Multi-Model Adaptive Switching Control
with Fine Controller Tuning

Simone Baldi ∗∗, 1 Giorgio Battistelli ∗ Daniele Mari ∗
Eedoardo Mosca ∗ Pietro Tesi ∗

∗ Dipartimento Sistemi e Informatica, DSI - Università di Firenze,
Via S. Marta 3, 50139 Firenze, Italy (email: 
{battistelli,mari,mosca,ptesi}@dsi.unifi.it).
∗ Computer Science Department, University of Cyprus (UCY),
75 Kallipoleos Street, 1678 Nicosia, Cyprus (email: 
sbaldi@cs.ucy.ac.cy).

Abstract: This paper addresses the problem of controlling uncertain plants by means of a
finite family of candidate controllers supervised by an appropriate switching logic. To this end,
a novel adaptive control scheme is introduced, whereby switching among pre-designed candidate controllers is suitably combined with an iterative control design procedure. These results are shown to be of practical relevance for on-line implementation of highly performing adaptive switching control schemes. Simulation results are presented.

Keywords: Control of uncertain plants; Adaptive control; Switching control.

1. INTRODUCTION

In recent years, adaptive switching control has emerged as an alternative to conventional continuous adaptation in order to control plants in presence of large model uncertainties. In switching control, a so-called supervisor switches-on at any time in feedback with the plant one controller from a family of candidate controllers using recorded plant I/O data. These data are processed in such a way to enable the supervisor of deciding whether or not the currently switched-on controller is adequate, and, in the negative, replace it by another candidate controller. Compared to conventional forms of adaptation, switching control offers the definite advantage that controller selection is carried out by means of logic-based switching rather than continuous tuning, thus allowing fast (discontinuous) adaptation of the control system. In this respect, (Morse [1995]) provides a general overview of the topic.

Adaptive switching control has been approached by several diversified techniques, within both model-free control (Fu and Barnish [1986], Safonov and Tsao [1997], Wang et al. [2007]) and model-based control (Narendra and Xiang [2000], Morse [1995], Zhivoglyadov et al. [2000], Pait and Kassab [2001], Hespanha et al. [2003], Baldi et al. [2010]). Although these contributions originate from fundamentally different approaches, the common idea is to have a finite family of pre-designed candidate controllers, so that, for each possible plant model in the plant uncertainty set, at least one of the controllers performs satisfactorily. However, the adoption of a finite number of candidate controllers may prevent from achieving optimal control performance because of possible detuning arising from the discrete nature of the controller family in contrast with the continuous nature of the plant uncertainty. Even more importantly, satisfactory trade-offs between the conflicting objectives of number of candidate controllers (hence memory/computational load) and desired performance need not even exist in some cases, especially if the uncertain plant uncertainty set is large.

The paper extends the approach considered in (Baldi et al. [2010]) so as to combine the positive features of switching and tuning: respectively, speed and accuracy of the control system response. Assuming that a family of $N$ pre-designed candidate controllers is available, a candidate controller is first selected via switching, in accordance with some suitable performance criterion; then, by means of an appropriate tuning mechanism, the parameters of the switched-on controller are adjusted so as to design an ($N + 1$)-th candidate controller, potentially yielding higher performance. It is to be pointed out that the idea of combining switching and tuning schemes for adaptive control is by no means new in the literature (Narendra and Xiang [2000]). However, despite the similarities, the approach developed in this paper differs from that of Narendra and co-authors, since the control design procedure is formulated as a parameter optimization problem in which the optimization is carried directly on the controller parameters, with no intermediate plant model identification effort.

2. PROBLEM SETTING

Let the “switched” system be represented as follows

$$ u(t) = P(u(t)) $$
$$ y(t) = C_{\sigma(t)}(r - y(t)) $$

where $t \in Z_+$, $Z_+ := \{0, 1, \cdots \}$, $P : u \mapsto y$ denotes the uncertain plant, and $\sigma(t)$ the subscript identifying the candidate controller connected in feedback to the plant at

