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Monsees, G.; Scherpen, J.M.A.

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Automobile Road Vibration Reproduction using Sliding Modes[§]

G. Monsees* and **J.M.A. Scherpen****
Control Laboratory, Faculty of Information Technology and Systems,
Delft University of Technology, P.O. Box 5031, 2600 GA Delft, The Netherlands

Abstract

Sliding mode controllers have a reputation for their robustness against parameter variations, modeling errors and disturbances. They have been successfully applied in several practical situations which demonstrated the potential of sliding mode control for other control problems. However research has mainly been focused on continuous-time sliding mode controllers. In practical applications, where the continuous-time system is sampled by the computer, it is often assumed that the sampling time is sufficiently fast to consider the sampled system as a continuous-time system.

This paper aims at providing an overview of the design procedure for discrete-time, output-based, sliding mode controllers, based on discrete-time models. The applicability of these controllers were suggested by the SCOOP project where extra robustness has to be gained by extending the controller setup by the sliding mode feed-back controller.

Keywords: Vibration replication, output tracking, discrete-time systems, sliding mode control, variable structure systems.

1 Introduction

The Seat COmfort Optimisation Procedure (SCOOP) aims to greatly improve seat design and hence help to lower the large medical and welfare costs associated with low back pain thereby alleviating the growing concern of the driving public towards vehicle comfort. To enhance the effectiveness of seat design procedures, new seat designs are tested in a laboratory environment. By reproducing standard road profiles on a simulation platform mounted seat, comfort and durability can be tested and compared. Presently, mission reproduction is limited by the accuracy of the available test hardware. To reduce costs and shorten the design process, more sophisticated control software that increases the effective limits of the already existing hardware should be employed. With the currently available mission reproduction software packages, several iterations are required to achieve high reproduction accuracy on a seat test system. Besides, the nonlinear characteristics of human perception suggest a fast convergence time for the controller algorithm. Therefore, reducing the number of iterations improves the objectiveness of the human subject who evaluates the seat performance. Thus, the goal is to develop a design strategy for controllers which reproduce, with high fidelity, on a test rig, target time histories that represent, for instance, vertical acceleration of automobiles, measured whilst performing road tests. Simultaneously, this controller should assure robust stability and performance despite modeling uncertainties and noisy environments.
Mission reproduction, or equivalently, the time waveform replication problem, is formulated as an output tracking problem with internal stability; i.e., we seek a controller which ensures the controlled system to be internally asymptotically stable and its output to tend asymptotically towards a desired trajectory. Precision

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*G.Monsees@ITS.TUDelft.nl

**J.M.A.Scherpen@ITS.TUDelft.nl

output tracking, one of the fundamental problems for control engineers, poses increasingly stringent performance requirements to be satisfied in a variety of applications, notably in the automobile, robotics and aerospace industries.
The design methodology we propose is to divide the controller design procedure into the three steps:

Step 1 **.** Model identification

Step 2 **.** Feed-forward controller design

Step 3 **.** Feed-back controller design

These three steps have been thoroughly studied within the SCOOP project, for which new techniques have been developed. For Step 1 and Step 2 we refer to [3], [9] and [10]. This paper is entirely focused on the third step. It is assumed that we already have obtained a proper (though not perfect) state-space model (Step 1) and a feed-forward control signal which leads to perfect tracking for the identified model. Worthwhile to mention is that, as can be found in [9], the system under pure feed-forward control already has a remarkable tracking performance. The goal of the Step 3 feed-back controller introduced in this paper, is to reduce the sensitivity to disturbances, parameter variations and of course, to increase the tracking performance.
We suppose that the system has the following form:

$$\begin{aligned} x[k+1] &= Ax[k] + Bu[k] + F(x,u,k) \\ y[k] &= Cx[k] \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^m$, $F(x,u,k) \in \mathbb{R}^n$, and consequently $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$. The vector $F(x,u,k)$ contains all nonlinearities and disturbances. Furthermore, we assume that the Step 1 identification procedure determines the matrices (A,B,C) exactly. This assumption does not result in any serious restrictions because any modeling errors can be thought of as disturbances which are incorporated in the vector $F(x,u,k)$. It is also assumed that $p \geq m$ (i.e. there are at least as many measured outputs as there are inputs), $\text{Rank}\{CB\} = m$ (i.e. each output has relative degree one), and that the triple (A,B,C) is both controllable and observable. The proposed control law is the superposition of a feed-forward term $u_{ff}[k]$ and a (sliding mode) feed-back term $u_{fb}[k]$, represented by:

$$u[k] = u_{ff}[k] + u_{fb}[k] \tag{2}$$

The feed-forward component $u_{ff}[k]$ is obtained by design Step 2, and is supposed to give perfect tracking of the nominal system (i.e. the model (A,B,C)) of the true system (1) and hence:

$$\begin{aligned} x^d[k+1] &= Ax^d[k] + Bu_{ff}[k] \\ y^d[k] &= Cx^d[k] \end{aligned} \tag{3}$$

