On Signalling and Debt Maturity Choice

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Abstract

The theoretical literature on a firm’s choice of debt maturity argues that a borrowing firm can signal its value in asymmetric information setting by borrowing short. This well-known fact is based on Flannery (1986). This paper questions the use of debt maturity as a signalling device. We demonstrate that Flannery’s (1986) signalling outcome is vulnerable on two accounts. First, the separating equilibrium established by Flannery is not driven by the incentive compatibility. Second, derivations of the separating equilibrium appear to be vulnerable due to the lack of the refinements of pooling equilibria. If correct constraints are provided, the parameter space for the separating equilibrium shrinks, moderating the signalling role of debt maturity.

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1. Introduction

The theoretical literature on a firm’s choice of debt maturity argues that a borrowing firm can signal its value in an asymmetric information setting by borrowing short. This well-known fact is based on a seminal paper by Flannery (1986). Flannery argues that high-quality firms may signal their type by issuing short-term debt when debt issue entails positive transaction costs. Flannery’s (1986) model is still very popular in the debt maturity literature and provides a critical theoretical reference to many recent empirical studies that deal with debt market segmentation across borrowers by means of debt maturity (Berger, et. al. 2004, Scherr and Hulburt, 2001, Guedes and Opler, 1996, Stohs and Mauer, 1996, Mitchell, 1993). However, these empirical studies provide little empirical evidence for the use of debt maturity as a signalling device.

This note, in line with recent empirical evidence, questions the theoretical relevance of debt maturity as a signalling device. The main argument we make is that Flannery (1986) is much too positive about the probability that a signalling equilibrium will result if a firm uses its debt maturity to signal its value. Flannery derives parameter restrictions for different types of equilibria (pooling and separating) by comparing the equity value of firms under different pooling possibilities and under a candidate separating equilibrium. However, the analysis of Flannery suffers from the lack of incentive compatibility constraints and of the refinements of pooling equilibria in deriving a signalling separating equilibrium. In this note we will show that adding correct constraints to the model by Flannery will shrink the parameter space for which a separating equilibrium exists. Consequently, the use of debt maturity as a signalling device is questionable.

The paper proceeds as follows. Section 2 provides a review of Flannery’s (1986) model with the focus on how the signalling separating equilibrium arises. Section 3 analyzes the weakness of the Flannery’s signalling outcome and accordingly sets out the relevance of incentive compatibility. Section 4 introduces a model extension, which aims to derive the correct signalling outcome based on the incentive
compatibility constraints and other relevant refinements. In this section, we also discuss the relevant implications of our analysis to Flannery’s work. Section 5 concludes the paper.

2. Review of Flannery’s (1986) model

In a seminal paper, Flannery (1986) examines debt maturity as a signalling instrument under asymmetric information. In effect, he analyzes the choice of debt maturity under several different settings of information and transaction costs, e.g. perfection information versus asymmetric information, zero transaction costs versus positive transaction costs and partial endogenous transaction costs. For the purpose of this paper, we focus on the case in which positive transaction costs and asymmetric information are assumed.

2.1 The model setup

Flannery considers a wealth-constrained entrepreneur who is endowed with a risky investment project, which lasts for two periods. The project can be financed with short-term (one period) or long-term (two periods) debt. If the investment project is carried out, all cash flows will occur at the end of period 2. All investment projects require an amount $D$ of investment, and hence external financing of $D$ is needed. During each period the project can increase in value with a probability $p$ and decrease in value with a probability $(1-p)$. The project’s value follows a binomial process as illustrated by figure 1. Since the time-state values $M_1$ through $M_5$ are identical for all projects, project’s “up” probability determines its probability of default on debt maturing at $t = 2$. At $t=0$, all lenders and the entrepreneur know that the project’s liquidation value at $t=2$ will be $M_3$ with probability $p^2$, $M_4$ with probability $2p(1-p)$ and $M_5 = 0$ with probability $(1-p)^2$. All the project’s time-state values except for $M_5$ are sufficient to repay $D$.

There are two types of projects that differ in their “up” probabilities. Let us denote $p_g$ and $p_b$ as the probability of success for the good quality project and the low quality project, respectively, or analogously for the Good firm and the Bad firm, respectively.
Under information asymmetries, lenders only know that \( \theta \) percent of all projects (and firms) are good, while the “up” probability of a certain project remains privately-known to the entrepreneur endowed with the project. Thus, lenders face a problem of pre-contract asymmetric information and adverse selection, where they fail to identify a particular borrower’s quality at any cost. Presumably, both types of firms can apply for short- or long-term debt, knowing that they have to pay extra transaction cost for short-term debt.

