DISCRETE-TIME SLIDING MODE CONTROL WITH A DISTURBANCE ESTIMATOR

G. Monsees* **J.M.A. Scherpen†**

Delft University of Technology
ITS, Control Laboratory
P.O. Box 5031, 2600 GA, Delft, The Netherlands
fax: +31 15 27 86679
*e-mail: G.Monsees@ITS.TUDelft.nl
†e-mail: J.M.A.Scherpen@ITS.TUDelft.nl

Keywords: Sampled Data Systems, Variable Structure Control, Automotive Systems.

Abstract

This paper presents a novel output-based, discrete-time, sliding mode controller design methodology. Output based controllers with and without disturbance estimation are presented. First several existing discrete-time reaching conditions are analyzed and compared. From these methods the linear reaching law is selected to form the basis for the output-based controller design.

1 Introduction

Sliding mode control is a well known robust control algorithm for linear- as well as nonlinear systems [2], [8], [9]. Continuous-time sliding mode control has been extensively studied and has been used in various applications. Much less is known of discrete-time sliding mode controllers. In practice it is often assumed that the sampling frequency is sufficiently high to assume that the controller is continuous-time [10]. Another possibility is to design the sliding mode controller in discrete-time, based on a discrete-time model, however stability has not yet been assured [1], [3].

To our knowledge, research in discrete-time sliding mode control has so far been focused on state-based sliding mode controllers. In this paper we introduce an output-based sliding mode controller, both with as without disturbance estimation.

The outline of the paper is as follows, in Section 2 the (state-based) discrete-time reaching laws which can be found in literature are introduced and compared. Based on the latter analysis a disturbance estimation based state-based sliding mode control is introduced. Using this information, Section 3 gives a design procedure for discrete-time, output-based, sliding mode controllers. The performance of the presented controllers is illustrated in the simulation study presented in section 4. Section 5 finally presents the conclusions.

2 Reaching Law Definitions

In the following subsections we explore the reaching laws which can be found in literature. One of these reaching laws (the linear reaching law) will be extended with a disturbance estimator. Finally the methods are compared in a simple simulation example.

We consider the system:

$$x[k+1] = Ax[k] + Bu[k] + F(x,u,k) \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $F \in \mathbb{R}^n$. The pair (A,B) is assumed to be controllable. The switching function is defined by:

$$\sigma_x[k] = Sx[k] \tag{2}$$

where $\sigma_x \in \mathbb{R}^m$ (the subscript x is used to indicate that this switching function is based on the state), hence $S \in \mathbb{R}^{m\times n}$. It is assumed that Rank$\{SB\} = m$, and S is known and results in stable dynamics in sliding mode (i.e. $\sigma_x[k] = 0$). It is well known that for the system (1) there exists an invertible transformation $T_\sigma \in \mathbb{R}^{n\times n}$ which brings the system in the following form [2],[8]:

$$\begin{aligned} x_1[k+1] &= A_{11}x_1[k] + A_{12}\sigma_x[k] + F_u(x,u,k) & (3)\\ \sigma_x[k+1] &= A_{21}x_1[k] + A_{22}\sigma_x[k] + B_2u[k] \\ &\quad +F_m(x,u,k) & (4)\end{aligned}$$

where $x_1 \in \mathbb{R}^{(n-m)}$, $A_{11} \in \mathbb{R}^{(n-m)\times(n-m)}$, $A_{12} \in \mathbb{R}^{(n-m)\times(m)}$, $A_{21} \in \mathbb{R}^{(m)\times(n-m)}$, and $A_{22} \in \mathbb{R}^{m\times m}$. The subscripts u and m in $F_u(x,u,k)$ and $F_m(x,u,k)$ stand for matched and unmatched respectively. Equation (4) will be used in the following sections to explore several existing reaching law definitions as well as a new estimation based sliding mode controller. The disturbance vector is supposed to be limited by:

$$|F_m[x,u,k]| < \hat{F}_m \tag{5}$$

Is will become clear in the following subsections that "true" sliding mode (i.e. $\sigma = 0$) cannot be achieved in discrete-time. Instead the system will be forced into the so called quasi sliding mode band defined by:

$$\|\sigma[k]\| < \Delta \tag{6}$$

for some bounded $\Delta \in \mathbb{R}$. For simplicity, reaching laws for single input systems will be studied ($m = 1$). However, they are straightforwardly extendable to multiple input systems.

