Measuring MRI noise
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Chapter 3

Theory

3.1 Lorentz forces acting on gradient coils

The gradient magnetic fields are generated by electric currents carried by the gradient coils. These gradient coils are positioned within the main magnetic field. Electric currents $I$ within a magnetic field experience a force (figure 3.1), the so-called Lorentz force:

$$\vec{F}_L = \int_\ell I \vec{d}\ell \times \vec{B}, \quad (3.1)$$

where $\vec{d}\ell$ is a wire element inside the magnetic field $\vec{B}$. The resultant force on the gradient coil structure deflects the structure (figure 3.2). With an alternating current, the varying Lorentz forces induce vibrations in the gradient coil structure. These vibrations are transferred to the rest of the scanner and the surroundings. Airborne vibrations are the acoustic noise under investigation in this thesis.

3.2 Sound

Sound is the part of the acoustic radiation spectrum that can be perceived. Acoustic waves propagate through a compressible medium. In air, this implies a variation of the pressure. The auditory system is capable of detecting these variations between 20 Hz and 20 kHz. When the acoustic pressure becomes too high, the auditory system may be damaged. Health regulations prescribe the permissible sound pressure levels (SPLs) (e.g., 29 CFR 1910.95, Occupational Noise Exposure). For plane waves, this acoustic pressure $p$ is a function of the specific acoustic impedance $Z_s$, and the associated particle velocity $u$:

$$p = Z_s u. \quad (3.2)$$
Figure 3.1: Part of the $X$-gradient coil from figure 2.5, with the electric currents generating the gradient magnetic fields depicted in grey. The broad white arrows depict the resultant Lorentz forces acting on the gradient coils.

Figure 3.2: Two of the vibrational modes in which the gradient structure can vibrate, the rest position is dotted. A: The cone-shape mode due to the $Z$-gradient vibrations, and B: the banana-shape mode due to $X$- or $Y$-gradient coil vibrations ($X$-gradient coil in this case, with a deflection due to a force distribution as in figure 3.1).
3.3. TRANSFER FUNCTION

Sound pressure levels are stated in decibels (dB), that is

$$\text{SPL} = 20 \log_{10} \frac{p_{\text{rms}}}{p_{\text{ref}}}$$

(3.3)

where $p_{\text{ref}}$ is $2 \times 10^{-5}$ Pa, and $p_{\text{rms}}$ is the root-mean-square (rms) pressure variation of the air (ANSI S1.13-1995 [R1999]; Kinsler et al. [2000]).

The vibration distribution over an MRI scanner is complex, and for that reason the sound pressure levels vary over locations within the scanner bore and inside the scanner room. Measuring the acoustic radiation of these vibrations at the location of the head of a subject is supposed to give a good indication of what subjects will endure during scanning.

3.3 Transfer function

The transfer function $H(\omega)$ of a system is defined as the ratio of the output spectrum $R(\omega)$ and the input spectrum $S(\omega)$,

$$H(\omega) = \frac{R(\omega)}{S(\omega)}$$

(3.4)

$H(\omega)$ is a complex function of $\omega$, which can be separated in a gain part (the modulus),

$$A(\omega) = |H(\omega)|,$$

(3.5)

and a phase part (the argument),

$$\phi(\omega) = \text{arg}(H(\omega)).$$

(3.6)

$H(\omega)$ can either be measured directly in the frequency domain through acquisition of $R(\omega)$ and $S(\omega)$ at the $\omega$ values of interest, by Fourier transform of the (im)pulse response in the time domain, or by indirect measurement of the response to broadband noise. The latter two methods assume a linear system, and miss nonlinear effects. Nonlinear components in the response do contribute to the rms measurements.

