Dipole & Quadrupole moments of the local cosmic velocity field


We have compared the bulk flow and velocity shear components of reconstructed peculiar velocity fields with respect to the expected values within a volume of $60\ h^{-1}\text{Mpc}$ centred at the Local Group. The velocity reconstructions were performed by means of the Zel’dovich approximation, linear theory and our implementation of the Least Action Principle, FAM. The velocity fields were extracted from mock galaxy catalogs that mimic as close as possible the real mass distribution up to $150\ h^{-1}\text{Mpc}$ from the Local Group from constrained $N$-body simulations of our nearby Universe. These catalogs were constructed to sample properly the LS region ($30\ h^{-1}\text{Mpc}$), and the large scale structure following the selection function from the PSCz catalog ($30\sim150\ h^{-1}\text{Mpc}$). Our mock catalogs successfully reproduce the main structures of our nearby cosmic region such as, the Local Supercluster, Great Attractor, Perseus-Pisces supercluster, Coma, Cetus. The reliability of the catalogs was assessed by computing their dipolar moment of the mass distribution. The velocity component’s comparison show that the 3 methods give similar results (provided a smoothing procedure) and consistent with the expected values within the $1\sigma$ errors. However, the FAM technique proved to give more accurate and reliable results along the whole range in distances than the other two methods, proving in this way to be the tool to study the dynamics of the Local Group and the Local Supercluster.
3.1 Introduction

Within the Gravity Instability [GI] scenario for the growth of cosmic structures, the peculiar velocity field of galaxies provides a direct and reliable probe of the matter distribution, under the natural assumption that these objects are unbiased tracers of the large scale, gravitationally induced, velocity field. Furthermore, since peculiar velocities are non-local and have contributions from many different scales and regions, their respective analysis provides information on regions not covered by the data, e.g. the Zone of Avoidance (e.g. Kolatt et al. 1996; Zaroubi et al. 1999), and on scales larger than the sampled regions (Hoffman et al. 2001).

The present large scale structures of our Universe and their motions are due to the growth of gravitational instabilities from an initially homogeneous Gaussian density field. This implies that peculiar velocities at early epochs were negligible. Under this assumption, the peculiar velocity of cosmic regions like our Local Group is mainly due to the gravitational pull of the mass distribution surrounding them.

An important characteristic of the peculiar velocity field is that if GI is valid, then the large scale velocity field is expected to be irrotational, i.e. $\nabla \times \mathbf{v} = 0$. Any vorticity mode would have decayed during the linear regime as the Universe expanded, and based on Kelvin’s circulation theorem the flow remains vorticity-free (irrotational) in the quasi-linear regime, even in the presence of nonlinearities, provided these are not so large as to cause dissipative processes (e.g. Peebles 1990; Dekel 1994). This property of the velocity field will prove to be very useful along this chapter.

The dipole anisotropy of the cosmic microwave background (CMB) radiation is generally interpreted as a Doppler effect due to the motion of the Sun with respect to the CMB rest frame. The velocity of the Local Group [LG, hereafter] is now well determined from COBE (Kogut et al. 1993), $627 \pm 22$ km s$^{-1}$ toward $l = 276 \pm 3^\circ$, $b = 30 \pm 3^\circ$. Less well known is the depth and degree to which nearby galaxies share in this motion and the scales and amplitudes of the mass fluctuations responsible for the flow. The boundaries of the LG are not very clear and they change from study to study. A common one is to consider a spherical volume of radius of 5 $h^{-1}$Mpc around the galaxy-pair-system: Milky way - Andromeda. A dynamical definition is given by determining the zero-velocity surface separating the LG from the local Hubble expansion flow. With a total mass of $\sim 3 \times 10^{12}$ $M_\odot$, and an internal velocity dispersion $\sim 60$ km s$^{-1}$ (Sandage 1986), such a region has a radius $\sim 1.8$ $h^{-1}$Mpc.

Provided that the acceleration is dominated by long wavelength modes, implying that linear theory is applicable, the velocity and acceleration vectors should be parallel with a constant of proportionality from which we may determine the density parameter $\Omega_m$. In biased theories of galaxy formation light does not trace mass, but on large scales at least the fluctuations on light should be proportional to the underlying fluctuations in mass, so one can still expects that the velocity and gravity vectors to be parallel, but now the constant of proportionality is the bias parameter $b$.

The analysis of peculiar velocities plays a major role in Cosmology and the formation of structure in the Universe. Some of the most important goals of peculiar velocity fields and their surveys are the confirmation of the gravitational instability picture, the determination of $\Omega_m$, to find whether initial fluctuations were Gaussian, if so, then if the power spectrum was scale-invariant, and the characterization of the mass distribution on very large scales. Because of shot noise, existing redshift surveys cannot account properly for fluctuations at distances larger than 150 $h^{-1}$Mpc. Nevertheless, large amplitude coherent peculiar velocities on very large scales can be detected at such distances with modest samples (e.g. Lauer & Postman 1994; Dekel 1994). This is one of the most important features of peculiar velocities, the detection of large scale flows (dipole) due to matter concentrations. Hence, the peculiar velocity field of galaxies and clusters provides a direct and reliable probe of the matter distribution, under the natural assumption that these objects are unbiased tracers of the large-scale, gravitationally induced, velocity field. In order to measure peculiar velocities of galaxies, observers use a variety of distance indicators. In general, these indicators relate two quantities, one of those is distance dependent (e.g. galaxy luminosity), and the other one is distance independent (e.g. galaxy rotational velocity). The best known examples of such relations are the Tully-Fisher (Tully & Fisher 1977) and Faber-Jackson (Faber & Jackson 1976) relations, although in the last decade many others
3.1. INTRODUCTION

Distance indicators have been employed to measure cosmological distances (e.g. SNIa, Cepheids, etc.). The availability of an increasing number of galaxy peculiar velocity catalogs, some of them with few thousand objects, have turned cosmic flows to one of the main probes used to study the large scale structure in the nearby Universe.

On scales large enough, where the dynamics is dominated by gravity and the deviations from homogeneity are small, the velocity field reflects the dynamical evolution of structure and the total underlying mass distribution, observed and unobserved. On more nearby and shorter scales, it can serve to help us to understand the dynamics of our own Local Group and to realize the role and influence of the large scale structure over the LG. One way to achieve these goals is by reconstructing or modelling the full dynamical structure in the local cosmic neighborhood.

Spatial variations in the mass distribution in our Universe induce (as a consequence) variations in the gravitational force field measured by an observer. These, in turn, induce a peculiar velocity field characteristic of this particular cosmic region, hence exhibiting corresponding spatial variations, which are manifested as a velocity shear.

Our Local Group is located in a very peculiar cosmic location in our Universe. Indeed, as it has been shown by different analysis of surveys (e.g. IRAS 1.2 Jy, MARK III, etc.), the LG is situated at a saddle point between two huge structures, from one side, the Great Attractor [GA], and at the opposite site the Perseus-Pisces supercluster [PP]. Furthermore, the LG is just at the edge of a bridge connecting these two structures, the Local Supercluster [LS]. Even more amazing is the fact that all these structures reside almost along the same plane, the supergalactic $x-y$ plane. This particular and unique situation gives a special flavour to the dynamics of the LG. Within this large scale mass configuration, the LG moves towards the direction of the GA.

The influence of the structures conforming the LS, more in concrete the Virgo Cluster, over the dynamics of the LG is of considerable importance. There is a net peculiar motion of the LG towards Virgo, known as the Virgocentric infall (see Pierce & Tully 1988). This motion accounts for ~ 46% of the total amplitude of the LG motion (Davis & Peebles 1983b). The LS has the shape of a wall located right in front of the LG at the $x-y$ supergalactic plane, and at a distance of about $15 \, h^{-1}\text{Mpc}$. The LS is similar in shape to a flattened ellipse (pancake), with the Virgo Cluster near its center its extent in the longest direction is about $40-50 \, h^{-1}\text{Mpc}$.

The fact that galaxies within a volume enclosing the LS seem to fall into Virgo is a mere coincidence. They are directed towards a rather central, wide region of the LS which coincides with the positions of the Virgo cluster and the Ursa Major cluster (Burstein et al. 1990). It has been measured that galaxies within the plane of the LS are falling towards this central region with velocities that increase with distance outward. The LG, which lies near the edge of the LS, has a velocity ~ $300 \, \text{km s}^{-1}$ to its centre, but only $150 \, \text{km s}^{-1}$ is directed towards the central region of the LS.

Lilje, Yahil, & Jones (1986), estimated that at the LG location the velocity shear had a value ~ $200 \, \text{km s}^{-1}$ with respect to the Virgo cluster. They argued that the source of this shear had to be a considerable mass concentration at a distance of ~ 3 times the distance to the Virgo cluster. This study uncovered the source of our local velocity flow, the Great Attractor, from a sample of peculiar velocities of galaxies within a radius of $60 \, h^{-1}\text{Mpc}$ around the LG. Hoffman et al. (2001) reconstructed the tidally induced component of the cosmic velocity field out to a distance of $60 \, h^{-1}\text{Mpc}$. Their results revealed that the Shapley supercluster is a major tidal source even at scales of ~ $140 \, h^{-1}\text{Mpc}$, although it is not the only relevant external perturbation. With a constructed toy model composed by a big mass concentration (Shapley supercluster), and two voids, they were able of fitting the observed results given by the Mark III catalog (Willick et al. 1997a) and SFI data (Haynes et al. 1999b).

Several methods for modelling peculiar velocity fields from galaxy positions have been proposed in the last decades. Most of these methods fail in reproducing the velocity fields even at mildly non-linear regions, being their predictions only valid at the linear regime. Hence, the success of the methods is limited by the results of linear theory or some first order approximation (Zel’dovich approx.). The necessity of an algorithm capable of dealing with the mildly non-linear regime and predicting/reconstructing/modelling bias-free peculiar velocity fields is of great importance. Such a method would help us to understand in a more consistent, systematic and complete way, the inter-
play between the different structures that shape the portion of Universe under study, our local cosmic
neighborhood. Furthermore, it would allow us to make more realistic estimates of the real velocity
field even at the mildly non-linear regime without loosing generality by applying large smoothing
procedures. A method that has proved to be able to overcome the problems mentioned above is the
Fast Action Minimization algorithm [FAM] (Nusser & Branchini 2000). This method is a very effi-
advantages over other methods are the capability to deal with a large number of objects, an efficient
computing gravity method (making it as a consequence a fast method), and the ability to deal with the
mildly non-linear structures (clusters and superclusters of galaxies).

In this Chapter we will address two main questions. The first one is how well we can construct
synthetic mock galaxy catalogs from specific purpose constrained N-body simulations aimed to mimic
as close as possible our nearby Universe. We will assert the degree of confiability by computing one of
the most characteristic statistical quantities, the gravity dipole. After having asserted the reliability of
our mock catalogs, we will ask ourself how well we can recover the main characteristics of the peculiar
velocity field of galaxies from the galaxy mocks even at the mildly non-linear regime, the bulk flow
and shear velocities. To do so, we will apply two standard reconstruction techniques: linear theory and
Zel’dovich approximation, together with the FAM technique presented in Chapter 2. We will compare
the estimated modeled velocity quantities with respect to those from the real velocity field. We will
show that the estimates derived with the FAM algorithm are in good agreement and closer to the real
measured bulk flow and shear velocities than the ones computed by means of the linear and Zel’dovich
approximations.

We will assume in this Chapter that the motion of our Local Group has been generated by gravita-
tional perturbations. Because of these reasons and in order to minimize errors like the “Kaiser e-
effect” (Kaiser 1987), we will simplify our approach by working in “real” space instead of redshift
space.

A very important quality of our analysis is that we attempt to model as realistically the actual
observations by using constrained simulations of our nearby Universe. For this reason, we will limit
our analysis up to distances of 60 \( h^{-1}\)Mpc from the LG location.

3.2 Theoretical framework

In linear perturbation theory, initial peculiar velocities are damped by the expansion of the universe,
and the peculiar velocity field \( \mathbf{v}(\mathbf{x}) \) is directly proportional to the gravitational acceleration due to the
matter distribution around the position \( \mathbf{x} \) (Peebles 1980). We can express such acceleration due to
matter with density contrast \( \delta(\mathbf{x}) \) in a given volume like:

\[
\mathbf{g}(\mathbf{x}) = \frac{3}{4\pi} \int \mathbf{d}x \delta(\mathbf{x}) \frac{\mathbf{x}' - \mathbf{x}}{|\mathbf{x}' - \mathbf{x}|^3}.
\]  

The acceleration \( \mathbf{g} \) is related to the peculiar velocity according to the relation (Peebles 1980):

\[
\mathbf{v}(\mathbf{x}) = \frac{1}{3} f H_0 \mathbf{g}(\mathbf{x}),
\]  

where \( H_0 \) is the Hubble constant measured in units of 100\( h \) km s\(^{-1}\) Mpc\(^{-1}\). The quantity \( f = d\ln D/d\ln a \) is the logarithmic derivative of the amplitude of the growing mode of density perturbations
with respect to the scale factor \( a \). This factor carries information about the underlying cosmological
model, and is related to the cosmological matter density parameter \( \Omega_m \), cosmological constant \( \Omega_\Lambda \)
and \( z \) (redshift). A common approximation is given by \( f(\Omega_m) \approx \Omega_m^{0.6} \) (Peebles 1980), while a more
complete approximation in the general case at \( z = 0.0 \) is given by Lahav et al. (1991)

\[
f(\Omega_m, \Lambda) = \Omega_m^{0.6} + \frac{\Omega_\Lambda}{70} \left( 1 + \frac{1}{2} \Omega_m \right).
\]  

In the present study, we will adopt this form for being the most complete approximation for \( f \).
3.2. THEORETICAL FRAMEWORK

Following Eqn. 3.2, the peculiar velocity field \( \mathbf{v}(\mathbf{x}) \), and the linear mass density contrast \( \delta(\mathbf{x}) \), are then related to one another according to the local differential relation (the continuity equation)

\[
\nabla \cdot \mathbf{v}(\mathbf{x}) = -H_0 f \delta(\mathbf{x}),
\]

which has as a solution or global (integral) representation, valid under the GI regime for an irrotational field

\[
\mathbf{v}(\mathbf{x}) = \frac{H_0 f}{4\pi b} \int d\mathbf{x}' \delta(\mathbf{x}') \frac{\mathbf{x}' - \mathbf{x}}{|\mathbf{x}' - \mathbf{x}|^3},
\]

where \( b \) is the “linear biasing” factor, which transforms the observed galaxy density into a mass density: \( \delta_{\text{gal}} = b \delta_{\text{mat}} \). Along this analysis we will make the assumption that \( b \) is constant, independent of position or smoothing scale. Notice that the peculiar velocity field is determined by the distribution of the matter with all its components, especially by the dominant dark matter component.

