Chapter 2
Which shape factor(s) best describe granules?

2.1 Summary
This study evaluates methods used for granule shape characterisation. The aim is to identify an optimal combination of shape factors to measure granule shape and roughness.

Granules were prepared from microcrystalline cellulose (MCC), $\alpha$-lactose, microfine cellulose (MFC), and dextrin, using a small-scale high-shear mixer. Granule shapes were analysed by measuring weight, by microscopic analysis, and by sedimentation analysis in olive oil. A comparison was made between (1) the aspect ratio, (2) the circularity, (3) a new projection shape factor defined in this paper, (4) the shape factor $e_R$, (5) the radial shape factor, (6) One-Plane-Critical-Stability (OPCS), (7) the Stokes' shape factor, and (8) a new mass shape factor defined in this paper. Besides the evaluation of the shape factors, (9) a roughness factor $R$ is defined.

When the shape of granules is evaluated there can be effects of both shape and surface roughness. The scale of scrutiny of the measurement determines whether or not a shape factor is affected by surface roughness. We chose to call a protrusion smaller than 60 $\mu$m a roughness and a larger protrusion a shape effect.

The best evaluation of granule shape is obtained when shape and roughness are described by different parameters. The aspect ratio, the mass shape factor, OPCS, $e_R$ and the radial shape factor could not significantly discriminate between some of the different shapes. The other three shape factors give distinct differences in value for all the different shapes used in this study. Of these factors, Stokes' shape factor was rejected because of the high viscosity of the oil needed, which results in a significant temperature dependency. Circularity and the new projection shape factor both work well for our granules. The combination of the roughness factor with the new projection shape factor will provide a good description of granule shape and roughness.

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2.2 Introduction

Granulation is a particle enlargement technique used in different industries. High shear granulation gives dense granules in short processing times. Pharmaceutical formulations often contain an active component mixed with fillers and other excipients. The properties of the granules depend mainly on those of the filler. Some fillers (such as microcrystalline cellulose or MCC) easily produce smooth spherical granules, but others yield granules that are rough or irregular.

This chapter describes part of a project to find which physico-chemical properties of materials lead to spherical granules. Such an understanding might allow modification of other materials or formulations such that they also give spherical granules. In our project it is necessary to quantify the shape and roughness of granules formed for proper evaluation of the produced granules. The granules we produce possess similar shapes; according to Hawkins the shapes of the particles possess a family likeness [1].

Examples of granules are shown in Figure 1. They are made of $\alpha$-lactose, dextrin, microcrystalline cellulose and microfine cellulose. The $\alpha$-lactose granule is almost spherical; the MCC granule is a distorted sphere, the dextrin granule is more elongated and the MFC granule is quite irregular. The surfaces of dextrin and MCC are smooth; that of $\alpha$-lactose is a little rougher, and that of MFC is quite rough. These observed differences are typical; based on these observations we decided to focus on the ‘shape’ or sphericity of the granules and on their roughness.

In general, the shape of particles is described by one or two shape factors. Many shape factors are used; in this paper we investigated which shape factors could best be used for describing our granules. The shape factors considered are:

(1) the aspect ratio
(2) the circularity
(3) a new projection shape factor
(4) the shape factor $e_R$
(5) the radial shape factor
(6) the one-plane-critical-stability (OPCS)
(7) the Stokes’ shape factor
(8) a new mass shape factor
(9) a new roughness factor.
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![Typical pictures of the different types of granules: a) α-lactose, b) dextrin, c) MCC, and d) MFC.](image)

‘Shape’ and ‘roughness’ are difficult properties to define and measure. A distinction is to be made between shape and roughness. Whalley [2] described that in practice there is at least an order of magnitude difference between shape and roughness. We have chosen a more arbitrary boundary. In the case of our pellets, the most logical protrusion would be the primary particles. We consider deviations of the size of a primary particle as roughness, and larger deviations as shape. The boundary we use is therefore 60 µm. Ideally a shape factor is not affected by protrusions smaller than this boundary, while the roughness factor is not affected by protrusions larger than this boundary.

Since it was our objective to investigate whether the shape factors could distinguish between different but similar shapes (family likeness), statistics were used to evaluate the performance of the different shape factors.

### 2.3 Theory

A shape factor is a number characterizing the shape of a particle. In many cases a shape factor is derived from a microscopic image of the particle, but it can also be obtained in other ways. All shape factors except for $e_\text{AR}$ mentioned in this paper have in common that they range from zero to unity, in which unity represents a perfect sphere. Shape factor $e_\text{AR}$ ranges from unity (representing a sphere) to negative infinity (representing extremely elongated or rough shapes).