(with the superscript d denoting desired). Of course, strictly spoken, only the outputs have to be tracked and consequently have *desired* trajectories. We assume however, that there exists a state trajectory which leads to the defined desired output and hence there exists a desired state trajectory as well.
If we now define the state tracking error by $e_x[x] = x[k] - x^d[k]$ and the output tracking error by $e_y[k] = y[k] - y^d[k]$, we can write for the error dynamics:

$$\begin{aligned} e_x[k+1] &= Ae_x[k] + Bu_{fb}[k] + F(x,u,k) \\ e_y[k] &= Ce_x[k] \end{aligned} \tag{4}$$

Obviously the desired state- and output tracking errors are zero for all times, the feed-back term $u_{fb}[k]$ can be employed to achieve this. A control technique known for its robustness properties is Sliding Mode Control, which has been extensively studied for linear- as well as nonlinear continuous-time systems in the past [1], [2], [4], [7], [8]. Much less is known of discrete-time sliding mode controllers. In practice it is often assumed that the sampling frequency is sufficiently high to assume that the controller is continuous-time [11]. Another possibility is to design the sliding mode controller in discrete-time, based on a discrete-time model. However, in the discrete-time domain, research has so far been focused on state-based sliding mode controllers. The purpose of our research is to introduce a design procedure for discrete-time, output based, sliding mode controllers which can be used in conjunction with a feed-forward controller to enhance both the tracking and robustness properties of the closed-loop system. The design of a sliding mode controller consists of two phases:

Step 3_I **.** Sliding surface design

Step 3_{II} **.** Actual controller design

Roughly spoken Step 3_I determines a subspace in state-space which results in stable system dynamics once the system is forced on it by the actual controller from Step 3_{II}. Once the system is confined to the sliding surface the system is said to be in sliding mode, the order of the system is then reduced by the number of inputs.
The two design phases will be introduced in the next two sections (Sections 2 and 3). After the theoretical introduction, the derived controller will be tested in Section 4 on the so called quarter car tracking problem. Finally Section 5 presents the conclusions.

2 Sliding Surface Design

As opposed to continuous-time sliding mode control, true sliding mode is in discrete-time no longer achievable. Except in the case of perfect model and disturbance knowledge, the closed-loop system cannot be maintained on the sliding surface. The goal in discrete-time sliding mode control is to bring the system as close as possible to the sliding surface. Ideally, the closed-loop system will move to, and subsequently stay in a boundary region, called the *Quasi Sliding Mode Band*, around the switching surface, defined by:

$$\|\sigma[k]\| < \Delta \qquad \forall\, k \geq k_s \tag{5}$$

where $\Delta \in \mathbb{R}$, k_s is some finite time instant and $\sigma \in \mathbb{R}^m$ is called the *switching function* or *sliding variable*. It is desirable to have Δ as small as possible. In that case, σ may be approximated by zero. This approximation is used in the design of the sliding surface, for which the same procedure can be used as in continuous-time sliding mode control [11].
The design procedure for the output based sliding mode control presented in this section is based on the design procedure given by Edwards and Spurgeon in [2] for continuous-time systems. Without any proof we repeat their procedure, which is now to be used in discrete-time.
The switching function is defined by:

$$\sigma_y[k] = S e_y[k] \tag{6}$$

By a nonsingular transformation the system (4) (without the disturbance vector $F(x,u,k)$) can be transferred to [2]:

$$\begin{bmatrix} e_{x_0}[k+1] \\ e_{x_1}[k+1] \\ e_{y_1}[k+1] \\ e_{y_2}[k+1] \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & A_{22} & A_{23} & A_{24} \\ 0 & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} e_{x_0}[k] \\ e_{x_1}[k] \\ e_{y_1}[k] \\ e_{y_2}[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ B_2 \end{bmatrix} u_{fb}[k] \tag{7}$$

$$e_y[k] = [0_{p\times(n-p)} \quad T] \begin{bmatrix} e_{x_0}[k] \\ e_{x_1}[k] \\ e_{y_1}[k] \\ e_{y_2}[k] \end{bmatrix} \tag{8}$$

where $e_{x_0} \in \mathbb{R}^r$, $e_{x_1} \in \mathbb{R}^{(n-p-r)}$, $e_{y_1} \in \mathbb{R}^{(p-m)}$, $e_{y_2} \in \mathbb{R}^m$, $T \in \mathbb{R}^{p\times p}$ is invertible, and $\mathrm{rank}(B_2) = m$. Defining $[S_1 \;\; S_2] = ST \quad (S_1 \in \mathbb{R}^{m\times(p-m)}$ and $S_2 \in \mathbb{R}^{m\times m}$), leads to:

$$\sigma_y[k] = S_1 e_{y_1}[k] + S_2 e_{y_2}[k] \tag{9}$$

The dynamics in sliding mode can be obtained by setting the above equation to zero, making $e_{y_2}[k]$ explicit and substituting this in the equations for $e_{x_0}[k+1]$, $e_{x_1}[k+1]$, and $e_{y_1}[k+1]$ (equation (7)) resulting in:

