for integration, even though it might not necessarily be good for firms. Obviously, Riordan (1990) and Olsen (1996) argue that an integration attempt should be carried-out whenever the benefits outweigh the costs. Their papers also face the same limitation as Grossman and Hart's paper. In their paper, a firm has the right to refuse integration. On the contrary, in our framework a division has no authority to refuse a mandated integration by the head office.

In contrast to the previously mentioned papers, Holmström and Tirole (1991) present a framework for evaluating the costs and benefits of integration that combines both the analysis of external and internal boundaries of the firm. They consider a downstream firm that needs to obtain an intermediate good. The principal of the firm may decide to integrate the firm with an external upstream firm, or to rely on the external market to supply the good. They assume that if integration is chosen, there will be a common owner. This owner will delegate all control rights to the head office. The head office will have to decide on the type of delegated-authority that should be given to divisional managers. There are three possibilities. The head office may fully delegate authority over trading decisions to managers. Thus, managers are allowed to trade externally if they cannot agree on the internal transfer price. The head office may also mandate internal trade between units, but still allows managers to freely negotiate the internal transfer price. Finally, the head office
may instead be very strict, in the sense that the head-office mandates trade to be internal and also determines the internal transfer price. This case corresponds to full centralization of authority. Thus, Holmstrom and Tirole (1991) analyze two types of decision. The first one is the decision on whether or not to integrate, and the second one is the decision on whether or not to delegate authority over trading decisions to unit managers. The former is an external boundaries of the firm analysis, while the latter is an internal boundaries of the firm analysis.

Holmstrom and Tirole (1991) show that integration allows better coordination across divisions. However, if integration occurs, the head-office may be tempted to mandate trade to be internal. This may create inefficiency when external trade gives a better deal than internal trade. The trade-off between this inefficiency and the benefits of better coordination will determine the optimality of integration.

Although Holmstrom and Tirole's paper analyzes firms' internal organization, it focuses only on the issue of delegation of authority to unit managers, but not on the issue of the design of divisional structure. In their integration case, it is not clear how the result will change when both, the upstream unit and the downstream unit, are integrated under a common manager, as compared to when both units are separated and each of them is managed by a manager. In contrast, our paper focuses on the design of divisional structure. In the paper, we provide a rationale for a firm to adopt either an integrated or a separated divisional structure.

As in Holmstrom and Tirole's paper, we also explicitly analyze the structure of decision making authority in firms. However, we use a different concept of authority, which is borrowed from Aghion and Tirole (1997). We define authority as the right to select actions affecting part or the whole of a firm. It can be distinguished into formal and real authority. The head-office has formal authority over the choice of projects to be implemented. However, the head-office is willing to delegate the decision making authority to divisional managers whenever divisional managers are better informed about the projects' prospects. In this case divisional managers have real authority. In contrast, Holmstrom and Tirole's paper does not make such a distinction. Their paper considers only formal authority. For example, in their full centralization setting, the head-office has formal authority and knows with certainty the best decision to follow. Thus, in this example formal and real authority reside in the hand of the head-office. In reality, it is often the case that although the head-office has formal authority, unit managers may be better informed. The head-office may then prefer to let divisional managers decide.

The framework of Aghion and Tirole (1997) is interesting, as it captures the notion of managerial initiatives. This can be explained as follows. Suppose that the head-office wants a divisional manager to implement a project that is chosen from a set of available
projects. To know which project should be implemented, the head-office and the manager will have to exert effort in the information acquisition. As a result, they will get informed about this project with some probabilities. Assume that their preference on the project to be implemented differs. If the head-office is informed about her preferred project, she can always ask the manager to implement this project. The manager cannot refuse it, because he has no formal authority. However, if the head-office is not informed but the manager is informed, the head-office may delegate the right to choose the project to the manager. The head-office will not overrule the manager’s project choice. In this kind of setting, higher monitoring effort of the head-office will imply a higher probability that the manager’s project choice will be overruled by the head-office. This lowers the manager’s incentive to take initiatives.

Our present paper intends to incorporate Aghion and Tirole’s framework into the analysis of costs and benefits of divisional integration vis-à-vis divisional separation. We assume that the decision to integrate or to separate divisions rests in the hands of the head-office. If integration is adopted, the head-office will appoint a common manager to run the integrated divisions. If, instead, separation is chosen, the head-office will assign a manager to run each division.

Integration will be beneficial for the head-office if it generates complementary (synergy) gains. Consequently, the larger these gains are, the more attractive integration for the head-office will be. For the appointed manager, integration entails costs and benefits. In an integrated structure, the manager will have to allocate effort on each division. If the effort that is exerted on divisions are substitutes in the costs of effort function, i.e. the marginal costs of effort exerted on a division is increasing in the effort exerted on another division, then having an integrated structure is costly for the appointed manager. However, integration enables the appointed manager to obtain higher rents, i.e. private benefits of control. Hence, the manager faces a trade-off between costs and benefits of integration. If the benefits exceed the costs, the manager will be motivated to exert high effort on divisions. The higher the complementary gains are, the higher the incentive of the head-office to monitor will be. This increases the probability that the manager will be overruled by the head-office. As a result, the manager will then be less motivated to exert effort. The overall impact on the managerial effort is ambiguous. There may or may not be an adverse effect of integration on the managerial effort.

The head-office might still prefer to integrate divisions, even though integration leads to an adverse effect on the managerial effort, if the benefits of synergy gains outweigh this adverse effect. However, if the adverse effect exceeds the benefits of synergy gains then the head-office prefers to separate divisions. This is an interesting result, as it tells

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4 Throughout this paper, we assume that the head-office is female and divisional managers are male.
that integration is not always warranted even though it generates synergy gains. It also shows that the decision to separate divisions acts as a commitment device. By separating complementary divisions, and thus foregoing the benefits of integration, the head-office can commit not to harshly monitor divisional managers. Managers will then be motivated to work hard. We also have a mirror case of the above story. It may also be the case that integration is preferable even though it does not bring synergy gains, as long as the positive effect on the managerial effort compensates the negative synergy gains. Here, the decision of integrating non-complementary divisions also acts as a commitment device to elicit the managerial effort.

To sum-up, our paper thus shows that the optimality of integrating or separating divisions is determined by the trade-off between synergy gains and the managerial effort inducement. In addition, we obtain other interesting auxiliary results. We show that if the head-office prefers to integrate divisions, then she will appoint a manager whose interests are the most congruent with hers to manage the integrated divisions. We also show that integration will become more attractive when divisions are more asymmetric in terms of their degree of interests congruence with the head-office. This last result prevails under the assumption that the head-office uses an information acquisition technology that is inferior than the one used by divisional managers. We will show that such a technology enables a manager with a low degree of interest congruence to take advantage from managers with a high degree of interest congruence. Integration solves this free-riding problem. Consequently, the more asymmetric divisions are, the more attractive integration will be.

The remainder of the paper is organized as follows. In section 2, we describe the model. In section 3, we solve the model for the optimal effort of the head-office and divisional managers. Then, in section 4 we compare the optimal managerial effort in different internal organization forms. In section 5, we compare the utilities obtained by the head-office in different internal organization forms. In section 6, we discuss the choice of a firm’s internal organization. In section 7, we discuss an extension of the model to the case of separable monitoring costs of the head-office. Finally, section 8 concludes.

2 The Model

Our model is based on Aghion and Tirole’s model (1997). We consider a firm consisting of a head-office, two divisions (D_1 and D_2); and two managers (M_1 and M_2). The head-office will have to decide on how to structure the divisions. There are two options available to the head-office (see Figure 1). The head-office can separate the two divisions and assigns a manager to each division. We call the resulting structure as a separated form. Alternatively, the head-office can merge the two divisions and assigns a common manager to manage both
Divisions. We call the resulting structure as an integrated form.

![Divisional Forms](image)

Figure 1: Divisional Forms

Divisions may be interdependent. We interpret this interdependence as payoffs externalities across divisions. We assume that these externalities can only be realized when there is coordination across divisions. Divisional coordination can be created by integrating the two divisions under a common manager.

Each division can undertake a project from \( n > 3 \) potential projects. We assume that there is no project overlap between the two divisions. Thus, each division has a different project portfolio, denoted respectively by \( i \in \{i_1, i_2, i_3, \ldots, i_n\} \) and \( j \in \{j_1, j_2, j_3, \ldots, j_n\} \). We adopt an incomplete contracting approach, and thus assume that the nature of the projects cannot be described ex-ante. Hence, they are not ex-ante contractible. The payoffs can only be verified ex-post of their realization. The head-office and divisional managers must acquire information to know which projects among \( n \) potential projects give non-negative profits and private benefits. Divisional managers and the head-office must exert effort, respectively \( e_1 \), \( e_2 \), and \( E \), to acquire information. In an integrated form, in which there is only a manager, \( e_1 \) and \( e_2 \) will denote effort levels exerted by the appointed manager on the two divisions.

