Meson-Baryon coupling constants in QCD sum rules
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4.1 The Vector Mesons

An important ingredient of the baryon-baryon interactions is the exchange of the members of vector-meson octet (\( \rho, \phi, \omega, K^* \)). Vector mesons play also a special role in the electromagnetic interaction of hadrons. The Vector-Meson Dominance (VMD) model [97] relates the hadronic electromagnetic current to the neutral vector-meson fields \( V_\mu = \rho_\mu, \omega_\mu \) and \( \phi_\mu \). In this context, the vector-meson–baryon coupling constants are among the most fundamental quantities that one would like to compute from QCD. The Lagrangian density for the interaction of a vector-meson with a spin-1/2 baryon is given by

\[
\mathcal{L}_{VBB} = -ig_{V}^{B} \bar{\psi} \gamma_\mu \psi V_\mu - \frac{f_{V}^{B}}{4m} \bar{\psi} \sigma_{\mu\nu} \psi (\partial_\mu V_\nu - \partial_\nu V_\mu), \tag{4.1}
\]

where the first term \( (g_{V}^{B}) \) is called the vector (electric) coupling and the second one \( (f_{V}^{B}) \) the tensor (magnetic) coupling. \( m \) is a scaling mass to take \( f_{V}^{B} \) dimensionless, conventionally taken to be equal to the proton mass.
The physical states $\phi$ and $\omega$ are mixtures of the unitary singlet and the octet states analogous to Eq. (3.2). We will assume ideal-mixing with the mixing angle $\theta_v = 35.3^\circ$, which is close to the experimental value, $\theta_v = 37.5^\circ$ [69]. This means that the $\phi$-meson is a pure $\bar{s}s$ state, and hence does not couple to the nucleon. As in Eq. (2.50), the couplings of the vector mesons to the baryon octet can be written in terms of the $NN\rho$ coupling constant and $\alpha_{v,m}$ [14], where $\alpha_v$ ($\alpha_m$) is the $F/(F+D)$ ratio of the vector (magnetic) coupling constants. VMD predicts $\alpha_v = 1$ via the universal coupling of the $\rho$-meson to the isospin current [98].

Our aim in this chapter is to calculate the vector and the tensor coupling constants of the vector mesons $\rho$ and $\omega$ to the $N$, $\Lambda$, $\Xi$, and $\Sigma$ baryons using the external-field QCDSR. For this purpose, we assume a constant background tensor field $Z_{\mu\nu}$ and evaluate the vacuum-to-vacuum transition matrix element of two baryon interpolating fields to construct the sum rules. We define the external vector-meson field as

$$Z_\mu = -\frac{1}{2}Z_{\mu\nu}x^\nu. \quad (4.2)$$

The background field can be decomposed into symmetric ($Z^{S}_{\mu\nu}$) and antisymmetric ($Z^{A}_{\mu\nu}$) parts. The antisymmetric part has been used to calculate the baryon magnetic moments [13, 52–54] while the symmetric part was used in Ref. [63] to determine the vector-meson couplings $g_{N}^{\rho}$ and $f_{N}^{\omega}$. In this work, we use a similar method to calculate the vector-meson–baryon coupling constants. We note that the sum rules for the antisymmetric part of the external field can be obtained from the sum rules for the baryon magnetic moments in Refs. [13,52–54], but the numerical results for the couplings cannot be obtained trivially, since they need an independent analysis taking into account the sum rules for the symmetric part of the external field as well. This analysis was made in Ref. [63] with the aim to calculate the $NN\rho$ and $NN\omega$ couplings. We find it useful to revisit these calculations for a couple of reasons. First, we make a more systematic analysis of the sum rules including the single-pole contributions which were not taken into account in Ref. [63]. Moreover, we extend the calculations to hyperons as well by calculating terms involving the quark mass in the sum rules. We compare our
results with VMD and a successful One Boson Exchange (OBE) model of the $NN$ and $YN$ interaction, the Nijmegen soft-core potential (NSC) [1–5, 16].

We follow an analysis similar to the one in Chapter 3 on scalar-meson–baryon coupling constants. We shall first consider the sum rules in the SU(3)-flavor symmetric limit to see if the predicted values for the meson-baryon coupling constants from the sum rules are consistent with the SU(3) relations. We show that this is indeed the case which leads to a determination of the $F/(F + D)$ ratio of the vector-meson octet. Furthermore, keeping track of these coupling constants with the SU(3) relations, we obtain the values of the other vector-meson–baryon coupling constants where we assume ideal-mixing. As we move from the $S = 0$ to the $S = −1$ and $S = −2$ sectors, the flavor-SU(3)-breaking occurs as a result of the $s$-quark mass and the physical masses of the baryons and mesons. We also consider the SU(3)-breaking effects for the sum rules to estimate the amount of breaking, individually for each coupling.