#### 2.3.1 Two-dimensional shape factors

Microscopic images are two-dimensional; therefore they only show part of the shape of the three-dimensional granule. The image is usually that of particles on a flat support with the particles lying on their most stable plane: what is seen is the largest projection area.

One of the earliest shape factors known is the aspect ratio ($\varphi_{AR}$), introduced by Schneiderhöhn [3]. It is defined as the ratio of the length of the minor and the major axis.
Equation 1 [3]
\[ \varphi_{AR} = \frac{b}{l} \]

The major axis \((l)\) is a straight line connecting the two most distant points of the projection area. The minor axis \((b)\) is a straight line perpendicular to the major axis connecting the two points with the largest distance between them. This shape factor only reflects the elongation of a granule, but it does not make a distinction between, e.g., squares and circles. Therefore, the usefulness of the aspect ratio is limited [4]. Nevertheless, it provides information about the compactness of the granule; whether it is a sphere or cube, or a more oblong shape.

Cox [5] described a shape factor called circularity, which is often referred to [6, 7]. This shape factor is based on the projected area of the granule \((A)\) and the overall perimeter of the projection \((P_{\text{rough}})\) according to:

Equation 2 [5]
\[ \varphi_{\text{circularity}} = \frac{4 \cdot \pi \cdot A}{P_{\text{rough}}^2} \]

Granule roughness is incorporated in this shape factor due to the incorporation of the overall perimeter. This perimeter is measured counting all pixels on the granule outline. Measuring perimeters causes problems which are often referred to as the ‘Coastline of Britain problem’ [1, 8]. The scale of measurement and the resolution of the picture greatly influence the value of the perimeter, thus influencing the value of circularity.

We have investigated a variation of the circularity, which we call the projection shape factor. A polygon is obtained by drawing 72 lines from the centre of gravity of the image to the particle outline, at 5 degrees intervals. Whether more or less angular samples (varying from 36 to 96 or even more) are used is an arbitrary choice. However, the outcome changes by this choice. The intended purpose of the measurements should determine the number of angular samples taken. The perimeter of the polygon is taken as the smooth perimeter of the particle, largely eliminating the effects of roughness [1].

Equation 3
\[ \varphi_{\text{projection shape factor}} = \frac{4 \cdot \pi \cdot A}{P_{\text{smooth}}^2} \]

In the results there is no difference in value of the projection perimeter whether or not roughness is included; therefore, the projection shape factor is not noticeably influenced by roughness.

Podczeck and Newton proposed a shape factor called \(e_R\) [4, 9]. This shape factor is a combination of circularity and the aspect ratio. Originally, it was defined to measure oblong granules with a certain roughness.
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\[ e_R = \frac{2\pi \cdot r_e}{P \cdot f} - \sqrt{1 - \left(\frac{b}{l}\right)^2} \]  
Equation 4 [9]

In this equation, \( P \) represents the perimeter of the granule, measured with all pixels on the granule outline; \( f \) is a correction factor and equals 1.008-0.231•(1-\( b/l \)); \( r_e \) is the mean radius of 360 radii, drawn from the centre of gravity to the perimeter of the granule. In the original version of \( e_R \), \( r_e \) was defined as the mean radius of 360 radii drawn from the centre of gravity towards the particle outline. In a modernised version [9] \( r_e \) is referred to as calculated from only 72 radii. Although in this paper we use the latest version of \( e_R \), [9] we think using 360 radii would be the optimal procedure, because roughness was especially incorporated in this shape factor. Increasing the number of radii did not result in a different value for \( e_R \), therefore this number is optimal in our case regarding the results and the amount of time needed for the calculations. Similarly to circularity versus projection shape factor, \( e_R \) can also be regarded without roughness, then 72 radii should be taken for calculating \( r_e \).

To obtain the radial shape factor, the centre of gravity of the projected area is determined, and the distance of each pixel on the perimeter of the granule to the centre of gravity is measured. These radii are normalised with their average. The radial shape factor is calculated by subtracting the standard deviation of these normalised radii from unity.

\[ \varphi_{radial} = 1 - \frac{1}{N-1} \left( \sum_{i=1}^{N} \left( \frac{r_i}{r_{av}} - 1 \right) \right)^2 \]  
Equation 5

The value of \( N \) (number of measurements) depends on the number of pixels on the granule outline, since all pixels are used. The radius from the centre of gravity to the specific point on the granule outline is represented by \( r_i \), the average radius is represented by \( r_{av} \). Irregular shapes will result in a low value of the shape factor. Surface roughness has only a small lowering effect. Since every radius has to have a single value (a re-entrant situation is not possible for the calculation), this shape factor is limited to compact shapes only [1].