It is well known that the peculiar velocities of nearby galaxies are not randomly oriented but they rather participate in a coherent flow (bulk motion) together with our LG, at least for a volume of radius \( \sim 60 \, h^{-1}\text{Mpc} \). However, there should also exist velocity components related to the linear and non-linear local dynamics of each region acting over the nearby galaxies and cluster of galaxies. In this way, we can understand peculiar velocities as a composition of several modes coming from the large scale distribution which will be reflected in a coherent flow and local contributions.

When expanding the peculiar velocity field in a multipolar decomposition, the first component is related to the Hubble expansion, this monopole term introduces a “breathing mode” which affects the Hubble expansion within the volume, hence having no measurable effect. The second term is the dipolar moment, this term presents a very coherent pattern within the sample that disappears when considering peculiar velocities relative to a central observer instead of absolute velocities. This term is known as the “bulk flow”. The next term in the velocity expansion is the quadrupole, which effects rapidly decrease with distance. This component of higher order in the decomposition is the outcome of the linear and non-linear configuration of the mass distribution. In the case of the peculiar location of the LG (as shown in Figs. 3.3, 3.4 & 3.5), the configuration between the GA and PP introduces a distortion pattern in the peculiar velocity field, plus the influence of the LS. This term is well known like the “shear” velocity. The other higher order terms will be related to the highly non-linear regime (e.g. cluster dynamics). The influence of these terms will be very local because of their strong dependence in distance. So, we can fit a model to the several components that conform the velocity field of the sort:

\[
v_i(\mathbf{x}) = V_{\text{bulk},i} + \xi_{ij} \hat{x}_j + O(\mathbf{x}),
\]

where \( i, j \) refer to the 3 Cartesian components and \( \hat{x}_j \) refers to the distance component along the \( j \) direction. \( V_{\text{bulk}} \) represents the bulk component, and \( \xi_{ij} \) is the second order velocity component, the other higher terms are included in \( O(\mathbf{x}) \).

Before proceeding, we want to make a distinction between the dipolar moment computed from the mass distribution and the one computed from the peculiar velocity field; we will call dipole to the quantity inferred from the mass distribution, while we will call bulk flow the one obtained from the velocities.

3.2.1 The velocity bulk flow

The Bulk flow is defined as the average streaming motion within certain volume. The motion direction and amplitude of this bulk flow on different scales are the simplest quantities to measure from peculiar velocity data. Nevertheless, they provide constraints on the power-spectrum of mass fluctuations. Theoretically, the mean square bulk velocity within a sphere of radius \( R \), is given by,

\[
\langle v^2(R) \rangle = \frac{H_0^2 \Gamma^2}{2\pi^2} \int_0^\infty P(k) W^2(kR) dk,
\]
where $P(k)$ is the mass fluctuation power spectrum and $\tilde{W}(kR)$ is the Fourier transform of a top-hat window of radius $R$. The \textit{rms} expected bulk velocity is then given by,

$$V_b(R) = \sqrt{\langle v^2(R) \rangle}.$$  \hspace{1cm} (3.8)

It is interesting to compare the fluctuations in the bulk flow velocity within a sphere of radius $R$ with the corresponding mass fluctuations within the same sphere,

$$\left\langle \left( \frac{\delta M(R)}{M(R)} \right)^2 \right\rangle = \frac{1}{(2\pi)^3} \int_0^\infty P(k) \tilde{W}^2(kR) dk.$$ \hspace{1cm} (3.9)

Both Eqns. 3.8 and 3.9 differ with two powers of $k$ in the integrand. This means that the peculiar velocity field on a given scale is more sensitive to components of the power spectrum on larger scales than is the density field, and thus is a useful probe of the large scale power. Therefore, the \textit{rms} velocity includes contributions from much larger scales than do the \textit{rms} mass fluctuations (dipole).

In practice, the bulk flow is computed from velocity surveys in the following way; let $v_i$ be the radial peculiar velocity of the $i$th galaxy in the sample and $\epsilon_i$ the uncertainty on that quantity. The bulk flow vector $V_b$ will be obtained from minimizing the merit function (e.g. Lynden-Bell et al. 1988)

$$\chi^2 = \sum_i w_i \left( \frac{v_i - V_b \cdot \hat{x}_i}{\epsilon_i} \right)^2,$$ \hspace{1cm} (3.10)

where $\hat{x}_i$ is the unit vector in the direction of the $i$th galaxy, $w_i$ is a weight given by the selection function of the sample, and $\epsilon_i$ is obtained from the distance indicator used in the catalog (e.g. Tully-Fisher, SNIa, Faber-Jackson relations, etc.)

3.2.2 The velocity shear

The second term in the expansion of the peculiar velocity field is conformed by 3 components and it can be expressed as:

$$\zeta_{ij} = \frac{\partial v_i}{\partial x_j} = \frac{\delta_{ij}}{3} + s_{ij} + \omega_{ij},$$ \hspace{1cm} (3.11)

in the matrix representation of this tensor we can understand them as 3 different components: the symmetric part, trace, and antisymmetric part. $\omega_{ij}$ represents the antisymmetric part, known as the \textit{vorticity} term, but because we have assumed that the peculiar field is irrotational, this term is equal to zero. The trace-free symmetric part $s_{ij}$ is the \textit{shear tensor}. $\Theta$ accounts for possible local isotropic perturbations about the Hubble expansion ($\delta_{ij}$ is the identity matrix), and it represents the \textit{divergence} of the velocity field $\Theta = \nabla \cdot \mathbf{v}$.

The gravitational source of $\zeta_{ij}$ is the tidal field determined by the structures surrounding the observer. In the case of the LG the possible structures responsible for this cosmic shear are mainly: the GA, PP, Coma, Cetus, the local void.

3.3 Setting the environment: Initial conditions

Because we are interested in studying how well we can model the main components of the peculiar velocity field of our cosmic neighborhood, we not only demand that the $N$–body simulations should resemble the statistical properties of our “real” Universe, but also should be able of reproducing the distribution and spatial configuration of matter at linear and weakly non-linear scales. The standard approach for producing the desired realizations is to assume a cosmological model and to use the appropriate primordial power spectrum to construct a random realization of the density field within a desired simulation volume and to evolve them via $N$–body machinery. This method was followed in
3.3. SETTING THE ENVIRONMENT: INITIAL CONDITIONS

Chapter 2. An alternative approach is to find a way of producing simulations which reproduce and mimic the matter distribution at large and intermediate scales of our real Universe. Our intention is to reproduce the matter distribution as close as possible up to considerable large volumes ($\geq 60 \, h^{-1}\text{Mpc}$) as revealed by the actual large scale surveys. This can be done by setting up the initial density fields by constrained realizations of Gaussian fields (Bertschinger (1987); Hoffman & Ribak (1991) and its implementation van de Weygaert & Bertschinger (1996)), thereby constructing the density and velocity fields in such a way that both agree with the observed large scale structures and the assumed cosmological model. This procedure is usually referred as “constrained simulations” [CS, hereafter]. For these reasons, we have adopted this technique for the present study. In the following subsections we will introduce the procedure of the CS while a more rigorous and complete mathematical description can be found at the end of this chapter (Appendix 3.A and 3.B) and Chapter 8.

3.3.1 Constrained Simulations

CSs have been performed in the past by several authors with the purpose of investigating several properties of our local Universe (Kolatt et al. 1996), the morphology of the density field and dynamics (Klypin et al. 2003), the coldness of the local flow (van de Weygaert & Hoffman 1999), the formation of the local galaxy population (Mathis et al. 2002), etc. Nonetheless, none of the previous studies have tackled the problem of the kinematics of the LG & LS and its origins, the origin of the bulk flow velocity, the structures responsible for this motion and their influence range.

Within the GI scenario, the primordial perturbations responsible for the large scale structure constitute a Gaussian Random field, which is defined by its power spectrum and a normalization power. It is assumed that in these random fields, the density peaks are the progenitors of objects like galaxies and clusters, while their minima will correspond to the centers of voids. The theory specifies only the statistical properties of the amplitudes and phases in the plane waves representation. Thus, for Gaussian fields the real and imaginary parts are independently normally distributed around zero with a variance given by half of the power spectrum, which implies that the phases are uniformly distributed.

This is being used in setting up the initial conditions for large $N$–body simulations. Nevertheless, there are many interesting problems where one is interested in generating special-purpose initial conditions, which are designed to obey some given constraints (see van de Weygaert & van Kampen 1993; van Haarlem & van de Weygaert 1993; van de Weygaert & Babul 1994). This is the case for the present Chapter where we are interested in modelling the main components of the velocity field of our Local Universe. A very successful tool for reconstructing the Large scale structure of our Universe and setting up the initial conditions that prevailed at the primeval Universe is the combined use of the Wiener filter (Wiener 1949; Rybicki & Press 1992; Lahav et al. 1994; Bond 1995; Fisher et al. 1995b; Zaroubi et al. 1995, 1999) and Constrained Realizations (Bertschinger 1987; Hoffman & Ribak 1991; van de Weygaert & Bertschinger 1996).

The present available redshift and radial velocity surveys provide us with relevant information that enables the reconstruction of the large scale mass distribution of our nearby Universe. An efficient algorithm for reconstructing such density and velocity fields from incomplete, sparse and noisy data from observations is provided by the formalism of the Wiener filter (WF, Lahav et al. 1994; Fisher et al. 1995b; Zaroubi et al. 1995, 1999). The application of the WF requires some model for the power spectrum that defines statistical properties of the perturbation field. In particular, this method holds on scales where the linear theory is valid and the underlying perturbation field is Gaussian.

The Wiener filter provides an optimal estimator of the underlying field in the sense of a minimum-variance solution given the data (e.g. peculiar velocity data) and an assumed prior model. The prior defines the data autocorrelation and the data-field cross-correlation matrices. In the case where the data are drawn from a random Gaussian field, the WF estimator coincides with the conditional mean field and with the most probable configuration given the data. In the case of Gaussian fields where quadratic entropy can be assigned, the WF also coincides with the maximum entropy solution (Zaroubi et al. 1995).

The WF is a very conservative estimator, in the absence of good data (regions where the data
is sparse and/or noisy) it attenuates the estimate toward its unbiased mean field. This means that by construction the WF suppresses some of the power spectrum that is otherwise predicted by the assumed model. As a consequence the WF often produces an estimated field that is much smoother than the typical random realization of the assumed power spectrum would be. Furthermore the WF estimator is not statistically homogeneous. A way of providing the missing power and regaining statistical homogeneity consistent with the data and the theoretical model is provided by the method of Constrained Realizations (CR) of Gaussian fields (Bertschinger 1987; Hoffman & Ribak 1991; van de Weygaert & Bertschinger 1996). The CR method provides a realization of the underlying field made of two parts. The first one is dictated by the data and the model (provided by the WF approach) and the second one is random in such a way that in those places where the WF suppresses the signal, the random component compensates for it.

The WF and CR are constructed assuming that linear theory is valid on all scales. Hence, in principle it can be done with any desired resolution, even on scales that at present lie in the non-linear regime. This means that the WF/CR algorithm provides a reconstruction of how the present-day structures would appear if the linear regime is still valid.

The Initial conditions were then generated by using data on scales where the linear regime is still valid, so we could apply it to recover the large scale fluctuations and to supplement them with fluctuations due to a random realization of a specific power spectrum on small scales. These fluctuations are then extrapolated back in time using linear theory to provide the reconstruction of the initial conditions.

The WF/CR approach has been applied to several data catalogs and with different purposes: to study the two-dimensional IRAS galaxy distribution (Lahav et al. 1994), the velocity potential out of POTENT (Ganon & Hoffman 1993), the COBE/DMR cosmic microwave background mapping (Bunn et al. 1994), etc.