### 4.2 Construction of the Sum Rules

We start with the correlation function of the baryon interpolating fields in the presence of a constant background tensor field $Z_{\mu \nu}$, defined by

$$i \int d^4 x \ e^{ip \cdot x} \left< 0 \left| \mathcal{F} \left[ \eta_B(x) \bar{\eta}_B(0) \right] \right| 0 \right> Z = \Pi(p) + g_q^V Z_{\mu \nu} \Pi_{Z}^{\mu \nu}(p) ,$$

where $g_q^V$ is the vector-meson–quark coupling constant and $\eta_B$ are the baryon interpolating fields which are chosen as in Eq. (3.9).

The external field contributes to the correlation function in Eq. (4.3) in two ways: first, it directly couples to the quark field in the baryon current. Second, it induces the following vacuum condensates:

$$\langle \bar{q} \sigma_{\mu \nu} q \rangle_Z = g_q^V \chi Z_{\mu \nu}^A \langle \bar{q} q \rangle ,$$

$$g_c \langle \bar{q} G_{\mu \nu} q \rangle_Z = g_q^V \kappa Z_{\mu \nu}^A \langle \bar{q} q \rangle ,$$

$$g \varepsilon_{\mu \nu \alpha \beta} \langle \bar{q} \gamma_5 G_{\alpha \beta} q \rangle_Z = i g_q^V \xi Z_{\mu \nu}^A \langle \bar{q} q \rangle .$$
\[
\langle \bar{q} \frac{1}{2} (\gamma_\mu \nabla_\nu + \gamma_\nu \nabla_\mu) q \rangle_Z = g^V_q \xi Z^S_{\mu\nu} \langle \bar{q} q \rangle,
\]

where \((\chi, \kappa, \xi)\) and \((\zeta, \phi)\) are the susceptibilities related to \(Z^A_{\mu\nu}\) and \(Z^S_{\mu\nu}\), respectively. These susceptibilities are defined in terms of the vector-meson–quark coupling constants \(g^V_q\), where we assume

\[
g^\omega_u = g^\omega_d = g^\omega_q,
\]

for the isospin \(I = 0\) \(\omega\)-current and

\[
g^\rho_u = -g^\rho_d = g^V_q,
\]

for the isospin \(I = 1\) \(\rho\)-current. For the couplings of the external field to the \(s\)-quark we assume

\[
g^\omega_s = g^\rho_s = 0,
\]

which is based on the OZI rule [99–101]. Note that Eq. (4.9) and Eq. (4.10) can be justified from the degeneracy and the equal decay constants of the \(\rho\)- and the \(\omega\)-mesons [9–11] using the Current Field Identities and the VMD.

At the quark level, the two-point correlator can be brought into the form as in Eq. (3.15) and Eq. (3.25), where \(S_q\) represents the quark propagator in the external tensor field \(Z_{\mu\nu}\);

\[
S_q(x) = \delta^{(0)}(x) + S_q^{(Z)}(x).
\]

\(S_q^{(0)}(x)\) is given in Eq. (2.14) whereas \(S_q^{(Z)}(x)\) is calculated as [13, 63]

\[
i S_q^{(Z)ab} = \langle 0 | T[q^a(x)q^b(0)] | 0 \rangle_Z
= g^V_q \left[ \frac{i \delta^{ab}}{32\pi^2 x^2} Z_{\mu\nu}(\hat{x} \sigma^{\mu\nu} + \sigma^{\mu\nu} \hat{x}) - \frac{\delta^{ab}}{8\pi^2 x^4} Z_{\mu\nu\xi} x^{\mu} x^{\nu} \hat{x} \right]
\]
4.2. Construction of the Sum Rules

\[ \delta_{ab} \frac{288}{\bar{q}q} Z_{\mu\nu} (\chi^2 \sigma^{\mu\nu} - 2x_\alpha x^\nu \sigma^{\alpha\mu}) - \frac{\delta_{ab}}{12} \zeta Z_{\mu\nu} x^\mu y^\nu \]

\[ - \frac{\delta_{ab}}{24} \chi Z_{\mu\nu} \sigma^{\mu\nu} \langle \bar{q}q \rangle - \frac{i\delta_{ab}}{48} (1 + \phi) \langle \bar{q}q \rangle Z_{\mu\nu} x^\mu x^\nu \]

\[ + \frac{\delta_{ab}}{576} \langle \bar{q}q \rangle Z_{\mu\nu} [x^2 (\kappa + \xi) \sigma^{\mu\nu} - x_\alpha x^\nu (2\kappa - \xi) \sigma^{\alpha\mu}] \]

\[ + \frac{i\delta_{ab}}{48} \phi \langle \bar{q}q \rangle Z_{\mu\nu} x^\mu y^\nu \] + \(O(Z^2)\).