Chapman et al. [10] have introduced the one-plane-critical-stability or OPCS, in which the value of the shape factor is based on the maximum angle between a horizontal plane and a tilted plane of maximum stability of the granule resting on that plane. The OPCS is calculated according to equation 6.

\[ \varphi_{OPCS} = 1 - \frac{2}{\pi} \arcsin \left( \max \left( \frac{|r_i|}{|r_{av}|} \right) \right) \]  
Equation 6 [10]

Here \( r_i \) are 32 vectors from the centre of gravity of the granule to the perimeter, dividing the projection area into 32 equal area segments.
2.3.2 Other shape factors

Not all shape factors are based on two-dimensional image-analysis. Pettyjohn and Christiansen [11] used Stokes’ shape factor, which is based on sedimentation of a particle. Stokes’ shape factor is derived from a force balance. During stationary settling of a particle in a fluid, gravitation is balanced by buoyancy and friction. The gravitational force depends on the mass of the granule. Consequently, when having a sphere the diameter considered is the mass diameter.

The sedimentation velocity of granules is compared to the theoretical settling velocity of spheres of the same mass and volume. The terminal velocity of the granules ($u_t$) is measured in a viscous fluid. The shape factor is defined as:

$$\varphi_{Stokes} = \frac{18 \cdot \eta \cdot u_t}{(\rho_g - \rho_f) \cdot g \cdot d_{mass}^2}$$  \hspace{1cm} \text{Equation 7 [11]}

Here the density of the granule and the density of the fluid are represented by $\rho_g$ and $\rho_f$ respectively, $g$ represents the gravitational acceleration, $\eta$ viscosity of the fluid, and $d_{mass}$ represents the mass equivalent sphere diameter or mass diameter, defined in Equation 8.

$$d_{mass} = \sqrt[3]{\frac{6 \cdot m_{pellet}}{\rho_g \cdot \pi}}$$  \hspace{1cm} \text{Equation 8}

A prerequisite for the use of Stokes’ shape factor is sedimentation in the laminar regime, i.e., the Reynolds number must be smaller than 0.3 [8, 12, 13]. Therefore, the viscosity ($\eta$) of the medium has to be high; a viscous fluid like a vegetable oil is recommended.

Another possible shape factor is the mass shape factor. This mass shape factor combines the mass with the projection of the granule, therefore, it might be a powerful tool in analysing shape. We define the mass shape factor as:

$$\varphi_{mass} = \frac{d_{mass}}{d_{projection}}$$  \hspace{1cm} \text{Equation 9}

Here $d_{projection}$ is defined as the projected area equivalent sphere diameter (projections of particles with a stable repose on a microscope slide), and $d_{mass}$ is calculated according equation 8.

2.3.3 Roughness

Finally, we would like to consider a measure for roughness. Barrett described the idea to express surface texture as the ratio between the true perimeter of a two-dimensional particle outline and a convex hull surrounding the particle outline [14]. Podczeck and Newton proposed a surface roughness factor $s$, [15].
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\[ S_r = \frac{2 \cdot \pi \cdot r_e}{P_m} \]  \hspace{1cm} \text{Equation 10}

\( P_m \) represents the perimeter of the granule, measured with all pixels on the granule outline; \( r_e \) is the mean radius of 360 radii, drawn from the centre of gravity to the perimeter of the granule. Unfortunately, the perimeter measurement is dependent on the pixel size, as explained above. Therefore, we have investigated the use of a roughness factor, which we define as

\[ R = 1 - \frac{P_{\text{smooth}}}{P_{\text{rough}}} \]  \hspace{1cm} \text{Equation 11}

Here \( P_{\text{smooth}} \) is the perimeter measured with 72 points on the outline of the granule at 5° intervals; \( P_{\text{rough}} \) is measured with 360 points. Note that here maximum roughness corresponds to a value of 1; a completely smooth surface corresponds to a value of 0.

2.3.4 Effect of roughness on shape factors

One of our aims is to separate the effects of shape and of roughness. Our granules have dimensions of about one millimetre; the primary particles are 10-60 micrometers. We consider irregularities of the size of the primary particles as roughness; only larger deviations are considered to contribute to shape.