2.1 Discretized Continuous-Time Reaching Law

A first attempt to come to a discrete-time sliding mode definition would of course be to discretize the continuous-time definition straightforwardly. A first order approximation of the derivative of σ_x leads to the following reaching law definition:

$$\sigma_x[k]\left(\sigma_x[k+1] - \sigma_x[k]\right) \leq -\eta|\sigma_x[k]| \tag{7}$$

for some $\eta > 0 \in \mathbb{R}$. The above definition does guarantee that the sliding surface is crossed in finite time. However, it does not guarantee a stable behaviour around the sliding surface. In fact, as long as the sliding surface is crossed within each sample period the above rule is satisfied. Unfortunately switching between plus infinity and minus infinity also suits the above proposed reaching law.

2.2 Sarpturk Reaching Law

A convergent discrete-time reaching condition was proposed by Sarpturk et. al. [5]:

$$|\sigma_x[k+1]| < |\sigma_x[k]| \tag{8}$$

This reaching law ensures that the sliding surface is reached monotonically. The following control law is proposed:

$$\begin{aligned} u[k] \;\; = \;\; & B_2^{-1}\left\{\left(\Phi - A_{22}\right)\sigma_x[k] - A_{21}x_1[k]\right. \\ & \left. -K\,\text{sign}(\sigma_x[k])\right\} \end{aligned} \tag{9}$$

($K \in \mathbb{R}^+$. $0 \leq \Phi < 1 \in \mathbb{R}$). Reaching law (8) implies an upper as well as a lower bound on the switching gain K, which can be found to be:

$$K \;>\; F_m(x,u,k) + \Phi\epsilon - \epsilon \tag{10}$$

$$K \;<\; F_m(x,u,k) + \Phi\epsilon + \epsilon \tag{11}$$

Using the knowledge that $|F_m(x,u,k)| < \hat{F}$ we obtain for K:

$$\hat{F} - (1-\Phi)\epsilon < K < -\hat{F} + (1+\Phi)\epsilon \tag{12}$$

From the above it is clear that for $\epsilon < \hat{F}$, the above condition can no longer be satisfied.

2.3 Gao Reaching Law

According to Gao [3], a discrete-time sliding mode controller should have the following properties:

I Starting from any initial state, the trajectory will move monotonically towards the switching plane and cross it in finite time.

II Once the trajectory has crossed the switching plane the first time, it will cross the plane again in every successive sampling period, resulting in a zigzag motion about the switching plane.

III The size of each successive zigzagging step is nonincreasing and the trajectory stays within a specified band.

To achieve discrete-time sliding mode, the following reaching law is proposed [3]:

$$\sigma_x[k+1] = \Phi\sigma_x[k] - K\text{sign}(\sigma_x[k]) \tag{13}$$

for some constant scalar $0 \leq \Phi < 1$. From equations (4) and (13) we can determine the control signal to be:

$$\begin{aligned} u[k] \;\; = \;\; & B_2^{-1}\left\{\left(\Phi - A_{22}\right)\sigma_x[k] - A_{21}x_1[k]\right. \\ & \left. -K\,\text{sign}(\sigma_x[k])\right\} \end{aligned} \tag{14}$$