Lorentz covariance and parity conservation implies that the correlation function can be written in terms of different Lorentz-Dirac structures,

\[ g_\nu^v \Pi^\mu^v (p) = \Pi_1^S (p_\mu \gamma_v + p_v \gamma_\mu) + \Pi_2^S \hat{p} p_\mu p_v + \Pi_3^S p_\mu p_\nu + \Pi_4^S \hat{p} (p_\mu \gamma_v + p_v \gamma_\mu) \]

\[ + \Pi_1^A (\hat{p} \sigma_{\mu\nu} + \sigma_{\mu\nu} \hat{p}) + \Pi_2^A \hat{p} (p_\mu \gamma_v - p_v \gamma_\mu) + \Pi_3^A \sigma_{\mu\nu}, \quad (4.14) \]

where \(\Pi_1^S, \Pi_2^S, \Pi_3^S\) and \(\Pi_4^S\) are related to the symmetric part of the external field and \(\Pi_1^A, \Pi_2^A, \Pi_3^A\) are related to the antisymmetric part of the external field. For the antisymmetric part of the external field, we construct the sum rules at the structure \((\hat{p} \sigma_{\mu\nu} + \sigma_{\mu\nu} \hat{p})\), which have also been used for the determination of the baryon magnetic moments [13, 52–54]. Using the quark propagator above, we obtain

\[ \Pi_1^{A(N)} = \frac{1}{16\pi^4} \left\{ \frac{g_\nu^v}{2} p^2 \ln(-p^2) - \frac{a_\nu^2}{18p^4} \left[ -(2g_\nu^u + 3g_\nu^d) + g_\nu^v (2\kappa - \xi) \right] \right. \]

\[ - \frac{g_\nu^v}{3} \frac{a_\nu^2}{p^4} \left[ p^2 + \frac{m_0^2}{8} \right] + \frac{b}{48p^2} (4g_\nu^u + g_\nu^d) \right\}, \quad (4.15) \]

\[ \Pi_1^{A(\Sigma)} = \frac{g_\nu^v}{16\pi^4} \left\{ \frac{p^2}{2} \ln(-p^2) - \frac{a_\nu^2}{18p^4} (-2 + 2\kappa - \xi) - \frac{\chi a_\nu^2}{3p^4} \left[ p^2 + \frac{m_0^2}{8} \right] \right. \]

\[ + \frac{b}{12p^2} (4g_\nu^u + g_\nu^d) \right\}, \quad (4.16) \]
\[ \Pi_1^{A(\Xi)} = \frac{g_v}{16\pi^4} \left[ \frac{a_q^2}{6p^4} + \frac{b}{48p^2} + \frac{3}{16p^2} m_s a_q (1 + f) \right], \]

\[ \Pi_1^{A(\Lambda)} = \frac{g_v + g_d^\prime}{16\pi^4} \left\{ -\frac{p^2}{12} \ln(-p^2) - \frac{a_q^2}{54p^4} \left[ 7 + 8f - \frac{1}{2} (2\kappa - \xi) (1 + 2f) \right] \\
+ \frac{\chi}{54} \frac{a_q^2}{p^4} \left[ p^2 + \frac{m_0^2}{8} (1 + 2f) \right] + \frac{8b}{243p^2} \\
+ \frac{8}{243p^2} m_s a_q [19 - 8(2\kappa - \xi)] \right\}, \]

for \( N, \Sigma, \Xi \) and \( \Lambda \), respectively. Here \( a_\bar{q} = -(2\pi)^2 \langle \bar{q}q \rangle \) and we have defined \( f = \langle \bar{q}q \rangle / \langle \bar{s}s \rangle - 1 \), which is a parameter that quantifies SU(3)-breaking in the vacuum condensates.

For the symmetric part of the Lagrangian, we construct the sum rules at the structures \( p_\mu \gamma^\nu + p_\nu \gamma^\mu \) (hereafter structure I) and \( \hat{p} p_\mu p_\nu \) (hereafter structure II), for reasons that will become clear below. The OPE sides at the structure I are calculated as:

\[ \Pi_1^{S(N)} = \frac{i}{8\pi^4} \left\{ \frac{1}{8} (2g_u^\nu + g_u^\nu) p^2 \ln(-p^2) + \frac{a_q}{3} \zeta (4g_u^\nu + g_d^\nu) \ln(-p^2) \\
- \frac{a_q^2}{6p^4} (2g_u^\nu + g_d^\nu) \right\}, \]

\[ \Pi_1^{S(\Sigma)} = \frac{i g_u^\nu}{8\pi^4} \left\{ \frac{1}{4} p^2 \ln(-p^2) + \frac{4a_q}{3} \zeta \ln(-p^2) - \frac{a_q^2}{3p^4} - \frac{a_q}{2p^2} (f + 1) m_s \right\}, \]

\[ \Pi_1^{S(\Xi)} = \frac{i g_u^\nu}{8\pi^4} \left\{ \frac{1}{8} p^2 \ln(-p^2) + \frac{a_q}{3} \zeta \ln(-p^2) - \frac{a_q^2}{6p^4} \right\}, \]
4.2. Construction of the Sum Rules

\[ \Pi_{S}(\Lambda)_{1} = \frac{i(g_{u}^{v} + g_{d}^{v})}{8\pi^{4}} \left\{ \frac{1}{8} p^{2} \ln(-p^{2}) + \frac{4}{9} \zeta a_{q} \ln(-p^{2}) - \frac{a_{q}^{2}}{18p^{4}} (4f + 3) \right\} \]

