Meson-Baryon coupling constants in QCD sum rules
Erkol, Güray

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CHAPTER 3

Scalar-Meson–Baryon Couplings in QCD Sum Rules

3.1 The Scalar Mesons

In terms of phenomenological one-boson exchange (OBE) models of two-baryon interaction, the scalar mesons play significant roles. In early OBE models for the $NN$ interaction, the exchange of an isoscalar-scalar “$\sigma$” meson with a mass of about 500 MeV was needed to obtain enough medium-range attraction and a sufficiently strong spin-orbit force. It was only later understood that the exchange of a broad isoscalar-scalar meson, the $\varepsilon(760)$ [71–73], simulates the exchange of such a low-mass “$\sigma$” [74].

Existence of the “$\sigma$” meson is expected also from chiral symmetry of QCD. The $\sigma$ meson appears as a chiral partner of the Nambu-Goldstone boson, $\pi$, and thus plays the role of the “Higgs boson” in chiral symmetry breaking. Recent analyses of $\pi - \pi$ scattering have revealed a broad resonance at the mass around 600 MeV, called $f_0(600)$ by the Particle Data Group [75], which is considered to play the role of $\varepsilon(760)$ in the boson exchange potential of the baryonic interactions. The $\varepsilon(760)$ is difficult to detect because it is broad and hidden under the
strong signal from the $\rho^0(770)$. There are strong arguments from chiral symmetry for the existence of such a light isoscalar-scalar meson approximately degenerate with the $\rho$ meson [76].

The structure and even the status of the scalar mesons, however, have always been controversial [77, 78]. In the quark model, the simplest assumption for the structure of the scalar mesons is the $^3P_0 \bar{q}q$ states. In this case, the scalar mesons might form a complete nonet of dressed $q\bar{q}$ states, resulting from e.g. the coupling of the $P$-wave $q\bar{q}$ states to meson-meson channels [79, 80]. Explicitly, the unitary singlet and octet states, denoted respectively by $\varepsilon_1$ and $\varepsilon_8$, read

$$\varepsilon_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3},$$
$$\varepsilon_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}. \quad (3.1)$$

The physical states are mixtures of the pure SU(3)-flavor states, and are written as

$$\varepsilon = \cos \theta_s \varepsilon_1 + \sin \theta_s \varepsilon_8,$$
$$f_0 = -\sin \theta_s \varepsilon_1 + \cos \theta_s \varepsilon_8. \quad (3.2)$$

For ideal-mixing holds that $\tan \theta_s = 1/\sqrt{2}$ or $\theta_s \simeq 35.3^\circ$, and thus one would identify

$$\varepsilon(760) = (u\bar{u} + d\bar{d})/\sqrt{2},$$
$$f_0(980) = -s\bar{s}. \quad (3.3)$$

The isotriplet member of the octet is $a_{0}^{\pm,0}(980)$, where

$$a_0^{0}(980) = (u\bar{u} - d\bar{d})/\sqrt{2}. \quad (3.4)$$

An alternative and arguably more natural explanation for the masses and decay properties of the lightest scalar mesons is to regard these as cryptoexotic $q^2\bar{q}^2$ states [81, 82]. In the MIT bag model, the scalar $q\bar{q}$ states are predicted around
1250 MeV, while the attractive color-magnetic force results in a low-lying nonet of scalar $q^2\bar{q}^2$ mesons [81–83]. This nonet contains a nearly degenerate set of $I = 0$ and $I = 1$ states, which are identified as the $f_0(980)$ and $a_0^{\pm,0}(980)$ at the $\bar{K}K$ threshold, where

$$
a_0^0(980) = (sd\bar{s} - su\bar{u})/\sqrt{2},$$
$$
f_0(980) = (sd\bar{s} + su\bar{u})/\sqrt{2},$$

(3.5)

with the ideal-mixing angle $\tan \theta_s = -\sqrt{2}$ or $\theta_s \simeq -54.8^\circ$ in this case. The light isoscalar member of the nonet is

$$
\varepsilon(760) = ud\bar{u}\bar{d}.
$$

(3.6)

The nonet is completed by the strange member $\kappa(880)$, which like the $\varepsilon(760)$ is difficult to detect because it is hidden under the strong signal from the $K^*(892)$ [77, 78]. In this work we shall use the nomenclature $(a_{0}^{\pm,0}, f_0, \sigma, \kappa)$ for the scalar-meson nonet, where one should identify $\sigma = \varepsilon(760)$.

One way to make progress with the scalar mesons is to study their role in the various two-baryon reactions ($NN$, $YN$, $YY$). Our aim in this chapter is to calculate the $NN\sigma$, $\Lambda\Lambda\sigma$, $\Xi\Sigma\sigma$ and $\Sigma\Sigma\sigma$ coupling constants, using the external-field QCDSR method. We will compare our results for the coupling constants with the values from a successful OBE model of the $NN$ and $YN$ interaction, the Nijmegen soft-core potential (NSC) [2, 4, 5, 15], which was originally derived from Regge-pole theory. (The coupling constants of this OBE model were analyzed from the point of view of the large-$N_c$ expansion of QCD in Ref. [84].)