In a separated form of internal organization, the two divisional managers will learn the payoffs of all possible projects with probabilities \( e_1 \) and \( e_2 \). With probabilities \( (1 - e_1) \) and \( (1 - e_2) \) they learn nothing. Acquiring information is a costly activity. Divisional managers have to incur costs of acquiring information, respectively \( \frac{(e_1)^2}{2} \) and \( \frac{(e_2)^2}{2} \):

In an integrated form of internal organization, \( e_1 \) and \( e_2 \) indicate the probabilities that the appointed manager is informed about the payoffs of the two divisions under his control. Hence, \( (1 - e_1) \) and \( (1 - e_2) \) denote the probabilities that the appointed manager learns nothing. In this internal organization form, costs of effort are interdependent. The total costs of effort of the appointed manager are \( \frac{(e_1)^2}{2} + \frac{(e_2)^2}{2} + \pm e_1 e_2 \). Parameter \( \pm \) represents
the degree of costs interdependency. We impose the following assumption on the value of parameter $\pm$

Assumption 1: $0 \leq \pm \leq 1$

Thus, we assume that effort levels are substitutes. Only when $\pm = 0$; effort levels are independent: If we have $\pm > 0$; then increasing managerial effort on the first division increases marginal costs of managerial effort on the second division. We have a perfect substitute when $\pm = 1$: This cost-substitutability exists because of, for instance, limited ability of the manager. We could also interpret $\pm$ as the degree of similarity in the organizational cultures of the two divisions. When $\pm$ is high, it means that the head-office faces costly integration, which may be due to the significant differences in organizational cultures of the two divisions.

The head-office learns the payoffs of all possible projects with probability $E$; and with probability $(1 - E)$ the head-office learns nothing. The costs of acquiring information for the head-office are $\frac{(E)^2}{2}$: For simplicity, the head-office’s costs of effort are assumed to be inseparable. Thus, these costs represent the total costs of acquiring information about the projects’ prospects of the two divisions. This implies that when she is informed (uninformed) about the projects’ prospects of the first division, she will also be informed (uninformed) about the projects’ prospects of the second division. We will later see how the results change when the head-office’s costs are separable.

We consider a particular case in which only two projects in each portfolio give non-negative profits and private benefits. Only one of these two gives non zero profits to the head-office. Similarly, only one of these two gives non zero private benefits to the manager in charge. With probability $\gamma = 2(0;1)$ the same project is preferred by both, the head-office and the manager. Hence, parameter $\gamma$ measures the degree of interest congruence between the head-office and the manager. If no project is undertaken, then the ‘status quo’ prevails. Profits and private benefits are normalized to zero. We allow for the congruence parameter between the first division and the head-office ($\gamma_1$) to be different from the congruence parameter between the second division and the head-office ($\gamma_2$).

The head-office’s profits in a separated form of internal organization. In this organizational form, divisions cannot realize potential externalities unless there is coordination.

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5 This assumption essentially says that the head-office has an inferior information acquisition technology as compared to divisional managers. A divisional manager, if he is appointed to run the integrated divisions, can have a more precise information acquisition technology. However, we assume that such a superior technology is costly. This notion is captured by the term $\pm$ in the costs of effort function of the manager. This particular setting will make the trade-offs in the model more explicit and will enable us to draw a sharp conclusion.
across divisions. However, creating coordination within a separated form of internal organization is costly. For simplicity, we assume that these costs are prohibitively high. If the preferred projects of the head-office are implemented, the head-office obtains $1 + \bar{\xi}_1 + \bar{\xi}_2$.

If instead the preferred projects of divisional managers are implemented, the head-office obtains $1 - \bar{\xi}_1 + \bar{\xi}_2$.

The head-office's profits in an integrated form of internal organization. In this organizational form, the two divisions are assigned to a common manager. A better coordination can be easily created. This coordination enables the head-office to realize potential externalities across divisions. We can think of these externalities as complementary gains accrued from combining divisions. We denote these gains by parameter $\bar{\rho}$.

**Assumption 2:** The size of $\bar{\rho}$ is equal for both divisions, and for simplicity we assume that it could take any value between $1 < \bar{\rho} < 1$.

Positive externalities ($\bar{\rho} > 0$) can be interpreted as a value-enhancing integration (synergistic), while negative externalities ($\bar{\rho} < 0$) can be interpreted as a value-destroying integration. There will be no externalities when $\bar{\rho} = 0$. If the preferred projects of the head-office are implemented, the head-office obtains $(1 + \bar{\rho}) \bar{\xi}_1 + \bar{\xi}_2$.

If instead the preferred projects of the manager are implemented, the head-office obtains $(1 + \bar{\rho}) - \bar{\xi}_1 + \bar{\xi}_2$.

Note that subscript $k$ indicates the manager who is appointed to manage both divisions in the integrated form. This manager could be either $M_1$ or $M_2$.

Managers' private benefits in a separated form of internal organization. Each divisional manager obtains private benefits $B_1$ and $B_2$; if their preferred projects are implemented. These private benefits could be in the forms of job satisfaction, perquisites, etc. Following Aghion and Tirole (1997), we assume that divisional managers are not motivated by monetary benefits and receive their reservation wages which are normalised to zero. If instead the preferred projects of the head-office are implemented, divisional managers obtain respectively $-1B_1$ and $-2B_2$.

Managers' private benefits in an integrated form of internal organization. If the preferred projects of the manager are implemented, the manager obtains $\bar{\xi}_1 + B_2$.

If instead the preferred projects of the head-office are implemented, the manager obtains $-\bar{\xi}_1 + B_2$.

For simplicity, we impose the following assumption.

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6 This assumption can be motivated by two reasons. Firstly, if managers are infinitely risk averse with respect to income risks, then the head-office should provide full insurance to managers, and thus should provide fixed wages which are set as high as managers' reservation wages. Secondly, in a world of incomplete contract, a contract specifying a monetary compensation cannot be designed. See also de Bijl (1996) for a similar analysis.
Assumption 3: $i_1 = i_2$ and $B_{i_1} = B_{i_2}$.

As for the structure of authority, we assume that the head-office retains formal authority. However, divisional managers may have real authority. Note that this distinction between formal versus real authority follows that of Aghion and Tirole (1997). Formal authority is defined as authority which results from an explicit or implicit contract allocating the right to decide on specific matters. Real authority refers to an effective control over decisions. Real authority could be the result of a superior possession of information. Managers might have real authority if they are informed, while the head-office ce is not. When this is the case, the head-office may delegate the decision making authority to managers.\footnote{The head-office ce will be better-off following managers’ suggestions because doing so yields positive payoffs, except of course when the interests of the principal and divisional managers are diametrically opposed. If the head-office ce opts for the ‘status-quo’ project, that is by refusing to follow managers’ suggestion, she gets zero.}

The timing of the model is depicted in Figure 2. In the first stage, the head-office ce chooses the firm’s internal organizational form. Then, in the second stage the head-office ce and managers simultaneously exert effort to acquire information about projects’ payoffs. In the third stage, divisional managers convey their information to the head-office ce. Subsequently, if the head-office ce is informed about projects’ payoffs, she will decide which projects should be implemented by divisional managers. Otherwise if the head-office ce is not informed, she is willing to accept the suggestion of divisional managers. If both the head-office ce and divisional managers are informed, then the head-office ce will exercise her formal authority by overruling managers’ suggestions, and forcing managers to implement her preferred projects. If both are not informed, then no project is going to be implemented (the status quo prevails) and payoffs and private benefits are normalized to zero. We assume that the information conveyed by divisional managers is hard information, in the sense that if it is communicated by the other party it can be easily and costlessly verified.\footnote{See Aghion and Tirole (1997) for the concept of hard and soft information.}

In the last stage payoffs and private benefits are realized.

![Figure 2: The Time Frame](image)

We will now write the expressions of the payoffs of the head-office ce and divisional managers in both divisional structures.
2.1 Separated Form of Internal Organization

The payoffs of the head-office can be expressed as,

\[ U_{sp}^h = 2E_{sp}^1 + (1 - E_{sp}) (e_{sp}^1 + e_{sp}^2) \frac{(E_{sp})^2}{2} \quad (1) \]

In which superscript 'sp' denotes the separated form. Note that, by assumption 3, we have that \( E_{1} = E_{2} \). Since they are the same, throughout this paper we will just suppress the subscripts. The first part represents the head-office's payoffs when she is informed (with probability \( E_{sp} \)) and asks the two divisional managers to implement her preferred projects. With probability \( (1 - E_{sp}) \) she is not informed, and she is willing to accept the projects that are proposed by divisional managers. Divisional managers are informed with probability \( e_{sp}^1 \) and \( e_{sp}^2 \). The head-office's payoffs are then discounted by, respectively \( \bar{1} \) and \( \bar{2} \), the degree of interest congruence between her and divisional managers.