(4.22)

At structure II, the OPE sides of the sum rules are given as:

\[ \Pi_{S}(N)_{2} = \frac{i}{4\pi^{4}} \left\{ \frac{1}{8} (2g_{u}^{v} + g_{d}^{v}) \ln(-p^{2}) + \frac{a_{q}}{3p^{2}} \zeta (g_{u}^{v} + g_{d}^{v}) + \frac{a_{p}^{2}}{3p^{6}} (2g_{u}^{v} + g_{d}^{v}) \right\} , \]

(4.23)

\[ \Pi_{S}(\Sigma)_{2} = \frac{ig_{u}^{v}}{4\pi^{4}} \left\{ \frac{1}{4} \ln(-p^{2}) + \frac{a_{q}}{3p^{2}} \zeta + \frac{2a_{q}^{2}}{3p^{6}} (f + 1)m_{s} \right\} , \]

(4.24)

\[ \Pi_{S}(\Xi)_{2} = \frac{ig_{d}^{v}}{4\pi^{4}} \left\{ \frac{1}{8} \ln(-p^{2}) + \frac{a_{q}}{3p^{2}} \zeta + \frac{a_{q}^{2}}{3p^{6}} \right\} , \]

(4.25)

\[ \Pi_{S}(\Lambda)_{2} = \frac{i(g_{u}^{v} + g_{d}^{v})}{4\pi^{4}} \left\{ \frac{1}{8} \ln(-p^{2}) + \frac{5a_{q}}{18p^{2}} \zeta + \frac{a_{q}^{2}}{9p^{6}} (4f + 3) \right\} + \frac{a_{q}}{12p^{4}} m_{s}(1 - 3f) \right\} . \]

(4.26)

In order to construct the hadronic side, we saturate the correlator in Eq. (4.3) with baryon states

\[ \Pi_{Z}^{\mu\nu}(p) = \left\langle 0|\eta_{B}|B\right\rangle \left\langle B|V B\right\rangle \left\langle B|\bar{\eta}_{B}|0\right\rangle \frac{p^{2} - m_{B}^{2}}{p^{2} - m_{B}^{2}} , \]

(4.27)

and define the vector-meson–baryon interaction by the following vertices:

\[ \Gamma_{\omega BB} \equiv \left\langle B|\omega B\right\rangle = \bar{u} \left( g_{B}^{\omega} \gamma_{\mu} + f_{B}^{\omega} \frac{i}{2m} \sigma_{\mu\nu} q^{\nu} \right) v \cdot \omega^{\mu} , \]

(4.28)
\[ \Gamma_{\rho BB} \equiv \langle B | \rho B \rangle = \bar{\nu} \left( g_B^\rho \gamma_\mu + f_B^\rho \frac{i}{2m} \sigma_{\mu\nu} q^\nu \right) \tau \nu \cdot \rho^\mu. \] (4.29)

The sum rules are obtained by matching the OPE side with the hadronic side and applying the Borel transformation. The sum rules for \( N, \Sigma, \Xi, \) and \( \Lambda \) are given as follows at the structure I:

\[
\begin{align*}
M^6 E_1^N L^{-4/9} & \left( 2g^V_u + g^V_d \right) + \frac{8M^4}{3} E_0^N L^{2/9} \zeta a_q \left( 4g^V_u + g^V_d \right) \\
&+ \frac{4}{3} a_q^2 L^{4/9} \left( 2g^V_u + g^V_d \right) \frac{e^{m^2/M^2}}{\lambda^2_N} = g^V_N + C_N M^2,
\end{align*}
\] (4.30)

\[
\begin{align*}
g^V_u & \left[ 2M^6 E_1^\Sigma L^{-4/9} + \frac{32M^4}{3} E_0^N L^{2/9} \zeta a_q g^V_u + \frac{8}{3} a_q^2 L^{4/9} \right] \\
&- 4m_s (f + 1) a_q M^2 \frac{e^{m^2/M^2}}{\lambda^2_\Sigma} = g^V_\Sigma + C_\Sigma M^2,
\end{align*}
\] (4.31)

\[
\begin{align*}
g^V_u & \left[ M^6 E_1^\Xi L^{-4/9} + \frac{8M^4}{3} E_0^N L^{2/9} \zeta a_q g^V_d \right] \\
&+ \frac{4}{3} (f + 1)^2 a_q^2 L^{4/9} \frac{e^{m^2/M^2}}{\lambda^2_\Xi} = g^V_\Xi + C_\Xi M^2,
\end{align*}
\] (4.32)