Stokes’ shape factor does not depend on roughness, since, at small Reynolds numbers, the drag on a particle is less than or equal to the body which encloses it \([8, 16]\). The aspect ratio, the radial shape factor, OPCS, and the mass shape factor use diameters of particles, these diameters are in a range that is near those of the granule, so they are hardly influenced by roughness. Circularity and \( \varepsilon_{R} \) use the perimeter measured by all pixels around the perimeter. Roughness is in these cases always included, whereas measuring the perimeter via 72 points on the particle outline, as was done with the projection shape factor, protrusions smaller than 60µm will have no effect on the outline of a 1 mm diameter pellet.

2.4 Materials and methods

2.4.1 Granulation

Different powders were used for granulation: microcrystalline cellulose (Pharmacel 101 (lot 60841), DMV International, Veghel, the Netherlands), \( \alpha \)-lactose (Pharmatose 450M (lot 047933), DMV International, Veghel, the Netherlands), microfine cellulose (Elcema P100 (lot 912111), Degussa AG, Frankfurt, Germany), and dextrin (Primogran W (lot 45P/EP), Avebe, Foxhol, the Netherlands). These powders were agglomerated in a small-scale high-shear mixer (Mi-Pro 250, Pro-C-epT, Zelzate, Belgium) using water as a binder fluid.
Wetting and mixing in the Mi-Pro were performed with the chopper at 1500 rpm and the impeller at 1000 rpm. Once the impeller and chopper were switched on, the water was added immediately, at the rates stated in Table 1. The used amounts of powder and amounts of binder fluid can also be found in Table 1. The powder sticking to the wall was removed manually when needed, up to three times per run. The produced granules were tray-dried at 50º C.

<table>
<thead>
<tr>
<th>Material</th>
<th>Amount powder (g)</th>
<th>Liquid addition rate (ml/min)</th>
<th>Amount of binder fluid (ml)</th>
<th>Granulation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α-Lactose</td>
<td>30</td>
<td>5</td>
<td>5</td>
<td>900</td>
</tr>
<tr>
<td>Dextrin</td>
<td>30</td>
<td>3</td>
<td>11.5</td>
<td>900</td>
</tr>
<tr>
<td>MCC</td>
<td>24</td>
<td>48</td>
<td>24</td>
<td>900</td>
</tr>
<tr>
<td>MFC</td>
<td>10</td>
<td>40</td>
<td>20</td>
<td>900</td>
</tr>
</tbody>
</table>

2.4.2 Measurement of granule shape

We used the shape factors defined in equations 1 to 9, and 11. Every batch was sieved; the fraction between 1.0 mm and 1.18 mm was taken out for further shape research. From this fraction 16 (measuring Stokes shape factor and the mass shape factor) or 100 (measuring two-dimensional shape factors) granules were taken. Granules were weighed and density was measured (see below). Granule size (mass diameter) was calculated assuming a spherical shape of the granule (Equation 8).

Microscopic pictures of the granules were taken with a Nikon DN 100 Digital Net camera. Pixel size equalled 2 µm in both directions. Pictures were examined with an image-analysis program. The aspect ratio and circularity were measured using Sigma Scan Pro 5 (Jandel Scientific, Erkrath, Germany). The projection shape factor, the radial shape factor, OPCS, and $\theta_R$ were determined using Matlab 6.1 (The MathWorks, Inc., Gouda, the Netherlands).

Granule size (projection diameter) was determined by calculating the diameter of the circle which has the same projection area as the granule.

Sedimentation measurements were carried out in a transparent column with a length of 1 meter and a diameter of 8 centimetres. The column was filled with olive oil, a vegetable oil with a density of 890 kg/m$^3$ and a viscosity 0.054 Pa·s (25°C). The viscosity of the oil was measured before every experiment by measuring the sedimentation time of standard spheres.

Marks were placed on the column to indicate distances. The sedimentation time of one granule at a time, was measured using a stopwatch. If the granule adhered to the column wall, the measurement was rejected.
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2.4.3 Density measurements
The method of measuring the density of the granules was performed according to the ‘kerosene replacement’ method described by Iveson et al. [17]. Instead of kerosene, we used the oil in which sedimentation took place. This results in the granule density as it appears in the oil. The densities and Reynolds numbers used for calculating Stokes shape factor are found in Table 2.