There are two divisional managers in charge of running divisions. Their payoffs are respectively,

\[ U_{sp}^{m1} = (E_{sp}^{-1} + (1 - E_{sp}) e_{sp}^1) B \frac{(e_{sp}^1)^2}{2} \quad (2) \]

\[ U_{sp}^{m2} = (E_{sp}^{-2} + (1 - E_{sp}) e_{sp}^2) B \frac{(e_{sp}^2)^2}{2} \quad (3) \]

Similarly because \( B_{11} = B_{22} \), throughout this paper we suppress the subscripts. When the head-office is informed, each manager will have to implement the preferred project of the head-office. Managers receive private benefits which are discounted by, respectively \( \bar{1} \) and \( \bar{2} \). However, when the head-office is not informed but managers are, then managers get their highest private benefits.

2.2 Integrated Form of Internal Organization

In an integrated form of internal organization, there is a common manager in charge of the two divisions. We use superscript 'in' to denote the integrated form. In a similar fashion as the previous case, we can express the payoffs of the head-office as,

\[ U_{in}^h = 2(1 + \Theta) E_{in}^1 + (1 + \Theta) (1 - E_{in}) (e_{in}^1 + e_{in}^2) \frac{(E_{in})^2}{2} \quad (4) \]

with subscript \( k \) indicates the manager who is appointed to manage the integrated divisions. This manager could either be \( M_{1} \) or \( M_{2} \). We assume that they are equally probable to be appointed. Thus, \( e_{in}^1 \) and \( e_{in}^2 \) indicate the exerted effort levels of the appointed manager.
on respectively the first division and the second division. Note that, as is mentioned before, \( \text{\&} \) denotes complementary gains accrued from integration.

The payoffs of the appointed manager can be expressed as,

\[
U_{m_k}^{\text{in}} = 2E_{\text{in}}^{-} k B_k + i_1 \text{E}_{\text{in}}^{-} e_{k1}^{\text{in}} + e_{k2}^{\text{in}} - (e_{k1}^{\text{in}})^2 - (e_{k2}^{\text{in}})^2 - i_2 \text{E}_{\text{in}}^{-} e_{k1}^{\text{in}} e_{k2}^{\text{in}} \tag{5}
\]

In an integrated form, since one of the two managers will be appointed, hence there will be three possible congruence parameter configurations, i.e. \( k = 1 \) \& \( 2 \), or \( k = 1 \) \& \( 2 \), or \( k = 1 = 2 \).

3 The Optimal Effort Levels of the Head-office and Manager(s)

We start with the case of separated form of internal organization. Taking the FOCs of expressions (1), (2), and (3) gives,

\[
E_{sp} = 2i_1 \left( e_{sp}^{-1} + e_{sp}^{-2} \right) \tag{6}
\]

\[
e_{sp}^{1} = \left( 1 - E_{sp} \right) B \tag{7}
\]

\[
e_{sp}^{2} = \left( 1 - E_{sp} \right) B \tag{8}
\]

Solving simultaneously the system of equations (6), (7), and (8) yields,

\[
E_{sp} = \frac{2i_1 \left( 1 - E_{sp} \right) B \left( 1 + \frac{1}{2} \right)}{1 - E_{sp} \left( 1 + \frac{1}{2} \right)} \tag{9}
\]

\[
e_{sp}^{1} = \frac{1 - 2i_1 \left( 1 - E_{sp} \right) B \left( 1 + \frac{1}{2} \right)}{1 - E_{sp} \left( 1 + \frac{1}{2} \right)} \tag{10}
\]

\[
e_{sp}^{2} = \frac{1 - 2i_1 \left( 1 - E_{sp} \right) B \left( 1 + \frac{1}{2} \right)}{1 - E_{sp} \left( 1 + \frac{1}{2} \right)} B \tag{11}
\]

Since \( e_{sp}^{1}, e_{sp}^{2}, \) and \( E_{sp} \) are probabilities, their values should be between \((0; 1)\). Using this information, we derive the following lemma.

Lemma 1: \( e_{sp}^{1}, e_{sp}^{2}, E_{sp} \) \((0; 1)\) for all admissible values of \( 1 \) and \( 2 \), if \( 0 < \left( 1 - \frac{1}{2} \right) \); and \( 0 < B \left( 1 - \frac{1}{2} \right) \).
Proof. From the model setting, we know that \( | \) > 0; \( \beta > 0; \) and \( \bar{\gamma} 1, \bar{\gamma} 2 (0; 1): \) Manipulating expression (9) to have \( 0 \leq 6 \varepsilon_{(1+2)} 1; \) we obtain, \( 0 < | 6 \beta 1 \frac{2}{(1+2)}: \) Similarly, manipulating expressions (10) and (11) to have \( 0 \leq 6 \varepsilon_{(1+2)} 1; \) we obtain \( 0 < \beta 1 \frac{2}{(1+2)} \bar{\gamma} 1, \beta > 0: \) Combining all of this information, we can straightforwardly derive Lemma 1. ■

Note that Lemma 1 does not imply that the values of \( | \) and \( \beta \) cannot be bigger than \( \frac{1}{2}. \) For some values of \( \bar{\gamma}, \) we can still have \( | ; \beta > \frac{1}{2} \) without violating \( e; E (0, 1) \): Since we are interested in knowing the effect of varying \( \bar{\gamma} \) on effort levels and the head-officer's payoffs, there is no loss of generality if we consider only the values of \( | \) and \( \beta \) which are valid for all admissible values of \( \bar{\gamma}: \)

Using expressions (6) - (11) we can establish the following proposition.

**Proposition 1:** In a separated form of internal organization;

(i) Effort levels of the head-officer and divisional managers are strategic substitutes.

(ii) An increase in effort levels of a divisional manager has no direct effect on effort levels of the other divisional manager. However, there is a positive indirect effect.

(iii) Effort levels of the head-officer are decreasing in the degree of interests congruence between the head-officer and a divisional manager. Effort levels of the manager will increase due to the strategic substitutability.

(iv) Effort levels of a divisional manager is increasing in the degree of interests congruence between the head-officer and his fellow manager.

(v) Effort levels of the head officer are increasing in the payoffs that can be generated by a division. Similarly, effort levels of a divisional manager are increasing in the size of his private benefits.

**Proof.** See appendix. ■

Point (i) tells that when the head-officer intensifies her monitoring efforts, divisional managers will exert lower effort levels. Monitoring has an adverse effect on managerial initiatives.

Point (ii) shows that an increase in effort levels of a manager will only have an indirect effect on effort levels of the other manager. This is because when a manager increases his effort levels, the head-officer will monitor less. This will increase the other manager's effort levels. As is shown in appendix, this can be easily checked using the following total derivative.
\[
\frac{\text{de}_2^\text{SP}}{\text{de}_1^\text{SP}} = \frac{\mu}{\alpha_2^\text{SP} \left( \text{de}_2^\text{SP} \right)} + \frac{\mu}{\alpha_1^\text{SP} \left( \text{de}_1^\text{SP} \right)} > 0 = 0
\]

This result is driven by the inferiority of the head-office’s monitoring technology. If the head-office has a more precise technology, in the sense that, her monitoring costs are separable, then this indirect effect will be absent.

Point (iii) tells that when the interests of the head-office and a divisional manager becomes more aligned, then the head-office will reduce her monitoring intensity. Due to the costs inseparability, a decrease in the head-office’s monitoring-exert levels will benefit the other manager too (point (iv)). Thus, basically there is a spillover-effect operating. Suppose that the degree of interests congruence of divisional managers are such that \( \gamma_1 > \gamma_2 \), and for an exogeneous reason \( \gamma_1 \) increases. Then, the second manager with a lower degree of interest congruence (\( \gamma_2 \)) could take advantage of this situation. We know that if \( \gamma_1 \) increases, the head-office’s monitoring-exert levels will decrease. This is of course beneficial for the second manager, as he will then also face a less stringent monitoring.

Finally, point (v) is intuitive. The bigger the size of the pie, the higher the incentive to exert exert.