### 3.3.2 Reconstructing the Initial Density Fields

#### 3.3.2.1 Parent catalog data

The WF/CR algorithm was applied to the MARK III catalog (Willick et al. 1997a). This catalog has been compiled from several data sets of spiral, elliptical and SO galaxies with the direct Tully-Fisher and the \( D_n - \sigma \) distances. The sample consist of \( \approx 3400 \) galaxies and provides radial velocities and inferred distances with fractional errors around \( 17 - 21\% \). It has an anisotropic and non-uniform density that is a strong function of the distance. The sampling of the density field is reliable out to \( \sim 60 \, h^{-1}\text{Mpc} \) in most directions outside the Galactic plane, and out to \( \sim 70 \, h^{-1}\text{Mpc} \) in the direction of the GA and PP. In some other directions the data extends out to \( \sim 80 \, h^{-1}\text{Mpc} \). The data has been corrected for the Malmquist biases. This sample enables a reasonable recovery of the dynamical fields with \( \sim 12 \, h^{-1}\text{Mpc} \) smoothing.

The cosmological model assumed is the currently popular flat low-density cosmological model \( \Lambda\text{CDM} \) with \( \Omega_{\Lambda} = 0.7, \Omega_0 = 0.3 \), where \( \Omega_{\Lambda} \) measures the cosmological constant \( \Lambda \) in units of the critical density and \( \Omega_0 \) is the cosmological density parameter. The Hubble constant is \( h = 0.7 \) (measured in units of \( 100 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \)) and the power spectrum is normalized by \( \sigma_8 = 0.9 \). The \( \Lambda\text{CDM} \) model assumed here is consistent with the newest and current observational constraints (WMAP Spergel et al. 2003). This model is also consistent with the radial velocity surveys including the MARK III.

Zaroubi et al. (1999) performed a detailed analysis of the Large scale structure reconstructed from the Mark III survey. The most robust features of the structures recovered from this catalog are the Great Attractor (GA), the Perseus-Pisces (PP) supercluster, the filamentary local supercluster connecting GA and PP, and the local void.

#### 3.3.2.2 Reconstruction of the density field

The WF/CR algorithm has been applied to the MARK III catalog with the assumed cosmological model. This was done on a \( 128^3 \) grid with a grid size of \( 2.5 \, h^{-1}\text{Mpc} \), thus reconstructing the density
Figure 3.1 — Initial constrained density fields for two catalogs of our sample (S1 & S2). The panels show projections along the 3 supergalactic planes. The top panel corresponds to the $x$-axis projection, the middle one to the $y$-axis, while the bottom one to the $z$-axis. The density fields have been smoothed with a Gaussian filter of $5 \, h^{-1}\text{Mpc}$ radius.
Figure 3.2 — Initial constrained density fields for the other two catalogs of our sample (S3 & S4). The panel distributions are the same as in Fig 3.1. The panels show only the central regions of the catalogs within a radius of $80 \, h^{-1}\text{Mpc}$ from the center (LG). The initial seeds of the main structures surrounding the LG can be better noticed. The density fields have been smoothed with a Gaussian filter of $5 \, h^{-1}\text{Mpc}$ radius.
and velocity field within a box of $320 \, h^{-1}\text{Mpc}$ on a side with the LG located at the center of the simulation box.

For statistical purposes we have made four WF/CR realizations. All of them have been produced for the same cosmological scenario. The variations among the CRs thus provide an interesting representation of the uncertainty in the reconstructing algorithm, and the CRs provide clear indications for the robustness of the various structures. Furthermore, this also will allow us to estimate the statistical significance of our results. In the regions where the data dominates over the noise ($\leq 70 \, h^{-1}\text{Mpc}$), the WF maps are characterized by a high S/N (Zaroubi et al. 1999), and there is little variation between the different CRs. At small scales ($< 12 \, h^{-1}\text{Mpc}$), the properties of the structures, depend on the nature of the prior model, hence the CRs exhibit high variations, consistent with the low S/N. All these can be better noticed in Figures 3.1 & 3.2. These figures show the linear density fields resulted from the outcome of the WF/CR algorithm after being extrapolated back in time (initial density fields). The 3 panels in each column represent projections along the 3 different supergalactic planes. Each density field has been convolved with a Gaussian kernel of $R_D = 5 \, h^{-1}\text{Mpc}$. The top panel of each figure corresponds to slides which coincide with the supergalactic $y-z$ plane. The middle one to the supergalactic $x-z$ plane, while the bottom panel to the $x-y$ supergalactic plane. All slides have been centered at the Local Group position. In all plots the lines correspond to regions of equal densities, the continues lines represent the high density regions while the broken lines to under-dense regions. The thick continues line indicates the mean density.

Figure 3.1 shows for simulations S1 & S2 the full planes $[-160, +160] \, h^{-1}\text{Mpc}$. In these plots can be noticed that the distribution and shape of structures at large scales ($> 80 \, h^{-1}\text{Mpc}$ from the LG, and correspond to the unconstrained random realization part of the method), differ from catalog to catalog. Still, their statistical properties will be similar because they are given by the same prior assumed cosmological model. Nevertheless, the central regions of the catalogs contain similar kind of structures which correspond to the constrained realization part of the method, these regions are strongly constrained by the data. This can be better noticed in Fig. 3.2 which shows the inner regions $([-80, +80] \, h^{-1}\text{Mpc})$ of the initial density fields for the simulations S3 & S4. The locations of the original seeds of the massive structures like the GA and PP superclusters do not change in spatial location substantially from catalog to catalog. The conspicuousness of the density peaks and voids are also very similar between the different catalogs.

Finally, a useful feature of the WF/CR reconstruction method is that it can relate two data sets that differ in many ways. As an example it can translate velocities into densities allowing comparisons between velocity and density data, even if the data have been sampled with different resolutions and not exactly the same positions (van de Weygaert & Bertschinger 1996; Zaroubi et al. 1999).

We are aware of the fact that all constraints are within a radius of $< 80 \, h^{-1}\text{Mpc}$ from the LG, and that structures smaller than $12 \, h^{-1}\text{Mpc}$ cannot be resolved properly in the maps as a consequence of the previous smoothing procedure on the MARK III data. Beyond the $> 80 \, h^{-1}\text{Mpc}$ radius the velocity constraints in the local volume still work out, as the velocity is a non-local constraint. Any similarly between the structures at these ranges ($< 12 \, h^{-1}\text{Mpc}$, or $\gg 80 \, h^{-1}\text{Mpc}$) with the real Universe will be a consequence of the random constrain procedure, e.g. mere chance.

### 3.4 $N$–body simulations

#### 3.4.1 The particle distribution

The evolution of the WF/CR initial density fields was performed by means of the public numerical code HYDRA (Couchman, Thomas, & Pearce 1995). This code uses an adaptive $P^3M$ code as a gravity solver, and a SPH algorithm to solve the hydrodynamic equations. Here we restrict ourselves to trace only the mass distribution of our Universe. Therefore, we have performed pure $N$–body analysis carrying out simulations of dark matter alone.

The initial particle positions were interpolated from a uniform grid which covers the whole computation box of side $L = 320 \, h^{-1}\text{Mpc}$. The amplitude of the initial displacements are proportional to
the computed WF/CR initial density fields. The initial velocities (peculiar velocities) are proportional to their respective displacements in accordance with the Zel’dovich approximation (Zel’dovich 1970).

Figure 3.3 — Particle distributions along the three Supergalactic Planes for the S4 simulation at the final output time (actual time). The left-column slides are 40 $h^{-1}$Mpc thick while the right-side plots are zooms of the central marked areas. The top panels correspond to projections along the supergalactic $x$-axis, the middle panels to projections along the supergalactic $y$-axis, while the bottom panels to the $z$-axis projection. The LG is located at the center of each computational box and marked by a circle. The position of the main structures like GA, PP, Coma, Cetus and Shapley are also indicated.
The four simulations were started at $z = 50$ and evolved up to actual time $z = 0$ (with $z$ being the redshift). We have started the simulations at that initial time to assure the validity and linearity of the initial input density fields.

The simulations were run with $128^3$ particles of equal mass, with a mass resolution of $6.4 \times 10^{11}h^{-1}M_\odot$. The force computation was done on a grid of $256^3$, and a spatial resolution of $25 h^{-1}$ kpc. Figure 3.3 shows for the S4 simulation slides of the 3 Cartesian projections centered along the 3 Supergalactic planes. The top panels correspond to projections along the supergalactic $x$-axis. The middle panels refer to projections along the $y$-axis, while the bottom panels to the $z$-axis projection. The left-hand boxes correspond to the full simulation box length ($320 h^{-1}$ Mpc), and a thickness of $40 h^{-1}$ Mpc. The right-hand plots are zooms of the central regions. The position of the LG is marked on the zooms by the central circles. One may clearly notice how well the $N$-body simulation traces the Local Supercluster as well as the presence of the GA, PP, and similar structures like Cetus and Coma. It is interesting to notice how it seems that the LS is connected with the GA and PP superclusters. Another interesting feature is the presence of a Shapley-alike structure located at the region where the real Shapley supercluster resides, although this region correspond to the weakly constrained part of the WF/CR algorithm.

The similarities and differences between the different simulations as a consequence of the WF/CR algorithm can be better noticed in Figure 3.4, where zooms of the central regions ($[-80, +80] h^{-1}$ Mpc) of the S1 & S4 simulations are presented. For comparison purposes the fields have been convolved with different Gaussian kernels of radius 2.5 $h^{-1}$ Mpc (S1 & S4 respectively). We have done this in order to resolve the small scale structure (S1) and the LG. Despite the different map resolutions, the similarities between the two simulations are quite remarkable; the location and geometry of the structures (peaks and voids) between the two set of maps are very similar. The prominence of the LS can be clearly seen in the $z$-projection of S4, and how this structure connects with the GA and PP superclusters. The presence of the Coma cluster can also be located at the top right of the $z$-projection, the Cetus region at the bottom central part of the same slides. We can also compare the smoothed S4 density fields with its initial density maps showed in Fig. 3.2. As can be noticed between the two set of plots, the linear regime remained almost intact, the location, prominence, shape and geometry of peaks and voids are quite similar between the 2 density fields. The final outputs of the four simulations have reproduced the Local Supercluster, the Great Attractor region, the Perseus-Pisces supercluster, the Coma cluster and the Cetus region.

The location of the LG in our nearby Universe is confirmed to be found at a saddle point of the density field between two big nearby surrounding structures, the Great Attractor and Perseus-Pisces superclusters. This situation is depicted in Figure 3.5 where surface plots of the same slides of Fig. 3.4 for the S4 simulation are presented. The plotted planes ($\pm 60 h^{-1}$ Mpc) coincide with the three supergalactic planes and have a thickness of $2.5 h^{-1}$ Mpc. The peaks correspond to high density regions (like the GA and PP structures). The location of the LG is at the center of the plots and marked in black. Notice that the location of the LG is very close to a minima in the three projected density field surfaces.

### 3.5 Observing the simulated Universe: Mock catalogs

Our aim is to study how well we can model the main characteristics of the full, real local cosmic peculiar velocity field. For these reasons we construct mock galaxy catalogs from the final output of the $N$-body simulations aimed to resemble the real mass distribution as revealed by galaxy surveys. For statistical purposes, we have produced four sets of mock catalogs, one set from each $N$-body realization. We follow the mock catalog procedure presented in Chapter 2 (Sect. 2.5.2). The only difference is that this new set of mock catalogs extends up to volumes of $150 h^{-1}$ Mpc. We will only mention here the main characteristics of the catalogs.

Each catalog consists of two different sampled regions. The first one (internal region) is a volume-limited sample and is aimed to trace the mass distribution within a region of $30 h^{-1}$ Mpc, mimicking the
Figure 3.4 — Final output projected density fields for simulations S1 & S4. The plots show zooms of the central regions of the simulations (±80 $h^{-1}$Mpc) centered at the location of the LG. The main structures (LS, GA, PP & Coma) have been also labeled for reference. The slides are 5 $h^{-1}$Mpc thick and coincide with the 3 supergalactic planes. The fields have been smoothed with Gaussian filters of $R_G = 2 & 5$ $h^{-1}$Mpc respectively for comparison purposes.
Nearby Galaxy Catalog of Tully [NBG] (Tully 1988). This catalog includes around 2800 galaxies and it can be considered as a fair volume-limited sample of our Local Supercluster and its surroundings. The second region is a flux-limited sample and it is aimed to trace the mass distribution of regions super exceeding the LS from 30 up to 150 $h^{-1}$Mpc. This region mimics the IRAS-PSCz galaxy catalog (Saunders et al. 2000). To do so, we have chosen objects beyond 30 $h^{-1}$Mpc from the real mass distribution according to the PSCz selection function used by Branchini et al. (1999, see Chapter 2, Sec. 2.5.2.2). The catalog-making algorithm can be described as follows:

1. We select all particles within a radius of 150 $h^{-1}$Mpc from the closest galaxy located at the geometrical centre of the parent $N$–body simulation.
2. We proceed to split the parent catalog into a “Volume” and “flux” limited regions.
3. For the volume-limited region NBG, we enclose all particles within a radius of 30 $h^{-1}$Mpc.
Because the total number exceeds the one of the NBG, we proceed to re-sample this region by a Monte Carlo selection process to match the observed number of galaxies.