$$\begin{bmatrix} e_{x_0}[k+1] \\ e_{x_1}[k+1] \\ e_{y_1}[k+1] \end{bmatrix} = \underbrace{\begin{bmatrix} A_{11} & A_{12} & \left(A_{13} - A_{14}S_2^{-1}S_1\right) \\ 0 & A_{22} & \left(A_{23} - A_{24}S_2^{-1}S_1\right) \\ 0 & A_{32} & \left(A_{33} - A_{34}S_2^{-1}S_1\right) \end{bmatrix}}_{=A_{sm}} \begin{bmatrix} e_{x_0}[k] \\ e_{x_1}[k] \\ e_{y_1}[k] \end{bmatrix} \tag{10}$$

The poles of the above system are given by:

$$\lambda(A_{sm}) = \lambda(A_{11}) \cup \lambda\left(\begin{bmatrix} A_{22} & \left(A_{23} - A_{24}S_2^{-1}S_1\right) \\ A_{32} & \left(A_{33} - A_{34}S_2^{-1}S_1\right) \end{bmatrix}\right) \tag{11}$$

(where $\lambda(A)$ returns the eigenvalues of the matrix A) It is well known that the eigenvalues of the sub-matrix A_{11} contains the open-loop zeros of the system (4) [2]. Therefore, in order to stabilize the closed-loop system, the open-loop system should be minimum-phase which is assumed to be the case in the remainder of this report.

Defining the matrix $M = S_2^{-1}S_1$, the problem of designing a sliding surface reduces to placing the eigenvalues of the following matrix within the unit circle:

$$\begin{bmatrix} A_{22} & (A_{23} - A_{24}M) \\ A_{32} & (A_{33} - A_{34}M) \end{bmatrix}$$

We choose S_2 such that $S_2 B_2 = I_m$, which again couples each entry of the switching function to one single input. The switching function in the original coordinates can then be found from:

$$S = [B_2^{-1}M \qquad B_2^{-1}]T^{-1} \tag{12}$$

3 Controller Design

In the following sections three different controller implementations are presented. The first, given in Section 3.1, is the most straightforward method which is called the direct linear controller. In Section 3.2 a direct linear controller in combination with a disturbance estimator is introduced and finally in Section 3.3 the combination of a direct linear controller and a reduced order state error observer is described.

3.1 Direct Linear Controller

In this section we will derive an output-based, discrete-time, sliding mode controller which steers the closed-loop system to the output-based sliding surface developed in Section 2. We again consider the system (4), transformed to the following form:

$$\begin{bmatrix} e_{x_o}[k+1] \\ e_{x_1}[k+1] \\ e_{y_1}[k+1] \\ e_{y_2}[k+1] \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & A_{22} & A_{23} & A_{24} \\ 0 & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} e_{x_o}[k] \\ e_{x_1}[k] \\ e_{y_1}[k] \\ e_{y_2}[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ B_2 \end{bmatrix} u_{fb}[k] + \begin{bmatrix} F_{x_o}(x,u,k) \\ F_{x_1}(x,u,k) \\ F_{y_1}(x,u,k) \\ F_{y_2}(x,u,k) \end{bmatrix} \tag{13}$$

As was already defined in Section 2, the switching function in the new coordinates is given by:

$$\sigma_y[k] = S_1 e_{y_1}[k] + S_2 e_{y_2}[k] \tag{14}$$

With the transformation:

$$T_\sigma = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & S_1 & S_2 \end{bmatrix} \tag{15}$$

We can now bring system (13) into the following form:

$$\begin{aligned} \begin{bmatrix} e_{x_o}[k+1] \\ e_{x_1}[k+1] \\ e_{y_1}[k+1] \\ \sigma_y[k+1] \end{bmatrix} &= \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{A}_{13} & \bar{A}_{14} \\ 0 & \bar{A}_{22} & \bar{A}_{23} & \bar{A}_{24} \\ 0 & \bar{A}_{32} & \bar{A}_{33} & \bar{A}_{34} \\ \bar{A}_{41} & \bar{A}_{42} & \bar{A}_{43} & \bar{A}_{44} \end{bmatrix} \begin{bmatrix} e_{x_o}[k] \\ e_{x_1}[k] \\ e_{y_1}[k] \\ \sigma_y[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_m \end{bmatrix} u_{fb}[k] \\ &+ \begin{bmatrix} F_{x_o}(x,u,k) \\ F_{x_1}(x,u,k) \\ F_{y_1}(x,u,k) \\ S_1 F_{y_1}(x,u,k) + S_2 F_{y_2}(x,u,k) \end{bmatrix} \end{aligned} \tag{16}$$