\[
\begin{align*}
M_0 & \quad M_1 & \quad M_2 & \quad M_3 & \quad M_4 & \quad M_5=0 \\
(1-p_i) & \quad (1-p_i) & \quad (1-p_i) & \quad p_i & \quad p_i & \quad p_i \\
M_2 & \quad M_3 & \quad M_4 & \quad M_5 & \quad M_6 & \quad M_7 \\
M_0 & \quad M_1 & \quad M_2 & \quad M_3 & \quad M_4 & \quad M_5 & \quad M_6 & \quad M_7 \\
\end{align*}
\]

Figure 1. Time-state profile of Flannery’s (1986) model (\( M \) indicates project and firm value)

In such a situation, the choice of debt maturity can play a role in signalling the true quality of firms to the lenders. Flannery argues that it may be in the interest of firms with good projects to borrow short in order to signal their superiority. If they do so, according to Flannery, a separating equilibrium results in which high quality firms issue short term debt while low quality firms issue long-term debt.
2.2 Alternative borrowing strategies

In deriving conditions for the separating equilibrium, Flannery derives the equity value for both types of firms under alternative borrowing strategies, following the binomial process as depicted in figure 1.

If a firm \( i, i \in \{g, b\} \) borrows long the valuation of its equity \((V_l)\) equals

\[
(1) \quad V_l = p_i^2(M_4 - DR_{li}) + 2p_i(1 - p_i)(M_4 - DR_{li}) + (1 - p_i)^2 0 - c
\]

\[
= p_i^2M_4 + 2p_i(1 - p_i)M_4 - p_i(2 - p_i)DR_{li} - c
\]

where \( R_{li} \) is the interest rate\( ^1 \) on long-term debt for firm \( i \) and \( c \) is the transaction cost of long-term debt.

If a firm \( i, i \in \{g, b\} \) uses short-run debt, the valuation \((V_s)\) equals

\[
(2) \quad V_s = p_i^2(M_3 - D) + p_i(1 - p_i)(M_4 - D) + p_i(1 - p_i)(M_4 - DR_{si}) - 2c
\]

\[
= p_i^2M_3 + 2p_iM_4(1 - p_i) - p_iD - p_i(1 - p_i)DR_{si} - 2c
\]

where \( R_{si} \) is the interest rate\( ^2 \) on short-term debt for firm \( i \), and \( 2c \) is the transaction cost of short-term debt.

Under asymmetric information, the decisions on the borrowing strategy of firms will result in two types of equilibria: separating and pooling. Under the separating equilibrium, loans are correctly priced and interest rates should be the same as in the case of perfect information. From Flannery’s (2) and (3) on his page 23, the short- and long-term interest rates are derived as:

\[ ^1 \text{More specifically, } R_i \text{ is the two-period interest factor} \]
\[ ^2 R_s \text{ is the one-period interest factor} \]
By substituting (3), and (4) in (1) and (2), respectively, the equity value for a firm using short or long-term debt can be simplified to:

\[ V_i^S = p_i^2 M_3 + 2 p_i (1 - p_i) M_4 + (1 - p_i)^2 M_5 - D - c = V_i^c - c \]

\[ V_{si}^S = p_i^2 M_3 + 2 p_i (1 - p_i) M_4 + (1 - p_i)^2 M_5 - 2c = V_i - 2c \]

Where \( V_i^S \) and \( V_{si}^S \) represent the equity value of a firm of type \( i \) under separation; in Flannery’s language, \( V_i \) refers to the “intrinsic” value that a firm of type \( i \) can achieve under the full information regime exclusive of the related transaction cost incurred.

Under the pooling equilibrium, from Flannery’s (4) on page 23, the long pooling interest rate \( R_{lp} \) is

\[ DR_{lp} = \frac{D - M_5 E (1 - p)^2}{2E(p) - E(p^2)} \]

which can be rewritten as

\[ R_{lp} = \frac{D - M_5 \left[ \theta(1 - p_g)^2 + (1 - \theta)(1 - p_b)^2 \right]}{D \left[ \theta(2 - p_g) p_g + (1 - \theta)(2 - p_b) p_b \right]} \]

and from Flannery’s (5) on page 24, the short pooling interest rate \( R_{sp} \) is
By substituting (7) and (8) in (1) and (2), respectively, the equity value for a firm \( i \) using short or long-term debt under a pooling strategy can be reduced to:

\[
V^p_{li} = \left[ p_i^2 M_3 + 2 p_i (1 - p_i) M_4 + (1 - p_i)^2 M_5 - D \right] + V^\text{mis}_i(i) - c = V_i + V^\text{mis}_i(i) - c
\]

\[
V^p_{si} = \left[ p_i^2 M_3 + 2 p_i (1 - p_i) M_4 + (1 - p_i)^2 M_5 - D \right] + V^\text{mis}_i(i) - 2c = V_i + V^\text{mis}_i(i) - 2c
\]