4. For the flux-limited region ($30 - 150 \, h^{-1}\text{Mpc}$), objects were selected accordingly to the following IRAS-PSCz Selection Function, (Branchini et al. 1999, Chapter 2, Sec 2.5.2.2).

$$\psi(x) = A x^{-2\alpha} \left( 1 + \frac{x^2}{x_s^2} \right)^{-\beta} \text{ if } x > x_s .$$  \hspace{1cm} (3.12)

where $x$ is the distance of the galaxy expressed in $h^{-1}\text{Mpc}$, and $\alpha = 0.53$, $\beta = 1.8$, $x_s = 10.9 \, h^{-1}\text{Mpc}$, and $x_s = 84 \, h^{-1}\text{Mpc}$ (Branchini et al. 1999). The total number of selected objects is in good agreement with the expected number of objects (Figure 3.7).

5. The zone of avoidance was treated in the same way that in Chapter 2.

We justify the first step in the catalog-making procedure by noting that the WF/CR initial density fields were constructed in such a way that the LG would be located at the center of the simulation box, and that the linear regime still prevails at actual time. Therefore, the position of the Local Group
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in the evolved density fields should also be located at the center. This can be noticed in Figures 3.3 & 3.4. Under these assumptions we have chosen the geometrical centers of the original simulation boxes to be the geometrical centers of our mock catalogs. Nevertheless, this assumption is not totally valid because of the fact that the region corresponding to the LG corresponds to the small scale part of the power spectrum, and as we have pointed out earlier, this part has been produced by random realizations. However, from Figures 3.3 & 3.4 we can see that the LG is located at the centre of the simulation box, so our assumption is not far from reality.

Figure 3.6 shows the outcome of this procedure, the projected spatial distribution of objects along the three supergalactic planes for one of our catalogs (S4). The centre of each projection corresponds to the location of an observer situated at the LG position. As can be noticed, the number of objects decreases with distance from $30 \, h^{-1}\text{Mpc}$ outward. The inner circle represents the transition part from volume to flux limited regions. It has been noticed before that there are several structures that expand beyond the $30 \, h^{-1}\text{Mpc}$ region, this is the case for the LS filament and others.

Figure 3.7 shows the average object counts as a function of distance for the four set of catalogs. As can be noticed, there is a good match between the theoretical expected counts (solid line) and the galaxy catalogs (histogram). The discontinuity at $30 \, h^{-1}\text{Mpc}$ where the transition from volume-limited to flux-limited regions can be clearly seen. The distribution of mock-galaxy objects projected along the sky can be noticed in Figure 3.8, where we have plotted in galactic coordinates the particle distribution of the same mock catalog shown in Fig. 3.6.

3.6 Testing the Mock Universe

In this section we want to assess the confidence of our $N$-body simulations and their corresponding mock catalogs. In particular, we are interested in checking that our Mock catalogs have been sampled properly and that they have accounted for the whole mass distribution. We have already seen that the simulations have been able of reproducing the cosmography of our nearby Universe. Nevertheless a more rigorous and quantitative analysis is necessary. Zaroubi et al. (1999) studied in detail and performed several statistical tests for the WF/CR method. In our case we will do so by estimating a dynamical characteristic of the galaxy density field, the dipole moment.
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We would like to make clear that while we will consider only the dipole moment from the mass distributions for testing the reliability of our catalogs, we do not assume that this alone provide a good description of the inhomogeneity around us. The dipole is a convenient statistics for which it seems to be reasonable to compare with the real observed mass distribution.

We will focus on 2 aspects of the gravity dipole, its amplitude and direction. The amplitude will give us information about how large the contributions from structures located at different distances from the central observer are, while the direction will point the region towards the central observer is moving.

The direction of the LG is that given by the dipole component of the cosmic microwave background radiation field. In principle, any sample of objects (catalog) that traces the mass distribution “fairly” should converge to this velocity and direction as the samples get deeper. This phenomenon is known as the dipole convergence, and the distance at which the shells’ dipole motions begin to match the LG motion is call the convergence depth. This distance ($R$ in Eqn. 3.13) is to be believed in the order of 150 – 200 $h^{-1}$Mpc.

3.6.1 The gravity dipole

The mass distribution of galaxies exerts a pull over an object due to gravity effects. The amplitude and direction of this pull is determined by the distribution, concentration and distances of galaxies around the observer’s vicinity.

The dipole moment provides an estimate of the gravitational acceleration acting on the LG, and hence of the large scale mass distribution around the LG. In linear theory, the peculiar acceleration dipole vector is given by the following expression:

$$g_D(x) = \frac{3\Omega_m H^2}{8\pi} \int_0^R dx' \delta(x') \frac{x'}{|x'|^3}. \tag{3.13}$$

In this equation, the inferred acceleration vector must be aligned with the LG velocity vector. But, while this is a necessary condition to apply, it is not sufficient, one must also be sure that the radius of the sample ($R$) of mass tracers is sufficiently deep that it does not miss any contribution from distant density fluctuations. This means that the dipole vector of the mass distribution must converge to its total value within the effective depth of the sample $R$.

The peculiar configuration of our cosmic vicinity imposes strong characteristics in $g_D$, this region which is enclosed within a volume of $\sim 40 – 50 \, h^{-1}$Mpc is responsible of at least 60% of the dipole.
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Since we do not have a continuous density field, but the positions of a finite number of objects in our galaxy flux-limited and full mass distribution catalogs, our estimate of the acceleration on the LG is given by a discrete form of Eqn. 3.13 in velocity units (relation valid only in the linear regime),

\[ g_D = \frac{f(\Omega_m, \Lambda)H_0}{4\pi n} \sum_i \frac{x_i}{\psi(x_i)|x_i|^3}, \]  

where \( n \) is the average number density of objects selected in the sample, and \( \psi(x_i)^{-1} \) is a weighting factor associated to the selection function evaluated at the distance \( x_i \). This weighting factor is equal to 1 for all objects inside the volume-limited region. Despite the discreteness of the Eqn. 3.14, the estimate of \( g_D \) does not loose generality in the sense that the weighting factor corrects for the relative number density of objects with respect to the number density in the “real” \( N \)-body mass distribution.

We have computed the gravity dipole from Eqn. 3.14 for the \( N \)-body and mock catalogs. Figure 3.9 shows the cumulative \( g_D \) computed for the \( N \)-body mass distribution (continues line) and for the mock galaxy distribution (dotted line) from the S4 simulation. We started the calculation from an initial radius of 5 \( h^{-1} \) Mpc and up to 150 \( h^{-1} \) Mpc. This was done due to two reasons, the first one is due to the lack of objects to compute the dipole inside this region (which introduce a large dispersion in the calculation), and the second and most important one is because we want to determine the origin and influence radius which is responsible for the movement of the LG. As it can be noticed, the estimate inside the inner zone \([0 \sim 30 h^{-1} \text{Mpc}]\) of the mock catalog matches the one from the full distribution showing an almost perfect sampling. Deviations from the \( N \)-body estimate start to arise around 30 \( h^{-1} \) Mpc and they become more pronounce at larger radii. This effect is due to the fact that these regions represent the tail of the selection function which does not account properly for all objects. We observe a rapid raising in the dipole in the first \( 10 \sim 30 h^{-1} \) Mpc, after which it remaining almost flat up to 60 \( h^{-1} \) Mpc. This quick increase \( (\sim 400 \text{ km s}^{-1}) \) means that the mass distribution enclosed in this region exerts most of the gravitational pull onto the LG. Indeed, as seen in the corresponding mass distributions maps of Figs. 3.4 & 3.6 and reflected in the corresponding velocity maps, the particular configuration between the Local Supercluster, the GA and PP structures over the LG are responsible of this movement, matching the observational results. Nevertheless, after remaining almost constant for a range of distances, the dipole amplitude continues raising as the volume increases, meaning that there are still mass concentrations at those distances which also play a major role into the dynamics of the LG. In the real world, a similar phenomenon is observed. Several authors (e.g. Rowan-Robinson et al. 2000; Plionis & Kolokotronis 1998; Basilakos & Plionis 1998) have pointed out that the Shapley su-
percluster is a very likely candidate responsible for this increase in amplitude which contributes almost
20 to 30% to the total cumulative dipole. This huge mass overdensity is located at \( \approx 140 \ h^{-1}\text{Mpc} \) in
the direction of the Hydra-Centaurus supercluster (GA). In all our catalogs the dipole amplitude does
continue raising at the limit of our catalogs (150 \( h^{-1}\text{Mpc} \)). This effect can be explained in terms of
the WF/CR initial density maps. The input velocity field used to generate the WF fields has imprinted
on itself the influence of the mass distribution at very large scales. Hence, registering a general flow
towards the direction of GA and Shapley (Hoffman et al. 2001). In this sense the evolution of the
WF/CR has successfully reproduced the observations in the sense that there might be still structures
influencing over the LG (e.g. Hoffman et al. 2001). Nevertheless, the region which corresponds to the
location of the Shapley concentration, corresponds to the weakly constrained part of the WF/CR algo-

rithm, so the real presence of this structure in our simulations is dubious, although we have measured
the same effect.

The speculation that the Shapley supercluster is not the major contributor to the motion of the LG
but that it is coming even from larger distances was proposed by Raychaudhury (1989) and Plionis & Valdarnini (1991). These authors speculated that large structures like the Horologium supercluster
still plays an important role over the LG. Nevertheless, Rowan-Robinson et al. (2000) used the PSCz
catalog to compute the dipole even at scales up to 300 \( h^{-1}\text{Mpc} \). They found that there is indeed a rise
in the dipole after 150 \( h^{-1}\text{Mpc} \) but it finds a convergence before the 200 \( h^{-1}\text{Mpc} \) with no evidence for
a significant contribution to the dipole, excluding in this way the speculation about the Horologium
supercluster. This result should be met with caution in the sense that the errors in the PSCz catalog and
its selection function increase considerable with distance, so that the reliability of the result at these
very large scales decreases accordingly.

### 3.6.2 Shot-noise effects

It has already been pointed out that the discreteness and sampling effects introduce uncertainties in the
calculation of the dipole. Furthermore, these effects introduce an additive dipole term, the \textit{shot-noise
dipole}. In order to correct for this extra dipole, we need to compute the corresponding shot-noise and
to correct for it our dipole estimates.

There are two alternative approaches to correct for shot-noise effects. The first one concerns to
generating several Monte Carlo realizations of mock catalogs and keeping the number of objects and
selection function unchanged, while the second one is by computing the shot-noise by the analytic
estimate of Strauss et al. (1992). Due to the fact that we are also interested in reconstructing their cor-
responding peculiar velocity fields, it will be very cumbersome and expensive in consuming computing
time to use the Monte Carlo approach. The latter would imply necessary to produce a large amount of
catalogs per set. We will therefore resort to the second method, supported also by the demonstration
by Basilakos & Plionis (1998), who showed that both methods seem to give equivalent results.

From the Appendix A of Strauss et al. (1992) it follows that the shot-noise is given by the expected
variance between the contributions to the acceleration from all galaxies on a shell at distance \( x \) and the
galaxies within the flux-limited sample at the same shell. In this case, the expected value is given by:

\[
\sigma^2 = \left( \frac{H_0 \beta}{4 \pi n_1} \right)^2 \sum_{\text{galaxies}} \frac{1}{\psi(x_i)} x_i^4 \left( \frac{1}{\psi(x_i)} + 1 \right),
\]

where \( n_1 \) is the average density of galaxies, \( \psi(x_i) \) is the selection function at the distance \( x \) of
the shell and \( \beta \) is the bias factor, that for our case is equal to \( f(\Omega_m, \Lambda) \). In Figure 3.9 the effects of shot-
noise are shown. The dashed line is the shot-noise contribution according to Eqn. 3.15 to the dipole.
This is almost zero at the inner 30 \( h^{-1}\text{Mpc} \) due to the fact that this region is volume-limited sampled.
The shot-noise starts to be noticed from the 30 \( h^{-1}\text{Mpc} \) outward. This effect is due to the fact that
the selection function depends strongly on distance, so as a consequence the shot-noise will increase
at larger distances. The gray long dashed line represents the dipole estimate corrected for shot-noise.
The agreement between the real and corrected mock dipole amplitudes, even at large distances, is quite
remarkable.
In general, the shot noise amplitudes represent almost the 10% – 15% of their total magnitudes for all catalogs in our sample, and they show the same behavior as in Fig. 3.9.

### 3.6.3 Dipole direction and convergence

The direction and convergence of the dipole will help us to assess the reliability of our samples. Therefore, we have computed the direction of the dipole by using Eqn. 3.14 as a function of depth. This is shown in the left panel of Figure 3.10. The directions for the cumulative \(N\)-body mass distribution (black diamonds) and for the mock galaxy catalogs (gray triangles) are shown in galactic coordinates.

![Figure 3.10 — Dipole directions and convergences for the S4 catalog. The left panel shows in galactic coordinates how the dipole wanders around when increasing the computing radius. The diamonds correspond to the full, \(N\)-body mass distribution, while the triangles to the mock galaxy catalogs. The right panel shows the angle \(\theta\) between the acceleration vector and the CMB dipole vector for the real mass distribution (continuous line), and for the mock galaxy catalogs (broken, gray line).](image)

The convergence of the direction of the dipole has been computed with respect to the one measured by COBE. The right panel of Figure 3.10 shows the angle \(\theta\) between the estimated dipole and the COBE one as a function of depth. These two plots refer to the same catalog shown in Fig. 3.9. Around the first \(20 – 30\ h^{-1}\)Mpc the angle \(\theta\) does not converge due to the big influence of the Local Supercluster over the LG (Virgo-centric infall). The sudden convergence at \(\sim 60 – 80\ h^{-1}\)Mpc means that the massive structures like the GA and PP supercluster have been enclosed. This result agrees with those obtained by Strauss et al. (1992) for the 1.2-Jy IRAS catalog. The slight rise at \(100\ h^{-1}\)Mpc and its convergence at \(\sim 130\ h^{-1}\)Mpc can be possibly interpreted like the enclose of structures similar to the Shapley concentration at this radius. In effect, as may be seen from the mock catalog particle distribution (Fig. 3.6) and from the density fields (Figs. 3.1 & 3.3, 3.4) the most prominent mass density structure beyond the GA location coincides along the same direction and \(\sim 110\ h^{-1}\)Mpc far from the LG. It is this Shapley-alike structure responsible for this effect.