We now proceed with the case of integrated form of internal organization. Taking the FOCs of expressions (4) and (5) gives,

\[
E^\text{in} = 2\left(1 + \delta \right) \left( 1 + \delta \right) i E^\text{in}_k + E^\text{in}_k \quad k = 1, 2
\]

\[
e^\text{in}_1 = \frac{1}{1 + \pm i} E^\text{in} \quad k = 1
\]

\[
e^\text{in}_2 = \frac{1}{1 + \pm i} E^\text{in} \quad k = 2
\]

We solve the optimal exert levels in two steps. In the first step we solve the manager’s exert allocation problem. The manager who is in charge of the two divisions will have to decide how to allocate his exert on both divisions. In the second step, we then solve for the optimal exert of the head-office and the manager. Solving (14) and (15) yields,

\[
e^\text{in}_1 = \frac{1}{1 + \pm i} E^\text{in} \quad k = 1
\]

\[
e^\text{in}_2 = \frac{1}{1 + \pm i} E^\text{in} \quad k = 2
\]

Substituting the above results into (13) we obtain,
\[ E^{in} = \frac{2\varepsilon (1 + \bar{\xi}) [1 + \varepsilon (1 - \gamma)\kappa] - \varepsilon B}{(1 + \varepsilon) \kappa B (1 + \bar{\xi})} \] (18)

Optimal effort levels exerted by the manager on the two divisions are,

\[ e^{in}_{k1} = \frac{\mu 1 + 2\varepsilon (1 + \bar{\xi})}{(1 + \varepsilon) \kappa B (1 + \bar{\xi})} \] (19)

\[ e^{in}_{k2} = \frac{\mu 1 + 2\varepsilon (1 + \bar{\xi})}{(1 + \varepsilon) \kappa B (1 + \bar{\xi})} \] (20)

The following lemma applies,

Lemma 2: \( e^{in}_{k1}; e^{in}_{k2}; E^{in} \in 2 ((0; 1) \) for all admissible values of \( \bar{\xi}; \kappa \); and \( \varepsilon \) if \( 0 < \varepsilon \leq 1 \) and \( 0 < B \leq 1 \):

Proof. From the model setting we know that \( \varepsilon > 0 \); \( B > 0 \); \( 1 \leq 6 \bar{\xi} \leq 6 \); \( 0 \leq \varepsilon \leq 1 \); and \( -1 \leq \gamma \leq 2 ((0; 1) \) : Manipulating expression (18) to have \( 0 \leq E^{in} \leq 1 \), we obtain \( \varepsilon \leq \frac{1}{1+\varepsilon} \) and \( B > \frac{(1+\varepsilon)}{\kappa} \). Next, using expression (19) to check for \( 0 \leq e^{in}_{k1} \leq 1 \) and \( 0 \leq e^{in}_{k2} \leq 1 \) yields \( 0 \leq 6 \bar{\xi} \leq 6 \frac{1}{1+\varepsilon} \kappa B; \varepsilon \leq \frac{1}{2}; \) and \( B > 0 \). It is then easy to check that Lemma 2 holds.

Combining Lemma 1 and Lemma 2 we can straightforwardly derive the following Lemma.

Lemma 3: \( e^{sp}_{k1}; e^{sp}_{k2}; E^{sp} \in 2 ((0; 1) \) for all admissible values of \( \bar{\xi}; \kappa \); and \( \varepsilon \) if \( 0 < \varepsilon \leq 1 \) and \( 0 < B \leq 1 \):

Using expressions (13)-(19), we can establish the following proposition.

Proposition 2: In an integrated form of internal organization;

(i) Effort levels of the head-office and the manager are strategic substitutes.

(ii) An increase in the manager’s effort levels exerted on a division has an ambiguous effect on his effort levels exerted on the other division.

(iii) Effort levels of the head-office are decreasing in the degree of interests congruence between the head-office and the manager.

(iv) Effort levels of the head-office are increasing in the size of the complementary gains (\( \bar{\xi} \)) accruing from integration. Effort levels of the manager will decrease due to the strategic substitutability.
(v) Effort levels of the manager are decreasing in the degree of costs substitution $(\pm)$. Monitoring effort levels of the head-office will increase due to the strategic substitutability.

(vi) Effort levels of the head-office are increasing in the payoffs that can be generated by a division. Similarly, effort levels of the manager are increasing in the size of his private benefits.

Proof. See appendix. □

Point (i), (iii), and (vi) in the above proposition are the same as point (i), (iii), and (v) in the previous proposition.

Point (ii) is obtained because there are two opposing effects of a change in $e_{k_1}^n$ (or $e_{k_2}^n$): On the one hand, an increase in $e_{k_1}^n$ ($e_{k_2}^n$) will decrease $e_{k_2}^n$ ($e_{k_1}^n$) due to the substitutability of the costs of effort. On the other hand, it will decrease $E^{in}$; which in turn will raise $e_{k_2}^n$ ($e_{k_1}^n$). As is shown in appendix, we can check it using the following total derivative.

$$\frac{de_{k_2}^n}{de_{k_1}^n} = \frac{\frac{\partial e_{k_2}^n}{\partial e_{k_1}^n} (E^{in} - E_{k_2}^n)}{\frac{\partial E^{in}}{\partial e_{k_1}^n}} + \frac{\frac{\partial e_{k_2}^n}{\partial e_{k_1}^n}}{\frac{\partial E^{in}}{\partial e_{k_1}^n}}$$

(21)

The net effect is unclear. It depends on the degree of costs of effort substitution $\pm$. If $\pm$ is sufficiently high, we have $\frac{de_{k_2}^n}{de_{k_1}^n} < 0$; otherwise we have $\frac{de_{k_2}^n}{de_{k_1}^n} > 0$. This means that the appointed manager might still be willing to increase effort on another division despite the presence of costs of effort substitution if the degree of substitution is sufficiently low. Otherwise, if the degree of substitution is too high, it might be optimal for the manager to concentrate only on a division and spend no effort on the other division.

A higher $\mathcal{R}$ (complementary gains) implies that the head-office's payoffs will be higher when the head-office is informed, but on the other hand she will have to forego a higher share of her payoffs when she is not informed. The first effect dominates the second effect. Because of strategic substitutability in effort levels, the manager will then be less willing to exert effort.

An increase in the degree of substitution $(\pm)$ in the manager's cost function implies that a higher effort spent on a division will further increase marginal costs of increasing effort on another division. Thus, the manager faces increasingly high coordination costs. As a result, a higher $\pm$ creates a dis-incentive for the appointed manager to exert effort on the two divisions.
4 Comparing Managerial Effort Levels

Suppose that manager 1 is the appointed manager in an integrated form, then we have  \( k = 1 \). We will now compare the optimal effort levels that manager 1 will exert on division 1 in an integrated structure (\( e_{k1}^{n} \)) with the optimal effort levels that he will exert on division 1 if instead the firm adopt a separated structure (\( e_{1}^{sp} \)). Subtracting \( e_{1}^{sp} \) from \( e_{k1}^{n} \) we obtain the following,

\[
\xi e = \frac{1}{i} \frac{2(1 + \bar{\omega}) B}{(1 + \bar{\mu}) i 2(1 + \bar{\mu})} - i \frac{(1 + 2\bar{\mu}) B}{1} - i \frac{(1 + 2\bar{\mu}) B}{1} - i \frac{(1 + 2\bar{\mu}) B}{1} \tag{22}
\]

in which \( \xi e = e_{k1}^{n} - e_{1}^{sp} \).

Suppose that integration does not bring complementary gains (\( \bar{\omega} = 0 \)) and there are no costs of effort substitution (\( \bar{\mu} = 0 \)), then we obtain the following,

\[
\xi e = \frac{\mu}{i} \frac{(1 + 2\bar{\mu}) B}{1} - i \frac{(1 + 2\bar{\mu}) B}{1} - i \frac{(1 + 2\bar{\mu}) B}{1} \tag{23}
\]

We observe that \( \xi e = 0 \) prevails only if managers are symmetric in their degree of interests congruence (\( \bar{\mu} = \bar{\mu} \)). While if they are not, and \( \bar{\mu} < \bar{\mu} \) (\( \bar{\mu} > \bar{\mu} \)), we have \( \xi e < 0 \) (\( \xi e > 0 \)). Thus, we can establish that,

**Proposition 3:** Suppose \( \bar{\omega} = 0 \) and \( \bar{\mu} = 0 \);

(i) If \( \bar{\mu} = \bar{\mu} \), then the manager will be indifferent between the two internal organizational forms.

(ii) If \( \bar{\mu} \neq \bar{\mu} \), then holding \( \bar{\mu} \) constant, \( \xi e \) is increasing in \( \bar{\mu} \); This implies that the manager will not be indifferent anymore.