1 S. Baldi is supported by the European Commission FPT-ICT-5-3.5, Engineering of Networked Monitoring and Control Systems, under the contract 257806 (AGILE).
The uncertain plant \( P \) consists of the following discrete-time LTI dynamic system
\[
P: A(d) y(t) = B(d) u(t),
\]
with input \( u(t) \in \mathbb{R} \), output \( y(t) \in \mathbb{R} \), and \( B(d)/A(d) \) the transfer function of \( P \), where \( A(d) \) and \( B(d) \) denote polynomials in the unit backward shift operator \( d \), with strictly Schur greatest common divisor (g.c.d.). A so-called supervisor handles the plant I/O records in order to generate the sequence \( \sigma \) giving rise to a switching controller of the form \( u(t) = C_\sigma(t) (r - y)(t) \), where \( r \) is a reference signal. Specifically, each candidate controller belongs to a family \( \mathcal{C} = \{C_i, i \in \mathcal{N}\} \), \( \mathcal{N} := \{1, 2, \ldots , N\} \), of one-DOF LTI controllers \( C_i \), with transfer functions \( C_i(d) := S_i(d)/R_i(d) \), \( i \in \mathcal{N} \), where \( S_i(d) \) and \( R_i(d) \) are polynomials with strictly Schur g.c.d. Accordingly, given \( \sigma(t) \), the plant input \( u(t) \) is given by \( R_\sigma(t)u(t) = S_\sigma(t)(r - y)(t) \). In the sequel, the linear time-varying feedback system (1) will be denoted by \((P/C_\sigma(t))\).

**Remark 1.** Following the lines of (Battistelli et al. [2010]), the analysis carried out herein can be extended to include the presence of disturbances in the loop. Such a case is not addressed here only for the sake of simplicity. \( \square \)

Let \(| \cdot |\) denote the Euclidean norm of a vector, and \( S(\mathbb{Z}_+) \) the space of all real-valued vector sequences on \( \mathbb{Z}_+ \). For any \( s \in S(\mathbb{Z}_+) \), and \( t_0, t \in \mathbb{Z}_+; t_0 \leq t \), we define \( s|_{t_0} := \{s(t_0), \ldots , s(t)\} \). If \( t_0 = 0 \), we simply write by \( s^t \).

Finally, \( \|s|_{t_0}^t\| := \sqrt{\sum_{k=t_0}^t |s(k)|^2} \).

**Definition 1.** The switched system \((P/C_\sigma(t))\) is said to be stable relatively to \( r \) or, shortly, \( r\)-stable if, for every input \( r \in S(\mathbb{Z}_+) \), there exist positive reals \( c_1 \) and \( c_2 \) such that
\[
\|s|^t \| \leq c_1 + c_2 |r|^t, \quad \forall t \in \mathbb{Z}_+ \tag{3}
\]
where \( z(k) := [u(k), y(k)]^T \).

Let \( P \) belong to a plant uncertainty set \( \mathcal{P} \). A pre-requisite for the adaptive control scheme is that the family \( \mathcal{C} \) be adequately chosen relatively to \( \mathcal{P} \). In this respect, given any \( \mathcal{P} \in \mathcal{P} \), let \( S(\mathcal{P}) \subseteq \mathcal{N} \) be the set of all indices \( s \in \mathcal{N} \) such that \((P/C_s)\) is internally stable.

**Definition 2.** The control problem is said to be feasible if \( S(\mathcal{P}) \neq \emptyset, \forall \mathcal{P} \in \mathcal{P} \).

To decide whether or not, and, in the affirmative, how to change the controller, the supervisor embodies a family \( \Pi := \{\Pi_i, i \in \mathcal{N}\} \) of test functionals such that, in broad terms, \( \Pi_i(t) \) quantifies the performance of the loop \((P/C_i)\) given the data up to time \( t \). In the hysteresis switching logic considered hereafter, at each step, one computes the least index \( i^* \) (in \( \mathcal{N} \)) such that \( \Pi_{i^*}(t) \leq \Pi_i(t), \forall i \in \mathcal{N} \).

Then, the switching index sequence \( \sigma \) is given by
\[
\sigma(t + 1) = \Pi(\sigma(t), \Pi(t)), \quad \sigma(0) = i_0 \in \mathcal{N} \quad \left\{ \begin{array}{ll}
l(i, \Pi(t)) = \begin{cases} i, & \text{if } \Pi_i(t) < \Pi_{i^*}(t) + h \\ i^*(t), & \text{otherwise} \end{cases} \\
\end{array} \right. \tag{4}
\]
where \( h > 0 \) is the hysteresis constant.