#### 3.6.3.1 Averaged Dipole results

The robustness of our results can be noticed from Figure 3.11 where the average cumulative \(\text{g}_D\) (top panels) and the average dipole direction (bottom panel) for the four samples are shown. The top-left panel shows the corresponding cumulative dipole for the full \(N\)-body mass distribution, while the top right panel for the mock galaxy catalogs. The gray shadows represent the \(\pm 1\sigma\) dispersion around the average values at each radius. The agreement between the two curves is quite remarkable, meaning that the mock catalogs have successfully traced the real mass distribution. The rapid increase in \(\text{g}_D\) for the first \(15\ h^{-1}\)Mpc is due to the gravitational attraction of the LS in combination with the GA influence, which is reflected after this radius. The magnitude of this increment varies from realization to realization as may be noticed from the large dispersion around the mean. The almost flat behavior
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Figure 3.11 — Average cumulative gravity dipole (continues line) for the 4 catalogs of our set (top panels), and average direction (bottom panel) at 150 $h^{-1}$ Mpc and 60 $h^{-1}$ Mpc for both samples in galactic coordinates. The top-left panel corresponds to the full mass distribution, while the top-right panel to the mock galaxy catalogs. The gray shadows represent the 1σ dispersion around the mean value.

after 40 $h^{-1}$ Mpc and up to 80 $h^{-1}$ Mpc means that there are no considerable structures which exert a strong influence over the LG. The sudden raise in the dipole after 80 $h^{-1}$ Mpc without convergence, even at the edge of the catalogs, imply the presence of big(s) structure(s) acting still over the LG. All catalogs show a dipole mass concentration in the density fields along the direction of the GA and Shapley concentration which in turn is responsible for this result. The bottom panel of Fig. 3.11 shows the positions of the average dipole directions computed at 2 different radius, at 60 and 150 $h^{-1}$ Mpc, projected in the sky in galactic coordinates of the N-body mass distribution (black square), and the mock galaxy dipole (gray diamond) and their corresponding error bars. The separation between the two quantities gets larger at larger radius. At 60 $h^{-1}$ Mpc both directions differ only 3°, while at 150 $h^{-1}$ Mpc the difference is ~ 8°, being this is an effect of the sampling procedure. The dipole at 60 $h^{-1}$ Mpc is ~ 12° far from the real dipole direction, while the one computed at the edge of the sample around 20°. These results agree with those obtained from the literature for the bulk velocity direction and the gravity dipole. Because our initial density fields were computed from real observed velocity fields, then it is expected that our estimates should coincide with those obtained from the velocity bulk flow.

In conclusion, we can say with a high degree of confidence, that our Mock catalogs trace very well the full and real mass distribution got from the N-body simulations, and that in principle we have taken into account possible effects that could influence any results obtained from these mock catalogs. Furthermore, the fact that the dipole estimates reproduce quite well the real observed dipole behavior even at very large scales (~ 150 $h^{-1}$ Mpc) justify the use of constrained simulations being extended by unconstrained regions.
3.7 The velocity field

The intricated and very peculiar location of the LG in our Universe is reflected in its corresponding peculiar velocity field. This field not only shows as well the complexity of the local cosmography, but also reveals the entangled dynamical state around the LG.

Figure 3.12 shows slides of the peculiar velocity fields corresponding to the 3 supergalactic planes and centered around the LG position (black central circle) for the S4 simulation. The left-column corresponds to the peculiar velocities at each particle position represented by arrows proportional to their magnitude and pointing towards their projected direction. As can be noticed the structures around the LG influence considerably the dynamics of the surrounding mass distribution. This can be noticed more clearly in the right-column panels. These plots are zooms of the regions enclosed in boxes at the left-hand panels. In these plots, the peculiar velocity field have been interpolated into a grid. The contours correspond to mass density contrast located along the same projections; the continuous line indicate overdensities, while the dotted lines underdense regions, and the thick black line corresponds to the mean density. In these representations, the local flows around the big mass concentrations are more clear. Furthermore, a more general dominant flow along the upper part of \( x - y \) direction can be recognized. This is the influence of the GA, and the movement and direction of the LG.

The inhomogeneities in the mass distribution (clusters and superclusters of galaxies) are the origin of the local flows. The configuration of those inhomogeneities also plays an important role. The clumpiness of the mass distribution distorts the amplitude and direction of the peculiar velocities. These distortions have the shape of a shear pattern. These shear patterns can be more easily recognized in the left-hand panels of Fig. 3.12 around the non-virialized structures.

In all panels of the Fig. 3.12, it can be noticed that the LG participates in a very general velocity flow (indicated by the size of the arrows), and it is very well defined towards the direction of the GA region. The influence of the PP supercluster can also be recognized at the lower right of the LG position in the \( x - y \) supergalactic plane. Even more, the LS also participates in this intricate game. It exerts a gravitational influence over the LG (indicated by the verticality component of the arrows), the Virgocentric infall. Another peculiarity of this scenario is the elongation of the LS along the same direction of the dominant flow, towards the GA location.

3.7.1 Modeling the peculiar velocity field

Several methods have been proposed to reconstruct/model the peculiar velocity fields. The common approach is to construct the peculiar velocity field from galaxy catalogs (mass densities) using the results of linear theory, or a higher order approximation (quasilinear) like the Zel’dovich approximation. In the present study, we will use these two methods and the advanced implementation of LAP (FAM) to model the peculiar velocity field.

The large scale mass distribution is within the linear regime. Therefore, the inferred bulk flow using linear, Zel’dovich or FAM methods should in principle reproduce the \( N \)-body bulk flow. In the more local neighborhood, the structures are within the weakly non-linear regime, so the reconstruction of the velocity shear component will tell us what kind of method performs better at this regime. At these scales, linear theory is not totally valid and Zel’dovich could in principle reproduce properly the main characteristics. In the case of FAM, as shown by Romano-Díaz, Branchini, & van de Weygaert (2004, see Chapter 2), this could be the ideal scenario to test it.

3.7.1.1 Linear Theory

Within linear theory, Eqn. 3.2 relates the velocity and gravity fields. The linear peculiar velocities are proportional to \( H_0 f(\Omega_m, A) g(x) \), where \( g(x) \) is the gravitational force field obtained from the particle distribution (Eqn. 3.14). Hence, we have used this relation to compute the linear peculiar velocity field of our catalogs.
Figure 3.12 — Peculiar velocity fields for the S4 simulation. The left column represents the velocities at the particle positions for $\pm 60\ h^{-1}\text{Mpc}$ regions around the LG along the 3 supergalactic planes. The right panels show zooms of the velocities interpolated onto a grid for the LS region, $\pm 30\ h^{-1}\text{Mpc}$. The position of the LG is marked by a black circle at the center of the maps.
3.8. RESULTS

3.7.1.2 FAM approach

We have reconstructed the peculiar velocity fields following the prescription of Romano-Díaz, Branchini, & van de Weygaert (2004) presented in Chapter 2 (Sec. 2.3.3). We have chosen \( N_f = 6 \) basis functions to parameterize the orbits, a tolerance parameter \( tol = 10^{-4} \) to search for the minimum of the action \( S \) and setting a softening parameter of 0.25 \( h^{-1}\text{Mpc} \) to smooth the gravitational force. We have checked that this choice of parameters is optimal in the sense that decreasing \( tol \) or increasing \( N_f \) does not modify the final results appreciably.

3.7.1.3 Zel’dovich approximation

Velocities in the Zel’dovich approximation can also be obtained by means of FAM (Nusser & Branchini 2000; Romano-Díaz et al. 2004). For this, we set the number of expansion basis functions \( N_f = 1 \), while keeping the tolerance parameter unchanged. The straight-line orbits are then of the form \( x_i(D) = x_{0,i} + DC_{i,1} \), where \( D \) is the linear growing mode; \( x_{0,i} \) is the position of the particle at actual time and the vectors \( C_{i,1} \) are the expansion coefficients with respect to which the action in FAM is going to be minimized, the subindex 1 indicates that the gravitational force field at present time is used for the calculation. These velocities should be similar to those that would be obtained by applying the PIZA method (Croft & Gaztanaga 1997).

3.8 Results

In order to extract the bulk flow and the shear component from the peculiar velocity field, we have followed Romano-Díaz, Branchini, & van de Weygaert (2004, see Chapter 2) in using the procedure by Kaiser (1991). This procedure computes the bulk velocity from a multipolar decomposition of the peculiar velocity field of the form of Eqn. 3.6 up to the quadrupolar expansion term.

Because we want to compare our results with respect to the real Universe, we will only present results concerning to the fully constrained part of our catalogs (60 \( h^{-1}\text{Mpc} \)).

3.8.1 Multipolar velocity decomposition

We decomposed the velocity field by means of a field Taylor expansion truncated at the quadratic term (Kaiser 1991, see also Chap. 2). The velocity field \( v \), is then modeled by the first two components, a bulk flow vector, \( \tilde{u}_i \), and a quadratic shear tensor contribution, \( \tilde{s}_{ij} \),

\[
v_i = \tilde{u}_i + \tilde{s}_{ij}x_j, \quad \text{where} \quad i, j = \{1, 2, 3\},
\]

in which \( i, j \) denotes the Cartesian component indices. The vectors \( \hat{x}_j \) represent the vector components along the Cartesian \( j \) direction of the spatial unity vector oriented along the object position vector \( x \). Using these notations, we can rewrite Eqn. 3.16 and express the \( i \)-component of the velocity of object \( n \) into a product of the vectors \( F_{n,i} \) and \( V_{H_i} \),

\[
v_{n,i} = \sum_{I=1}^{4} F_{n,I}(x)V_{H_i},
\]

in which the data 4-vector \( F_J \) and the velocity field component 4-vector \( V_{H_i} \) are defined as

\[
F_{n,I} = \{1, x\hat{x}_1, x\hat{x}_2, x\hat{x}_3\}
\]

\[
V_{H_i} = \{\tilde{u}_i, \tilde{s}_{i1}, \tilde{s}_{i2}, \tilde{s}_{i3}\}.
\]

The “dipolar” bulk flow components \( \tilde{u}_i \) and “quadrupolar” velocity shear components \( \tilde{s}_{ij} \) can then be obtained by solving for the vectors \( V_{H_i} \) on the basis of a fitting analysis (to be precise, \( \tilde{s}_{ij} \) also includes
that the selection function only weights galaxy masses but not their respective velocities. But for our region of interest the distributions match each other very well. The left panel in Figure 3.13 shows the amplitude of the cumulative bulk flow computed from the S4 catalog for the N–body mass distribution (continues line) and the mock galaxy catalog (dotted line). As in the case of the gravity dipole, we have avoided the inner 5 h^{-1}Mpc of the catalogs. The discrepancy between the two bulks in the first 15 h^{-1}Mpc is due to sampling errors (low number of galaxies picked up at the core of the sample). Nevertheless, there is a very good agreement between the two quantities for the following 60 h^{-1}Mpc. Still, there is a small discrepancy between the two quantities which increases with distance. This result is caused by selection effects and sparse sampling, in particular by shot-noise. For these reasons, we have computed and corrected for the shot-noise contribution as described in Section 3.6.2. The shot-noise amplitude is not so large at these scales. For this particular catalog, at the edge of the 60 h^{-1}Mpc it is only ~ 20 km s^{-1}. The dot-dashed line in the left panel of Fig. 3.13 represents the mock estimate after being corrected by shot-noise effects. The agreement between the 2 distributions gets better; although some discrepancies beyond 60 h^{-1}Mpc remain. This is ascribed to the fact that the selection function only weights galaxy masses but not their respective velocities. But for our region of interest the distributions match each other very well.

The right panel of Fig. 3.13 shows how the bulk flow directions move around the sky when increasing the volume of the sample. Apart from some disperse points which correspond to the first 15 h^{-1}Mpc, all directions are located around a common area and converge around l = 270°, b = 11°. This direction is just less than 10° far from its respective gravity dipole direction. The fact that the bulk flow directions are concentrated around a given area, imply that the main structures responsible for the bulk flow are located/aligned around the same direction in the sky.

The average results show in a general way what has been learned from the analysis of the individual catalog. The left panel in Figure 3.14 presents the average bulk flows for the N–body mass distribution (broken line), and the galaxy mock catalogs corrected for shot-noise (continuous line). The gray shadow represents the ±1σ dispersion around the mean mock bulk flow. We have excluded the inner 5 h^{-1}Mpc to avoid non-linear effects and sparse sampling. As in the individual catalog, the mean mock bulk flow distribution follows the same behavior than the corresponding N–body flow. Both quantities match each other almost perfectly within the volume-limited region. Differences arise in the flux-limited region, although this mismatch is only ~ 20 km s^{-1}. Nevertheless, both bulk flow amplitude estimates are consistent with each other within the 1σ dispersion error.

