**Conclusion**

*Summary.* This thesis falls into three main parts. The first part claims that the hypotheses in the Bayesian scheme offer a better control over the inductive assumptions inherent to predictions. The second part adds to this by showing that spaces of hypotheses prove very useful in encoding specific aspects of the predictions in a prior probability. The third part illustrates the use of the Bayesian scheme in solving some problems in the philosophy of science, in particular problems concerning scientific method.

Let me run through the chapters in some more detail. As for the first part, chapter 1 presents inductive inferences as logical, using a representation of observations in a cylindrical algebra, a representation of beliefs in terms of a probability assignment over this algebra, and the probability axioms alongside Bayesian updating as the inference rules. I distinguish between a Carnapian and a Bayesian scheme for generating the inductive predictions. Chapter 2 concerns the nature of statistical hypotheses in the Bayesian scheme. The hypotheses can be associated with specific sets in the observation algebra, so that the two schemes can be treated on equal footing. In chapter 3 I argue that the Bayesian scheme has a specific advantage over the Carnapian scheme. The hypotheses offer a natural control over the inductive assumptions underlying the predictions. Where the Carnapian scheme leaves the assumptions implicit, the Bayesian scheme brings them within conceptual grasp.

The Bayesian scheme invites two different discussions in the two other parts of the thesis. The second part concerns the use of the Bayesian scheme in capturing inductive predictions that are sensitive to analogy and independence. In chapter 4, in particular, I discuss the specific class of analogical predictions based on explicit similarity. After providing a system of Carnapian prediction rules, I define the Bayesian scheme that underlies this system. This latter scheme offers some insights into the system of prediction rules, and leads up to a further exploration in chapter 5. This chapter employs the scheme to develop a general model of analogical predictions, but unfortunately it fails to achieve full generality. Chapter 6, finally, employs the same scheme to model predictions for nodes in a Bayesian network. It further shows how the notion of inductive dependence can be incorporated in the Bayesian scheme. More generally, the second part of this thesis illustrates that the Bayesian scheme,
by using hypotheses, allows for a better expression of inductive assumptions in a way that stands quite apart from the advantage stressed in the first part: transformations in the hypotheses space allow for the definition of priors that are difficult to define otherwise.

The third part of the thesis considers the Bayesian scheme in relation to three venerable problems in the philosophy of science. Chapter 7 shows that the Bayesian scheme offers a solution to the logical part of the problem of induction, but also that it offers nothing on the epistemological part of the problem. Chapter 8 concerns the problem of inductive inference and theory change. It is shown that the Bayesian scheme suggests a natural place for changes in the inductive assumptions, namely in a change of the statistical hypotheses. It further develops the formal tools to ensure that such changes remain as conservative as possible. Finally, chapter 9 concerns the use of purely theoretical concepts and distinctions in inductive inference, and thus relates to the problem of underdetermination. It shows that such distinctions can indeed be useful, and suggests a further exploration of this fact in a Bayesian model of abductive inference. On the whole the third part claims that the Bayesian scheme is not only suitable for an interesting inductive logical exercise, but that it provides insight into actual scientific methods.

The bigger picture. Let me start by noting that from its conception onwards, probabilistic inductive logic has developed rather slowly, and never really picked up speed. I can see two reasons. First, without meaning to be disloyal to the old masters, there is what I call the curse of Carnap. While the Carnapian framework has certainly been a step forward in studying inductive inference as part of a formal system, both the inherent empiricist view on language and the notion of logical probability have not always been helpful in the development of this system. It may even be conjectured that a failure to disentangle logic from epistemology is the main cause for the problematic development of inductive logic, certainly in comparison to the mature discipline of deductive logic.

As a second reason for the slow development of inductive logic, it appears that Carnapian logic has never really been connected to the main use of probabilistic inductive inference in science, namely in statistical inference. An exception to this is the statistical treatment of \( \lambda \gamma \) rules in Festa (1993), which has been a strong source of inspiration for the present thesis. In the larger discussion on scientific methodology, however, Popper had statisticians such as Fisher, Neyman and Pearson on his side, whereas Carnap failed to find fruitful common ground for his logical framework and the tradition of Bayesian statistics.
Apart from that, Carnapian logic has been connected to conceptual problems in the philosophy of science only to a very limited extent. Inductive logic has therefore remained a rather isolated discipline, immersed in its own problems, and at best gesturing towards applications to statistics and scientific method more generally.