Let \( \mathcal{J} \) denote the class of all possible switching sequences \( \sigma \) giving rise to the switched system \((P/C_\sigma(t))\). Consider the assumptions:

**A1.** For each \( \sigma \in \mathcal{J} \) and \( i \in \mathcal{N} \), \( \Pi_i(t) \) admits a limit (even infinite) as \( t \to \infty \);

**A2.** For each \( \sigma \in \mathcal{J} \), there exist integers \( \mu \in \mathcal{N} \) such that \( \Pi_i(\cdot) \) is bounded.

**HSL Lemma** (Morse et al. [1992]) Let \( z \) denote the vector-valued sequence of \( I/O \) plant data, and \( \sigma \) the switching sequence resulting from (1) and (4). Then, for any initial condition and reference \( r \), if A1 and A2 hold, there is a finite time \( t_f \in \mathbb{Z}_+ \), after which no more switching occurs. Moreover, \( \Pi_{\sigma(t_f)}(\cdot) \) is bounded. \( \square \)

3. **INFEERENCE VIA REFERENCE-LOOP IDENTIFICATION**

As anticipated, adaptive switching control has been approached within both model-free control and model-based control. In a recent paper (Baldi et al. [2010]), it is shown that it is possible to combine the positive features of model-free and model-based approaches in the following sense: by reducing the time duration of learning transients after start-up in case \( \mathcal{P} \) be tightly approximated by a nominal model distribution \( \mathcal{M} \); by also preserving, as in model-free control, stability irrespective of \( \mathcal{M} \), viz. irrespective of possible plant-model mismatches, thus retaining guaranteed stability properties under the only assumption of problem feasibility. The aim of this section is to briefly sum up the main results of (Baldi et al. [2010]), to which the reader is referred for a more detailed analysis.

We assume that a finite family \( \mathcal{M} = \{M_i, i \in \mathcal{N}\} \) of strictly causal LTI dynamic models \( M_i \) is available, \( M_i(d) := B_i(d)/A_i(d) \), \( i \in \mathcal{N} \), where \( B_i(d) \) and \( A_i(d) \) are polynomials with strictly Schur g.c.d.. The candidate controllers \( C_i \) are chosen so as to satisfy the feasibility condition, while the \( M_i \)'s, along with the associated \( C_i \)'s, form a finite family \( \mathcal{M}_R \) of internally stable reference-loops, each designed to fulfill desirable prescriptions (which, in general, need not be optimal relatively to any specific performance index). Given an unknown plant \( P \in \mathcal{P} \), the aim is to carry out a reference-loop identification task, viz., select a candidate controller \( C_{\sigma^*} \) in such a way that \((P/C_{\sigma^*})\) behave as closely as possible to one of the candidate reference-loops in \( \mathcal{P} \).

Hence, roughly speaking, the ideal goal of the switching supervisor, can be envisaged as follows. Given an uncertain plant \( P \in \mathcal{P} \), find an index \( i \in \mathcal{N} \) such that:

i) \((P/C_{\sigma_i^*})\) be stable;

ii) The behavioural data produced by \((P/C_{\sigma^*})\) in response to \( r \) be as close as possible to the ones produced by \((M_i/C_{\sigma^*})\) in accordance to the reference-loop identification criterion
\[
\sigma := \arg \min \sup_{i \in \mathcal{N}, r \neq 0} \frac{\| (P/C_\sigma) r - (M_i/C_{\sigma^*}) r \|}{\| (M_i/C_{\sigma^*}) r \|} \tag{5}
\]
where, by simplicity, no time-indication is shown.

**Remark 2.** As elaborated in (Van den Hof and Schrama [1995], Mosca and Agnoloni [2001]), the reason for using the percentage criterion (5) is essentially that, in case
of large plant uncertainties, we can have a different cost associated to each reference-loop.

Hereafter, it is shown how the criterion (5) leads, via the notion of a virtual reference, to a simple implementable solution. To maintain continuity, we defer comments on numerical computations to Sect. 4.2.