In these expressions, \( V^p_{li} \) and \( V^p_{si} \) denote the equity values of firm \( i \) under for long and short pooling equilibria; \( V^\text{mis}_i(i) \) and \( V^\text{mis}_i(i) \) measure a firm \( i \)'s “misinformation value” arisen from such long and short pooling equilibria. These “misinformation values”, according to Flannery, stem from the fact that in a pooling debt market equilibrium insiders and outsiders differ in their assessment of the firm’s “up” probabilities. In other words, \( V^\text{mis}_i(i) \) and \( V^\text{mis}_i(i) \) represent the differences in equity values of a firm of type \( i \) between a pooling strategy and a separating strategy.

From his (7) and (9) on page 25 of Flannery’s article, these misinformation values are derived as:

\[
V^\text{mis}_i(i) = (D - M_5) \frac{\left[ 2(E(p) - p) + (p^2 - E(p)^2) \right]}{2E(p) - E(p)^2}
\]
\[ (12) V_{i}^{mis} = (D - MS) \frac{(1 - p_{b}) [(E(p) - p_{b}) + p_{b}E(p) - E(p)^{2}]}{E(p) - E(p)^{2}} \]

and \( V_{i}^{mis}(g) < V_{i}^{mis}(b) < 0 \)

\( V_{i}^{mis}(b) > V_{i}^{mis}(b) > 0 \)

Combining equity values of both types of firms under alternative borrowing strategies, Flannery constructs a matrix indicating the net benefits from alternative choices, see his table 1 on page 29. To keep track of our subsequent analysis, we present here the equity value corresponding to each borrowing plan by Good firms and Bad firms.

Table 1. Alternative borrowing strategies in Flannery’s (1986) model

<table>
<thead>
<tr>
<th>Good firms’ choice</th>
<th>Borrowing Short</th>
<th>Borrowing Long</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1: Short Pooling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good firms:</td>
<td>( V_{sg}^P = V_{sg} + V_{sg}^{mis}(g) - 2c )</td>
<td></td>
</tr>
<tr>
<td>Bad firms:</td>
<td>( V_{sb}^P = V_{sb} + V_{sb}^{mis}(b) - 2c )</td>
<td></td>
</tr>
<tr>
<td><strong>2: Separating</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good firms:</td>
<td>( V_{sg}^S = V_{sg} - c )</td>
<td>( V_{sg}^I = V_{sg}^I - 2c )</td>
</tr>
<tr>
<td>Bad firms:</td>
<td>( V_{sb}^S = V_{sb} - 2c )</td>
<td>( V_{sb}^I = V_{sb}^I - c )</td>
</tr>
<tr>
<td><strong>3: Separating (?)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good firms:</td>
<td>( V_{sg}^S = V_{sg} - c )</td>
<td></td>
</tr>
<tr>
<td>Bad firms:</td>
<td>( V_{sb}^S = V_{sb} - 2c )</td>
<td></td>
</tr>
<tr>
<td><strong>4: Long Pooling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good firms:</td>
<td>( V_{sg}^L = V_{sg} + V_{sg}^{mis}(g) - c )</td>
<td></td>
</tr>
<tr>
<td>Bad firms:</td>
<td>( V_{sb}^L = V_{sb} + V_{sb}^{mis}(b) - c )</td>
<td></td>
</tr>
</tbody>
</table>
2.3 Derivation of a signalling equilibrium.

The key concept underlying Flannery’s analysis of the behaviours of Good and Bad firms is that Good firms take the first move, and accordingly Bad firms react as a response to the Good firms’ choice. There are only two possible responses by Bad firms, either following the strategy of Good firms or choosing an alternative one. In equilibrium, firms of both types should opt for the strategy that offers them the highest value, given the other firm type’s reaction.

If Good firms borrow long, Bad firms will always follow, resulting in a long pooling equilibrium. This situation is straightforward from the table as one compares cells (3) and (4) for Bad firms’ equity value ($V_{sb}^l < V_{lb}^l$). This outcome constitutes result II of Flannery on his page 29. If Good firms borrow short, the reaction of Bad firms is unclear beforehand as can be seen from cells (1) and (2) for Bad firms’ equity value ($V_{sb}^p < or > V_{lb}^s$). Hence a pooling or a separating outcome may result. Moreover, the choice of Good firms toward borrowing long or short is also indeterminate.

Flannery argues that a separating equilibrium may emerge when Good firms are willing to bear the added transaction cost associated with short-term debt and at the same time Bad firms are unwilling or unable to mimic such a costly behaviour. Thus, such a separation is warranted under certain conditions.