\[
\begin{align*}
(g^V_u + g^V_d) & \left[ M^6 E_1^\Lambda L^{-4/9} + \frac{32}{9} M^4 E_0^N L^{2/9} \zeta a_q + \frac{4}{9} (4f + 3) a_q^2 L^{4/9} \right] \\
&+ \frac{2}{3} m_s (1 - 3f) a_q M^2 \frac{e^{m^2/M^2}}{\lambda^2_\Lambda} = g^V_\Lambda + C_\Lambda M^2,
\end{align*}
\] (4.33)

and at the structure II:

\[
\left[ M^6 E_0^N L^{-4/9} \left( 2g^V_u + g^V_d \right) + \frac{8M^4}{3} \zeta L^{2/9} a_q (g^V_u + g^V_d) \right]
\] (4.34)
4.2. Construction of the Sum Rules

\[
+ \frac{4}{3} a_q^2 L^{4/9} (2 g_u^V + g_d^V) \left[ \frac{e^{m^2_{\lambda}/M^2}}{\tilde{\lambda}_N^2} \right] = g_N^V + C_N M^2, \tag{4.35}
\]

\[
g_u^V \left[ 2 M^6 E_0^\Sigma L^{-4/9} + \frac{8 M^4}{3} \zeta L^{2/9} a_q g_u^V + \frac{8}{3} a_q^2 L^{4/9} \right]
\]

\[- 4 m_s (f + 1) a_q M^2 \left[ \frac{e^{m^2_{\lambda}/M^2}}{\tilde{\lambda}_N^2} \right] = g_S^V + C_S M^2, \tag{4.36}
\]

\[
(g_u^V + g_d^V) \left[ M^6 E_0^\Xi L^{-4/9} + \frac{10}{9} M^4 L^{2/9} \xi a_q + \frac{4}{9} (4 f + 3) a_q^2 L^{4/9} \right]
\]

\[+ \frac{2}{3} m_s (1 - 3 f) a_q M^2 \left[ \frac{e^{m^2_{\lambda}/M^2}}{\tilde{\lambda}_N^2} \right] = g_\Lambda^V + C_\Lambda M^2, \tag{4.37}
\]

which we use for the determination of the vector couplings \( g \).

The sum rules involving the antisymmetric part of the external field can easily be derived from the magnetic moment sum rules in Refs. [13, 52–54]. We use Eqs. (4.9)-(4.11) with the sum rules at the structure \((\hat{\rho} \sigma_{\mu \nu} + \sigma_{\mu \nu} \hat{\rho})\), which were also used for the determination of the magnetic moments. We obtain:

\[
\left\{ \begin{aligned}
4 M^6 E_1^N L^{-4/9} g_u^V + \frac{4}{9} a_q^2 L^{4/9} [- (2 g_u^V + 3 g_d^V) + g_u^V (2 \kappa - \xi)] \\
+ \frac{b}{6} M^2 L^{-4/9} (4 g_u^V + g_d^V) - \frac{8}{3} \chi a_q^2 L^{-4/9} g_u^V \left[ M^2 - \frac{m_0^2 L^{-4/9}}{8} \right] \right\} \frac{e^{m^2_{\lambda}/M^2}}{\tilde{\lambda}_N^2}
\right. 
\]

\[= (g_N^V + f_N^V) + C_N M^2, \tag{4.38}
\]
Chapter 4. Vector-Meson–Baryon Couplings in QCD Sum Rules

\[
g_V^u \left\{ 4M^6 E_1^\Sigma L^{-4/9} + \frac{4}{9} a_q^2 L^{4/9} \left[ -(2 + (2\kappa - \xi)) + \frac{2b}{3} M^2 L^{-4/9} \right] \right\} \frac{e^{m_\Sigma^2 / M^2}}{\tilde{\lambda}_\Sigma^2} = (g_\Sigma^V + f_\Sigma^V) + C_\Sigma M^2 ,
\]

\[
g_V^u \left\{ -\frac{4}{3} a_q^2 L^{4/9} + \frac{b}{6} M^2 L^{-4/9} + 24 m_s a_q (1 + f) M^2 \right\} \frac{e^{m_\Sigma^2 / M^2}}{\tilde{\lambda}_\Sigma^2} = (g_\Sigma^V + f_\Sigma^V) + C_\Sigma M^2 ,
\]

\[
(g_V^u + g_d^V) \left\{ \frac{2}{3} M^6 E_1^\Lambda L^{-4/9} - \frac{4}{27} a_q^2 L^{4/9} [7 + 8f - \frac{1}{2}(2\kappa - \xi)(1 + 2f)] \right\}
\]

\[
- \frac{4}{9} \kappa a_q^2 L^{-4/9} \left[ M^2 - \frac{m_0^2 L^{-4/9}}{8} \right] (1 + 2f) + \frac{2b}{9} M^2 L^{-4/9}
\]

\[
- \frac{2}{9} m_s M^2 a_q [19 L^{-4/9} - 8(2\kappa - \xi)] \right\} \frac{e^{m_\Lambda^2 / M^2}}{\tilde{\lambda}_\Lambda^2} = (g_\Lambda^V + f_\Lambda^V) + C_\Lambda M^2 .
\]

In the above sum rules, the continuum contributions are included by the factors

\[
E_n^B \equiv 1 - (1 + x_B + \ldots + \frac{x_B^n}{n!}) e^{-x_B} ,
\]

with \( x_B = \sigma_0^B / M^2 \), where \( \sigma_0^B \) is the continuum threshold. We have included the single-pole contributions with the factors \( C^{(i)}_B \).