In terms of OBE models for two-baryon interactions, the dominant contribution to $\Lambda\Lambda$ interaction comes from the scalar $\varepsilon(760)$ meson exchange [85, 86]. The recent identification of $^6_{\Lambda\Lambda}He$ and the measurement of the $\Lambda\Lambda$ pair suggest that the binding energy of $\Lambda\Lambda$ ($\Delta B_{\Lambda\Lambda} \simeq 1.0$ MeV) is considerably smaller than the binding energy of $NN$ [87]. This is in contrast to the outcome of the earlier measurement which is $\Delta B_{\Lambda\Lambda} \simeq 4.7$ MeV [88]. This issue has also been examined within the framework of Nijmegen OBE potential model D (NHC-D) [3]. In this
model, the pseudoscalar octet $\pi, \eta, \eta', K$, the vector octet $\rho, \phi, \omega, K^*$ and the scalar singlet $\varepsilon$ are the exchanged mesons and the coupling constants are fitted to data while other physical properties of the particles are taken from experiment. The estimated value of $\Delta B_{\Lambda\Lambda}$ in this model implies a rather strong attractive $\Lambda\Lambda$ interaction. However, in the NSC potential models, where there is a scalar nonet instead of a scalar singlet, we have much weaker attractive potentials than in the case of NHC-D in the $\Lambda\Lambda$ systems. In this framework, since the scalar $\varepsilon(760)$ exchange plays the most crucial role in $\Lambda\Lambda$ interactions, it is necessary to determine the $\Lambda\Lambda\varepsilon$ coupling constant in a model-independent way in order to understand the role of $\varepsilon$ exchange in the strangeness $S = -2$ sector.

In the SU(3)-flavor symmetric limit, one can classify the meson-baryon coupling constants in terms of two parameters, the $N N a_0$ coupling constant, $g_{N N a_0}$ and the $F/(F + D)$ ratio of the scalar octet, $\alpha_s$ [14] as in Eq. (2.50). Considering the mixing between the singlet and the octet members of the scalar nonet, one obtains for $g_{B B \sigma}$ and $g_{B B f_0}$,

$$
g_{B B \sigma} = \cos \theta_s g_1 + \sin \theta_s g_{B B \varepsilon S},
$$

$$
g_{B B f_0} = -\sin \theta_s g_1 + \cos \theta_s g_{B B \varepsilon S},
$$

(3.7)

where $g_1 = g_{B B \varepsilon 1}$ is the flavor singlet coupling, and $\theta_s$ is the scalar mixing angle.

We shall first consider the sum rules in the SU(3)-flavor symmetric limit to see if the predicted values for the meson-baryon coupling constants from the sum rules are consistent with the SU(3) relations. We show that this is indeed the case which leads to a determination of the $F/(F + D)$ ratio of the scalar octet. Furthermore, keeping track of these coupling constants with the SU(3) relations, we obtain the values of the other scalar-meson–baryon coupling constants. For this purpose, we assume ideal-mixing and make the analysis in both $q\bar{q}$ and $q^2\bar{q}^2$ pictures for the scalar mesons. As we move from the $S = 0$ to the $S = -1$ and $S = -2$ sectors, the flavor SU(3)-breaking occurs as a result of the $s$-quark mass and the physical masses of the baryons and mesons. We also consider the SU(3)-breaking effects for the sum rules to size the amount of breaking, individually for each coupling.
This chapter is organized as follows: in section 3.2 we present the formulation of QCDSR with an external isoscalar-scalar field and construct the relevant sum rules. We give the numerical analysis of the sum rules and discuss the results in section 3.3. Finally, we arrive at our conclusions for this chapter in section 3.4.

### 3.2 Construction of the Sum Rules

In the external-field QCDSR method one starts with the correlation function of the baryon interpolating fields in the presence of an external constant isoscalar-scalar field \( \sigma \), defined by the following:

\[
\Pi_{B\sigma}(p) = i \int d^4x \, e^{ip.x} \langle 0 \left| \mathcal{T}[\eta_B(x)\bar{\eta}_B(0)] \right| 0 \rangle_{\sigma},
\]  