The large errors (spikes) around the first 15 h^{-1}Mpc are the result of the position and strength (influence) of the LS in the catalogs. This is also the case for the GA and PP supercluster, where the errors are more pronounced around this region (40 – 60 h^{-1}Mpc). In the case of the directions, they...
3.8. RESULTS

Figure 3.13 — Amplitude and direction of the bulk flow velocity for the S4 catalog. The continuous line in the left panel represents the bulk flow of the \(N\)-body velocity distribution, while the dotted line the one of the mock galaxy catalog. The dot-dashed line represents the bulk flow corrected for shot-noise. The directions (same range in distances than in the left panel) of the \(N\)-body and corrected bulk flows are shown in the right panel.

Also show a very good agreement. The right panel of Fig. 3.14 illustrates that both bulk flow directions point toward the same direction and just with a relative mismatch of \(\sim 5^\circ\). With respect to the direction of the gravity dipole, they are off by \(\sim 20^\circ\). This misalignment implies that there is still a considerable influence from the mass distribution beyond 60 \(h^{-1}\)Mpc. This result is also shifted with respect to the real bulk direction estimated from the Mark III catalog by \(\sim 18^\circ\).

Figure 3.14 — Average amplitudes and directions of the cumulative \(N\)-body and mock bulk flows. The lines in the left panel represent the average bulk flows, while the gray shadow indicates the 1\(\sigma\) dispersion around the mean. The right panel shows the average directions of the bulk flow computed at the edge of the samples.

In general, these results from both samples at a radius of 60 \(h^{-1}\)Mpc agree with previous studies using the WF/CR method (Hoffman et al. 2001) and with the estimates from real velocity catalogs at these scales (e.g. MARK III, SFI catalogs, etc). Furthermore, our bulk flow directions are also consistent with the non-zero vectors of the bulk flow \((l, b) = (280, 0) \pm 30^\circ\) (Dekel 1999, and references therein).

3.8.3 The reconstructed velocity bulk

We have reconstructed the peculiar velocity field with the 3 methods listed in Section 3.7.1: linear theory, Zel’dovich approximation and FAM. We extracted the velocity bulk flow from the velocity field in the same way as for the \(N\)-body and mock bulk flow velocities.
Figure 3.15 shows the comparison between the \( N \)-body (continuous line), mock (dotted line) and shot-noise corrected FAM (\( V_{b,FAM} \), dot-dashed line) cumulative bulk flows for one catalog. Clearly, FAM over-estimates the bulk flow, it has suffered a considerable shift toward higher amplitudes. This effect is related to the FAM approach and can be corrected for. When reconstructing the velocity field the FAM technique assumes that inhomogeneities that could influence the dynamical behavior of the sample are contained within the sample itself. The centre of mass \( c.o.m \) of the mass distribution will therefore coincide with the geometrical centre of the sample. If this is not the case, a spurious bulk component is introduced due to the relative anisotropic mass distribution around the central observer. This will exert an extra pull over the system. For this particular catalog there is a relative displacement of \( \approx 10 \ h^{-1}\)Mpc between the \( c.o.m \) and the geometrical centers in the mock catalog. Furthermore, the average direction of the \( c.o.m \) points toward the direction where the GA and Shapley concentration are located. Juszkiewicz et al. (1990) showed that the contribution due to centre displacements is given by \( H_0 f(\Omega_m, \lambda) x_{c.o.m.}/3 \), where \( x_{c.o.m.} \) is the position of the centre of mass with respect to the geometrical centre. The offset in the bulk flow magnitude in our whole sample is in average \( 120 \ \text{km s}^{-1} \) and in the direction of the GA region. The corrected \( V_{b,FAM} \) is the 3dot-dashed line. The agreement between this corrected quantity and the \( N \)-body bulk distribution is remarkable. The bump in \( V_{b,FAM} \) at \( \approx 10 \ h^{-1}\)Mpc coincides with the location of the LS where FAM cannot reconstruct the velocity field properly due to the fact that this is a non-linear region and the lack of objects available for the reconstruction. At these regions and due to the degeneracy in orbits, FAM chooses from all possible solutions the one which is closest to the one given by linear theory, so that FAM cannot predict properly velocities in this regime (Nusser & Branchini 2000; Branchini et al. 2002; Romano-Díaz et al. 2004).

![Figure 3.15](image.png)

In general, the average results for the four catalogs strengthen the previous results. FAM is able to predict properly the main feature of the peculiar velocity field. The top-left panel in Figure 3.16 shows the mean results for \( V_{b,FAM} \), together with the \( V_{b,Mock} \) for comparison. The gray-shadowed area represents the \( \pm 1\sigma \) dispersion error around the FAM mean. The FAM bulk flow follows the same mock bulk flow distribution. This can be better noticed from the upper frame of the bottom-right panel. This frame shows the fractional difference between the mock bulk and the FAM one. As it can be noticed, there is a very good agreement between the 2 quantities. The only major difference is at short scales (\( 10 - 15 \ h^{-1}\)Mpc, region where is located the LS), caused by the failure in FAM in reproducing properly the movements at this non-linear scales, plus the lack of objects sampling such region.

In the same figure, we also display the average results from linear theory and \( V_{b,Mock} \) (top-right panel), together with its dispersion (gray shadows). In this case, we see that the \( V_{b,Lin} \) presents several
spikes along the whole distribution. These spikes are the regions where linear theory is not longer valid (cluster of galaxies). These differences are even more clearly shown in the corresponding fractional-difference panel. The most noticeable difference is around the LS position, while other subsequent spikes are due to considerable mass concentrations, like the GA and PP.

In the case of the Zel’dovich approximation (bottom-left panel), there is a clear mismatch between \( V_{b,Zel} \) and the Mock one. The fractional difference indicates that Zel’dovich overestimates the bulk flow by almost 30\%, even after subtracting the contribution from the \( c.o.m \) offset. This mismatch is inherent to the method itself, a consequence of it being a kinematic model. The pull that is exerted over the LG and companying structures is just ruled by the potential well of the mass distribution without taking into account the mutual gravity interaction of the different structures of the system. This introduces a spurious extra bulk flow component into the velocity field.

The bulk flow directions are very similar to each other. Figure 3.17 shows the directions at 60 \( h^{-1}\text{Mpc} \) for all bulk flow estimates. In particular from the zoom-in we may notice that all directions lie within an area of 20 square degrees. The Zel’dovich direction is the farthest from the rest. In general, all directions are consistent with the Mark III bulk direction, and they point towards the real direction of the bulk flow computed at this distance \((l, b) = (280, 0) \pm 30^\circ\) (Dekel 1999). The bulk directions are also consistent with the gravity dipole direction from the mock catalogs (black circle), differing by \( 17^\circ \pm 5^\circ \). We can conclude that our estimates within the linear regime are consistent, the
bulk flow and dipole directions do coincide with each other. The three velocity field models, FAM, linear and Zel’dovich successfully reproduce the direction of the bulk flow. The linear and Zel’dovich directions are within a distance of $\leq 8^\circ$ from the mock one, FAM is only $4^\circ$. With respect to direction, FAM appears to be the best performing method.

### 3.8.3.1 Smoothing Effects

Non-linear effects can also be an important factor. They introduce noise in the different $V_b$ estimates (e.g. Figs. 3.15 & 3.16). Because our selection procedure follows a Monte Carlo sampling prescription, it could be the case that several particles residing at very dense and clumpy regions with relatively high peculiar velocities have been chosen. At these regions, errors in the velocity estimates are introduced by all methods due to their failure to reconstruct peculiar velocities. In order to clarify this point, we have smoothed the real and reconstructed peculiar velocity fields, convolving the fields with top-hat filters of $R_{TH} = 2$ & $5 \, h^{-1}\text{Mpc}$. The average cumulative results for all fields are shown in Figure 3.18. The left panel show the unsmoothed $V_b$ estimates. The central panel displays bulk flows from smoothed velocities with $R_{TH} = 2$, while the right panel exhibits those with $5 \, h^{-1}\text{Mpc}$. The results show an improvement for the inner cores of the catalogs, the LS region where all small scale structures have been erased. The linear bulk flow estimate is the one who improves best with this smoothing procedure, all spikes have been erased and the distribution coincides better with the real one. The only difference is the central inner region where estimates remain too large. The Zel’dovich quantity does not improve significantly in the sense that the amplitudes are still over-estimated by the same amount than in the unsmoothed case. For the FAM estimate, the only improvement is with respect to the LS, which coincides now with the mock bulk flow estimate. We also computed the relative misalignment angle $\theta$ from the smoothed bulk flows results with respect to the unsmoothed case. No significant changes in the bulk flow directions and convergences were noticed. The directions have only been
shifted by \( \sim 2\) closer from their original position. These results show that our bulk flow estimates are not influenced substantially by local non-linearities (except in the case of the \( V_{b, \text{Lin}} \)) in the mass distribution.

Figure 3.18 — Smoothing effects on \( V_b \) for all methods. The left panel corresponds to the unsmoothed case, the center panel to velocities smoothed with a top-hat filter of \( 2 \, h^{-1}\text{Mpc} \) radius and the right panel to velocities smoothed with a \( 5 \, h^{-1}\text{Mpc} \) radius.

### 3.8.3.2 Shell contributions

We have also investigated what the contributions to the total \( V_b \) are from individual shells of thickness \( (5 \, h^{-1}\text{Mpc}) \). In this exercise, we have divided the total volume in 30 shells and isolated the velocities of the objects enclosed in each of these shells. For each one of these subsamples we have computed the 3 Cartesian components of their shell-bulk flow. Figure 3.19 shows the average bulk shell Cartesian contributions got from the \( N \)–body velocities (continues line), mock catalogs (dotted line) and FAM velocities (dashed line). For comparison the gray dot-dashed line represents the \( N \)–body full average cumulative bulk flow. The gray shadows depict the \( \pm 1\sigma \) dispersion errors computed from the FAM Cartesian contributions.

The agreement between the \( N \)–body, mock and FAM shell Cartesian contributions is very good. All Cartesian-group distributions follow each other very close for almost the whole range in distances. While the \( y \) & \( z \) Cartesian components remain almost unchanged for the whole range in distances, the behavior along the \( x \) component presents more radical changes. This component not only encloses the largest amplitude from the 3 components, but also decreases constantly with radius. Worthy to
notice is the fact that this Cartesian bulk shell behavior follows the one prescribed by the full N–body bulk distribution (gray dot-dashed line). This means that most of the gravity pull exerted over the LG comes from structures aligned along the x component. The spikes along the FAM x–bulk shell distribution represent the location of massive structures that are acting over the LG. Perhaps the most notorious spikes are those located at \( \sim 40 \, h^{-1}\text{Mpc} \) (GA), \( \sim 50 \, h^{-1}\text{Mpc} \) (PP). At those spikes the FAM components slightly deviates from the mock distribution. Notice as well that there is still a significant contribution of \( \sim 300 \, \text{km s}^{-1} \) at the edge of the sample. These results are in good agreement with those obtained by Hoffman et al. (2001) and Dekel (1999).

We have also looked for non-linear effects in the shell contributions. As before, we convolved the different peculiar velocity fields with a top-hat kernel of \( R_{TH} = 2 \& 5 \, h^{-1}\text{Mpc} \). The bulk distributions remain almost intact for the whole distance range.

### 3.8.4 The velocity Shear

A more demanding comparison between the different reconstructed peculiar velocity fields is the 2nd term in the multipolar velocity decomposition, the velocity shear. The estimate of this term will tell us how well the different techniques deal with the quasilinear regime of our local cosmic neighborhood.

We have computed the velocity shear from the peculiar velocities through the decomposition method exposed in Chapter 2. The matrix representation of the traceless shear tensor is then diagonalized to obtain the eigenvectors (directions) and eigenvalues (amplitudes). We have re-ordered these vectors in such a way that their eigenvalues will be in decreasing order and the sum of all of them will be equal to zero. Hence, the shear eigenvalues follow the following properties:

\[
\begin{align*}
1) & \quad s_1 > s_2 > s_3 \\
2) & \quad s_1 + s_2 + s_3 = 0 \\
3) & \quad s = (s_1^2 + s_2^2 + s_3^2)^{1/2}
\end{align*}
\]

The quadrupole moment of the velocity field informs us about the configuration of the mass distribution. The eigenvalues will tell us how significant the influences of the structures surrounding the LG are. We can visualize these 3 eigenvalues like the 3 main axis of a velocity ellipsoidal, with directions given by their corresponding eigenvectors. These vectors are linearly independent of each other, defining in this way a shear reference frame. The \( s_1 \) eigenvalue is known like the dilational (stretching) mode of the shear velocity and it corresponds to the largest axis of the velocity ellipsoidal. It points towards the direction of the maximum stretch of the ellipsoidal. The \( s_3 \) eigenvalue corresponds to the smallest axis of the ellipsoidal, and it is known like the compressional mode. Its eigenvector corresponds to the direction where the velocity ellipsoidal is being compressed (like by expanding structures, e.g. voids), and its perpendicular to the dilational eigenvector. The \( s_2 \) eigenvalue corresponds to the middle mode, and determines the “morphology” of the shear.