where $\bar{A}_{11} = A_{11}$, $\bar{A}_{12} = A_{12}$, $\bar{A}_{13} = \left(A_{13} - A_{14}S_2^{-1}S_1\right)$, $\bar{A}_{14} = A_{14}S_2^{-1}$, $\bar{A}_{22} = A_{22}$, $\bar{A}_{23} = \left(A_{23} - A_{24}S_2^{-1}S_1\right)$, $\bar{A}_{24} = A_{24}S_2^{-1}$, $\bar{A}_{32} = A_{32}$, $\bar{A}_{33} = \left(A_{33} - A_{34}S_2^{-1}S_1\right)$, $\bar{A}_{34} = A_{34}S_2^{-1}$, $\bar{A}_{41} = S_2A_{41}$, $\bar{A}_{42} = (S_1A_{32} + S_2A_{42})$, $\bar{A}_{43} = \left(S_1A_{33} + S_2A_{43} - S_1A_{34}S_2^{-1}S_1 - A_{44}S_1\right)$, $\bar{A}_{44} = \left(S_1A_{34}S_2^{-1} + A_{44}\right)$, and $I_m = S_2B_2$ (by design choice presented in the Section 2). Defining the reaching law as:

$$\sigma_y[k+1] = \Phi\sigma_y[k] \tag{17}$$

where $\Phi \in \mathbb{R}^{m\times m}$ is, for simplicity, chosen as a diagonal matrix with all entries $0 \leq \phi_i < 1$. From equation (16) and (17) the feed-back control term can be concluded to be:

$$u_{fb}[k] = \left(\Phi - \bar{A}_{44}\right)\sigma_y[k] - \bar{A}_{43}e_{y_1}[k] - \bar{A}_{41}e_{x_o}[k] - \bar{A}_{42}e_{x_1}[k] - S_1F_{y_1}(x,u,k) - S_2F_{y_2}(x,u,k) \tag{18}$$

Obviously the previous control law is not implementable. The disturbance and modeling error components $F_{y_1}(x,u,k)$ and $F_{y_2}(x,u,k)$ where assumed to be unknown, but also the state-components $e_{x_o}[k]$ and $e_{x_1}[k]$ are not known. Therefore, the unknown parts are omitted resulting in the control law:

$$u_{fb}[k] = \left(\Phi - \bar{A}_{44}\right)\sigma_y[k] - \bar{A}_{43}e_{y_1}[k] \tag{19}$$

From a thorough stability analysis [6], it can be concluded that designing a stable sliding surface according to the procedure given in Section 2 and choosing the stable feedback matrix Φ, may not automatically lead to a stable closed-loop system. And thus, after designing the sliding surface S and the feedback matrix Φ, the closed loop stability has to be checked and possibly the design procedure has to be repeated.

3.2 Direct Linear Controller with Disturbance Estimation

Applying the derived control law of the previous section (equation (19)) to the system (7), $\sigma_y[k+1]$ can be determined to be:

$$\sigma_y[k+1] = \Phi\sigma_y[k] + A_{41}e_{x_0}[k] + A_{42}e_{x_1}[k] + F_m(x,u,k) \tag{20}$$

If we compare the above with the desired $\sigma_y[k+1] = \Phi\sigma_y[k]$ then we see that the error is given by $A_{41}e_{x_0}[k] + A_{42}e_{x_1}[k] + F_m(x,u,k)$, hence the unknown terms $e_{x_0}[k]$ and $e_{x_1}[k]$ could be considered as disturbances just like the disturbance $F_m(x,u,k)$. Therefore we can employ the disturbance estimator introduced by us in [5]. An estimate of the disturbance ($\tilde{d}[k] \in \mathbb{R}^m$) can be obtained from:

$$\tilde{d}[k] = \tilde{d}[k-1] + \sigma_y[k] - \Phi\sigma_y[k-1] \tag{21}$$

Variable	Model	System	unit
m_c	200	300	[kg]
m_w	33	30	[kg]
c_c	9000	7000	[N/m]
c_w	20.000	22.000	[N/m]
d_c	1200	1100	[Nsec/m]

Table 1: Variable values of the model and the system.

However, as opposed to the state-based case where $\tilde{d}[k]$ is used to estimate the disturbance vector $F_m(x,u,k)$ only, in this case $\tilde{d}[k]$ ideally represents the disturbance vector $F_m(x,u,k)$ plus the terms $A_{41}x_0[k] + A_{42}x_1[k]$. The control law then becomes:

$$u[k] = B_2^{-1}\left\{(\Phi - A_{44})\,\sigma_y[k] - A_{23}y_1[k] - \tilde{d}[k]\right\} \tag{22}$$

As can be found in [5], the estimated disturbance at time k is in fact the disturbance at time $k-1$. However, one could assume, if the sampling frequency is sufficiently high, that the term $F_m(x,u,k) + A_{41}e_{x_0}[k] + A_{42}e_{x_1}[k]$ is slow enough. In the simulation example this proves to be the case.