We note that this control law is identical to the control law presented in the previous section (equation (9)), however the conditions on the switching gain K are different. It can be found that if the switching gain satisfies the following rule then the three rules (I, II and III) are satisfied [3]:

$$K \geq \frac{1+\Phi}{1-\Phi}\hat{F} \tag{15}$$

Which is minimized by choosing $\Phi = 0$ and $K = \hat{F}$, resulting in:

$$\Delta_{\Phi=0} = 2\hat{F} \tag{16}$$

2.4 Linear Reaching Law

Another way of defining the reaching law can be found in for example [4] or [6] given by:

$$\sigma_x[k+1] = \Phi\sigma_x[k] \tag{17}$$

Notice that this reaching law is similar to the reaching law in the previous section (equation (13)), however the switching part has been omitted. Again using equation (4) leads to the control law:

$$u_l[k] = B_2^{-1}\left\{\left(\Phi - A_{22}\right)\sigma_x[k] - A_{12}x_1[k]\right\} \tag{18}$$

where $0 \leq \Phi < 1$. The discrete-time sliding mode band can be found to be:

$$\Delta = \frac{1}{1-\Phi}\hat{F} \tag{19}$$

Like the controller developed in the previous section, the discrete-time sliding mode band is minimized with $\Phi = 0$ leading to $\Delta = \hat{F}$.

2.5 Linear Reaching Law with a Disturbance Estimator

As can be seen in the previous sections, the quasi sliding mode band is rather large. The smallest quasi sliding mode band is obtained with the linear reaching law where $\Phi = 0$, but still the quasi sliding mode band has the same magnitude as the disturbance. To overcome this problem we introduce a disturbance estimator. Define $\tilde{d}[k] \in \mathbb{R}$ by:

$$\tilde{d}[k] = \tilde{d}[k-1] + \sigma_x[k] - \Phi\sigma_x[k-1] \tag{20}$$

where $\tilde{d}[k]$ is the estimation of the disturbance vector projected on σ_x by which we mean that ideally $\tilde{d}[k] = SF(x,u,k)$. The control law (18) should then be changed to:

$$u[k] = B_2^{-1}\left\{(\Phi - A_{22})\,\sigma_x[k] - A_{21}x_1[k] - \tilde{d}[k]\right\} \qquad (21)$$

Theorem 2.1 *The closed-loop system formed by the controllable system (1) and controller (21) with disturbance estimator (20) will converge to the discrete-time sliding mode band:*

$$\Delta = \frac{1}{1-\Phi}\delta_F \qquad (22)$$

where δ_F is the maximum rate of change of the disturbance vector defined by:

$$|F_m(x,u,k+1) - F_m(x,u,k)| < \delta_F \qquad \forall \quad k \qquad (23)$$

Proof : *Substituting equation (4) and (21) into the expression for $\tilde{d}[k]$ (equation (20)) results in:*

$$\tilde{d}[k] = F_m(x,u,k-1) \qquad (24)$$

Then starting from some arbitrary bounded $\tilde{d}[0]$ we can write:

$$\sigma_x[1] = \Phi\sigma_x[0] + F_m(x,u,0) - \tilde{d}[0] \qquad (25)$$

For the next time instant we can find:

$$\sigma_x[2] = \Phi\sigma_x[1] + F_m(x,u,1) - F_m(x,u,0) \qquad (26)$$

For an arbitrary time $k > 1$ we can find $\sigma_x[k]$ to be:

$$\sigma_x[k] = \Phi^{k-1}\sigma_x[1] + \sum_{i=0}^{k-2}\Phi^i F_m(x,u,i+1) - \sum_{i=0}^{k-2}\Phi^i F_m(x,u,i) \qquad (27)$$

Since we know that $|F_m(x,u,k+1) - F_m(x,u,k)| < \delta_F$, the above equation is bounded by:

$$\sigma_x[k] < \Phi^{k-1}\sigma_x[1] + \sum_{i=0}^{k-2}\Phi^i\delta_F \qquad (28)$$

Since $0 \le \Phi < 1$, the above upper-bound can be approximated for sufficiently large k by:

$$\sigma_x[k] < \frac{1}{1-\Phi}\delta_F \qquad (29)$$

From which Δ instantly follows. □

We note that the above disturbance observer is based on the same idea as presented by Su et. al [7] where the disturbance is recovered from the equation:

$$F(x,u,k) = x[k+1] - Ax[k] - Bu[k] \qquad (30)$$

However in this paper, the disturbance estimator is based on the switching function. It can therefore be easily extended to output-based sliding mode controllers where there is no full knowledge of the system state. This is presented in Section 3.