### 4.3 Analysis of the Sum Rules

To proceed to the numerical analysis, we arrange the RHS of the sum rules in the form

\[
f(M^2) \equiv F + C_B^{(i)} M^2 ,
\]
and fit the LHS to $f(M^2)$. We determine the $g$ couplings from the sum rules in Eqs. (4.30)-(4.33), while we subtract these sum rules from the ones in Eqs. (4.38)-(4.41) in order to obtain the sum rules for the $f$ couplings. For the vacuum parameters, we adopt $a_q = 0.51 \pm 0.03$ GeV$^3$, $m_0^2 = 0.8$ GeV$^2$ [94] as in Chapter 3 and the average values of the susceptibilities $\chi = -4.5$ GeV$^{-2}$, $\kappa = 0.4$ and $\xi = -0.8$ [13, 52, 102]. We take the renormalization scale $\mu = 0.5$ GeV and the QCD scale parameter $\Lambda_{QCD} = 0.1$ GeV.

We shall first consider the sum rules in the SU(3)-flavor symmetric limit, where we take $m_q = m_s = 0$ and $f = 0$. In this limit we also set the physical parameters of all the baryons equal to the ones of the nucleon; $m_B = m_N = 0.94$ GeV, $\tilde{\lambda}_B^2 = \tilde{\lambda}_N^2 = 2.1$ GeV$^6$, $s_0^B \equiv s_0^N$. In the SU(3)-limit, we choose the Borel window $0.8$ GeV$^2 \leq M^2 \leq 1.4$ GeV$^2$ which is commonly identified as the fiducial region for the nucleon mass sum rules as emphasized in Chapter 3. For the vector-meson–quark coupling constant we adopt the value

$$g^V_q = g^\rho_q = 3.7$$

(4.44)
as estimated from the Nambu-Jona-Lasinio model of Ref. [103], which was used to successfully reproduce the $\rho \pi \pi$ coupling constant.

In order to determine the values of the vector couplings from the sum rules in Eqs. (4.30)-(4.37), one needs to know the value of the susceptibility $\zeta$. The value of this susceptibility is unknown, however we note that, if $\zeta$ is negligibly small, then the sum rules at structures I and II are consistent with each other and have the nice feature that $g^\rho_N / g^\rho_N = 1/3$, which agrees well with the OBE potential model [15] and the VMD model [97] results. Motivated with this feature, we first analyze the sum rules in Eqs. (4.30)-(4.33) for $\zeta = 0$ and then discuss the deviations for arbitrary $\zeta$ values. We present the Borel mass dependence of vector and tensor coupling constants of $\rho$ and $\omega$ to the nucleon in Fig. 4.1 and to the hyperons in Fig. 4.2, for the average values of the vacuum parameters. It is observed that single-pole contributions are quite important especially in the case of the sum rules for the tensor couplings. Taking into account the uncertainties in $s_0^B$ and $a_q$, the predicted values for the coupling constants of the $\rho$- and $\omega$-mesons
to the baryons read:

\[
\begin{align*}
g^o_N &= 7.2 \pm 1.8, \\
g^o_\Sigma &= g^o_\Xi = g^o_\Lambda = 4.8 \pm 1.2, \\
g^p_N &= g^p_\Sigma = 2.4 \pm 0.6, \\
g^p_\Lambda &= f^p_\Lambda = 0, \\
f^p_N &= 7.7 \pm 1.9, \\
f^p_\Sigma &= -2.2 \pm 0.6, \\
f^p_\Lambda &= f^p_\Xi = 2.3 \pm 0.4, \\
f^e_\Xi &= f^e_\Sigma = -5.0 \pm 1.0, \\
f^e_\Lambda &= -5.7 \pm 1.0.
\end{align*}
\]

Our next concern is to investigate the SU(3) relations for the vector-meson–
baryon interactions and see if the coupling constants above as obtained from
QCDSR are consistent with these relations. For this purpose, we calculate
the coupling constants in Eq. (4.45) with the central values of the parameters:
\( a_q = 0.51 \text{ GeV}^3 \) and \( s_0^B = 2.3 \text{ GeV}^2 \). We assume the ideal-mixing angle \( \theta_s \simeq 35.3^\circ \).