On-line implementation of (5) is impossible without using pre-routing schemes, which, in general, have to be ruled out since they typically cause large and long-lasting learning transients. A way for side-stepping such a difficulty hinges upon the use of the so-called virtual reference tool, as introduced by (Safonov and Tsao [1997]). At each time, and for each index \( i \in \bar{N} \), one solves in real-time with respect to \( v_i \) the following equation,

\[
v_i(t) = C_i^{-1}(d)(u(t) + y(t)), \quad t \in \mathbb{Z}_+\]

provided that the \( C_i \)'s be causal, and stably causally invertible (CSCI). As can be seen, \( v_i^o \) equals the input sequence that reproduces the \( I/O \) sequence \( \{u^o, y^o\} \) of the plant \( \mathcal{P} \) fed-back by the controller \( C_i \). In other terms, if \( (\mathcal{P}/C_{\sigma(i)}) \) is intended as the linear (time-varying) transformation (1) mapping \( r \) into \( z \), we find \( z = (\mathcal{P}/C_{\sigma(i)})r = (\mathcal{P}/C_i)v_i \).

Each candidate reference-loop \((\mathcal{M}_i/C_i)\), driven by the corresponding \( v_i \), is given by

\[
y_i(t) = M_i(u_i(t)), \quad u_i(t) = C_i(v_i - y_i(t)) \quad (7)
\]

Accordingly, by letting \((\mathcal{M}_i/C_i) v_i := [u_i, y_i]^T\), the reference-loop identification criterion (5) can be modified in the following on-line implementable form

\[
\sigma := \arg \min_{i \in \bar{N}, v_i \neq 0} \frac{\| (\mathcal{P}/C_i)v_i - (\mathcal{M}_i/C_i)v_i \|}{\| (\mathcal{M}_i/C_i)v_i \|}. \quad (8)
\]

A simple switching control scheme based on (8) is described hereafter. Let \( z_i(t) := [u_i(t) \ y_i(t)]^T \). By recalling that \( z(t) = [u(t) \ y(t)]^T \), a convenient test functional related to the identification criterion (8) is as follows

\[
\Pi_i(t) := \max_{\sigma_i} \pi_i(t) \quad (9)
\]

\[
\pi_i(t) := \frac{\| (z - z_i)^T \|}{\alpha + \| z_i \|^2}, \quad t \in \mathbb{Z}_+, \ i \in \bar{N}, \quad (10)
\]

where \( \alpha > 0 \) accounts for possible non-zero initial states. A more effective procedure for handling generic unknown initial conditions and getting rid of \( \alpha \) is addressed in (Baldi et al. [2010]). Let the switching sequence \( \sigma \) be selected in accordance with (4), with test functionals (9)-(10). As can be easily verified, the transfer function of the LTI system mapping \( z_i \) into \( z - z_i \) is given by

\[
\Omega_{\sigma_i}(d) := \left[ -S_i(d) \quad R_i(d) \right] \frac{1}{\chi_{\sigma_i}(d)} \frac{A_i(d)B_i(d)}{B_i(d)A_i(d)} \quad (11)
\]

where \( \chi_{\sigma_i} := A R_i + B S_i \) is the characteristic polynomial of \((\mathcal{P}/C_i)\); \( B_i := B - B_i \), \( A_i := A_i - A \). Therefore, under problem feasibility, assumption A2 in the HSL Lemma is satisfied. Consequently, because of the max operator in (9), which guarantees assumption A1, it follows that the switching stops in a finite time and the HSL Lemma holds. Further, from boundedness of \( \Omega_{\sigma} \) it is possible to prove r-stability of the switched system (1) (Baldi et al. [2010]).

Theorem 1. Consider the switched system \((\mathcal{P}/C_{\sigma(i)})\), with \( \sigma \) selected in accordance with (4), and test functionals (9)-(10). Then the HSL lemma holds and, under problem feasibility, for any initial condition and reference \( r \), \((\mathcal{P}/C_{\sigma(i)})\) is r-stable.

\[\square\]

4. FINE CONTROLLER-TUNING

The switching scheme built via the adoption of (9)-(10) allows one to consider adaptive control systems in which both the issues of robustness and performance can be taken into account. In fact, while stability is guaranteed under the only feasibility condition, performance can be achieved by designing the nominal model distribution \( \mathcal{M} \) dense enough in \( \mathcal{P} \).