3.3 Direct Linear Controller with State-Error Observer

The controller obtained in Section 3.1 (equation (19)), estimates the unknown terms by zero. Since these terms represent the error this is a plausible choice, since the error is supposed to converge to zero. However, one could imagine that there is a better estimation for the error states. A possible solution for this was introduced by us in [6]. A reduced order observer (in fact the order of the observer is $n-p$) is used to reconstruct the error states $e_{x_o}[k]$ and $e_{x_1}[k]$. The observed error-states will be represented by $\hat{e}_{x_o}[k]$ and $\hat{e}_{x_1}[k]$. The control law resulting from these considerations is given by:

$$u_{fb}[k] = \left(\Phi - \bar{A}_{44}\right)\sigma_y[k] - \bar{A}_{43}e_{y_1}[k] - \bar{A}_{41}\hat{e}_{x_o}[k] - \bar{A}_{42}\hat{e}_{x_1}[k] \tag{23}$$

The reduced order state error observer is given by:

$$\begin{bmatrix} \hat{e}_{x_o}[k+1] \\ \hat{e}_{x_1}[k+1] \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ 0 & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \hat{e}_{x_o}[k] \\ \hat{e}_{x_1}[k] \end{bmatrix} + \begin{bmatrix} \bar{A}_{13} & \bar{A}_{14} \\ \bar{A}_{23} & \bar{A}_{24} \end{bmatrix} \begin{bmatrix} e_{y_1}[k] \\ \sigma_y[k] \end{bmatrix} - L(\sigma_y[k+1] - \Phi\sigma_y[k]) \tag{24}$$

where $L \in \mathbb{R}^{(n-p)\times(n-p)}$ is a design matrix which ensures, if properly chosen, convergence to the actual state error.

4 Quarter Car Tracking Problem

Figure 1 pictures two typical tracking problems in the automotive industry. The right hand pictures displays a chair placed on several actuators where the task of the controller is to replicate some prescribed test profiles with high accuracy, the left hand figure presents a similar problem for a whole car. In this section we present simulation results for the latter problem, where only one quarter of the test platform and car are simulated. The task of the controller is to reproduce a measured road profile (the reference signal) and give the car on the base exactly the same accelerations in every successive test, regardless of any disturbances and parameter variations. A schematic test setup is presented in figure 2 for which we can obtain the linear, continuous-time, model:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

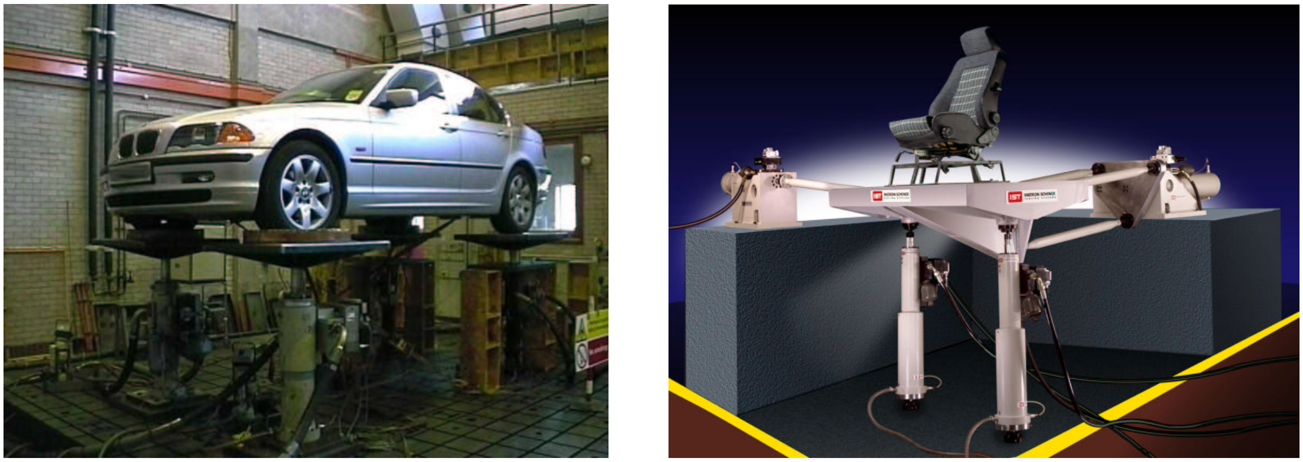

Figure 1: Automotive tracking problems.

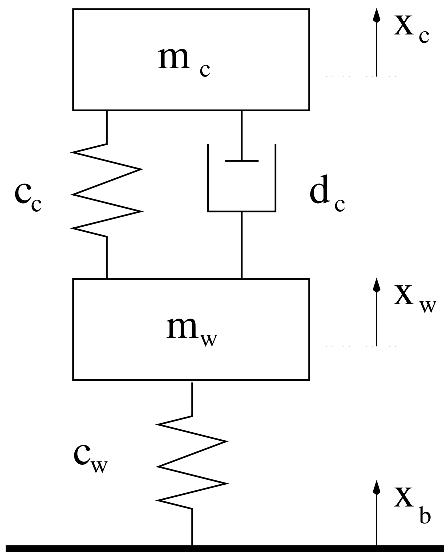

Figure 2: Mechanical diagram of the Quarter Car Model.