The \( F/(F+D) \) ratios, \( \alpha_v \) and \( \alpha_m \), can directly be calculated via the relations

\[
\begin{align*}
g^o_\Sigma - g^o_N &= \frac{-2 \alpha_v}{1 - 2 \alpha_v}, \\
f^o_\Sigma - f^o_N &= \frac{-2 \alpha_m}{1 - 2 \alpha_m}.
\end{align*}
\]
4.3. Analysis of the Sum Rules

Figure 4.2: The Borel mass dependence of $\rho$ and $\omega$ couplings to hyperons for $a_\eta = 0.51$ where we also take $m_B = m_N = 0.94$ GeV, $\lambda_B^2 = \lambda_N^2 = 2.1$ GeV$^6$ and $s_0^B = s_0^N = 2.3$ GeV$^2$ in the SU(3)-flavor limit.

With straightforward algebra, the values of the $F/(F+D)$ ratios $\alpha_v, m$ and the octet and the singlet couplings, $g_v^m$ and $g_1^m$, respectively are determined as

\[
\alpha_v = 1, \quad \alpha_m = 0.18, \quad g_v^\rho \equiv g_N^\rho = 2.3, \quad g_m = f_N^\rho = 7.6, \quad g_1^\rho = 5.6, \quad g_1^m = -1.8.
\]

(4.48)

Inserting $\alpha_v, m, g_v^m$ and $g_1^m$ into the SU(3) relations we observe that the coupling constants as determined from QCDSR are consistent with SU(3). This also gives $g_N^\rho = f_N^\rho = 0$, which is justified by the non-strange content of the nucleon and by the ideal-mixing scheme. In Table 4.1, we give all the vector-meson–baryon coupling constants, obtained from these relations.

In Fig. 4.3, we present the dependence of $\alpha_v = F/(F+D)$ on the susceptibility $\zeta$ for the sum rules at structures I and II, at $M^2 = 1$ GeV$^2$ and for the average values of the other vacuum parameters. We observe that, the sum rules are rather sensitive to a change in the value of $\zeta$, since it appears in the coefficient of dimension 3 operator. The sum rule at structure I shows a more reliable behavior. In Fig. 4.4, the dependence of $g_N^\rho / g_N^\omega$ on the susceptibility $\zeta$ for the sum rules at the structures I and II is given. For $|\zeta| > 1$, the terms in the sum rules
Figure 4.3: The dependence of $\alpha_v = F / (F + D)$ on the susceptibility $\zeta$ for the sum rules at the structures I and II, at $M^2 = 1$ GeV$^2$ and for the average values of the other vacuum parameters.

Figure 4.4: Same as Fig. 4.3 but for the dependence of $g^N / g^N_0$. 
4.3. Analysis of the Sum Rules

Table 4.1: The vector-meson–baryon coupling constants in the SU(3)-limit for the average values of the vacuum parameters.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$NNM$</th>
<th>$\Lambda\Lambda M$</th>
<th>$\Xi\Xi M$</th>
<th>$\Sigma\Sigma M$</th>
<th>$\Lambda\Sigma M$</th>
<th>$\Sigma N\Lambda$</th>
<th>$\Lambda N\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>$g$</td>
<td>6.9</td>
<td>4.6</td>
<td>2.3</td>
<td>4.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f$</td>
<td>−2.2</td>
<td>−5.7</td>
<td>−4.9</td>
<td>2.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>$g$</td>
<td>0</td>
<td>−3.3</td>
<td>−6.5</td>
<td>−3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f$</td>
<td>0</td>
<td>−4.8</td>
<td>−3.8</td>
<td>6.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>$g$</td>
<td>2.3</td>
<td>2.3</td>
<td>4.6</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f$</td>
<td>7.6</td>
<td>−4.9</td>
<td>2.7</td>
<td>6.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^*$</td>
<td>$g$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>−2.3</td>
<td>−4.0</td>
</tr>
<tr>
<td></td>
<td>$f$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.9</td>
<td>−5.9</td>
</tr>
</tbody>
</table>

Involving $\zeta$ dominate. In order to avoid the pole in $\alpha_v$ (for the structure I) on the negative $\zeta$-plane, we concentrate on the region $0 \leq \zeta \leq 1$ GeV$^{-1}$, where we obtain $5.2 \leq g_\omega^{\Lambda} \leq 12.7$ and $1.7 \leq g_\rho^{\Lambda} \leq 5.2$. This implies that away from $\zeta = 0$, $g_\rho^{\Lambda}/g_\omega^{\Lambda}$ tend to increase for the sum rules at structures I, which gets as high as 0.5 and the value of $\alpha_v$ gets as low as 0.8. These results disagree with those from OBE potential model [15] and the VMD model [97], which give $g_\rho^{\Lambda}/g_\omega^{\Lambda} = 1/3$ and $\alpha_v = 1$. This observation is in support of the assumption that $\zeta$ is negligibly small.