For Good firms to prefer short-term debt over long-term debt (and hence the resulting long pooling equilibrium), the value of Good firms with short-term debt at a separating strategy (cell 2) should be higher than that of Good firms under a long pooling strategy (cell 4):

$V_{sb}^s > V_{lb}^p$
This leads to Flannery’s condition (14) on page 30: \(-c > V_{i}^{mis}(g)\) or \(c < |V_{i}^{mis}(g)|\)

since \(V_{i}^{mis}(g) \leq 0\)

Reacting to the choice of borrowing short by Good firms, Bad firms may be induced to self-select long-term debt rather than misrepresenting Good firms at a short pooling strategy. This outcome arises if the value of Bad firms with long-term debt at a separating strategy (cell 2) is higher than the value of Bad firms with short-term debt at a pooling strategy (cell 1):

\[ (14) \quad V_{sb}^S > V_{sb}^P \]

This comes down to Flannery’s condition (12) on page 29: \(V_{s}^{mis}(b) < c\)

The combination of (13) and (14) thus simplifies to:

\[ (15) \quad V_{s}^{mis}(b) < c < |V_{i}^{mis}(g)| \]

Flannery concludes that a separating equilibrium exists if condition (15) is satisfied. Intuitively, the condition guarantees that it is in Good firms’ interest to incur the cost \(c\) by borrowing short rather than suffering a loss \(V_{i}^{mis}(g)\) in a long pooling strategy. At the same time, it is prohibitively expensive for Bad firms to incur the cost \(c\) and receive \(V_{s}^{mis}(b)\) in return at a short pooling strategy. As a result, Good firms opt for costly short-term debt and Bad firms borrow long, and a separating signalling equilibrium emerges.

For a comparison purpose with the analysis we will provide in section 3, we rewrite condition (15) by fitting the definitions for \(V_{lg}^{mis}\), \(V_{sb}^{mis}\), \(R_{IP}\) and \(R_{sp}\) given by our expressions (11), (12), (7) and (8), respectively. This reduces to the following condition denoted as C1
(16) \( C_1 \leq c \leq C_h \)

Where

\[
C_l = D - (1 - p_g)\delta M_5 - p_b D - p_b(1 - p_b)DR_p
\]

\[
C_h = p_g (2 - p_g)DR_p - D + (1 - p_g)^2 M_5
\]

\( C_l \) and \( C_h \) represent the lower boundary and the upper boundary of cost level, respectively. As argued by Flannery, condition \( C_1 \) provides the threshold values for the extra transaction cost of short-term debt such that both Good and Bad firms prefer a separating strategy over a pooling strategy. The implicit assumption here is that firms of both types are identified as a “pooled” type and thus are charged a pooling interest rate once they resign from a separating strategy. Flannery concludes that if \( C_1 \) holds, a separating equilibrium arises and hence a signalling mechanism works: Good firms borrow costly short-term debt to signal their superiority while Bad firms always borrow long-term debt.

3. The relevance of incentive compatibility constraints

In this section, we attempt to reconsider Flannery’s model with respect to his separating equilibrium. We prove that Flannery omits the incentive compatibility constraints in deriving his separating equilibrium. In other words, the condition of Flannery under which his separating equilibrium arises is not sufficient, since it does not satisfy the incentive compatibility requirement. Consequently, his conclusion regarding the signalling outcome is incorrect on this account.

3.1 The Incentive Compatibility Constraints and a Separating Equilibrium.

We repeat the setup of Flannery’s model in a way that allows us to pinpoint the necessity of incentive compatibility constraints.
There are two types of firms, good \((g)\) and bad \((b)\) seeking funds in the debt market to finance their investment project. Denote type of firm \(i \in \{g, b\}\) these firms differ in their “up” probabilities – probabilities of success of their project \(p_i\), assuming that \(p_g > p_b\). Under the asymmetric information setting, the market only knows ex-ante the overall distribution of firms, i.e. a fraction \(\theta\) of firms are Good firms. The strategy \(t\) of a firm \(i\) is the debt maturity that it chooses; a firm can choose \(t \in \{l, s\}\), i.e. long-term debt and short-term debt, so a firm’s choice is denoted as \(t_i = l\) or \(t_i = s\). A priori, firms know that they have to pay extra transaction cost \(c\) if they issue short-term debt. The interest rate that a firm has to pay depends on the belief that the market has for the type firm it faces. Denote the market’s belief with respect to Good firms is \(\rho\). Upon observing a strategy \(t\), i.e. the debt maturity chosen by the firm, the market induces the belief about the type of firm \(\rho = \rho(t)\) and accordingly sets the interest rate for that type.