2.6 Comparison in Simulation Results

In this section we present a simple simulation example (also considered in [3]) to compare the Gao reaching law, the linear reaching law and the linear reaching law with disturbance estimation. The system matrices of system (1) are given by:

$$A = \begin{bmatrix} 1.2 & 0.1 \\ 0 & 0.6 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (31)$$

The switching function (2) is defined by $S = [5 \quad 1]$ and the disturbance $F[k]$ is defined by:

$$F[k] = \begin{bmatrix} 0 \\ 0.2\sin\left(\frac{k}{4\pi}\right) \end{bmatrix} \qquad (32)$$

The control laws for the simulation results in Figure 1 are taken

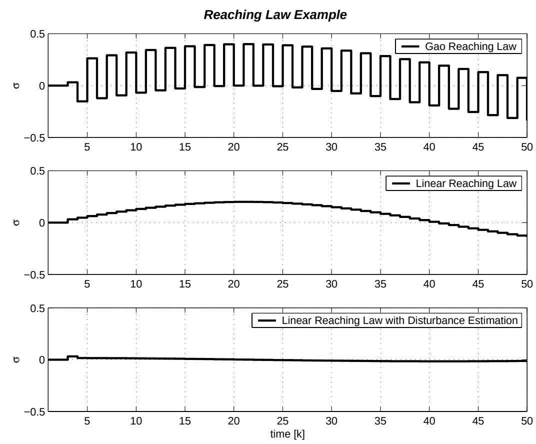

Figure 1: The simulation results ($\sigma[k]$) for the Gao reaching law (top plot), the linear reaching law (middle plot), and for the linear reaching law with disturbance estimation (lower plot).

as equation (14) (top figure), equation (18) (middle figure), and equation (20) together with equation (21) (lower figure), all controllers with $\Phi = 0$. Clearly the figures show that the Gao reaching law results in significant chattering, which is absent for the linear controllers. The disturbance estimation controller gives demonstrates the best results.

3 Output Based Controller Design

In this section we develop an output based discrete-time sliding mode controller. First a stable sliding surface will be designed. Then a linear controller which steers the system to the sliding surface based on the linear reaching law will be constructed. Finally, a linear controller with disturbance estimation is introduced.

3.1 Output Based Surface Design

In this section we give a design procedure based on the assumption that perfect sliding mode (i.e. $\sigma[k] = 0$) can be achieved. From the previous section it can be concluded that in general this is not the case, however it still results in a useful procedure [10]. The design procedure for the output based sliding mode control presented in this section is similar to the procedure given by Edwards et. al. in [2] for the continuous-time case. Without any proof we repeat their procedure, which is now to be used in discrete-time for the system:

$$\begin{array}{rcl} x[k+1] & = & Ax[k] + Bu[k] \\ y[k] & = & Cx[k] \end{array} \tag{33}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, and the matrices (A, B, C) of appropriate size. It is assumed that Rank$\{CB\} = m$, $p \geq m$, and that the triple (A, B, C) is both controllable as observable. The switching function is defined by:

$$\sigma_y[k] = Sy[k] \tag{34}$$

By a nonsingular transformation the system can be transferred to:

$$\begin{aligned} \begin{bmatrix} x_0[k+1] \\ x_1[k+1] \\ y_1[k+1] \\ y_2[k+1] \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & A_{22} & A_{23} & A_{24} \\ 0 & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} x_0[k] \\ x_1[k] \\ y_1[k] \\ y_2[k] \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ B_2 \end{bmatrix} u[k] + \begin{bmatrix} F_{x_0}(x,u,k) \\ F_{x_1}(x,u,k) \\ F_{y_1}(x,u,k) \\ F_{\sigma}(x,u,k) \end{bmatrix} \end{aligned} \tag{35}$$