Next, we turn to the effect of SU(3)-flavor breaking, where we allow $m_s = 0.15$ GeV and $f = −0.2$, keeping $m_u = m_d = 0$. We also restore the physical values for the masses, overlap amplitudes and the continuum thresholds of the baryons [see Eq. (3.47)]. The corresponding Borel windows are chosen as in Eq. (3.48). We follow a procedure similar to the one in the SU(3)-flavor conserving case and fit the LHS’s of the sum rules to the function in Eq. (4.43) in the Borel windows from Eq. (3.48). Taking into account the uncertainties in $s_0^B$ and $\alpha_q$, the predicted values for the coupling constants of the $\rho$- and $\omega$-mesons to $\Lambda$, $\Xi$, and $\Sigma$ with the SU(3)-flavor breaking effects read:

$$
g_\omega^{\Lambda} = 2.9 \pm 1.1, \quad g_\omega^{\Xi} = 1.1 \pm 0.7, \quad g_\omega^{\Sigma} = 3.1 \pm 1.2, \\
f_\omega^{\Lambda} = −4.0 \pm 0.8, \quad f_\omega^{\Xi} = 2.4 \pm 0.6, \quad f_\omega^{\Sigma} = 7.0 \pm 1.6.
$$
We observe that the SU(3)-breaking effects modify the couplings by $30 - 50\%$ which indicates a large breaking. While the $\Sigma \Sigma \omega$ and $\Sigma \Sigma \rho$ coupling constants increase with SU(3)-breaking effects, the other coupling constants tend to decrease.

## 4.4 Discussion and Conclusions

In this chapter, we have calculated the vector-meson–baryon coupling constants which play significant roles in OBE models of the $YN$ and $YY$ interactions, employing the external-field QCDSR method. The main uncertainties in the results stem from the undetermined QCD parameters. Although the values of the susceptibilities $\chi$, $\xi$ and $\kappa$ are relatively better known from magnetic-moment calculations, $\zeta$ is undetermined. In this framework, we have first made the analysis by taking $\zeta$ negligibly small, which produces couplings in agreement with the others in the literature. Then, we have analyzed the sum rules for arbitrary $\zeta$ and observed that the results are sensitive to a change in this susceptibility. In this respect, an independent determination of the susceptibility $\zeta$ is desirable.

The coupling constants can be determined in terms of the vector-meson–quark coupling constant in this method. To compare our results with the others in the literature and stay as model-independent as possible, we find it useful to give the following ratios of the coupling constants in the SU(3)-limit for the average values of the vacuum parameters,

$$\frac{f^\rho_N}{g^\rho_N} = 3.8, \quad \frac{f^{\omega^0}_N}{g^{\omega^0}_N} = -0.3,$$

(4.50)

which compares well with the results from VMD,

$$\frac{f^\rho_N}{g^\rho_N} \equiv \frac{F^\rho_2}{F^\rho_1} = 3.3, \quad \frac{f^{\omega^0}_N}{g^{\omega^0}_N} \equiv \frac{F^s_2}{F^s_1} = -0.1,$$

(4.51)

where $F^s_1$ ($F^\nu_1$) and $F^s_2$ ($F^\nu_2$) are the isoscalar (isovector) electric and magnetic form factors of the nucleon, respectively, at zero momentum transfer. The result is not totally surprising, because a similar scheme to the one of electromagnetic coupling has been assumed for the vector-meson–baryon interaction. These ra-
4.4. Discussion and Conclusions

The ratios are very close to those from the NSC \(NN\) potential model [15], which are \(f_{\rho}^{\rho}/g_{\rho}^{\rho} = 4.2\) and \(f_{\omega}^{\omega}/g_{\omega}^{\omega} = 0.3\). Our value for the vector \(NN\rho\) coupling constant, with the choice of the quark-\(\rho\) coupling constant in Eq. (4.44), agrees with the one from the recent Nijmegen extended-soft-core (ESC) potential model [16], which is \(g_{\rho}^{\rho} = 2.8\). The ESC model gives \(g_{\rho}^{\rho}/g_{\omega}^{\omega} = 1/4\), that is, a value for the \(NN\omega\) coupling constant larger than what we have obtained from QCDSR. Given the relation from the SU(3) symmetry,

\[
g_{\omega}^{\omega} + \sqrt{2}g_{\phi}^{\phi} = 3g_{\rho}^{\rho},
\]

for the ideal-mixing holds and \(\alpha_v = 1\), the main reason for this is the sizable \(NN\phi\) coupling in NSC potential models, which is simply \(g_{N}^{\phi} = 0\) in the QCDSR. Such a large value for the \(NN\omega\) coupling constant as in the ESC or \(3P_0\) models [16] requires a quark-\(\omega\) coupling constant that is about 50\% larger than what we have adopted in Eq. (4.44). Our value of the \(F/(F+D)\) ratio for the vector coupling, which is \(\alpha_v = 1\), agrees with the value given in NSC89 [4]. Our value for \(\alpha_m\), which is \(\alpha_m = 0.18\), is about half of the values obtained in NSCa-f [5] and NSC89 [4], which are \(0.37 \leq \alpha_m \leq 0.45\) and \(\alpha_m = 0.28\), respectively.