Figure 3.20 shows the average 3 eigenvalues and the shear eigenvalue amplitude \( s \) for the \( N \)–body velocity fields (black lines), and the Mock velocity fields (gray lines). The 3 Mock velocity shear components successfully reproduce the behavior of the shear pattern in the \( N \)–body velocity fields. The velocity field of the mock catalogs trace fairly the \( N \)–body peculiar velocity field. The mock estimates can be used as genuine representations of the real values. In this plot we present the calculations performed until the edge of the sample radius for displaying purposes. The shear converges to zero much faster than the corresponding bulk flow due to its stronger dependence in distance \( (x^{-3}) \). It can be noticed that the gross of the shear influence comes from structures located within 60 \( h^{-1}\text{Mpc} \), and more specific within 40 \( h^{-1}\text{Mpc} \). Although there is still some influence \( (< 10 \, \text{km s}^{-1} \, \text{Mpc}^{-1}) \) from structures located between 60–80 \( h^{-1}\text{Mpc} \). The influence of the LS can be clearly seen by the high values at regions \(< 15 \, h^{-1}\text{Mpc} \).

We have computed the value of the 3 cumulative eigenvectors and eigenvalues for all our catalogs with the 3 modeling methods (FAM, Zel’dovich & linear). The left-column panels in Figure 3.21 show the behavior of the eigenvalues for the 3 reconstruction techniques, plus the Mock distributions for
3.8. RESULTS

Figure 3.20 — Average eigenvalues for the 4 N-body velocity fields (black lines), and Mock velocity fields (gray lines) as a function of the computation distance.

Comparison purposes. The right-column panels show the angular misalignment $\theta$, measured between the mock eigenvectors and those from the reconstruction algorithms at each computing radius. The gray shadows represent the $1\sigma$ dispersion errors around the mean values. The darkest gray corresponds to the FAM errors, the half-gray scale the Zel’dovich errors, while the light gray depicts the errors from linear theory. The top panels correspond to the dilational mode, the central to the middle one, and the bottom to the compressional mode. In all panels the linear technique consistently fails in reproducing the expected results. The linear eigenvalue’s distributions are full of spikes and deviations from the expected ones; these spikes/deviations are due to the presence of virial motions (linear regime not longer valid) caused by non-linear structures like clusters of galaxies present in the catalogs. This corresponding relative eigenvector’s convergences show the same behavior. The large discrepancies in the directions at given positions are correlated with the peaks presented along the eigenvalue distributions, located at the same positions. In general, linear results are barely consistent with the expected values, specially at LS scales. These results make linear theory not very suitable for studying the dynamics of the LG and LS environment. The Zel’dovich approximation gives in general better results than linear theory. Although it underestimates the eigenvalues for regions where the LS and GA ($20 - 40 \, h^{-1}\, \text{Mpc}$) are present, they are still consistent within the errors. With respect to its eigenvectors, they show in general an average constant deviation of $10^\circ$ for each eigenvector from the mock ones (but again, consistent with the errors). This is a consequence of the method itself (as stated before) of being a kinematic approach. The method suffers from severe problems are regions where the flow is not longer laminar (e.g. cluster of galaxies). This is also reflected in the broad error shadows. The Zel’dovich approximation fails mainly in reproducing the right directions of the non-linear movements, but its performance at middle large scales ($\sim 30 - 40 \, h^{-1}\, \text{Mpc}$) its better than plain linear theory. In the case of FAM, its eigenvalues cannot reproduce properly the expected behavior for the inner regions of the catalogs ($< 20 \, h^{-1}\, \text{Mpc}$). This region corresponds to the non-linear regime where FAM (like the other methods) fails in reproducing the corresponding velocities properly. Beyond this radius, the FAM estimates are remarkably good in comparison with the expected ones. This situation is also reflected in the eigenvector’s convergence. The direction converges rapidly after the inner region, remaining very close to the Mock directions ($\leq 5^\circ$). Furthermore, the better agreement between the FAM and expected quantities is reflected in the error shadows, these are smaller than those given by the other two methods and hence, overshadowed.

With respect to the velocity ellipsoid, the maximum stretch ($s_1$ eigenvector) points to $(l, b) = (327, 14)^\circ \pm (15, 8)^\circ$ and its antipode (very close to the one predicted by Lilje, Yahil, & Jones 1986).
CHAPTER 3: Dipole & Quadrupole moments of the local cosmic velocity field

Figure 3.21 — Average shear components for all catalogs and modeling techniques. The Left-column panels represent the 3 eigenvalue amplitudes, while the right-column panels show the convergence of the corresponding eigenvectors with respect to the Mock ones. The gray shadows represent the 1σ dispersion error around the averages.

This indicates a relative offset with respect to the bulk velocity of 43°. This misalignment depends on the configuration of the quadrupolar and dipolar mass distributions. It also indicates that we are still missing contributions from mass concentrations beyond this radius. The contraction (s3) is pointing towards (l, b) = (103, 2)° ± (18, 10)°. It is interesting to notice that the expansion points towards the direction of the Hydra-Centaurus cluster (GA).

The last point to consider for the velocity ellipsoid is smoothing effects. Figure 3.22 shows the cumulative eigenvalue’s amplitudes (s) of the unsmoothed velocities (left panel), and the $R_{TH} = 5 \, h^{-1} \text{Mpc}$ smoothed velocities (right panel). The gray shadows indicate the ±1σ dispersion errors. The darkest gray represents the errors from the FAM technique, the middle-gray scale to the Zel’dovich, and the lightest gray to linear theory. Linear theory estimates improve as a result of erasing the small scale signal in the velocities. Nevertheless, it fails in reproducing the virial motions of the LS, but the overall smoothed results are consistent with the expected values. The Zel’dovich approximation un-
3.9. CONCLUSIONS

The reliability of the final mass distributions and their corresponding mock catalogs was tested by measuring their dipolar moment and comparing them with respect to the real observed estimates. Results showed that our simulations and mock catalogs successfully reproduce the mass distribution even beyond the constrained dynamical range of the catalogs.

The comparison between the computed gravity dipole for the $N$–body mass distributions and for the Mock galaxy distributions, showed that it is necessary to correct the mock catalogs due to sampling effects for shot-noise effects. These effects become stronger at large distances and can represent more than 10% of the dipole amplitude. The amplitude and direction of the cumulative Mock dipole and the one from the $N$–body mass distribution are very close to each other and follow the same average behavior, at $60 \ h^{-1}\text{Mpc}$ they are located at $(l,b) = (277,14)^\circ \pm (21,11)^\circ$ and with an amplitude of...
542 ± 52 km s\(^{-1}\), being these results in good agreement with those from the literature (e.g. Rowan-Robinson et al. 2000). The evolution of the WF/CR has successfully reproduced the observations in the sense that there might be still structures influencing over the LG at very large scales (e.g. Hoffman et al. 2001).

We have compared the bulk flows from the full \(N\)-body velocity distribution and the one from the Mock catalog velocities. The results showed that both estimates follow the same behavior for almost the whole range in distances. We have limited our analysis up to 60 \(h^{-1}\)Mpc for having similar regions (constrained regions) between the 4 catalogs. Although we have corrected the object distributions for shot-noise effects, we cannot correct for the velocities because of our ignorance of the surroundings around the objects. This means that the objects could be located at a highly non-linear region or in a middle of a void and there is no way to know this in order to correct for. From the selection function, we know that we are dealing with objects that are brighter than a certain limit, but these objects could be located anywhere. A way to overcome this problem, is by increasing the number of chosen objects by modifying the selection function. But because our aim is to produce realistic mock catalogs following (in this case) the real distribution of objects in the PSC\(_{2}\) catalog, we are limited by this constraint. The position of the computed bulk flows are located in the sky around \((l,b) = (294,3)^\circ ± (19,14)^\circ\), having an offset of less than 18\(^\circ\) from the dipole direction and with an amplitude of 330±62 km s\(^{-1}\). Both results are consistent with observations at this same radius (Dekel 1999; Zaroubi 2002). The fact that there exists a relative misalignment with respect to the real dipole and that the dipole magnitude up to this radius represents ~ 55% of the total LG bulk amplitude, implies that structures beyond this radius still play an important role in the dynamics of the LG. Between the \(N\)-body and Mock dipoles, there is an offset of 4\(^\circ\), attributed to the sampling procedure. Nevertheless, the mock catalog velocities represent our best estimate of the real velocity field given a selection function, under the hypothetical case that we can measure the 3D spatial object velocities. The shear components of these two velocity fields show also some interesting features. Both estimates are very close to each other and their directions are comparable. Furthermore, we have measured a significant influence beyond 60 \(h^{-1}\)Mpc, evidenced by the amplitude of the eigenvalues at these radii.

The modeling of the peculiar velocity field has been performed by using 3 different algorithms: linear theory, Zel’dovich approximation and the FAM technique. We observed the following:

The linear bulk flow proved to be highly noise dominated, especially at the regions of the LS, GA, and PP where prominent spikes are present in both amplitude and direction. At these regions, linear theory is not longer valid for being non-linear. Nevertheless, these effects can be overcome by applying a smoothing algorithm with a radius of 5 \(h^{-1}\)Mpc. These smoothed results were shown to be in good agreement with the expected values, although there was a large discrepancy at the LS region. The corresponding eigenvalues and eigenvectors suffered from the same effects at non-linear regions. Smoothed results proved to be in better agreement, both eigenvectors and eigenvalues are in general closer to the expected values. Nevertheless, at the LS regions, the method fails completely. As in the case of the bulk flow, the linear shear estimates are just barely consistent with the expected values.

In the case of the Zel’dovich and FAM reconstructions, an extra correction had to be done because of the shift between the geometrical centre and c.o.m.. This effect introduce an extra-spurious bulk flow because of the different apparent mass distributions when reconstructing the velocity field by means of the FAM technique. Even after these corrections, the Zel’dovich bulk flow had consistently larger amplitudes than the expected estimates. The reason for this is that the method overestimates the velocities in regions where the flow is not longer laminar. These over-predictions have a cumulative effect that is reflected in the bulk flow. The corresponding Zel’dovich bulk direction was much better determined, with an offset of only 9\(^\circ\) with respect to the mock one. The eigenvalues always underestimated the Mock values by almost 20% of their total value, but consistent within the 1\(\sigma\) dispersion errors. The average estimates did not show a significant improvement when smoothing apart from the LS region where the amplitudes decreased and converged to the expected ones. The Zel’dovich velocity ellipsoid is slightly distorted with respect to the expected one. There is a misalignment of 8\(^\circ\) between the Zel’dovich shear frame and the Mock one.

On the other hand, FAM estimates are in general much closer to the mock estimates than the other
2 reconstruction techniques. Its bulk amplitude is very similar to the Mock one. There is a discrepancy at small radius, the result of the LS influence. The robustness of the method is noticed by the fact that there are no significant changes in the results even when applying a smoothing procedure, plus the fact that errors are considerable small ≈ 60 km s$^{-1}$. The direction of its bulk flow is remarkably close to the Mock direction, with a misalignment angle of $3^\circ$, closer than those obtained from the linear and Zel'dovich approaches. The shear components also follow the expected behavior for regions excluding the LS. If smoothing is applied, then the FAM LS estimates improve considerably. In the case of the eigenvectors, they also remain very close to the Mock ones. The FAM velocity ellipsoidal has a shift of only $4^\circ$ with respect to the expected one.

We have proved that all methods employed in this study fail in reproducing the LG dynamics for being within the non-linear regime. A smoothing algorithm is necessary in order to overcome this effect, not in all cases results are completely satisfactory (linear theory and Zel'dovich approx.). In the mildly non-linear regime, the 3 methods give acceptable results and comparable to the expected ones within the errors. FAM performs slightly better than the other two methods. It properly reproduces the dynamical behavior of the LS environment and presenting tighter relations with respect to the expected values. At linear scales, the 3 reconstruction techniques give more similar results (provided extra-processing in the peculiar velocity field, like smoothing), and very close to the expected ones. Given these results, we can conclude that the FAM method is the most suitable technique to study the dynamics of the LG & LS cosmic environment. Furthermore, this study showed as well, that the success of FAM will depend on the kind of sample used.

As a last point, the fact that our results match those obtained from observations of our real Universe, is one success of employing the WF/CR algorithm. Even though at scales that have not being strongly constrained due to the lack of good data, the method is able to reproduce the main characteristics of the mass distribution and its corresponding peculiar velocity field, the dipole and bulk flow.

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3.A  Constrained Realizations

Bertschinger (1987) realized that a constraint realization of Gaussian random field $f$ is given by the sum of the analytically calculated mean field $\bar{f}$ and the random residual field $F$,

$$f(\mathbf{r}) = \bar{f}(\mathbf{r}) + F(\mathbf{r}).$$  \hspace{1cm} (A3-1)

Hoffman & Ribak (1991) presented a straightforward method for the construction of constrained realizations of Gaussian fields based on the defining work of Bertschinger (1987). They realized that, for any constraint that is a linear functional of the field, the problem can be solved exactly and in a simple way, without involving iterations.

van de Weygaert & Bertschinger (1996) exposed in a very detailed way the CR methodology and explored its benefits and advantages on particular realizations. Within this implementation, a peak or dip in the density field can be characterized by a set of 21 physical constraints, including its scale, position and orientation, density, velocity and velocity gradient.

In this appendix we follow the CR prescription of van de Weygaert & Bertschinger and expose its main concepts.