$$y[k] = [0_{p\times(n-p)} \quad T] \begin{bmatrix} x_0[k] \\ x_1[k] \\ y_1[k] \\ y_2[k] \end{bmatrix} \tag{36}$$

where $x_0 \in \mathbb{R}^r$, $x_1 \in \mathbb{R}^{(n-p-r)}$, $y_1 \in \mathbb{R}^{(p-m)}$, $y_2 \in \mathbb{R}^m$, $T \in \mathbb{R}^{p\times p}$ is invertible, and rank$(B_2) = m$. Defining $[S_1 \quad S_2] = ST \quad (S_1 \in \mathbb{R}^{p\times(p-m)}$ and $S_2 \in \mathbb{R}^{p\times m})$, leads to:

$$\sigma_y[k] = S_1 y_1[k] + S_2 y_2[k] \tag{37}$$

The dynamics in sliding mode can be obtained by setting the above equation to zero, making $y_2[k]$ explicit and substituting $y_2[k]$ in the equations for $x_0[k+1]$, $x_1[k+1]$, and $y_1[k+1]$ (equation (35)) resulting in:

$$\begin{bmatrix} x_0[k+1] \\ x_1[k+1] \\ y_1[k+1] \end{bmatrix} = A_{sm} \begin{bmatrix} x_0[k] \\ x_1[k] \\ y_1[k] \end{bmatrix} \tag{38}$$

with:

$$A_{sm} = \begin{bmatrix} A_{11} & A_{12} & \left(A_{13} - A_{14}S_2^{-1}S_1\right) \\ 0 & A_{22} & \left(A_{23} - A_{24}S_2^{-1}S_1\right) \\ 0 & A_{32} & \left(A_{33} - A_{34}S_2^{-1}S_1\right) \end{bmatrix} \tag{39}$$

The poles of the above system are given by:

$$\lambda(A_{sm}) = \lambda(A_{11}) \cup \lambda\left(\begin{bmatrix} A_{22} & \left(A_{23} - A_{24}S_2^{-1}S_1\right) \\ A_{32} & \left(A_{33} - A_{34}S_2^{-1}S_1\right) \end{bmatrix}\right) \tag{40}$$

where $\lambda(A)$ returns the eigenvalues of the matrix A. It is well known that the eigenvalues of the sub-matrix A_{11} contains the open-loop zeros of the system (33) [2]. Therefore, in order to stabilize the closed-loop system, the open-loop system should be minimum-phase which is assumed to be the case in the remainder of this paper.

Defining the matrix $M = S_2^{-1}S_1$, the problem of designing a sliding surface reduces to placing the eigenvalues of the following matrix within the unit circle:

$$\begin{bmatrix} A_{22} & (A_{23} - A_{24}M) \\ A_{32} & (A_{33} - A_{34}M) \end{bmatrix} \tag{41}$$

We choose S_2 such that $S_2 B_2 = I_m$ in which way each component of the switching function is associated with exactly one input . The switching function in the original coordinates can then be found from:

$$S = [B_2^{-1}M \quad B_2^{-1}]T^{-1} \tag{42}$$

In this section it has been shown how a stable sliding surface can be designed. The next section introduces a controller which will steer the system into discrete-time sliding mode, by which we mean that the system is as close as possible to the sliding surface.