**Proof.** It is straightforward from expression (23). ■

This implies that even if there are no complementary gains and costs interdependence, there might still be differences in the equilibrium effort levels exerted by the manager in the two different organizational structures, unless when \( \bar{\mu} = \bar{\mu} \) It depends crucially on the relative degree of interests congruence (\( \bar{\mu} \) vis-à-vis \( \bar{\mu} \)).

This result can be explained as follows. In a separated form, the manager can take advantage from a high degree of interest congruence between the head-office and his fellow manager, even though his interests might not be well-aligned to those of the head-office (his

\[^{9}\text{We have the mirror image of this case when the appointed manager is manager 2.}\]
own - is low). A high degree of interest congruence induces less monitoring by the head-office. Since by construction, the head-office's monitoring costs are inseparable, it implies that the manager will enjoy less monitoring as well. This is a kind of positive spillovers enjoyed by a manager in a separated form of internal organization. Thus, a manager with a low - will be more willing to exert effort in a separated form of internal organization than in an integrated form.

Now, suppose that \( \hat{\alpha} \leq 0 \) and \( \pm \geq 0 \). We know that \( \hat{\alpha} \) and \( \pm \) influence \( \xi \) through \( e_{k1}^n \) only. Proposition 2 shows that \( \frac{\partial \xi}{\partial \hat{\alpha}} < 0 \) and \( \frac{\partial \xi}{\partial \pm} < 0 \). Thus, \( \xi \) is decreasing in \( \hat{\alpha} \) and \( \pm \). It is then obvious that if \( \hat{\alpha} > 0 \) and \( \pm > 0 \); and whenever \( \hat{\alpha}_1 \neq \hat{\alpha}_2 \), we obtain \( e_{k1}^n < e_{k1}^{sp} \). If instead we have \( \hat{\alpha}_1 > \hat{\alpha}_2 \), then the sign of \( \xi \) will depend on the relative size of \( \hat{\alpha}; \pm \); and \( \hat{\alpha}_1 \).

To have a more comprehensive view on the relationship between, on the one hand, synergy (coordination) gains (\( \hat{\alpha} \)); the degree of cost substitution (coordination costs) (\( \pm \)) and the degree of interests congruence (\( - \)) and, on the other hand, managerial effort levels (\( e^n \) and \( e^{sp} \)) we carry out numerical examples. We fix some parameter values. Lemma 3 tells us that the valid range of \( \hat{\alpha} \) and \( \pm \) for all admissible values of \( - \) are \( 0 \leq \hat{\alpha} \leq \frac{1}{4} \) and \( 0 \leq \pm \leq \frac{1}{2} \). Thus, let us fix \( \hat{\alpha} = \frac{1}{4} \) and \( \pm = \frac{1}{2} \); and consider different values of \( - \) within the admissible range (\( 0 \leq \hat{\alpha} \leq \frac{1}{4} \)).

If managers are symmetric in their degree of interests congruence (\( \hat{\alpha}_1 = \hat{\alpha}_2 \)), it implies that there will be no positive spillovers on the other manager resulting from a high degree of interests congruence of a manager. If instead managers are asymmetric (\( \hat{\alpha}_1 \neq \hat{\alpha}_2 \)), then the relative size of \( \hat{\alpha}_1 \) to \( \hat{\alpha}_2 \) will influence managerial effort levels. We will, therefore, perform two numerical examples, the first one is for the case of \( \hat{\alpha}_1 = \hat{\alpha}_2 \), and the second one is for the case of \( \hat{\alpha}_1 \neq \hat{\alpha}_2 \). If \( \hat{\alpha}_1 = \hat{\alpha}_2 \) occurs; then it does not really matter which manager should be appointed to run the integrated organizational form because they are identical. This is not the case when \( \hat{\alpha}_1 \neq \hat{\alpha}_2 \).

Plugging the values for \( \hat{\alpha}; \pm \); \( \hat{\alpha}_1 \), and \( \hat{\alpha}_2 \) into (22) we obtain \( \xi \) as a function of \( \hat{\alpha} \) and \( \pm \). We depict the level curves of \( \xi \) at \( \xi = 0 \) for different values of \( \hat{\alpha}_1 \) and \( \hat{\alpha}_2 \). These curves thus represent 'iso-effort levels' curves showing combinations of \( \hat{\alpha} \) and \( \pm \) which give the same value of managerial effort in different internal organization forms (\( \xi = 0 \)).

If Managers are Symmetric (\( \hat{\alpha}_1 = \hat{\alpha}_2 \))

Figure 3 below depicts the 'iso-effort-levels' curves at \( \xi = 0 \).
The lower curve is valid for $\bar{\beta}_1 = \bar{\beta}_2 = 1$; while the upper curve is valid for $\bar{\beta}_1 = \bar{\beta}_2 = 0$. The dashed lines indicate that for all combinations of $\bar{\alpha}$ and $\bar{\beta}$ lying on the left-hand side (right-hand side) of the level curves we have $\xi e^\alpha > 0$ ($\xi e^\alpha < 0$). We can then establish the following results.

**Proposition 4:** If the two managers’ interests are equally aligned to the head-office’s interests ($\bar{\beta}_1 = \bar{\beta}_2$), then it is the case that,

(i) If $\bar{\alpha} > 0$ prevails; then managers will always exert lower effort levels in an integrated form of internal organization ($\xi e^\alpha < 0$).

(ii) If $\bar{\alpha} < 0$ prevails; then managers might or might not exert lower effort levels in an integrated form of internal organization, depending on the relative size of $\bar{\alpha}$ and $\bar{\beta}$.

(iii) If $\bar{\alpha} < 0$ prevails and $\bar{\beta}$ are increasing (approaching the lower ‘iso effort-levels’ curve of Figure 3), then a separated form will become increasingly attractive for the manager. If $\bar{\beta}$ are decreasing (approaching the upper ‘iso effort-levels’ curve of Figure 3), then an integrated form will become increasingly attractive for managers.

This result can be explained intuitively as follows. In an integrated form, a higher and positive $\bar{\alpha}$ will be beneficial for the head-office because it reflects synergy gains accruing from a better coordination between the two complementary divisions. Thus, it induces the head-office to monitor more. Unfortunately, it will not motivate the appointed manager.
to work hard. Furthermore, when $\pm$ is also positive; it will even be worse for the manager. As a result, the appointed manager will exert lower effort in an integrated form.

A negative $\beta$ on the other hand, will not be preferred by the head-office, because this means that integration creates negative externalities. The appointed manager, however, will be benefited, because a negative $\beta$ lowers the head-office’s incentive to monitor. This will motivate the manager to exert higher effort levels in an integrated form, unless $\pm$ is sufficiently high.

Another interesting result is that, a higher value of the degree of interests congruence ($\bar{\gamma}$) makes a separated form more attractive for managers. We know that a higher value of $\bar{\gamma}$ implies a larger utility for managers, and thus will motivate managers to work harder. However, the effect of a higher $\bar{\gamma}$ on the managerial effort differs in size for different organizational forms. In an integrated form, the effect of $\bar{\gamma}$ will be smaller because there is a countervailing effect arising from a positive degree of costs of effort substitution ($\pm$). In a separated form, this countervailing effect does not exist. Consequently, if we have $j|\beta| < \pm$ then a manager will tend to exert higher effort levels in a separated form than in an integrated form. When $\gamma$ increases, a separated form will become more attractive for managers, unless of course when $j|\beta| > \pm$.

If Managers are Asymmetric ($\bar{\gamma}_1 \neq \bar{\gamma}_2$)

Next, we will do the same analysis for the case of $\bar{\gamma}_1 \neq \bar{\gamma}_2$. Figure 4 below illustrates the results of the numerical examples. The lower level curve is valid for $\bar{\gamma}_1 = 0$ and $\bar{\gamma}_2 = 1$; while the upper level curve is valid for $\bar{\gamma}_1 = 1$ and $\bar{\gamma}_2 = 0$. As in the previous case, we have $\zeta E^a < 0$ and $\zeta E^a < 0$ in, respectively, the left-hand side area and the right-hand side area of the level curves.
Figure 4: Managerial Effort Levels (−₁, 0, = 0.25, B = 0.5)

We obtain the following results,

**Proposition 5:** If the two managers’ interests are not equally aligned to the head-office’s interests (−₁), then it is the case that,

(i) If ® > 0, ± > 0; and −₁ < −₂ prevail, then managers will always exert lower effort levels in an integrated form of internal organization (±e < 0).