3.4 Harmonic distortion

To obtain the transfer, the ratio of response and stimulus spectra suffices if the transmission is linear. Due to nonlinearities in systems, harmonic distortion occurs, thereby introducing frequencies in the output signal which are not present in the input signal. These additional frequencies are easily shown with a Fourier transform, but they are not considered in the evaluation of the transfer function.
Chapter 3

The ratio of the power contribution of these harmonics and the power of the fundamental frequency, is called the total harmonic distortion (THD):

\[
\text{THD} = 20 \log_{10} \sqrt{\frac{\sum_{n=2}^{N} a_n^2}{a_1^2}},
\]

(3.7)

where \(a_1\) and \(a_n\) are the amplitudes of the fundamental and the \(n\)th harmonic, respectively. The power of signals is related to the square of the amplitude of sinusoidal signals.

During measurement of the response, not only the response due to the input signal is measured, but also the noise. Especially at low signal-to-noise ratios, the noise may contribute significantly to the measured signal. The number to represent all contributions to the response, which are not at the fundamental frequency, is the total harmonic distortion plus noise (THD + N):

\[
\text{THD+N} = 20 \log_{10} \sqrt{\frac{\text{TP} - a_1^2}{a_1^2}},
\]

(3.8)

where TP is the total power in the response.

3.5 Sound pressure transfer function

The sound pressure level (SPL) is based on the root-mean-square of the sound pressure. Time-averaging the sound pressure does not discriminate between frequencies like the Fourier transform, but integrates over frequencies. These frequencies may be the overtones caused by harmonic distortion, which should be taken into account when evaluating the transfer function, and also the noise in the signal. Sound pressure levels are calculated with this time-average of the sound pressure \(p_{\text{rms}}\) (equation 3.3); for the determination of the transfer function, this \(p_{\text{rms}}\) should be related to the rms value of the input signal:

\[
H'(\omega) = A'(\omega) = \frac{p_{\text{rms}}(I(\omega))}{I_{\text{rms}}(\omega)}
\]

(3.9)

where \(p_{\text{rms}}(I(\omega))\) is the rms value of the sound pressure due to the gradient current of a single frequency \(\omega\), and \(I_{\text{rms}}(\omega)\) is the rms value of that same current. In decibels, it is stated as

\[
L_A = \text{SPL}(I(\omega)) - 20 \log_{10} I_{\text{rms}}(\omega).
\]

(3.10)

Figure 3.3 shows an example where a single frequency signal gives a response that contains the same frequency and some of its harmonics. Evaluating the transfer as
3.5. SOUND PRESSURE TRANSFER FUNCTION

Figure 3.3: Sound pressure transfer. A: A single frequency input signal has an amplitude spectrum as in B. The response to such a signal may be distorted and produce a sound as in C (harmonic distortion of 0 dB), with an amplitude spectrum as in D. The transfer at the input signal frequency is 1 Pa/V, while the total sound pressure in this case is 3 dB higher. E: The sound pressure transfer function sums all harmonic distortion and projects that onto the input frequency: the Response' signal amplitude is 1.4 times higher to give a 3 dB higher transfer.

in section 3.3, would lead to a transfer of 1 Pa/V. However, the harmonic distortion of 0 dB (viz., as much energy in the harmonics as in the fundamental) contributes significantly to the sound pressure of the response (figure 3.3C). Neglecting this contribution leads to a transfer function that cannot predict the sound nor the sound pressure accurately. The sound pressure transfer function is capable of predicting the sound pressure level as it also takes harmonic distortion into account. With an input signal containing more frequencies, all responses can be superposed. Phase information is not relevant, as by calculating the sound pressure (which is time averaging), phase information is lost.

If the input signal is a frequency modulated signal as described in section 3.6, then the transfer function can be determined with consecutive rms values of output and input signal. From the time signal it is known which frequency was presented to the system; with this information the frequency response can be determined.
In this method, the frequency resolution of the transfer function of the system is determined by the sweep rate and the time over which the rms values are calculated. One of the standard intervals for SPL calculations is 125 ms which is used throughout this thesis.