3.2 Linear Reaching Law

From equation (35) and the linear reaching law $\sigma_y[k+1] = \Phi\sigma_y[k]$ (where all eigenvalues of $\Phi \in \mathbb{R}^{m\times m}$ are within the unit circle) the control input can be obtained as:

$$\begin{array}{rcl} u[k] & = & B_2^{-1}\left\{(\Phi - A_{44})\,\sigma_y[k] - A_{43}y_1[k]\right. \\ & & \left.-A_{41}x_0[k] - A_{42}x_1[k] - F_m(x,u,k)\right\} \end{array} \tag{43}$$

In the previous equation the variables $y_1[k]$ and $\sigma_y[k]$ can be measured, however the variables $F_m(x,u,k)$, $x_0[k]$, and $x_1[k]$ are unknown. Therefore, like the continuous-time output based sliding mode controllers [2], the unknown terms are omitted from the control law leading to:

$$u[k] = B_2^{-1}\left\{(\Phi - A_{44})\,\sigma_y[k] - A_{43}y_1[k]\right\} \tag{44}$$

3.3 Linear Reaching Law with Disturbance Estimator

Applying the derived control law of the previous section (equation (44)) to the system (35), $\sigma_y[k+1]$ can be determined to be:

$$\sigma_y[k+1] = \Phi\sigma_y[k] + A_{41}x_0[k] + A_{42}x_1[k] + F_m(x,u,k) \tag{45}$$

If we compare the above with the desired $\sigma_y[k+1] = \Phi\sigma_y[k]$ then we see that the error is given by $A_{41}x_0[k] + A_{42}x_1[k] + F_m(x,u,k)$, hence the unknown terms $x_0[k]$ and $x_1[k]$ could be considered as disturbances just like the disturbance $F_m(x,u,k)$. Therefore we can employ the disturbance estimator of Section 2.5. Again $\tilde{d}[k] \in \mathbb{R}^m$ is given by:

$$\tilde{d}[k] = \tilde{d}[k-1] + \sigma_y[k] - \Phi\sigma_y[k-1] \tag{46}$$

However, as opposed to the state-based case where $\tilde{d}[k]$ is used to estimate the disturbance vector $F_m(x,u,k)$ only, in this case

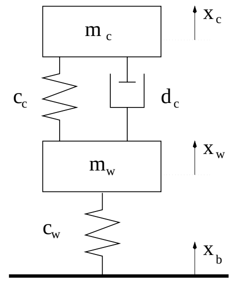

Figure 2: Mechanical diagram of the Quarter Car Model.

$\tilde{d}[k]$ ideally represents the disturbance vector $F_m(x, u, k)$ plus the terms $A_{41}x_0[k] + A_{42}x_1[k]$. The control law the becomes:

$$u[k] = B_2^{-1} \left\{ (\Phi - A_{44})\, \sigma_y[k] - A_{23}y_1[k] - \tilde{d}[k] \right\} \quad (47)$$

In the state-based case it was assumed that the rate of change of the disturbance vector $F_m(x, u, k)$ was limited. However, in the output-based case $\tilde{d}[k]$ is used to estimate the term $F_m(x, u, k) + A_{41}x_0[k] + A_{42}x_1[k]$. Still one could assume, if the sampling frequency is sufficiently high, that the term $F_m(x, u, k) + A_{41}x_0[k] + A_{42}x_1[k]$ is slow enough. In the simulation example this proves to be the case.

4 Simulation Example

As a simulation example we have chosen the so called Quarter Car. It represents one quarter of a vehicle placed on a moving base. To improve reproducibility of test procedures for cars as well as durability tests of new developed cars, one would like to reproduce predefined road-profiles exactly. Therefore the goal of the controller is to reproduce a measured road profile and hence give the car on the base exactly the same accelerations in every successive test. A schematic test setup is presented in Figure 2 for which we can obtain the linear model:

$$x[k+1] = Ax[k] + Bu[k] \quad (48)$$

$$y[k] = Cx[k] \quad (49)$$

The system state, input and output are given by:

$$x = [x_c \quad x_w \quad \dot{x}_c \quad \dot{x}_w]^T \qquad y = [\dot{x}_c \quad \ddot{x}_c] \qquad u = x_b$$