(ii) If −₁ is increasing such that −₁ > −₂; then we may have either ±e < 0 or ±e > 0; depending on the size of ® and ±

(a) For ® > 0; we have ±e < 0 if ® and ± are big enough, otherwise ±e > 0.
(b) For ® < 0; we have ±e > 0; unless ± is sufficiently high:

If −₁ prevails, then the relative size of − will influence the managerial effort. As is mentioned before, a manager with a lower − in a separated form obtains a windfall benefit from the fact that the second manager has a higher −: A higher − induces less monitoring, which is good for managers. Consequently, if − of a manager increases, then an integrated form will become increasingly attractive for a manager with a high value of − relative to the other manager, but not for a manager with a low value of −: The manager with a low value of − would prefer to have a separated form of organization, because he can free-ride on the high value of − of his fellow manager.

Also, a manager with a very high value of − relative to the other manager will be more willing to tolerate a small adverse effect of positive synergy gains (® > 0) and a sufficiently small degree of costs of effort substitution on his incentive to exert effort. The positive effect of a high value of − relative to the other manager will be sufficiently strong enough to offset the negative effect of small and positive values of ® and ±

Note that if the appointed manager is a manager with a low value of − relative to the other manager, then he will only be willing to exert higher effort in an integrated form if ® < 0 (©© is sufficiently big) or if ± is sufficiently low.

If the head-office decides to adopt an integrated form of internal organization, then a manager with a high degree of interests congruence, or a manager who shares more or less a similar organizational value with the head-office will be appointed by the head-office (see also Enz, 1988).

10 With a sufficiently small ± the manager will still increase his effort levels exerted on both divisions if − increases. This is shown in the comparative statics results in Table 2 in appendix.
5 The Payoffs of the Head-Office

Suppose that $\alpha = 0$ and $\pm = 0$; then if $\beta_1 = \beta_2$ prevails, there will be no difference between internal organizational forms. However, if $\beta_1 \neq \beta_2$ prevails, then the head-office will not be indifferent anymore. Her payoffs obtained from managing an internal organizational form will depend on the relative degree of interests congruence.

Now, let us allow for $\alpha > 0$ and $\pm > 0$: We again perform two numerical examples. The first one is for the case of symmetric managers ($\beta_1 = \beta_2$) and the second one is for the case of asymmetric managers ($\beta_1 \neq \beta_2$). We use the same parameter values as before, and plug them into expressions (4) and (1). Subtracting (1) from (4) we will obtain $(U_{in} - U_{sp})$ as a function of $\alpha$ and $\pm$. Let us denote $(U_{in} - U_{sp})$ as $U^h$. We will use $U^h = 0$ as the benchmark, and then show the level curves of $U^h$. These plots represent 'iso-payoffs' curves for the head-office.

If Managers are Symmetric ($\beta_1 = \beta_2$)

Figure 5 below depicts the 'iso-payoffs' curves at $U^h = 0$. Note that when $\alpha$ increases then the 'iso-payoffs' curve will shift rightward.

We can establish the following result.

Proposition 6: If the two managers' interests are equally aligned to the head-office's interests ($\beta_1 = \beta_2$), then it is the case that,
(i) If \( \mathcal{R} < 0 \) prevails, then the head-office will always prefer a separated form to an integrated form of internal organization.

(ii) If \( \mathcal{R} > 0 \) prevails, then the head-office's incentive to integrate divisions will be increasing, unless the degree of costs of effort substitution (\( \mathcal{E} \)) is sufficiently high. In order to keep integration remains attractive for the head-office, \( \mathcal{R} \) has to be sufficiently high.

(iii) If \( \mathcal{A} \) are increasing, then a separated form of internal organization will be increasingly preferred to an integrated form of internal organization.

The area to the left of the iso-payoffs curves represents the case of \( \zeta \, U^a < 0 \); and the area to the right of the iso-payoffs curves represents the case of \( \zeta \, U^a > 0 \). In general, if integration does not bring complementary gains (\( \mathcal{R} < 0 \)), there will be no incentive for the head-office to integrate the two divisions.

If \( \mathcal{E} \) is sufficiently large, an integration becomes less attractive if the gains from integration (\( \mathcal{R} \)) are relatively low. A manager in an integrated form of internal organization will be confronted with a certain degree of costs of effort substitution (\( \mathcal{E} \)), while a manager in a separated form will not be. A positive \( \mathcal{E} \) creates a dis-incentive to exert effort for a manager in an integrated form.

If \( \mathcal{A} \) are increasing, managers will be motivated to exert higher effort levels (see propositions 1 and 2). However, due to the presence of parameter \( \mathcal{E} \) in an integrated form, the effect of increasing \( \mathcal{A} \) on the managerial effort will be more pronounced in a separated form than in an integrated form. Thus, in order to take advantage of increasing \( \mathcal{A} \), it will be better for the head-office to choose a separated form:

If \( \mathcal{E} \) becomes higher, a separated form will become more attractive, unless of course if \( \mathcal{R} \) is very high. It is obvious that when we have \( \mathcal{E} = 0 \) and \( \mathcal{R} > 0 \), an integrated form will always be preferred by the head-office.

If managers are asymmetric (\( \alpha_1 \neq \alpha_2 \))

Figure 6 below depicts the case of \( \alpha_1 \neq \alpha_2 \)
Figure 6: The Payoffs of the Head-office ($\bar{\beta}_1 \neq \bar{\beta}_2$; $\| = 0.25; B = 0.5$)

We can establish the following result.

Proposition 7: If the two managers' interests are not equally aligned to the head-office's interests ($\bar{\beta}_1 \neq \bar{\beta}_2$), then it is the case that,

(i) If $\bar{\beta}_1 > \bar{\beta}_2$ ($\bar{\beta}_1 < \bar{\beta}_2$) prevails, then in general the head-office would like to integrate (to separate) the two divisions: However, there might still be some exceptions depending on the relative size of $\bar{\beta}$s: If $\bar{\beta}_1 > \bar{\beta}_2$ ($\bar{\beta}_1 < \bar{\beta}_2$) and the relative difference is big, an integrated (separated) form can still dominate a separated form even if $\bar{\beta} < 0$ ($\bar{\beta} > 0$).

Thus, in general a decision to integrate divisions will be influenced by the size of synergy gains. However, it does not mean that when $\bar{\beta} > 0$ it is always better to integrate, or when $\bar{\beta} < 0$ it is always better to separate divisions. It also depends on the size of $\|$: When the appointed manager in an integrated form and the head-office have aligned interests, it could still be profitable to integrate even though $\bar{\beta} < 0$: The benefit of the aligned interests will be sufficiently high to outweigh the small costs of integration ($\bar{\beta} < 0$). Of course this will not happen if $\|\bar{\beta}$ is high, as the manager will then be less interested in exerting higher effort.

On the other hand, if the appointed manager has a much lower $\|\bar{\beta}$ than the other manager, then in general an integration will be less preferred by the head-office. This is because this manager will work less hard in an integrated form (see result 2), it is better
for him to be in a separated form and enjoy the spillover benefits of a higher \( - \) of his fellow manager. The adverse effect of a low \( - \) on the exerted effort in an integrated form makes an integration less attractive for the head-office, unless the benefits of integration (\( \delta \)) are sufficiently high to outweigh this adverse effect. This, of course, implies that if the head-office decides to integrate divisions, she will pick a manager with the highest degree of interests congruence.

6 The Choice of Internal Organization

If \( \delta = 0; \pm = 0; \) and \( \bar{\gamma} = \bar{\epsilon} \); then the head-office will be indifferent between the two internal organizational forms. Integration and separation will give the same payoffs for the head-office. If \( \bar{\gamma} \neq \bar{\epsilon} \); then the head-office will not be indifferent anymore between the two forms.

If instead \( \delta \neq 0 \) and \( \pm \neq 0 \) prevail. Higher gains (\( \delta \)) are good for the head-office. However, in our model higher gains also induces the head-office to monitor more, which will make a manager in an integrated form reluctant to work hard. This is bad for the head-office. Yet, if the benefits of integration are still sufficiently high to offset this adverse effect, the head-office might still prefer an integrated form to a separated form.

If Managers are Symmetric (\( \bar{\gamma} = \bar{\epsilon} \))

Figure 7 below depicts the choice of internal organization when \( \bar{\gamma} = \bar{\epsilon} \).

![Figure 7: The Choice of Internal Organization (\( \bar{\gamma} = \bar{\epsilon}; \delta = 0.25; \beta = 0.5 \))](image)

From our previous discussion we know that, a high \( \gamma \) will be more preferred by a manager in a separated form. In addition, we also know that a high \( \pm \) will not be preferred
by a manager in an integrated form. Thus when \( \bar{\alpha} \) and \( \pm \) are high, we have downward pressures on the managerial effort. If \( \bar{\alpha} \) is not high enough to outweigh the downward pressures, then it is better for the head-office to separate the two divisions.