The system state, input and output represent the following physical quantities:

$$x = [x_c \quad x_w \quad \dot{x}_c \quad \dot{x}_w]^T \qquad y = [\dot{x}_c \quad \ddot{x}_c] \qquad u = x_b$$

The variables x_c, x_w, and x_b represent the car, wheel and base displacement respectively. The system matrices are given by:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{c_c}{m_c} & \frac{c_c}{m_c} & -\frac{d_c}{m_c} & \frac{d_c}{m_c} \\ \frac{c_c}{m_w} & -\frac{c_c+c_w}{m_w} & \frac{d_c}{m_w} & -\frac{d_c}{m_w} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{c_w}{m_w} \end{bmatrix}^T$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{c_c}{m_c} & \frac{c_c}{m_c} & -\frac{d_c}{m_c} & \frac{d_c}{m_c} \end{bmatrix}$$

where m_w is the mass of the wheel, m_c is one quarter of the mass of the car, c_w is the wheel stiffness, c_c the suspension stiffness and d_c the suspension damping. The model is used for the controller design.

The designed controllers are tested on the "system" for which the parameters differ considerably from the nominal system, as can be seen in Table 1. The presented continuous-time model is discretized with a sampling time $T_s = 5ms$.

The total control action $u[k]$ is taken as the sum of a feed-forward term $u_{ff}[k]$ and a sliding mode feed-back term $u_{fb}[k]$, hence:

$$u[k] = u_{ff}[k] + u_{fb}[k]$$

The feed-forward term is computed such that it gives optimal tracking for the nominal system. The feed-back part will be used to compensate for modeling errors, therefore the switching function is defined as:

$$\sigma_{e_y}[k] = S\left(y[k] - r[k]\right)$$

where the signal $r[k]$ is the reference, or target, signal. Simulations are presented for the feed-forward controller only (figure 3), the output based sliding mode controller without observed error state or disturbance estimation (figure 4), the output based sliding mode controller with disturbance estimation (figure 5), and the output-based sliding mode controller with a state-error observer (6). For the sliding mode controller $\Phi = 0$ and $S = [0.123 \quad 0.00058]$ are used. The reduced order observer gain L is given by:

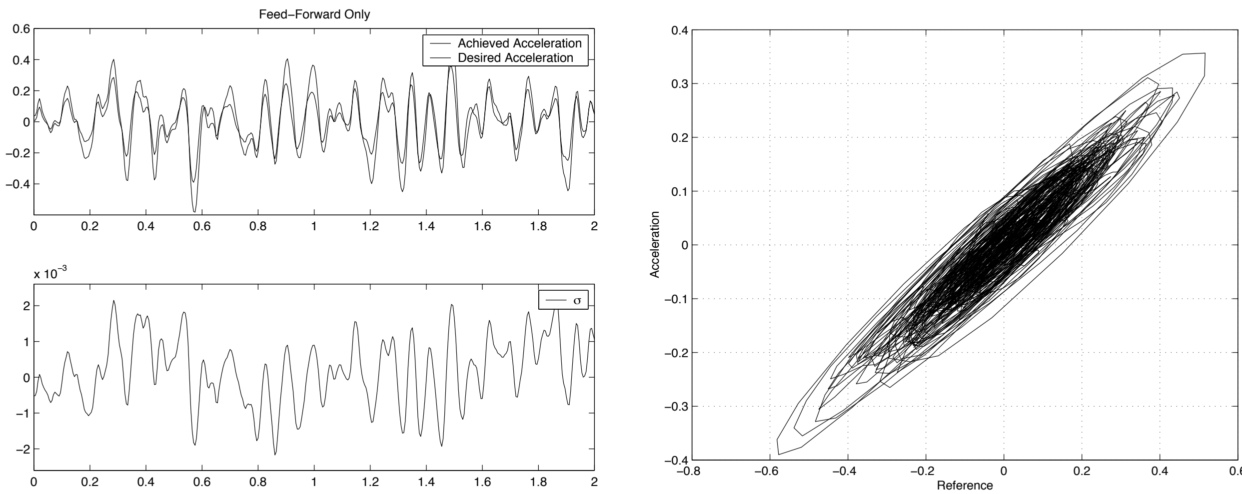

Figure 3: Simulation results with feed-forward controller only.

$L = [-20.28 \quad -190.25]$ which results in a dead-beat observer (all observer poles at zero).