This situation is considered as a signalling game with debt maturity choice. The core of a signalling game is to solve for a separating equilibrium (Bolton&Dewatripont, p. 103). By definition, in a separating equilibrium, the observed signal, which is the choice of debt maturity under this framework, should exactly reflect the firm’s type; that is each type of firms chooses a different debt maturity \(t_g \neq t_b\). Observing the signal, the market can thus infer the type of firm: \(\rho(t_g) = 1\) and \(\rho(t_b) = 0\). In a pooling equilibrium, the same strategy is chosen by both types, \(t_g = t_b\), so the market infers every firm as a “pooled” type \(\rho(t) = \theta\). Payoffs of a firm \(i\) are dependent upon its strategy and the respective belief of the market. Denote these payoffs as \(\pi_i(t_i, \rho(t_i))\).

In essence of Flannery’s analysis, payoffs are the equity value of the borrowing firm, which is determined by the chosen debt maturity, the related transaction cost it has to pay and the interest rate required by the market.

For a separating equilibrium to be incentive compatible, the following conditions should hold.
There conditions are known as the incentive compatibility constraints (ICCs). The left hand sides indicate the true payoff of a firm $i$ given its own borrowing strategy, while the right hand sides reflect the firm’s putative payoff given its mimicking strategy designed for the other firm type. Fitting the beliefs of the market as previously specified, these ICCs are formulated as:

$$\pi_g(t_g, \rho(t_g)) \geq \pi_g(t_b, \rho(t_b))$$

$$\pi_b(t_b, \rho(t_b)) \geq \pi_b(t_g, \rho(t_g))$$

ICCs ensure firms to be honest about their type in separation, i.e. the ICCs induce firms to prefer their own strategy rather than coveting a choice of the other type.

3.2 An Omission in Flannery’s Analysis and the Correction

We now turn to derive a missing point in Flannery’s analysis with respect to his conditions for the separating equilibrium. In the framework of Flannery’s model, there are two candidates for a separating equilibrium: one entails Good firms borrowing short and Bad firms borrowing long, while the other involves the inverse strategy. Intuitively, the latter candidate separating equilibrium cannot occur given that short-term debt is costly and therefore, Bad firms will only accept the strategy that coincides with their full information strategy – borrowing long. Consequently, only the former candidate is qualified as a candidate for a separating equilibrium.

Flannery claims that a separating equilibrium exists if the value of firms under a separating equilibrium is higher the value under a pooling equilibrium as derived in our (13) and (14):
where $V_{sg}^S$ and $V_{sb}^S$ denote the value of Good firms and Bad firms under separation, respectively with Good firms borrowing short and Bad firms borrowing long; $V_{lg}^P$ and $V_{sb}^P$ denote value of Good firms under a long pooling strategy and of Bad firms under a short pooling strategy, respectively.

In terms of payoff functions that we have specified, this condition can be rewritten as:

\[
\begin{align*}
\pi_s(s,1) & \geq \pi_s(l,\theta) \\
\pi_b(l,0) & \geq \pi_b(s,\theta)
\end{align*}
\]

Flannery assumes that if a firm deviates from a separating equilibrium, the firm will be assigned to a pooling equilibrium, and hence inducing the market belief $\rho = \theta$. His argument holds if a firm chooses its strategy only as a best-response to the other firm type’s strategy. In the context of a signalling model, this argumentation however, appears to be invalid. Equilibrium requires that the strategy of each firm type is profit-maximizing, given the strategy of the other firm type and given the beliefs of the market (Bolton & Dewatripont, 2005, p. 102). In essence of this model, the market consequently sets the interest rate based on such a belief. Therefore, it is vital to consider the market’s beliefs in deriving a separating equilibrium.

As discussed, the separating equilibrium, if existent, entails Good firms borrowing short and Bad firms borrowing long. Upon observing a deviating action from the separating equilibrium, the market will infer that such an action would only be set by the other type of firms. Hence, the market will conclude that it faces the other type with certainty. Now the correct conditions for the separating equilibrium to arise are:
These conditions are indeed the ICCs required for a separating equilibrium. The ICCs prevent Good firms from pretending to be risky and also deter Bad firms from mimicking Good firms.

### 4. A model extension on the signalling outcome with the choice of debt maturity

We move on in this section with verifying the signalling outcome of Flannery’s (1986) model. To do so, we provide a simple extension based on the same setting as has been defined in the previous section. This extension is imperative in attaining the feasible signalling separating outcome, which is the main purpose of a signalling model. As analyzed previously, the Flannery’s analysis regarding his signalling outcome ends at the conditions that drive a separating equilibrium. Yet, these conditions have been proven as incorrect incentive constraints. Further, the occurrence of a signalling outcome requires more restrictions than just the incentive compatibility constraints. Theoretically, the occurrence of a signalling separating equilibrium should satisfy two requirements, in which one ensures the incentive compatibility while the other guarantees the existence of separation. The first requirement has been fulfilled by introducing the correct incentive compatibility constraints to the Flannery’s separating equilibrium. For the second requirement, we need to validate the existence of a signalling separating equilibrium by ruling out possible pooling equilibria.