Table 2: The average values used for determining Stokes’ shape factor and the mass shape factor

<table>
<thead>
<tr>
<th>Material</th>
<th>(d_{mass}) (mm)</th>
<th>Re (-)</th>
<th>granule density (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)-Lactose</td>
<td>1.036 ± 0.084</td>
<td>0.050</td>
<td>1456</td>
</tr>
<tr>
<td>Dextrin</td>
<td>1.279 ± 0.103</td>
<td>0.089</td>
<td>1382</td>
</tr>
<tr>
<td>MCC</td>
<td>1.127 ± 0.075</td>
<td>0.062</td>
<td>1506</td>
</tr>
<tr>
<td>MFC</td>
<td>0.869 ± 0.085</td>
<td>0.019</td>
<td>1520</td>
</tr>
</tbody>
</table>

2.4.4 Statistical analysis
For the measurements of the 100 granules statistical analysis was performed using t-tests by using SPSS 11.0 for Windows. A normal distribution of shapes was assumed. Due to the relatively low amount of granules a high confidence level was chosen. A t-test provides an estimate on whether the results obtained with two different materials are significantly different when using a certain shape factor. For this to be the case the t-value has to be above the critical value (in our case 3.34 for a 99.9% confidence level, two tailed t-test).

2.5 Results and discussion
Microscopic pictures were made of all granules. Figure 1 shows some typical pictures. The picture shows that lactose granules are spherically shaped, but a little rough, whereas MCC granules are distorted spheres and smooth. Surprisingly, granules of dextrin also had a smooth surface, but these granules are elongated. Finally, MFC granules are rough and small granules form irregular agglomerates.

The values of the different shape factors and roughness factor \(R\) are given in Figures 2 and 3. In Table 3 statistics are shown. We looked at the values of each shape factor for the same four types of pellets, to see whether there was a significant difference between the results obtained by the different pellets. If not, the shape factor was rejected.
2.5.1 Aspect ratio, circularity, projection shape factor and $e_R$

The aspect ratio only measures elongation of a granule. There was no significant difference in shape observed between MFC and dextrin; and between $\alpha$-lactose and MCC. This is probably caused by the fact that only four points on the perimeter of each granule are taken into account. The aspect ratio insufficiently discriminates between the different granules, in spite of the differences in shape [4]. Therefore, the aspect ratio is not considered as a suitable method for determining the shape of these types of granules.

Circularity, on the other hand, is easily measured, and several granules can be measured simultaneously. This shape factor is the two-dimensional equivalent of the surface area to volume ratio. Circularity was able to measure differences between all four shapes. Also the projection shape factor, the result of the conversion of circularity into a shape factor that measures shape only, is able to distinguish the different shapes.

The shape factor $e_R$ was originally meant to be a combination between roughness and shape [4]. However, this shape factor did not provide a significant difference between $\alpha$-lactose and dextrin.

| Table 3: Statistical evaluation of the different two-dimensional factors |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| T-values                        | $\alpha$-lactose vs dextrin | $\alpha$-lactose vs MCC | $\alpha$-lactose vs MFC | dextrin vs MCC | dextrin vs MFC | MCC vs MFC |
| Aspect ratio                    | 5.028            | 0.878*          | 3.850            | 4.443          | 1.448*          | 3.173          |
| Radial shape factor             | 4.254            | 2.549*          | 8.604            | 7.144          | 5.107           | 11.087          |
| OPCS                            | 0.643*           | 9.818           | 11.284           | 8.512          | 11.239          | 22.303          |

Values marked with an asterix* are not significantly different (99.9% confidence level)

2.5.2 Radial shape factor and OPCS

The radial shape factor shows only a small difference in the obtained values, which indicates fairly spherical granules. There was no significant difference found upon comparison of $\alpha$-lactose with MCC. This shape factor might be interesting when granules with large differences in shape are analysed.
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The OPCS clearly shows differences in shape, although the variation is rather large. Roughness is not incorporated in this factor. There was no significant difference found upon comparison of \(\alpha\)-lactose with dextrin.

![Figure 2: All shape factors compared: Aspect ratio (AR), Circularity (Circ), Projection shape factor (PSF), \(e_R\) (eR), Radial shape factor (RSF), One-plane-critical-stability (OPCS), Stokes' shape factor (SSF), and Mass shape factor (MSF); where a value of 1 is spherical. Error bars represent the standard deviations.]