In fact, define

\[
\beta := \max_{\mathcal{P} \in \mathcal{P}} \min_{i \in \bar{S}(\mathcal{P})} \| \Omega_{\sigma_i} \|\infty \quad (12)
\]

and let \( \mathcal{M}(\beta) \) denote a corresponding nominal model distribution. As can be seen, the smaller \( \beta \) the smaller (for any possible plant in \( \mathcal{P} \)) \( \min_{i \in \bar{N}} \pi_i(t) \). Consequently, the smaller \( \beta \) the closer (for any possible plant in \( \mathcal{P} \)) the behavior of the final closed-loop \((\mathcal{P}/C_f)\) to the behavior of the reference-loop \((\mathcal{M}_f/C_f)\).

However, further aspects arise concerning (12). In many cases, it may in fact be difficult to achieve a desired \( \beta \), while retaining a moderate memory/computational load. To see this, consider the problem of controlling an oscillatory uncertain system (Astrom et al. [1998]), whose transfer function is given by

\[
M_s(s) = \frac{9}{(s + 1)(s^2 + s + 9)}, \quad (13)
\]

where \( \gamma \in [0.3, 3.5] \). For each \( \gamma \), the transfer function of the corresponding tuned controller \( C_{\gamma} \) was selected as the one among all PI (proportional-integral) controllers \( C_{\gamma} \),

\[
C_{\gamma}(s) = Kp_{\gamma} + \frac{K_{\gamma}}{s}, \quad (14)
\]

satisfying the weighted \( \mathcal{H}_\infty \) mixed-sensitivity criterion (Kwakernaak [1991]):

\[
c_{\gamma}^{w} = \arg \min_{\mathcal{P} \in \mathcal{P}} \max_{\omega} \frac{1}{1 + M_2(\gamma_1 + \gamma_2)(\gamma_1 + \gamma_3)}(1 + V_3(\gamma_1 + \gamma_2)(\gamma_1 + \gamma_3)^2), \quad (15)
\]

where \( V(s) := 1/(1 + s/\omega_n) \), \( \omega_n := 1.88 \) rad/sec. Three different continuous-time controllers \( C_{\gamma} \) were designed relatively to the nominal plant models \( M_s(\gamma) \) corresponding to the following three values: \( \gamma_1 = 0.3, \gamma_2 = 1 \) and \( \gamma_3 = 3.5 \). The discrete-time \((dt)\) nominal models \( M_d \) and related dt candidate controllers \( C_i \) are the ones resulting from the use of an input zero-order holder with sampling time \( T_s \) equal to 0.1 s, and the subscript \( i \) corresponds to \( \gamma_i \), \( i \in \bar{N} \). In particular, the dt controllers have transfer functions \( C_i(d) = Kp_i + K_{\gamma_i} T_s/(1 - d) \). The coefficients \( Kp_i \) and \( K_{\gamma_i} \) are reported in Table 1, along with the corresponding stability intervals.

In all simulation reported hereafter the hysteresis constant \( h \) and \( \alpha \) are set equal to 0.002, and the reference \( r(t) \) is a square-wave of amplitude \( \pm 2.5 \) and period 50 sec. Assume that \( \gamma = 1.5 \) (only \( C_2 \) and \( C_3 \) stabilize the plant), and let \( \sigma(0) = 1 \). Fig. 1 shows that \( C_2 \) is switched-on as the final controller right after start-up. However, \((\mathcal{M}_{1.5}/C_2)\) does not behave as desired, its closed-loop...
response being indeed drastically different compared to that of the reference-loop ($M_2/C_2$).

To achieve definite performance improvements, one is forced to increase the number $N$ of candidate reference-loops. To this end, we have constructed $M(\beta)$ model distributions for various values of $\beta$ covering the whole uncertainty set $\mathcal{P}$. For each $N$, the models have been logarithmically distributed over the uncertainty interval of $\gamma$. Table 2 indicates that $N > 80$ is needed so as to achieve $\beta < 0.1$.

In the sequel, an alternative procedure for enhancing the performance of the switching control scheme is considered.

### 4.1 An Iterative Control Design Scheme

Let $C_f$ denote the controller selected according to (9)-(10). According to data, $C_f$ is therefore recognized as the controller such that the closed-loop behaves as closely as possible to one of the $N$ candidate reference-loops. Consequently, among all candidate reference-loops, $(M_f/C_f)$ yields the reference-loop behavior more likely to be achievable by designing a new controller.