This thesis hopes to improve the prospects for inductive logic, both as a separate discipline and as a formal tool for solving problems in methodology and the philosophy of science. With respect to inductive logic as a separate discipline, it proposes a reorientation of the field by pushing two points: the logical perspective, and the Bayesian scheme. The logical perspective obviates the need for a notion of logical probability, and puts strong emphasis on the fact that inductive inference must be valid inference. It further emphasises that inductive logic must make explicit the assumptions underlying inductive inference. This is where the second point becomes effective. The Bayesian scheme employs statistical hypotheses, which are seen to provide access to underlying assumptions. They enlarge the expressive force of inductive logic, and provide a new take on some well-known problems. Thus, while the first part of this thesis simply describes the reorientation of inductive logic, and shows some conceptual advantages of it, the second part shows that this reorientation also results in a better treatment of internal questions. It turns out that certain problems of traditional inductive logic can be solved more easily within a Bayesian scheme.

The use of inductive logic in philosophy of science is illustrated in the third part. It is here suggested that the Bayesian scheme can provide insight into, and to a certain extent solutions for, some problems concerning scientific method. However, much is left to be done in this last research area. First, I suspect that the Bayesian scheme connects naturally to statistical procedures as used in the sciences, but an argumentation for this has not yet been produced. Moreover, it may be noted that there is still a yawning gap between the above schemes and the abundant use of Fisherian and Neyman-Pearson statistics in most of the social sciences. Future research will be directed towards a better understanding of these statistical techniques, and where possible to a reformulation of them in terms of valid statistical inferences. As a second line of development, besides this debate on statistics, I expect that the Bayesian scheme can contribute to many more philosophical debates in the philosophy of science, apart from the ones discussed here.

*The need for both observations and theory.* In the philosophical debate about the theory-ladenness of observations, I expect Bayesian inductive logic to be particu-
larly helpful. It may be noted that this thesis employs a rather naive framework for the observations, which are supposed to enter the Bayesian schemes as clear-cut and numbered packages of independently obtained information. This seems in direct opposition to the widely shared view that observations are partly determined by theory, in particular that they cannot be described or processed unless we already presuppose some theoretical framework. This point becomes all the more pressing for observations within a scientific experiment, as they are usually processed elaborately before being subjected to statistical analysis. In short, there are strong assumptions inherent to taking observations as clear-cut packages: we assume an unshakeable observation language. However, I submit that the Bayesian scheme contains the conceptual ingredients for a more nuanced view on observations than has been suggested until now. The first part of this thesis makes clear that in the Bayesian scheme, the partition functions as a pair of glasses for looking at the observations. The idea is to take the observations \( Q^q_t \) as referring to the raw material of the observation, or in other words the unrefined stimulus. The partition of hypotheses, which determines the impact of the raw observation on beliefs, then concerns the theoretical side of the observations.

This view on the Bayesian scheme, and on inductive inference within it, emphasises that there is not much that observations can convey all by themselves. They always presuppose a theoretical framework, and it is in this sense misguided to hope for objective inductive knowledge. The choice of a partition is similar to the choice of a language in a Carnapian inductive logic, and as Friedman (2004) argues, this choice may again be seen as a relativised and dynamic variant of the Kantian synthetic a priori. The difference is that within the context of Bayesian logic, the choice is within conceptual and formal grasp. On the other hand, as with the Carnapian language choice and the Kantian synthetic a priori, the assumptions underlying inductive knowledge do not convey much by themselves either: statistical partitions usually leave all possibilities open. Once one is provided with these assumptions, the observations are fully responsible for the result of the inductive inferences. It is therefore equally misguided to conclude that in inductive inference, anything goes. As elaborated in the third part of this thesis, inductive knowledge is best seen as a co-production of the observer and the observed, which interact on the strict interface of a logical scheme. The eventual value of the result, in many cases the accuracy of the predictions, thereby depends on making correct observations, on using a proper logic, and finally on the truth of the inductive assumptions.
Choosing inductive assumptions. I want to conclude with some philosophical remarks on the three elements of observation, logic and assumption, starting with the last. I must admit that it is rather disappointing that in this thesis the matter of choosing inductive assumptions has been left aside completely. As may be recalled from chapter 1, the motivation for this disregard is that choosing inductive assumptions is deemed an epistemological issue, or an issue closer to scientific practice, while this thesis is focused exclusively on the logical aspect of inductive inference. Here I want to briefly consider this epistemological aspect after all.