#### 4.1 Ruling out the Conceivable Pooling Equilibria

In the framework of Flannery’s model, there are two candidates for a pooling equilibrium, a pooling possibility where both types of firms borrow short-term debt and a pooling possibility where both types of firms borrow long-term debt.
A pooling equilibrium only exists only if it is upheld by an out-of-equilibrium belief (Bolton and Dewatripont, 2005, p.105). For a certain pooling possibility, if there exists a condition under which Bad firms tend to deviate from a pooling possibility no matter what the market believes, such a pooling strategy is not qualified as a pooling equilibrium.

We first consider the short pooling possibility. Bad firms deviate from the short pooling strategy and choose long-term debt instead, if the following condition holds.

\[ (19) \pi_b(l,0) \geq \pi_b(s,\theta) \]

Observing a deviating action from the short pooling possibility, the market may infer a type as Bad firms \( \rho(t_l=0) = 0 \). This is indeed the least favourable out-of-equilibrium belief of the market with respect to Bad firms. If Bad firms obtain a higher equity value under this belief, they do so in all other beliefs. Therefore, the short pooling equilibrium does not exist if condition (19) is satisfied. Note that in Flannery’s analysis, this condition is treated as an incentive constraint for Bad firms at his separating equilibrium (condition (12) on his page 29). We have proven that this condition precludes the short pooling possibility and thereby justifying the existence of the separating equilibrium.

We now look at the long pooling possibility. Assuming the same out-of-equilibrium belief as in the short pooling case, that is \( \rho(t_s=0) = 0 \), Bad firms will not deviate from this pooling, given that they certainly achieve a higher equity value at the long pooling possibility than deviating from the pooling, i.e. they choose short-term debt instead.

\[ (20) \pi_b(s,0) \leq \pi_b(l,\theta) \]

This condition always holds true, supporting the existence of the pooling equilibrium.
Proof

It is straightforward to see that (20) always holds:

\[ \pi_b(s, 0) \leq \pi_b(l, 0) : \text{Short-term debt is more costly than long-term debt at separation} \]

and \( \pi_b(l, 0) \leq \pi_b(l, \theta) : \text{For Bad firms long-term debt at separation is more profitable than at a pooling possibility. So } \pi_b(s, 0) \leq \pi_b(l, \theta) \text{ is always true.} \)

4.2 Refining the Long Pooling Equilibrium.

We now carefully consider the long pooling equilibrium with respect to its stability. It is argued that some pooling equilibria may not be stable under certain refinements and thus should be ruled out. In this analysis, we apply the so-called Intuitive Criterion (IC) (Cho and Kreps, 1987) to refine the long pooling equilibrium. More specifically, we conduct conditions under which the long pooling equilibrium does not satisfy the Intuitive Criterion. By definition, under the IC, if a firm i could not benefit from the out-of-equilibrium action no matter what beliefs were held by the market, the market’s belief must put zero probability on that type (Rasmusen, 2001).

The long pooling equilibrium is characterized as follows. Both types choose long-term debt \( t_g = t_b = l \). Rationally, the market infers every firm as a “pooled” type \( \rho(t_i) = \theta \) and hence charges the pooling loan rate of \( R_p \) to all borrowing firms. An out-of-equilibrium belief that upholds this pooling is \( \rho(t_i = s) = 0 \). Under this belief, no firm will deviate from the long pooling equilibrium to switch to short-term debt, given the signalling cost they must pay and the lower payoff they will obtain \((\pi_i(s, 0) \leq \pi_i(l, \theta))\). Therefore, the long pooling equilibrium pooling exists under the market’s belief as specified.
By introducing the *Intuitive Criterion*, we will consider whether or not such a belief is reasonable. If a certain condition exists such that the specified belief is not intuitive, the long pooling equilibrium does not survive the *Intuitive Criterion* and thus will be precluded.