To this end, let $C_\theta$ denote a controller in a given parametrized controller class, where (with obvious meaning of symbols) $\theta \in \mathbb{R}^{n_\theta}$ denotes the vector of controller parameters. By letting $C(\theta, d)$ be the transfer function of $C_\theta$, the virtual reference $v_\theta$ becomes

$$v_\theta(t) = C(\theta, d) u(t) + y(t). \quad (15)$$

As in (6), $v'_\theta$ equals the input sequence that reproduces the I/O sequence $(u', y')$ of the plant $\mathcal{P}$ fed-back by $C_\theta$, viz. $z = (\mathcal{P}/C_\sigma(\cdot)) r = (\mathcal{P}/C_\theta) v_\theta$. Let now

$$y_\theta(t) := W_r(d) v_\theta(t) \quad (16)$$

and

$$u_\theta(t) := C(\theta, d) (1 - W_r(d)) v_\theta(t) \quad (17)$$

where

$$W_r(d) := \frac{M_f(d)C_f(d)}{1 + M_f(d)C_f(d)} \quad (18)$$

is the complementary sensitivity function of $(M_f/C_f)$. Assuming available a batch of data $z^i$, the controller selection can be therefore obtained through the minimization, with respect to the controller parameters, of the following criterion

$$\pi(\theta, t) = \| (z - z_\theta)^i \|^2, \quad \theta \in \mathbb{R}^{n_\theta}, \quad (19)$$

where $z_\theta := [u_\theta \ y_\theta]^i$. In view of (18), the above optimization criterion simply amounts to finding the controller parameter vector $\theta$ such that $(\mathcal{P}/C_\theta)$ behaves, as closely as possible, to the desired reference model $W_r$.

The solution can be obtained based on an iterative gradient-descent approach

$$\theta_{j+1} = \theta_j - \eta_j H_j^{-1} \frac{\partial \pi}{\partial \theta}(\theta_j, t) \quad (20)$$

initialized from $\theta_0 := \tilde{\theta}_f$, where $\tilde{\theta}_f$ denotes the vector of parameters of $C_f$. As usual, $H_j$ is some appropriate positive definite matrix, e.g. the Hessian of $\pi(\theta_j, t)$, while $\eta_j$ is a positive scalar which determines the step size.

As can be easily checked, the gradient $\partial \pi / \partial \theta$ is given by

$$\frac{\partial \pi}{\partial \theta}(\theta_j, t) = -\frac{2}{\|z_\theta\|^2} \sum_{k=0}^{t} \left\{ [z_\theta(k) + \pi(\theta, t) z_\theta(k)] \frac{\partial z_\theta}{\partial \theta}(\theta, k) \right\} \quad (21)$$

where $z_\theta(k) := z(k) - z_\theta(k)$ and,

$$\frac{\partial z_\theta}{\partial \theta}(\theta, k) := \left[ \frac{\partial u_\theta}{\partial \theta}(\theta, k) \ \frac{\partial y_\theta}{\partial \theta}(\theta, k) \right]. \quad (22)$$

Based on (15)-(18), the gradient $\partial \pi / \partial \theta$ can be therefore computed from collected data as follows

$$\frac{\partial z_\theta}{\partial \theta}(\theta, k) = \begin{bmatrix} (1 - W_r(d)) \frac{\partial C}{\partial \theta}(\theta, d) u(k) \\ -W_r(d)C^{-2}(\theta, d) \frac{\partial C}{\partial \theta}(\theta, d) u(k) \end{bmatrix}. \quad (23)$$

Remark 3. The above minimization procedure has similarities with the iterative feedback tuning (IFT) approach of (Hjalmarsson et al. [1998]), since optimization is carried out directly on the controller parameters, with no intermediate plant model identification effort. The main difference is that, here, thanks to the virtual reference variable $v_\theta$, it is not required that $C_\theta$ be connected in feedback with $\mathcal{P}$ in order to update the controller parameters. A detailed comparison between IFT and the approach taken here lies beyond the scope of the present paper, and it is a problem worthy of further consideration.