The Variance Accounted For (VAF) of the output, defined by:

$$\text{VAF}(y, r) = \left(1 - \frac{\text{variance}(y - r)}{\text{variance}(y)}\right)$$

which can be computed from the simulation results, is given for each controller setup by:

$$\text{VAF}_{ff}(y, r) = \begin{bmatrix} 73.3 \\ 81.6 \end{bmatrix} \qquad \text{VAF}_{smc}(y, r) = \begin{bmatrix} 86.8 \\ 96.1 \end{bmatrix}$$
$$\text{VAF}_{smcdes}(y, r) = \begin{bmatrix} 99.9 \\ 99.6 \end{bmatrix} \quad \text{VAF}_{smcseo}(y, r) = \begin{bmatrix} 100.0 \\ 99.7 \end{bmatrix}$$

where the subscript $_{ff}$ stands for the feed-forward controller only, $_{smc}$ for the direct linear controller, $_{cdes}$ for the direct linear controller with disturbance estimator, and $_{smcseo}$ for the direct linear controller with state error observer.

The figures and the computed VAF clearly demonstrate the improvements made by the (output-based) sliding mode controllers. The performance of the output-based sliding mode controller with the reduced order state-error observer is nearly perfect.

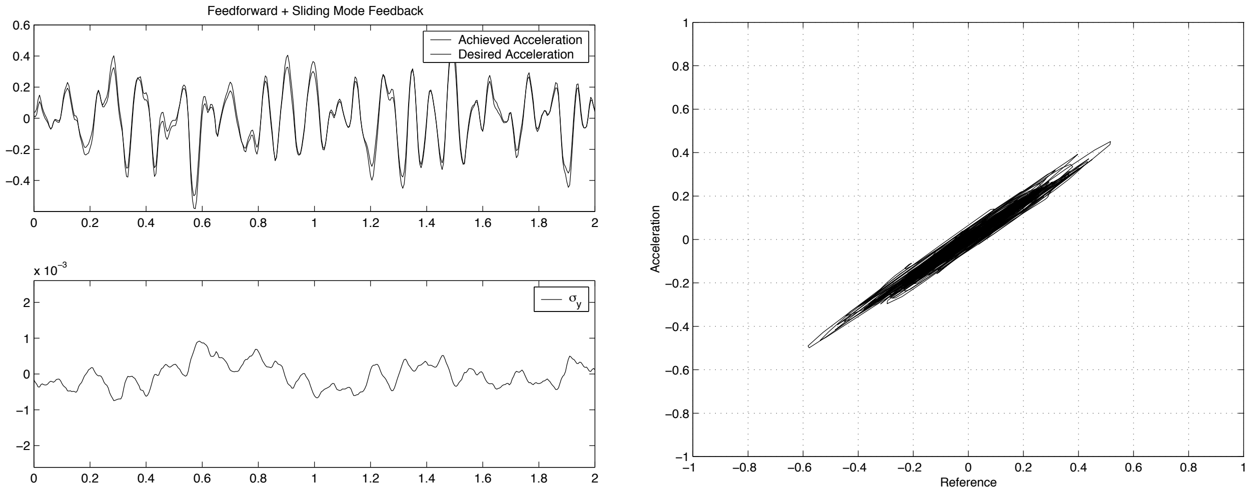

Figure 4: Simulation results for the direct linear controller.

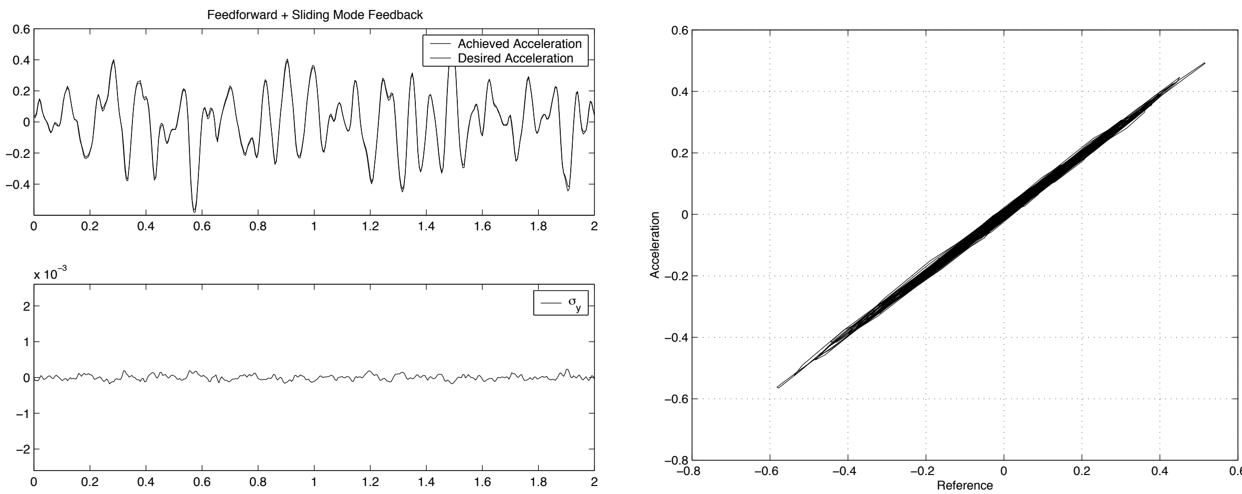

Figure 5: Simulation results for the direct linear controller with disturbance estimation.