It can first be noted, in particular with respect to science, that choosing inductive assumptions is related to the activity of conjecturing and model building. In other words, asking for the origin of inductive assumptions leads us to the context of discovery, the realm of supposedly irrational and intuitive scientific reasoning. Now I do not think that this side of scientific reasoning is irrational, and like most other philosophers I think it is also a perfectly respectable subject for further research. However, it seems to me that this research is not served best by a restriction to philosophical methods. It also requires empirical research into actual reasoning, which may be accessed by studying the history of science, and perhaps also by performing psychological experiments. The epistemological part of inductive inference is thus moved into the domain of the sciences themselves. Note further that the findings of these empirical studies are likely to differ from the schemes presented in this thesis. After all, sailors do not use fluid mechanics to determine the optimal positions of the sails on their boat. They just follow the rules of sailing. In the same way, actual scientific reasoning will quite probably be a dense network of ad hoc rules rather than a neat logical scheme. It is only by writing down the rules in terms of a Bayesian scheme that we can reconstruct and investigate the inductive assumptions underlying the reasoning.

Spinoza resolves Cartesian doubts. As for the second element of inductive inference, namely making correct observations, I can only offer the kind of basic trust, perhaps best known from Spinoza, that human cognition is by its very nature attuned to the world. This reliabilist trust is obviously not supposed to apply to all convictions, in which case the inductive schemes of this thesis would all become irrelevant. The trust applies only to the raw material of observations, that is, the direct sensory input. And I hold that for this raw material, the reliabilist ideas are in fact rather natural. The starting point of the argument for this is that the cognitive system of a human body is part of this world,
or rather, fully submerged in it just as tables and chairs are. In the way in which it interacts with the world on the level of unrefined stimuli, it does not differ in any fundamental sense from tables and chairs, although it is of course much more complex. But if that is so, then to say that human cognition, on the level of unrefined stimuli, is structurally at variance with the world is like saying that a certain type of chair cannot be fitted into space, suspends gravitation, or something of that sort. Under the assumption that human beings are nothing special, in the sense that they are as any other object part of this world, whatever this world consists in, it becomes hard to imagine what doubting the unrefined stimuli amounts to.

*Logic as metaphysics?* This brings me to the last element of inductive logic, namely that of using proper inference rules. Here I briefly discuss the their epistemic status. It will be clear by now that I think it sadly misguided to aim for a logical scheme that somehow also provides the correct inductive assumptions. The force of a logical scheme is exactly that it provides only the criteria for valid inference, and avoids the whole matter of truth. When it comes to probabilistic inductive logic, I am therefore emphatically against the slogan that probability is the guide of life. On itself, probability cannot tell us anything about life, if only for the simple fact that it is merely a formal tool.

It may be objected that, considered as a formal tool, the logical scheme reveals something synthetic after all. The idea behind this is that there must be something to the logical scheme that ensures its applicability to the world we live in. One may argue that logic is not just a game of symbolic manipulation, but that it really concerns the world, and that it somehow reveals invariances in its structure. Now I am not sure whether there is indeed some structure to the world that makes Bayesian updating the valid inference rule for it, or whether this validity derives only from the form that we choose for assumptions and conclusions. If the former is the case, Bayesian inductive logic does indeed not just accommodate the representation of inference, but it is also a branch of metaphysics. But this, to my mind, stretches the reach of the Bayesian scheme a bit too far.