### 4.2 Implementation Issues

Numerical computation of the virtual references in (6) and (15) requires that both the $C_i$’s and $C_\theta$ be CSCI. While the results considered in this paper hinge upon such an assumption, it can be shown that the same conclusions
hold true for possibly non-stably invertible controllers provided that modified virtual references be appropriately defined (Delghani et al. [2007]).

Similarly, numerical computation of (17) and (23) may not be feasible if the gradient of \( C_\theta \) and/or \( C_\phi \) are unstable. Appropriate procedures for coping with this situation are discussed in detail in (Hjalmarsson et al. [1998]). However, as elaborated next in more detail, though the developments of this section can be generalized so as to cover the case where \( dC_\theta/d\theta \) and/or \( C_\phi \) are unstable, stability of the map from \( u_\theta \) to \( u_\phi \) in (17) is required to extend Th. 1 to the case where switching and tuning are combined. A possible way for allowing the use of unstable controllers is to let \( C(\theta, d) := C_\theta^*(d)C_\phi^*(\theta, d) \), where \( C_\theta^*(d) \) is a fixed transfer function containing all unstable poles of \( C_\theta \). Accordingly, provided \( C^*(\theta, d) \) is stable, (17) can be replaced by \( u_\phi(t) = C_\theta^*(d)C^*(\theta, d)(1 - W_\theta(t))v(t) \), which yields a stable mapping. Enforcement of \( C_\theta^*(d) \) stability can be approached in many ways, e.g. by resorting to constrained optimization routines, or by the use of penalty functions (Bertsekas [1996], Syman [2005]).

5. AN ADAPTIVE SWITCHING CONTROL SCHEME WITH FINE CONTROLLER TUNING

The controller design procedure outlined above, though iterative, does not require that \( C_\phi \) be connected in feedback with \( P \) in order to update the controller parameters. As shown next, such a feature makes it possible to improve the performance of the switching scheme of Sec. 3, while retaining guaranteed stability properties.

Given \( t_\gamma \in Z_+ \), \( t_\gamma > 0 \), let \( C_\gamma \) be the controller resulting from the minimization of \( \pi(\theta, t_\gamma) \) by means of (20), where:

\[
\begin{align*}
\theta_0 := & \theta_{\sigma(t_\gamma)}^\prime, \quad \sigma(t_\gamma) \in \tilde{N} \\
W_\gamma(d) := & \frac{M_{\sigma(t_\gamma)}(d)C_{\sigma(t_\gamma)}(d)}{1 + M_{\sigma(t_\gamma)}(d)C_{\sigma(t_\gamma)}(d)} \\
\theta_\gamma := & \theta_{\sigma(t_\gamma)}^\prime, \quad \Delta \in Z_+ 
\end{align*}
\]

In (24), \( \theta_{\sigma(t_\gamma)}^\prime \) denotes the vector of parameters of \( C_{\sigma(t_\gamma)} \), while \( \Delta \) determines the number of iterations.

Let now \( C(\theta, d) \) denote the transfer function of \( C_\theta \). Then, the transfer function \( M(C_\theta, d) \) of the nominal model \( M_\theta \) corresponding to \( C_\theta \) can be therefore obtained from the open-loop transfer function of \( (M_{\sigma(t_\gamma)}C_{\sigma(t_\gamma)}) \), i.e.

\[
M(C_\theta, d)C(\theta, d) = M_{\sigma(t_\gamma)}(d)C_{\sigma(t_\gamma)}(d).
\]

Integration of the tuning scheme into the switching one is hence simply achieved by adding the \((N + 1)-th\) reference-loop \( (M_{N+1}/C_{N+1}) := (M_\theta/C_\theta) \) to the initial family of candidate reference-loops \( \mathcal{A} \), which therefore becomes \( \mathcal{A}_e := \{(M_\ell/C_\ell), \ell \in \tilde{N} + 1\} \). Clearly, problem feasibility is not destroyed by the introduction of an additional candidate reference-loop. Consequently, following the same lines as in Sec. 3, one concludes that stability of the switched system continues to hold provided that \( (M_{N+1}/C_{N+1}) \) is internally stable.

**Lemma 1.** Given an arbitrary \( t_\gamma \in Z_+ \), \( t_\gamma > 0 \), let \( C_{N+1} \) be the controller resulting from the minimization of \( \pi(\theta, t_\gamma) \) by means of (20)-(24). Furthermore, let \( M_{N+1} \) denote the corresponding nominal model computed via (25). Then, provided that \( C_{N+1} \) be stable and CSCI, the reference-loop \( (M_{N+1}/C_{N+1}) \) is internally stable.