### 3.6 Frequency sweep response

To retrieve the transfer information for all frequencies of interest separately, a linear frequency sweep can be used. The instantaneous frequency $\omega$ of a linear frequency sweep is determined by the starting frequency $\omega_0$ (rad/s), the sweep rate $\beta$ (rad/s²), and the time $t$ (s):

$$\omega(t) = \omega_0 + 2\beta t.$$  \hspace{1cm} (3.11)

The input signal $s(t)$ is then described by

$$s(t) = A \cos((\omega_0 + \beta t)t),$$  \hspace{1cm} (3.12)

and the response $r(t)$ for a linear system (no distortion or noise) by

$$r(t) = A_r(\omega) \cos((\omega_0 + \beta t)t + \varphi(\omega)),\hspace{1cm} (3.13)$$

with $\varphi_\omega$ the frequency dependant phase shift. In this ideal case, the gain and phase part of the transfer function would be described by

$$\text{gain}(\omega) = \frac{A_r(\omega)}{A} = |H(\omega)|,$$

$$\text{phase}(\omega) = \varphi(\omega) = \arctan \frac{\text{Im}(H(\omega))}{\text{Re}(H(\omega))},$$ \hspace{1cm} (3.15)

where the transfer function $H(\omega)$ is given by equation 3.4. Linearity assumes that $A_r(\omega)$ is always proportional to the stimulus amplitude $A$, independent of the magnitude of $A$. The analytic Fourier transform of a frequency sweep signal is given in appendix B.1, and practical Fourier analysis is described in section 4.3.1.

### 3.7 Digitizing signals

In order to accurately represent all frequencies in a time signal, the sample rate must be higher than twice the highest frequency $\nu$ in the time signal. This is the Nyquist frequency:

$$f_{\text{Nyquist}} \geq 2\nu.$$  \hspace{1cm} (3.16)

If a frequency sweep is used for a fixed bandwidth, then a higher sweep rate takes less time to cover the bandwidth:

$$t = \frac{2\pi f_{\text{BW}}}{\beta}.\hspace{1cm} (3.17)$$
3.8. WINDOWING

The acquisition time \( t \) must be matched with equation 3.16 and the sweep rate \( \beta \).

The Discrete or Fast Fourier Transform (DFT or FFT) returns as many points in the spectrum as are available in the time domain. Less time results in a lower frequency resolution after Fourier transformation. An in advance defined frequency resolution in combination with the sample rate sets the time signal length \( t \). The time signal length in combination with the highest frequency sets upper boundaries for the set of sweep signal parameters \( (\omega_0, \beta) \).

3.8 Windowing

Practical Fourier analysis (see section 4.3.1) is performed on a finite part \( s_T \) within the time window \( T \) of a continuous signal \( s(t) \). For a sampled signal (interval \( dt \)), this implies a limited number \( N \) of sample points, \( N = T/dt \). In principle, the Fast Fourier Transform assumes that the periodic repetition of \( s_T \) with period \( T \), gives the time signal \( s(t) = \sum_{n=-\infty}^{\infty} s_T(t - nT) \). The Nyquist criterion requires that beginning and end of \( s_T(t) \) connect smoothly, otherwise aliasing occurs. This smooth continuation is usually realized with the application of a proper time window \( w(t) \) (Harris [1978]). Multiplicative weighting time signals is equivalent to convolving the spectrum of the time signal with the Fourier transform \( W(\omega) \) of the time weighting function \( w(t) \):

\[
H(\omega) = \frac{R(\omega) * W(\omega)}{S(\omega) * W(\omega)} \approx \frac{R(\omega)}{S(\omega)}. \tag{3.18}
\]

For stationary signals, windowing affects stimulus and response similarly and does not affect the transfer function. Nonstationary signals that vary sufficiently smoothly provide a similar result, viz., that \( H(\omega) \) does not depend on the window \( W(\omega) \).

Thus, apart from spectral broadening as a result of the frequency sweep, the convolution with \( W(\omega) \) broadens the spectral lines of the Fourier transform of the time signal. This might result in the masking of neighboring frequency peaks. By choosing the right window, both these effects should be optimized.