2.5.3 Stokes' shape factor and the mass shape factor

To obtain reliable values of Stokes' shape factor, sedimentation has to take place in a laminar flow regime, so the Reynolds number has to be below 0.3 [8, 12]. In Table 2 the average Reynolds numbers, mass diameters, and granule densities as they appeared in the olive oil are stated. According to the Reynolds numbers, sedimentation did indeed take place in the laminar regime. However, temperature changes drastically change viscosity. Therefore, the temperature has to be kept constant. In general Stokes' shape factor does reflect shape effects, [12] indeed for all four granules a different shape value was obtained.

The mass shape factor could not provide a difference between MCC and lactose. The mass shape factor is based on the mass and the 2D surface area of a granule. Therefore it only covers the shape of a granule and does not take roughness into account. A disadvantage of this shape factor is that (variations in) the internal porosity of granules may disturb its accurate measurement. Although this disturbance was limited by measuring the reliability density, internal isolated pores cannot be measured, thereby limiting the efficacy of the shape factor.
Both Stokes’ and the mass shape factor have the disadvantage of being time-consuming measurements, whereas a method using only image analysis is very fast.

### 2.5.4 Roughness factor R

The obtained values of the roughness factor are presented in Figure 3. MFC granules have a high roughness. MCC and dextrin granules have similar roughness factors. A rough granule has an increased surface area compared to granules with a smooth surface. To determine roughness, the simplest way is the roughness factor R.

![Roughness factor R](image)

**Figure 3:** Roughness factor R, a value of 1 is maximum roughness, a value of 0 means no roughness at all. Error bars represent the standard deviations.

### 2.6 Conclusions

This paper compares eight methods used to describe the shape of granules. Next to the shape factors, a roughness factor R is proposed. It was our aim to find a shape factor that could distinguish granules both with regard to their shape (sphericity) as well as regarding their roughness.

The scale of scrutiny of the measurement determines whether or not a shape factor is affected by surface roughness. We chose to call a protrusion smaller than 60 µm a roughness and a larger protrusion a shape effect. Granule shape affects all shape factors studied.

The aspect ratio could well be used for extrusion-spheronization, because oblong granules can be expected in that process. The same applies to $e_R$, which has been created to compare elongated granules with a certain roughness. Both circularity
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and the projection shape factor can be used. The radial shape factor might be a powerful tool for quite irregular shapes; whereas OPCS is remarkably suitable to distinguish shapes with small deviations from sphericity. The Stokes’ shape factor could also distinguish between the shapes of the tested granules, however, it is a time-consuming measurement. The mass shape factor would be more useful for materials with equal porosity.

Which shape factor is the best choice in our case, where we aim for smooth granules having the least amount of surface area compared to their volume, so they are easily coated? The aspect ratio, OPCS, $e_R$ and the radial shape factor could not significantly discriminate between some of the different shapes. The other two shape factors give distinct differences in value for all different shapes used in this study. Stokes’ shape factor has two major disadvantages. First of all this measurement required the use of oils with a high viscosity which results in a large temperature dependency. Moreover, the measurement is rather time consuming. Circularity and the projection shape factor both work well for our granules. The roughness factor can be used to evaluate roughness. The combination of the roughness factor with the projection shape factor will give a good indication for granule shape and roughness in case the least surface area for a certain volume is desired. It is important to note that if a different goal could lead to a different conclusion.

2.7 Symbols

$A$ surface area of the projection of the granule (m$^2$)

$b$ length of a line perpendicular to the major axis, connecting the two points with the longest distance between them (m)

$d_{\text{mass}}$ diameter of a sphere with the same mass and primary particle density as the granule (m)

$d_{\text{projection}}$ diameter of a circle with the same projection area as the granule (m)

$f$ correction factor (-)

$g$ gravitational acceleration (m/s$^2$)

$l$ length of a line connecting the two most distant points of the projection area of a granule (m)

$m_{\text{granule}}$ granule mass (kg)

$N$ number of radii (-)

$P_{\text{rough}}$ perimeter of the projection of the granule measured by using 360 radii from the centre of gravity to the granule outline (m)

$P_{\text{smooth}}$ perimeter of the projection of the granule measured by using 72 radii from the centre of gravity to the granule outline (m)

$r_{\text{av}}$ average radius (m)

$r_e$ mean radius of 72 radii (m)

$r_i$ radius length (m)

$u_t$ terminal settling velocity (m/s)
Greek symbols

\( \varphi \)  
shape factor (-)

\( \eta \)  
viscosity (Pa·s)

\( \rho_f \)  
density of the sedimentation fluid (kg/m\(^3\))

\( \rho_g \)  
density of the granule (kg/m\(^3\))

2.8 References


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