We see that if \( \bar{\alpha} \) and \( \pm \) are such that we are in region III of the above graph, we have a conflicting situation. The head-office would like to have an integration, but an integrated form will not induce high managerial effort. Yet, an integration is still preferred by the head-office. If we are in region II, the head-office prefers to have a separated form, and this form of organization will also motivate the managers to work hard. Region II becomes smaller when \( \bar{\alpha} \) decreases. Finally, if we are in region I, there will be again a conflicting case. The manager will work harder only in an integrated form. However, the head-office prefers to sacrifice effort inducement and choose to separate divisions.

If Managers are Asymmetric (\( \bar{\alpha}_1 \neq \bar{\alpha}_2 \))

The graph can be explained as follows. In region I, the head-office prefers to have a separated form even though it does not motivate managers to work hard. In region II, the head-office still prefers a separated form, and now managers will be motivated to work hard in a separated form. In region III, the head-office will choose an integrated form, even though this organizational form is not conducive for the alleviation of managerial effort. In region IV the head-office will adopt an integrated form of internal organization. Manager 1 will also work harder in this internal organizational form.

If \( \bar{\alpha}_1 < \bar{\alpha}_2 \), then the left-hand side curve will shift leftward, and the right-hand side curve will shift rightward. As a result region II becomes bigger, and region IV may disappear if
is small relative to . The opposite happens when is high relative to . Figure 8 shows the highest possible relative to . The more congruent the interests of manager is, the more attractive an integrated form will be for the head-office.

An integrated form will also become more attractive for the head-office when is positive and large. However, there is an adverse effect of a positive value of on the managerial effort.

Note that if and is relatively high compared to , then the head-office might still be willing to tolerate an integration with negative externalities (), in order to take advantage of the higher value of . On the contrary, if and is relatively small compared to then an integrated form will not be chosen, unless is positive and large.

Summary:

We can summarize our results for both cases ( and ) in the following proposition.

Proposition 9: The optimal choice of internal organization is determined by the trade-off between the benefits of integration and the managerial effort elicitation. In addition we find that:

(i) If (no spillover-effect); then the lower the value of are, the more attractive integration will become.

(ii) If (there is a spillover-effect); then the higher the asymmetry in terms of degree of interest congruence between the two managers is, the more attractive integration will become.

(iii) If integration is optimal, the head-office will pick a manager with the highest degree of interests congruence to run an integrated form of internal organization.

(iv) Integration (separation) of divisions can be used as a commitment device by the head-office. For instance, by tolerating a 'non-synergistic' integration, provided that the head-office's losses are not too big, the head-office can commit not to intensely monitor managers.

7 An Extension: The Separability of the Head-office's Monitoring Costs

If we assume that the head-office is able to separate the monitoring costs, then expression (6) and (13) will become simpler,
\[ E_{1}^{sp} = (1 \ i \ e_{1}^{sp-1}) \] and \[ E_{2}^{sp} = (1 \ i \ e_{2}^{sp-2}) \]

\[ E_{1}^{in} = i \ i \ e_{k1}^{in-1} (1 + \bar{a}) \] and \[ E_{2}^{in} = i \ i \ e_{k2}^{in-1} (1 + \bar{a}) \]

This implies that in a separated form there will be no effect of an increase in \( e_{1}^{sp} \) on \( e_{2}^{sp} \). The following total derivative will be zero.

\[
\frac{de_{2}^{sp}}{de_{1}^{sp}} = \mu \left( \frac{e_{2}^{sp}}{e_{1}^{sp}} \right)_{\bar{a}=0} + \mu \left( \frac{e_{2}^{sp}}{e_{1}^{sp}} \right)_{\bar{a}=0}
\]

In an integrated form, we will then have a negative effect of an increase in \( e_{1}^{sp} \) on \( e_{2}^{sp} \):

\[
\frac{de_{2}^{in}}{de_{1}^{in}} = \mu \left( \frac{e_{2}^{in}}{e_{1}^{in}} \right)_{\bar{a}=0} + \mu \left( \frac{e_{2}^{in}}{e_{1}^{in}} \right)_{\bar{a}<0}
\]

With straightforward manipulations we can compare managerial effort in a separated form and in an integrated form,

\[
\xi e^{en} = e_{k1}^{in} i \ e_{1}^{sp} = \frac{B \ (1 \ i \ (1 + \bar{a}))}{(1 + \bar{a}) i \ -B i \ (1 + \bar{a}) i} \ B \ (1 \ i \ i)
\]

If \( \bar{a} = 0 \) and \( \bar{a} = 0 \) occur, then the exerted effort levels in the two organizational forms are the same.

\[
\xi e^{en} = \frac{B \ (1 \ i \ i)}{1 i \ -B i} i \ B \ (1 \ i \ i)
\]

It is obvious that the relative size of \( \bar{a} \) does not matter. Actually, the case of separability of the head-office’s monitoring costs is analytically equivalent to the case of equal \( \bar{a} \)s for the two managers when monitoring costs are inseparable. Thus, the separability of monitoring costs implies that the relative degree of interest congruence does not matter. Hence, we have analogous results as in proposition 4.

8 Concluding Remarks

In this paper we study the design of a firm’s internal organization. We consider a firm consisting of a head-office, two divisions, and two managers. The head-office can integrate
the two divisions and appoint one manager to run the integrated-divisions. Alternatively, the head-office can separate the two divisions and appoint a manager to run each division.

In our model, the head-office has formal authority over the choice of projects to be implemented. However, the head-office is willing to delegate the decision making authority to divisional managers whenever divisional managers are better informed about the projects’ prospects. In this case divisional managers have real authority.

We show that the head-office’s optimal choice of internal organization depends on the trade-off between synergy gains of integration and the elicitation of managerial effort. This trade-off occurs because the head-office concerns more about the firm’s total payoffs, while the managers concern more about their own private benefits. Their interests are not necessarily aligned.

The firm’s total payoffs are likely to increase when coordination between divisions can be created. One way of creating and sustaining coordination is by integrating divisions under a common manager. If the potential coordination benefits are large, an integrated form of internal organization will become appealing for the head-office. The head-office will also be more motivated to monitor managers when coordination benefits are large. However, there is an adverse effect of monitoring. Managers will have less incentives to take initiatives and to exert effort because it is more likely that they will be overruled by the head-office. On the contrary, when integration does not bring significant benefits, there will be less interests for the head-office to monitor.\footnote{Of course given that the integration is not beneficial, the head-office might not choose to integrate divisions if the negative externalities are too big.} It will motivate managers to exert higher effort, because their chance of getting their own way without being overruled is higher. As a matter of fact, by tolerating a ‘not-profitable’ integration, provided that the head-office’s losses are not too big, the head-office can commit not to intensely monitor the managers. This might be useful to induce managers to exert higher effort in an integrated form of internal organization. Higher managerial effort and smaller monitoring effort are expected to outweigh the losses due to the value-destroying integration.

We also show that when manager 1 has a high relative to manager 2; he might still be willing to tolerate a small positive effort although it induces a more intense head-office monitoring. This is because the high implies that the manager will still be able to obtain big private benefits from the preferred project of the headquarter. Thus, being overruled does not make that much different from not being overruled.

Managerial effort levels are also determined by the degree of costs of effort substitution. The costs of effort substitution represents coordination costs that have to be incurred by a manager in an integrated form of organization. These coordination costs could be the result of the differences in organizational cultures of the divisions or managers’ limited
time and ability. For instance, a manager might be more able to handle a certain kind of job and not the other. Consequently, when he has to focus on two jobs there may be a trade-off. If he spends more time and effort on the job that he knows well, he will have to put less attention on the second job. Given that he is not that familiar with the second job, it becomes then costlier for him to handle this job because he has to put more attention and time when both his ability and time are limited.

Finally, managerial effort levels are also influenced by the degree of interests congruence between managers and the head-office. The higher the congruence parameter is, the less the monitoring effort of the head-office will be, and thus the more willing the managers will be to exert effort.

There are two caveats of the paper. The rst one is that in this paper we assume that a manager who is appointed to run an integrated form of internal organization is picked from the existing managers. In the model setting, there are two managers, thus the appointed manager could be one of them. As a matter of fact, the head-office can also hire an outsider to become the manager of the integrated-divisions. To justify our setting, we essentially assume that the degree of interests congruence \( \bar{\theta} \) of an outside manager is not known to the head-office. This is not the case with inside managers. The head-office knows exactly how their interests are aligned. In this kind of setting, it is better for the head-office to pick a manager that she knows well to run the integrated-divisions. The second one is that in the model we assume that formal authority always resides on the hand of the head-office. In fact, we can also consider the case where the head-office delegates formal authority to divisional managers (see Aghion and Tirole (1997)). This paper follows Baker, Gibbons, and Murphy (1999) in assuming that the head-office always has formal authority.