For both types of firms, the only option to deviate from the long pooling is to choose short-term debt, which imposes the transaction cost on firms. As to Good firms, by deviating they wish to convince the market to believe in their true quality. So, deviation is a desirable choice for Good firms if the following condition holds:

\[(21) \quad \pi_g(s, 1) \geq \pi_g(l, \theta)\]

As for Bad firms, by deviating they wish to fool the market into believing them as a Good type. Bad firms will be indifferent about pooling and deviating if the following condition holds:

\[(22) \quad \pi_b(s, 1) \leq \pi_b(l, \theta)\]

The combination of (21) and (22) is referred as the *Intuitive Criterion* that justifies the stability of the long pooling equilibrium. Effectively, if (21) and (22) hold simultaneously, Good firms are able to convince the market that they are indeed better off by the out-of-equilibrium action while Bad firms are not. In order to support a deviating action by good firms, the reasonable out-of-equilibrium belief of the market should be \(\rho(t_i = s) = 1\). In other words, the out-of-equilibrium belief as specified \(\rho(t_i = s) = 0\) in the pooling definition appears to be unreasonable. Hence, the long pooling equilibrium fails to meet the *Intuitive Criterion* and should be precluded. It should be noted that, in his analysis, Flannery considers our condition (21) (or his condition (14) analogously) as the incentive constraint for Good firms to choose short-term debt over long-term debt.
4.3 The Feasible Signalling Outcome

The feasible incentive compatible separating equilibrium will result if the separating equilibrium is both incentive compatible and feasible. In other words, the conditions implied by the Incentive Compatibility Constraint and the conditions that rule out the pooling equilibria including the Intuitive Criterion should be satisfied. So, combining the conditions (17) to (22), exclusive of (20)\(^3\), we establish the following result.

**Result 1:** A signalling separating equilibrium under which Good firms signal by borrowing costly short-term debt while Bad firms borrow long-term debt will arise under the following condition:

\[
\begin{align*}
&\pi_g(s,1) \geq \pi_g(l,0) \\
&\pi_g(l,0) \geq \pi_g(s,1)
\end{align*}
\]

The first equation is retrieved from the condition under the Intuitive Criterion for Good firm, inducing Good firms to resign from the long pooling equilibrium. The second equation is obtained from the condition under the Incentive Compatibility Constraint for Bad firms, restricting the incentive of Bad firms’ to mimic the Good type.

**Proof:** Appendix A

Since payoffs denote the firm equity values, condition (23) can be specified as:

\[
\begin{align*}
&S_g \geq V_{lg}^S \\
&S_b \geq V_{lb}^S \geq V_{mimic}^S
\end{align*}
\]

\(V_{sg}^S\) and \(V_{lb}^S\) are the equity values for Good firms and Bad firms under the separating equilibrium where Good firms borrow short and Bad firms borrow long. \(V_{lg}^P\) is the

\(^3\)Condition (20) is unbinding irrespective of other parameters. So, it is excluded.
equity value for Good firms under the long pooling equilibrium and $V_{sb}^{mimic}$ is the putative equity value of Bad firms as they mimic the Good type in the long pooling strategy. Fitting all the relevant terms defined in section 2, the system can be simplified to the following condition denoted as \(C2\):

\[
(25) \quad C2 \quad C_d \leq c \leq C_h
\]

where

\[
C_d = (D-M_h) \frac{(p_g-p_s)(1-p_s)}{p_s}
\]

\[
C_h = p_s(2-p_g)DR_p - D + (1-p_s)^2 M_5
\]

Condition \(C2\) implies that costs of short-term debt should be higher than a certain threshold to make it unattractive for Bad firms to mimic Good firms at a signalling separating equilibrium, and should be lower than another threshold to motivate Good firms to incur a costly signalling behaviour.

**4.4 The Impact of Reconsidering the Signalling Outcome**

We have proven that in the presence of asymmetric information on firm quality, and of transaction cost for short-term debt, a signalling outcome with Good firms borrowing short will occur if \(C2\) holds. In this section, we will examine to what extent our condition \(C2\) differs from the condition \(C1\) under which Flannery claims that a financial signalling equilibrium will be attained. For a comparison purpose, we rewrite both conditions here:

\[
C1 \quad C_l \leq c \leq C_h \quad \text{and} \quad C2 \quad C_d \leq c \leq C_h
\]

where

\[
C_h = p_g(2-p_g)DR_p - D + (1-p_g)^2 M_5
\]
Both conditions converge with respect to their upper boundary, i.e. $C_h$. Regarding the lower boundaries, we have $0 \leq C_f \leq C_d$, irrespective of other parameters.

**Proof**: Appendix B

Therefore, a parameter space under $C_2$ is clearly more restrictive than that under $C_1$. As a consequence, the introduction of $C_2$ restricts the occurrence of the separating signalling outcome as suggested by Flannery.

In summary, our extension brings out an important implication to the signalling outcome of the Flannery’s model. The conditions for the occurrence of a signalling equilibrium now become more stringent with respect to the lower threshold of the extra cost of short-term debt. Thus, it will be less likely that debt maturity can be used as a signalling instrument than argued by Flannery (1986).