Note that the same conclusions of Lemma 1 hold true if \( C_{N+1}(d) = C_{\sigma(t_\gamma)}^u(d)C_{N+1}^u(d) \), where \( C_{\sigma(t_\gamma)}^u(d) \) contains all the unstable poles of \( C_{\sigma(t_\gamma)} \), and with \( C_{N+1}^u(d) \) stable.

Given \( (M_{N+1}/C_{N+1}) \), the control scheme is simply modified by adding at some instant \( t_+ \in Z_+ \), \( t_+ > t_\gamma \), the test functional corresponding to \( (M_{N+1}/C_{N+1}) \) into the switching logic. Notice that, in practice, to fairly compare \( \pi_{N+1} \) with all the other \( \pi_i \)'s, all candidate test functionals are reset at time \( t_+ \), i.e.

\[
\Pi_i(t) := \max \pi_i|_{t_+}^t \quad (26)
\]

\[
\pi_i(t) := \frac{|| (z - z_i)|_{t_+}^t ||^2}{\alpha + || z_{t_+}^i ||^2}, \quad t \in Z_{t_+}, \quad i \in \tilde{N} + 1 \quad (27)
\]

where \( Z_{t_+} := \{t_+, t_+ + 1, \ldots\} \).

5.1 Example (Continued)

Consider the same example of Sec. 4.1. Adopting the same notations of Sec. 5, let \( t_\gamma := 80 \text{s} \), and \( t_+ := 100 \text{s} \). Before time \( t_+ \), the supervisor switches among the three pre-designed candidate controller, and selects \( C_2 \). Accordingly, the controller design procedure starts at time \( t_\gamma \) by minimizing the loss function \( \pi(\theta, t_\gamma) \) based on recorded data \( z^* \) and reference model \( W_r \) corresponding to the reference-loop \((M_2/C_2)\), where

\[
M_2(d) = \frac{0.014d(1 + 3.533)(1 + 0.256d)}{(1 - 0.905d)(1 - 1.820d + 0.905d^2)} \quad (28)
\]

\[
C_2(d) = \frac{0.428(1 - 0.8411d)}{1 - d} \quad (29)
\]

The controller to be designed is chosen to be of the form

\[
C(\theta, d) = \theta^a + \frac{\theta^b}{1 - d} \quad (30)
\]

initialized from \( \theta_0 = [\theta_0^a \theta_0^b] = [\theta_2 = [0.360 0.068]^\prime \] (see Table 2). Fig. 2 shows that the parameter vector approaches \( \theta = [0.240 0.045]^\prime \) quite rapidly. Accordingly, a fourth reference-loop, \((M_4/C_4)\), is built with

\[
C_4(d) := \frac{0.285(1 - 0.8411d)}{1 - d} \quad (31)
\]

and \( M_4 \) obtained from (25),

\[
M_4(d) = \frac{0.0021d(1 + 3.533d)(1 + 0.256d)}{(1 - 0.905d)(1 - 1.820d + 0.905d^2)} \quad (32)
\]

At time \( t_+ \), all test functionals are reset and \( \pi_4 \) is inserted into the switching logic. As shown in Fig. 3, \( C_4 \) is soon switched on as the final controller, the behavior of \((P/C_4)\) being pretty close to the one dictated by the reference model \( W_r \) (in fact, \( M_4 \) matches almost perfectly with the discrete-time plant corresponding to \( \gamma = 1.5 \)).

6. CONCLUSIONS

We considered the problem of controlling an uncertain plant by means of a finite family of candidate controllers supervised by an appropriate switching logic. It was shown
that, although improved performance can be obtained in principle by increasing the number of the candidate controllers, computational aspects may force the designer to work with a limited candidate set. To this end, a novel, provably correct, adaptive switching control scheme has been introduced, wherein the use of pre-designed controllers is combined with a data-based controller design procedure. By on-line generating new candidate controllers, the proposed switching control scheme proves to compare favorably to a pure switching-based mechanism, thus resulting of practical relevance for on-line implementation of highly performing adaptive control systems.

REFERENCES