The variables x_c, x_w, and x_b represent the car, wheel and base displacement respectively. The system matrices are given by:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{c_c}{m_c} & \frac{c_c}{m_c} & -\frac{d_c}{m_c} & \frac{d_c}{m_c} \\ \frac{c_c}{m_w} & -\frac{c_c + c_w}{m_w} & \frac{d_c}{m_w} & -\frac{d_c}{m_w} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{c_w}{m_w} \end{bmatrix}^T$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{c_c}{m_c} & \frac{c_c}{m_c} & -\frac{d_c}{m_c} & \frac{d_c}{m_c} \end{bmatrix}$$

where m_w is the mass of the wheel, m_c is one quarter of the mass of the car, c_w is the wheel stiffness, c_c the suspension stiffness and d_c the suspension damping. The nominal model is used for the controller design. The designed controllers are tested on the system for which the parameters differ considerably from the model, as can be seen in Table 1.

Variable	Model	System	unit
m_c	200	300	[kg]
m_w	33	30	[kg]
c_c	9000	7000	[N/m]
c_w	20.000	22.000	[N/m]
d_c	1200	1100	[Nsec/m]

Table 1: Variable values of the Model and the real system.

The total control action $u[k]$ is taken as the sum of a feed-forward term $u_{ff}[k]$ and a sliding mode feed-back term $u_{sm}[k]$, hence:

$$u[k] = u_{ff}[k] + u_{fb}[k] \quad (50)$$

The feed-forward term is computed such that it gives optimal tracking for the model. The feed-back part will be used to compensate for modeling errors, therefore the switching function is defined as:

$$\sigma_{e_y}[k] = S\,(y[k] - r[k]) \quad (51)$$

where the signal $r[k]$ is the reference, or target, signal. Simulations are presented for the feed-forward controller only (Figure 3), the linear output based sliding mode controller, equation

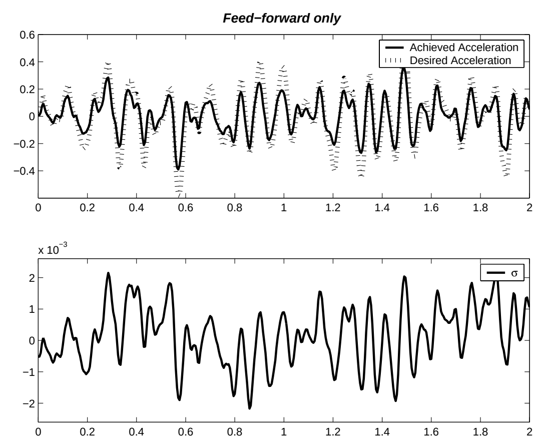

Figure 3: Simulation results with feed-forward controller only.

(44) (Figure 4), and the output based sliding mode controller with disturbance estimation, equations (47) and (46) (Figure 5). For both sliding mode controllers $\Phi = 0$ and $S = [0.123 \quad 0.0058]$ which yields the poles in sliding mode $p_1 = 0.9634$ and $p_2 = 0.8984$. The Variance Accounted For (VAF) of the output, defined by:

$$\text{VAF}(y, r) = \left(1 - \frac{\text{variance}(y - r)}{\text{variance}(y)} \right) \quad (52)$$

which can be computed from the simulation results, is given

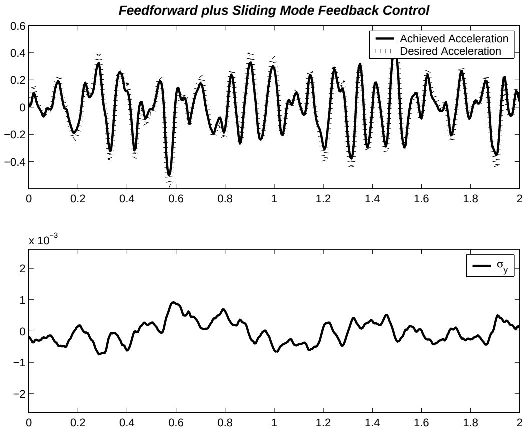

Figure 4: Simulation results for the linear reaching law.