5. **Conclusions**

This paper verifies the signalling role of debt maturity by means of investigating the well-know signalling model “Asymmetric Information and Risky Debt Maturity Choice” by Flannery (1986). We emphasize the relevance of incentive compatibility and of the refinements of pooling equilibria in deriving a separating equilibrium. In addition to a review on Flannery’s (1986) model, we have first demonstrated that the analysis of Flannery suffers from an important omission in that he does not consider the incentive compatibility constraints for his separating equilibrium. We have addressed this drawback by adding the incentive compatibility constraints to Flannery’s model. We have later argued that a signalling outcome is not yet warranted given our correct incentive compatibility constraints. We then provided an extension to the Flannery’s model where we focused on the occurrence of a signalling
outcome. We do so by considering the existence of separating equilibria in addition to the incentive compatibility constraints. We proved that the condition, for which a signalling separating equilibrium arises, is more restrictive than suggested by Flannery.

References


APPENDIX A: Proof of Result 1

The feasible incentive compatible separating equilibrium requires the following conditions to hold

\[
\begin{align*}
(17) & \pi_g(s,1) \geq \pi_g(l,0) \\
(18) & \pi_b(l,0) \geq \pi_b(s,1) \\
(19) & \pi_b(l,0) \geq \pi_b(s,\theta) \\
(21) & \pi_g(s,1) \geq \pi_g(l,\theta) \\
(22) & \pi_b(s,1) \leq \pi_b(l,\theta)
\end{align*}
\]

Recall that (17) and (18) refer to the ICCs required for a separating equilibrium; (19) guarantees the non-existence of a certain pooling equilibrium, (21) and (22) represent the IC, which justify the stability of the long pooling equilibrium.

Given that \( \pi_g(l,\theta) \geq \pi_g(l,0) \), i.e. for Good firms, the equity value generated at the long pooling possibility is always greater than the value of pretending as Bad firms at the separating possibility. Therefore, condition (17) always holds if condition (21) holds. As a result, condition (17) drops out of the system.

Similarly, we have \( \pi_b(s,1) \geq \pi_b(s,\theta) \) since the reverse holds for Bad firms, i.e. Bad firms choose mimicking Good firms at the separating possibility rather than staying at the short pooling possibility. Therefore, condition (19) always holds if condition (18) holds. As a result, condition (19) drops out of the system.

In addition, we also have \( \pi_b(l,\theta) \geq \pi_b(l,0) \), i.e. for Bad firms; the equity value generated at the long pooling possibility is always greater than the true value at a separating possibility. Thus, condition (22) is unbinding as long as condition (18) holds. As a result, condition (22) drops out of the system.

Excluding (17), (19) and (22), the system can now be reduced to:

\[
\begin{align*}
\pi_g(s,1) & \geq \pi_g(l,\theta) \\
\pi_b(l,0) & \geq \pi_b(s,1)
\end{align*}
\]
APPENDIX B: Proof $0 \leq C_I \leq C_d$

\[ C_I = D - (1 - p_b)^2 M_S - p_b D - p_b (1 - p_b) D R_{SP} \]
\[ C_d = (D - M_s) \frac{(p_g - p_b) (1 - p_b)}{p_g} \]

Where
\[ R_{SP} = \frac{D - D(\theta p_g + (1 - \theta) p_b) - M_S \left[ \theta (1 - p_g)^2 + (1 - \theta)(1 - p_b)^2 \right]}{D \left[ \theta (1 - p_g) p_g + (1 - \theta)(1 - p_b) p_b \right]} \]

One can see that $R_{SP}$ is a decreasing function of $\theta$. Given that $\theta$ is the proportion of Good firms in the market, a rise in $\theta$ will reduce the level of riskiness of the entire pool of borrowers. Accordingly, the pooling interest rate charged by the market will decrease.

Since $C_I$ is a decreasing function of $R_{SP}$, $C_I$ increases in $\theta$.

With $\theta = 0$, $R_{SP} = \frac{D - D p_b - M_S (1 - p_b)^2}{D (1 - p_b) p_b}$, leading to $C_I$

\[ C_I = D - (1 - p_b)^2 M_S - p_b D - p_b (1 - p_b) D \frac{D - D p_b - M_S (1 - p_b)^2}{D (1 - p_b) p_b} = 0 \]

With $\theta = 1$, we have $R_{SP} = \frac{D - D p_g - M_S (1 - p_g)^2}{D (1 - p_g) p_g} = \frac{D - M_S (1 - p_g)^2}{D p_g}$, $C_I$ will converge to $C_d$

\[ C_I = D - (1 - p_b)^2 M_S - p_b D - p_b (1 - p_b) D \frac{D - M_S (1 - p_g)^2}{D p_g} = (D - M_S) \frac{(p_g - p_b) (1 - p_b)}{p_g} = C_d \]

So $0 \leq C_I \leq C_d$, is proved.