Appendix:

Proof of Proposition 1:

It is straightforward to see from expressions (6), (7), and (8) that \( \frac{\partial E_{sp}}{\partial e_{sp}} < 0; \frac{\partial E_{sp}}{\partial e_{sp}^2} < 0; \) and thus explain point (i) of the proposition.

Checking other derivatives we obtain the following,

\[
\frac{d e_{sp}}{d e_1} = \frac{\mu}{\partial E_{sp}} \frac{\partial E_{sp}}{\partial e_{sp}} + \frac{\mu}{\partial E_{sp}} \frac{\partial E_{sp}}{\partial e_{sp}^2} + \frac{\partial E_{sp}}{\partial e_{sp}} \frac{\partial E_{sp}}{\partial e_{sp}^2} + \frac{\partial E_{sp}}{\partial e_{sp}^2} \frac{\partial E_{sp}}{\partial e_{sp}^2}
\]

(24)

We have \( \frac{\partial e_{sp}}{\partial e_1} = 0 \) and using point 1 above we can establish that \( \frac{\partial E_{sp}}{\partial e_{sp}} > 0 \): Hence we know that \( \frac{de_{sp}}{de_1} > 0 \).

\[
\frac{\partial E_{sp}}{\partial e_1} = \frac{b (2i - j)}{b (2i - j - 2) + b (1)}
\]

(25)

Using Lemma 1 and ignoring the case of \( b \leq \frac{1}{2} \), we can verify that \( \frac{\partial E_{sp}}{\partial e_1} < 0 \): Strategic
subsitutability in e¤ort levels implies that \( \frac{\partial e_{sp}}{\partial \psi_1} > 0 \) and \( \frac{\partial e_{sp}}{\partial \psi_1} > 0 \). A nalagously, we have the same results for increasing \( \bar{\psi}_2 \).

Finally, taking the derivatives of expressions (9) and (10) to check the e¤ect of increasing \( \psi \) and \( B \) results in,

\[
\frac{\partial E_{sp}}{\partial \psi} = \frac{2i \cdot B \left( \bar{\psi}_1 + \bar{\psi}_2 \right)}{\left( i \cdot B \left( \bar{\psi}_1 + \bar{\psi}_2 \right) i \cdot 1 \right)^2}
\]

\[
\frac{\partial E_{sp}}{\partial B} = \frac{2i \cdot 2 \left( \bar{\psi}_1 + \bar{\psi}_2 \right) i \cdot 1}{\left( i \cdot B \left( \bar{\psi}_1 + \bar{\psi}_2 \right) i \cdot 1 \right)^2}
\]

\[
\frac{\partial \bar{e}_{sp}}{\partial \psi} = \frac{B \cdot 2 \left( \bar{\psi}_1 + \bar{\psi}_2 \right) i \cdot 2B}{\left( i \cdot B \left( \bar{\psi}_1 + \bar{\psi}_2 \right) i \cdot 1 \right)^2}
\]

\[
\frac{\partial \bar{e}_{sp}}{\partial B} = \frac{1 \cdot 2i}{\left( i \cdot B \left( \bar{\psi}_1 + \bar{\psi}_2 \right) i \cdot 1 \right)^2}
\]

Using Lemma 1 and taking \( \psi < \frac{1}{2} \) we know that \( \frac{\partial E_{sp}}{\partial \psi} < 0; \frac{\partial E_{sp}}{\partial B} > 0; \frac{\partial \bar{e}_{sp}}{\partial \psi} < 0; \) and \( \frac{\partial \bar{e}_{sp}}{\partial B} > 0; \)

By symmetry we know that \( \frac{\partial \bar{e}_{sp}}{\partial \psi} < 0; \) and \( \frac{\partial \bar{e}_{sp}}{\partial B} > 0; \)

Table 1 below summarizes all the above results.

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Table 1: Separated Form

**Proof of Proposition 2:**

It is straightforward to see from expressions (13), (14), and (15) that \( \frac{\partial E_{in}}{\partial \psi_1} < 0; \frac{\partial E_{in}}{\partial \psi_2} < 0; \) and thus explain point (i) of the proposition.

Cheking other derivatives we obtain the following,

\[
\frac{\partial e_{in}}{\partial \psi_k} = \frac{\mu \cdot \frac{\partial E_{in}}{\partial \psi_k} \|}{\mu \cdot \frac{\partial E_{in}}{\partial \psi_k} \|} + \frac{\partial e_{in}}{\partial \psi_k} \| \>{0}
\]

\[
\frac{\partial e_{in}}{\partial \psi_k} = \left( 1 + \| \right) \bar{e}_{sp1} \| i \pm
\]
The net effect is unclear. It depends on the magnitude of \( \pm \) For \( \pm \) sufficiently high, \( \delta E_k^{in} < 0 \); otherwise \( \delta E_k^{in} > 0 \): The case of an increase in \( e_{k1}^{in} \) is analogous.

Taking the derivative of expression (18) w.r.t. \( \$ \) we can analyze the effect of changes in \( \$ \) on the head-ofce's monitoring effort.

\[
\frac{\partial E_{in}}{\partial \$} = \frac{2 \bar{k} B \left( 1 + \bar{\$} \right) \left( 1 + \pm \right)}{(2^{-\bar{k}^1} B \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right))^2}
\]

(32)

Because the denominator is clearly positive, we just need to check the numerator. To have a positive sign for the numerator, we need that \( 1 + \pm > \bar{\$} \). By lemma 3 we know that the RHS will be smaller than 1; so the above condition will always be satisfied. This means that \( \frac{\partial E_{in}}{\partial \$} > 0 \):

It can be straightforwardly inferred from (19) and (20) that \( \frac{\partial E_{k1}^{in}}{\partial \bar{\$}} < 0 \). By point (i) of the proposition we know that \( E_{in} \) will increase.

Now we check for the effect of changes in the degree of interest congruence (\( \bar{\$} \)) on the head-ofce's effort levels, by taking the following derivative.

\[
\frac{\partial E_{in}}{\partial \bar{\$}} = 2 \bar{k} B \left( 1 + \bar{\$} \right) \frac{2 \bar{k} \left( 1 + \pm \right) \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right)}{(2^{-\bar{k}^1} B \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right))^2}
\]

(33)

The denominator is clearly positive. The numerator is positive is \( 2 \bar{k} \left( 1 + \bar{\$} \right) > 1 \) is satisfied. By lemma 3 we know that \( 2 \bar{k} > \frac{1}{2} \). Hence, for all admissible values of \( \$ \), i.e. \( \$ > \frac{1}{2} \), the above condition will not be satisfied. Thus, we obtain \( \frac{\partial E_{in}}{\partial \bar{\$}} < 0 \):

Finally, similar to the case of a specialized form of internal organization, the head-ofce's and the manager's effort levels are increasing in own payoffs and private benefits. This follows from checking the following derivatives,

\[
\frac{\partial E_{in}}{\partial B} = 2 \bar{k} \left( 1 + \bar{\$} \right) B \left( 1 + \bar{\$} \right) \frac{2 \bar{k} \left( 1 + \pm \right) \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right)}{(2^{-\bar{k}^1} B \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right))^2}
\]

(34)

\[
\frac{\partial E_{in}}{\partial \bar{\$}} = 2 \bar{k} \left( 1 + \bar{\$} \right) \left( 1 + \pm \right) \frac{\bar{k} \left( 1 + \pm \right) \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right)}{(2^{-\bar{k}^1} B \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right))^2}
\]

(35)

\[
\frac{\partial E_{k1}^{in}}{\partial B} = \frac{(1 + 2 \bar{k} \left( 1 + \bar{\$} \right) \left( 1 + \pm \right) \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right)}{(2^{-\bar{k}^1} B \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right))^2}
\]

(36)

\[
\frac{\partial E_{k1}^{in}}{\partial \bar{\$}} = 2 \left( 1 + \bar{\$} \right) B \left( 1 + \bar{\$} \right) \frac{\bar{k} \left( 1 + \pm \right) \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right)}{(2^{-\bar{k}^1} B \left( 1 + \bar{\$} \right) i \left( 1 + \pm \right))^2}
\]

(37)

Using Lemma 3, it can be easily shown that \( \frac{\partial E_{in}}{\partial B} < 0, \frac{\partial E_{in}}{\partial \bar{\$}} > 0, \frac{\partial E_{k1}^{in}}{\partial B} > 0; \) and \( \frac{\partial E_{k1}^{in}}{\partial \bar{\$}} < 0 \): Table 2 below summarizes the results.
Table 2: Integrated Form

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References


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