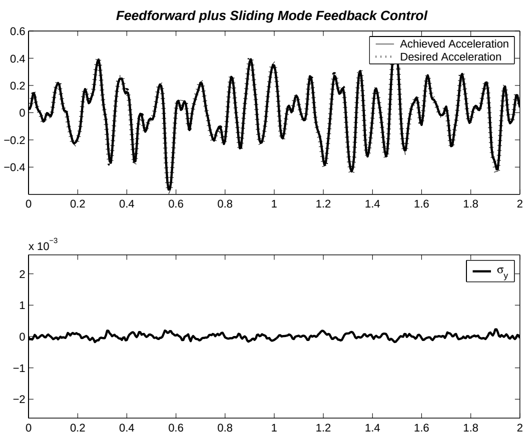

Figure 5: Simulation results for the linear reaching law with disturbance estimation.

for each controller setup by:

$$\mathrm{VAF}_{ff}(y,r) = \begin{bmatrix} 73.3 \\ 81.6 \end{bmatrix} \qquad \mathrm{VAF}_{lsm}(y,r) = \begin{bmatrix} 86.8 \\ 96.1 \end{bmatrix}$$
$$\mathrm{VAF}_{lsmde}(y,r) = \begin{bmatrix} 99.9 \\ 99.6 \end{bmatrix}$$

where the subscript $_{ff}$ stands for the feed-forward controller only, $_{lsm}$ for the feed-forward combined with the output-based linear sliding mode controller, and $_{lsmde}$ for the feed-forward controller combined with the output-based linear sliding mode controller with disturbance estimation.

The figures and the computed VAF clearly demonstrate the improvements made by the (output-based) sliding mode controllers. For the controller with disturbance estimation the tracking is nearly perfect, even though there are matched as well as unmatched disturbances.

5 Conclusions

A brief overview on state-based sliding mode approaches was presented. It was shown that, in the discrete-time case, the linear sliding mode controllers perform better then discontinuous controllers. Chattering is totally eliminated and the quasi sliding mode band is minimized. An even further improvement is to include the proposed disturbance estimation.

These observations were then used to design an output-based sliding mode controller, both with as without disturbance estimation. A practical simulation study then demonstrated the applicability of the proposed discrete-time, output-based, sliding mode controllers. Even in the case of severe modeling errors, nearly perfect tracking was reproduced by the sliding mode controller with disturbance estimation.

Acknowledgements

This work was supported by Brite-Euram under contract number BRPR-CT97-0508 and project number BE-97-4186.

References

[1] A. Bartoszewicz. Discrete-time quasi-sliding-mode control strategies. *IEEE Transactions on Industrial Electronics*, August 1998.

[2] C. Edwards and S.K. Spurgeon. *Sliding Mode Control, Theory and Applications*. Taylor & Francis Ltd, 1998.

[3] W. Gao, Y. Wang, and A. Homaifa. Discrete-time variable structure control systems. *IEEE Transactions on Industrial Electronics*, 42(2):117–122, April 1995.

[4] S. Hui and S.H. Źak. On discrete-time variable structure sliding mode control. *Systems & Control Letters*, 38:283–288, 1999.

[5] S.Z. Sarpturk, Y. Istefanopulos, and O. Kaynak. On the stability of discrete-time sliding mode control systems. *IEEE Transactions on Automatic Control*, 32(10):930–932, October 1987.

[6] S.K. Spurgeon. Hyperplane design techniques for discrete-time variable structure control systems. *International Journal on Control*, 55(2):445–456, 1992.

[7] W. Su, S. V. Drakunov, and Ü. Özgüner. An $o(t^2)$ boundary layer in sliding mode for sampled-data systems. *IEEE Transactions on Automatic Control*, 45(3):482–485, March 2000.

[8] V. Utkin, J. Guldner, and J. Shi. *Sliding Mode Control in Electromechanical Systems*. Taylor & Francis Ltd, 1999.

[9] V.I. Utkin. *Sliding Modes in Control Optimization*. Springer-Verlag, 1992.

[10] K. D. Young and Ü. Özgünter. Sliding mode: Control engineering in practice. pages 150–162, 1999.