Consider a random homogeneous and isotropic Gaussian field $F(\mathbf{r})$ with zero mean which is defined by its power spectrum $P(k)$. This field is subject to a set of $M$ constraints,
\[ \Gamma = \{ C_i(r) \}_{i=1}^{M} \equiv C_i[f;r] = c_i; \quad i = 1, \ldots, M. \]  
(A3-2)

The constraints are consequently imposed by forcing the field \( C_i[f;r] \), \( (i = 1, \ldots, M) \), a functional of the field \( f(r) \) as well as a function of the point \( r \), to have the specific value \( c_i \) at the given position. The constraints \( C_i \) are assumed to be linear functionals, clear examples are the values of the field itself at the point \( r_i \), the derivative of the field \( f(r) \) or a convolution over \( f(r) \) with some function \( g(r) \),

\[
C_i[f;r] = f(r_i) = c_i \\
C_j[f;r] = \frac{\partial}{\partial r} f(r) |_{r_j} = c_j \quad (A3-3) \\
C_k[f;r_k] = \int g(r - r_1) f(r) dr = c_k
\]

Since we have limited the field \( f(r) \) to those that obey the set of \( M \) constraints \( \Gamma \), the probability of possible realizations of our \( f(r) \) is given by the conditional probability distribution function,

\[
P[f(r)|\Gamma] = \frac{P[f(r),\Gamma]}{P[\Gamma]} = \frac{P[f]}{P[\Gamma]}, \quad (A3-4)
\]

where \( P[\ldots] \) is the multivariate Gaussian of the appropriate variables. This comes by virtue of the central limit theorem, the distribution of a Gaussian random field will approach normality, and the multivariate distribution \( P \) is multivariate Gaussian:

\[
P_N = \frac{\exp \left[ -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} f_i(M^{-1})_{ij} f_j \right]}{([2\pi]^N (detM)]^{1/2}} \prod_{i=1}^{N} df_i, \quad (A3-5)
\]

where \( M^{-1} \) is the inverse of the covariance matrix \( M \), the generalization of the variance \( \sigma^2 \) in a one-dimensional normal distribution. \( M \) is completely determined by the autocorrelation function \( \xi(r) \) if the field is a Gaussian random field,

\[
M_{ij} \equiv \langle f(r_i) f(r_j) \rangle, \quad (A3-6)
\]

where the brackets \( \langle \ldots \rangle \) denotes ensemble averages. Since \( f \) is a \( N \)-dimensional vector, the covariance matrix \( M \) can be written as:

\[
M = \langle ff^t \rangle, \quad (A3-7)
\]

with \( f^t \) being the transpose of \( f \).

Because the constraints \( C_i \) are linear functionals, the central limit theorem also assures them to have a Gaussian probability distribution when applied on a Gaussian field \( f(r) \). The second equality in Eqn. A3-4 comes from the fact that the constraints are linear functionals of \( f \), so that the joint probability space for \( f \) and \( \Gamma \) is the same as the probability space for \( f \).

The conditional probability distribution function can be described as a shifted Gaussian around the ensemble mean field \( \bar{f}(r) \) (van de Weygaert & Bertschinger 1996), defined as

\[
\bar{f}(r) = \langle f(r)|\Gamma \rangle = \xi_i(r) \xi_i^{-1} c_i, \quad (A3-8)
\]

where summation over repeated indexes is assumed. Hence, \( \bar{f}(r) \) is the “most likely” field satisfying the constraints and it equals the ‘average density profile’ obtained by BBKS. The term \( \xi_i \) in the previous equation is given by

\[
\xi_i(r) = \langle f(r) C_i \rangle, \quad (A3-9)
\]

and it represents the cross-correlation between the field and the \( i \)th constraint \( C_i[f;r] \), while

\[
\xi_{ij} = \langle C_i C_j \rangle, \quad (A3-10)
\]
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is the \(ij\)th element of the constraints’ correlation matrix, and \(C_i\) is evaluated at \(\mathbf{r}_i\). In the case where the constraints \(C_i\) involve only the field itself, both the correlation matrix \(\xi_{ij}\) and \(\xi_i(\mathbf{r})\) can be written in terms of the two-point autocorrelation function \(\xi(\mathbf{r})\), that is:

\[
\xi_{ij} = \langle f(\mathbf{r}_i)f(\mathbf{r}_j) \rangle = \xi(|\mathbf{r}_i - \mathbf{r}_j|), \tag{A3-11}
\]

\[
\xi_i(\mathbf{r}) = \langle f(\mathbf{r})f(\mathbf{r}_i) \rangle = \xi(|\mathbf{r}_i - \mathbf{r}|), \tag{A3-11}
\]

The residual field \(F(\mathbf{r})\) is defined as the difference between the Gaussian field \(f(\mathbf{r})\) satisfying the constrain set \(\Gamma\) and the mean field \(\bar{f}(\mathbf{r})\):

\[
F(\mathbf{r}) \equiv f(\mathbf{r}) - \bar{f}(\mathbf{r}). \tag{A3-12}
\]

Note that the residual field \(F\) is a random Gaussian field because it is the difference between two Gaussian fields. This residual field provides random noise which is added to the signal \(\bar{f}(\mathbf{r})\), which is completely fixed by the imposed set of constraints \(\Gamma\). In terms of the residual field \(F\), the constraint points are expressed as \(F(\mathbf{r}) = C_i[f; \mathbf{r}_i] = c_i; i = 1, \ldots, M\). Any particular constrained realization can be written then as the sum of the analytically calculated mean field \(F\) (via Eqn. A3-8), and the random residual field \(F(\mathbf{r})\):

\[
f(\mathbf{r}) = \bar{f}(\mathbf{r}) + F(\mathbf{r}) = \xi_i(\mathbf{r})\xi_i^{-1}c_j + F(\mathbf{r}). \tag{A3-13}
\]

The “crucial” point in this algorithm is that

\[
C_i[F] = C_i[f - \bar{f}] = C_i[f] - C_i[\bar{f}] = c_i - c_i = 0, \tag{A3-14}
\]

the statistical properties of the residual field are all independent of the numerical values of the constraints \(c_i\), and for any particular choice of the constraints a realization of the residual can be straightforward constructed.

The construction of a constrained realization of the field \(f(\mathbf{r})\) it can be done in five stages (Hoffman & Ribak 1991; van de Weygaert & Bertschinger 1996)

1. Create a random, unconstrained realization of the field \(\bar{f}\), which is a homogeneous and isotropic Gaussian random field whose statistics is determined solely by the power spectrum \(P(k)\).

2. Calculate for this particular realization \(\bar{f}\), the values \(\bar{c}\) of the constraints \(\{C_i(\mathbf{r})\}_{\mathbf{r}_i}, i = 1, \ldots, M\). These variables can be looked upon as defining another set of constraints, \(\bar{\Gamma} = \{\bar{c}\}\). This a posteriori set of constraints is evaluated at the positions of the original constraints and has the values of this specific realization.

3. Calculate for this “random” constrained set \(\bar{\Gamma}\) the corresponding mean field expected as if the set was chosen initially,

\[
\bar{f} = \langle \bar{f} | \bar{\Gamma} \rangle = \xi_i(\mathbf{r})\xi_i^{-1}\bar{c}_j. \tag{A3-15}
\]

4. Evaluate the residual field \(\bar{F}\) of the realization from the given particular realization and the calculated mean field \(\bar{f}\) as

\[
\bar{F}(\mathbf{r}) = f(\mathbf{r}) - \bar{f}(\mathbf{r}). \tag{A3-16}
\]

The residual field \(\bar{F}\) thus generated is the residual field of a particular realization restricted to the desired constraints, \(\Gamma\).

5. Evaluate the desired mean field \(\bar{f}\), according to Eqn. A3-8 and add it to the residual field \(\bar{F}(\mathbf{r})\) to obtain a particular realization of the desired constrained Gaussian random field

\[
f(\mathbf{r}) = \bar{f}(\mathbf{r}) + \xi_i(\mathbf{r})\xi_i^{-1}(c_j - \bar{c}_j). \tag{A3-17}
\]
The constructed field \( f(\mathbf{r}) \) obeys the imposed constraints and replaces the unconstrained field \( \hat{f}(\mathbf{r}) \). Note that there is a one-to-one correspondence between the trial field \( \hat{f}(\mathbf{r}) \) and the constructed one \( f(\mathbf{r}) \). Furthermore, the ensemble of realizations produced by this algorithm properly samples the sub-ensemble of all realizations constrained by \( \Gamma \). The algorithm is exact and involves the creation of only one random unconstrained realization and the calculation of the mean field under the given constraints. There is no restriction over the number of constraints and these can be in a very large number, and they can be imposed on the field itself or on any linear functional of it.

### 3.B The Wiener Filter

The common use of the Wiener Filter [WF] is for noise suppression. Several authors (e.g. Lahav et al. 1994; Fisher et al. 1995b; Zaroubi et al. 1995, 1999) implemented the WF to reconstruct the density field from the observed radial velocities and to interpolate or extrapolate the reconstruction to regions of poor sampling. The latter can be done in real space, to interpolate into the Zone of Avoidance or to extrapolate into large distances, or it can be used in a reciprocal space, such as the Fourier space or spherical harmonic space to increase the resolution of the data.

The WF multiplying the data to obtain the estimator is schematically \( P_k/(P_k + \sigma^2) \) (where \( P_k \) is the power spectrum and \( \sigma \) the error). This means that the filter attenuates the estimator to zero in regions where the noise dominates. The reconstructed mean field is thus statistically inhomogeneous. In order to recover statistical homogeneity the use of Constrained Realizations is necessary. In the CR the random realizations of the residual from the mean are generated such that they are statistically consistent both with data and with the prior model (Hoffman & Ribak 1991). In regions (real or any reciprocal space) dominated by “good” quality data, the CRs are dominated by the data, while in the limit of no data the realizations are practically unconstrained.

The general application of the WF/CR method to the reconstruction of large scale structure is described in detail in Zaroubi et al. (1995), see also Fisher et al. (1995b). Here we limit to the description and application of the approach to radial velocity data as stated in Zaroubi et al. (1999).

The data for the WF/CR analysis are given as a set of observed radial peculiar velocities \( u^0_i \) sampled at positions \( \mathbf{r}_i \) with estimated errors \( \epsilon_i \) which are assumed to be uncorrelated. The peculiar velocities are assumed to be corrected for systematic errors. The observed velocities are related to the true underlying velocity field \( v(\mathbf{r}) \), or its radial component \( u_i \) at \( \mathbf{r}_i \), by the following expression:

\[
\mathbf{u}_i^0 = v(\mathbf{r}_i) \cdot \hat{\mathbf{r}}_i + \epsilon_i \equiv u_i + \epsilon_i. \tag{B3-18}
\]

It is assumed that the peculiar velocity field \( v(\mathbf{r}) \) and the density fluctuation field \( \delta(\mathbf{r}) \) are related via the linear gravity instability theory. Under the assumption of a specific theoretical prior for the power spectrum \( P(k) \) of the underlying density field, the WF minimum-variance estimator of the reconstructed velocity field can be written as:

\[
v_{WF}(\mathbf{r}) = \langle v(\mathbf{r}) u_i^0 \rangle (u_i^0 u_j^0)^{-1} u_j^0, \tag{B3-19}
\]

where the angle brackets denote an ensemble average. The term \( \langle u_i^0 u_j^0 \rangle \) is the two-point radial velocity correlation function. The cross-correlation term \( \langle v(\mathbf{r}) u_i^0 \rangle \) is calculated from the two-point velocity correlation tensor (Zaroubi et al. 1999). The WF mass-density fluctuation field \( \delta_{WF}(\mathbf{r}) \) is given by an analogous expression:

\[
\delta_{WF}(\mathbf{r}) = \langle \delta(\mathbf{r}) u_i^0 \rangle (u_i^0 u_j^0)^{-1} u_j^0, \tag{B3-20}
\]

where the first term corresponds to the cross-correlation matrix of the density and radial velocity. Note that in linear theory the WF reconstruction of the velocity and density fields is equivalent to first reconstructing one of those fields and then solving the Poisson equation for the other.

The formalism of constrained realizations allows one to create a typical realization of the residual from the WF mean field Hoffman & Ribak (1991). The method is based on creating random realizations, \( \delta(\mathbf{r}) \) and \( \bar{v}(\mathbf{r}) \), of the underlying fields that are statistically consistent with the assumed power
spectrum and the data via linear theory, and a proper set of errors $\tilde{e}_i$. The velocity random realization is then “observed” like the actual data to yield a mock velocity data set $\tilde{u}_i^0$. Constrained realizations of the dynamical fields are then obtained by

$$v_{CR}^0(r) = \tilde{v}(r) + (v(r)u_i^0)(u_j^0)^{-1}(u_j^0 - \tilde{u}_j^0)$$ \hspace{1cm} (B3-21)

and

$$\delta_{CR}^0(r) = \tilde{\delta}(r) + (\delta(r)u_i^0)(u_j^0)^{-1}(u_j^0 - \tilde{u}_j^0).$$ \hspace{1cm} (B3-22)

The average of the CRs is the WF field. Their scatter about the WF field is the uncertainty that it is attributed to the field. The uncertainties are evaluated from the CRs for the purpose of evaluating errors in quantities that are computed later from the WF fields. It is worthy to mention that this type of reconstruction is designed to treat the random errors in an optimal way, thus stressing only robust structures and avoiding fake structures.