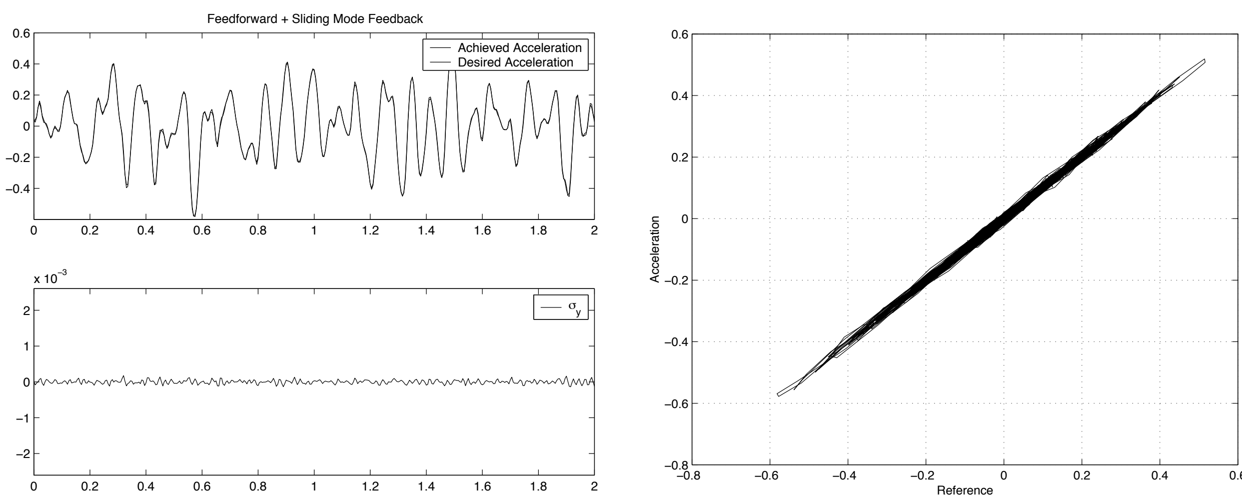

Figure 6: Simulation results for the direct linear controller with state-error observer.

5 Conclusion

This paper presented a controller design strategy to achieve reproduction of time trajectories with high accuracy, fast convergence and good robustness properties. The design procedure consists of three design phases (1 - System identification, 2- Feed-forward signal generation, and 3 - Feed-back controller design), where this paper is focused on the last phase, i.e. feed-back controller design.
Because of its robustness properties, sliding mode control was selected for the feed-back part. Over the last decade some research was done in the state-based, discrete-time, control area. However, the field of output-based discrete-time sliding mode control theory was, to our knowledge, not explored at all. The theory presented in this chapter forms a bases, however one should remember that the way from theory to practical applicability is not necessarily straightforward. The simulation results clearly show the potential of the theory, but there is still work to be done. Future research has to extend the design procedure to:

- practical test should point out the actual applicability of the derived control theory,
- the design procedure should be capable of handling uncontrollable but stabilizable systems,
- the design procedure should be capable of handling non-minimum phase systems.

References

[1] R. A. DeCarlo, S. H. Żak, and G. P. Matthews. Variable structure control of nonlinear multivariable systems: A tutorial. *Proceedings of the IEEE*, 76:212–232, March 1988.

[2] C. Edwards and S.K. Spurgeon. *Sliding Mode Control, Theory and Applications*. Taylor & Francis Ltd, 1998.

[3] K. George, M. Verhaegen, and J. M. A. Scherpen. Stable inversion of MIMO linear discrete time non-minimum phase systems. *The 7th IEEE Mediterranean Conference on Control and Automation, Haifa, Israel*, pages 267–281, 1999.

[4] J.Y. Hung, W. Gao, and J.C. Hung. Variable structure control: A survey. *IEEE Transactions on Industrial Electronics*, Vol. 40:2–21, February 1993.

[5] G. Monsees and J.M.A. Scherpen. Discrete-time sliding mode control with a disturbance estimator. *Submitted*.

[6] G. Monsees and J.M.A. Scherpen. Output tracking using a discrete-time sliding mode controller with reduced-order state-error estimation. *To appear at the 1st IFAC Symposium on System Structure and Control*, 2001.

[7] V. Utkin, J. Guldner, and J. Shi. *Sliding Mode Control in Electromechanical Systems*. Taylor & Francis Ltd, 1999.

[8] V.I. Utkin. *Sliding Modes in Control Optimization*. Springer-Verlag, 1992.

[9] M. Verhaegen and J. de Cuyper. State space modeling and stable dynamic inversion for trajectory tracking on an industrial seat test rig. *Submitted*.

[10] D. Westwick, K. George, and M. Verhaegen. Non-linear identification of automobile vibration dynamics. *The 7th IEEE Mediterranean Conference on Control and Automation, Haifa, Israel*, pages 724–737, 1999.

[11] K. D. Young and Ü. Özgünter. Sliding mode: Control engineering in practice. pages 150–162, 1999.