(3.8)

where \( \eta_B \) are the baryon interpolating fields which are chosen as \([11]\)

\[
\begin{align*}
\eta_N &= \varepsilon_{abc} [(u^T_a C \gamma_\mu u_b) \gamma_5 \gamma^\mu d_c], \\
\eta_\Xi &= \varepsilon_{abc} [(s^T_a C \gamma_\mu s_b) \gamma_5 \gamma^\mu u_c], \\
\eta_\Sigma &= \varepsilon_{abc} [(u^T_a C \gamma_\mu u_b) \gamma_5 \gamma^\mu u_c], \\
\eta_\Lambda &= (2/3)^{1/2} \varepsilon_{abc} [(u^T_a C \gamma_\mu s_b) \gamma_5 \gamma^\mu d_c - (d^T_a C \gamma_\mu s_b) \gamma_5 \gamma^\mu u_c],
\end{align*}
\]

for \( N, \Xi, \Sigma \) and \( \Lambda \), respectively. Here \( a, b, c \) denote the color indices, and \( T \) and \( C \) denote transposition and charge conjugation, respectively. For the interpolating field of each octet baryon, there are two independent local operators, but the ones in Eq. (3.9) are the optimum choices for the lowest-lying positive parity baryons (see e.g. Ref [89, 90] for a discussion on negative-parity baryons in QCDSR).

The external sigma field contributes to the correlation function in Eq. (3.8) in two ways: first, it directly couples to the quark field in the baryon current and second, it modifies the condensates by polarizing the QCD vacuum. In the presence of the external scalar field there are no correlators that break the Lorentz invariance; however, the correlators already existing in the vacuum are modified
by the external field:

\begin{align}
\langle \bar{q}q \rangle_\sigma & = \langle \bar{q}q \rangle + g_\sigma^q \chi_\sigma \langle \bar{q}q \rangle, \\
\langle g_c \bar{q} \sigma \cdot Gq \rangle_\sigma & = \langle g_c \bar{q} \sigma \cdot Gq \rangle + g_\sigma^q \chi_G \sigma \langle g_c \bar{q} \sigma \cdot Gq \rangle,
\end{align}

where only the responses linear in the external-field are taken into account. Here, $g_\sigma^q$ is the quark-$\sigma$ coupling constant and $\chi$ and $\chi_G$ are the susceptibilities corresponding to quark and quark-gluon mixed condensates, respectively. In Eq. (3.10), $\langle \bar{q}q \rangle$ represents either $\langle \bar{u}u \rangle$ or $\langle \bar{d}d \rangle$, as we have assumed that $\langle \bar{u}u \rangle \simeq \langle \bar{d}d \rangle$ and the responses of the up and the down quarks to the external isoscalar field are the same. Note that, here we assume ideal-mixing in the scalar sector, that is, we take the sigma meson without a strange-quark content. Therefore, the sigma meson couples only to the $u$- or the $d$-quark in the baryon, where we take $g_\sigma^u = g_\sigma^d$ and $g_\sigma^s = 0$.

In the Euclidian region, the OPE of the product of two interpolating fields can be written as follows:

\begin{equation}
\Pi_{B\sigma}(p) = \sum_n C_n^\sigma(p) O_n,
\end{equation}

where $C_n^\sigma(p)$ are the Wilson coefficients and $O_n$ are the local operators in terms of quarks and gluons. In order to calculate the Wilson coefficients, we need the quark propagator in the presence of the external sigma field. In coordinate space the full quark propagator takes the form:

\begin{equation}
S_q(x) = S_q^{(0)}(x) + S_q^{(\sigma)}(x),
\end{equation}

where $S_q^{(0)}(x)$ is given in Eq. (2.14) and

\begin{equation}
\begin{aligned}
i S_q^{(\sigma)ab} & \equiv \langle 0 | T[q^a(x)\bar{q}^b(0)] | 0 \rangle_\sigma \\
& = g_\sigma^q \sigma \left[ -\frac{\delta^{ab}}{4\pi^2 x^2} - \frac{1}{32\pi^2} \frac{\lambda^n_{ab}}{2} g_c G^n_{\mu\nu} \sigma^{\mu\nu} \ln(-x^2) \right]
\end{aligned}
\end{equation}
3.2. Construction of the Sum Rules

\[-\frac{\delta^{ab}(g_c^2G^2)}{2^9 \times 3\pi^2} x^2 \ln(-x^2) + \frac{i \delta^{ab}}{48} \langle \bar{q}q \rangle \hat{x} - \frac{\delta^{ab} \chi}{12} \langle \bar{q}q \rangle + \frac{i \delta^{ab} \chi^2}{2^7 \times 3^2} \langle g_c \bar{q} \sigma \cdot G q \rangle \hat{x} - \frac{\delta^{ab} \chi^2}{192} \chi G \langle g_c \bar{q} \sigma \cdot G q \rangle \] + O(\sigma^2).

Here, \( G^{\mu\nu} \) is the gluon tensor and \( g_c^2 = 4\pi\alpha_s \) is the quark-gluon coupling constant squared. Note that, in the quark propagator above, we have included the terms that are proportional to the quark masses, \( m_q \), since these terms give non-negligible contributions to the final result as far as the strange quark mass is considered.

Using the quark propagator in Eq. (3.12), one can compute the correlation function \( \Pi_{B\sigma}(p) \). The Lorentz covariance and parity implies the following form for \( \Pi_{B\sigma}(p) \):

\[ \Pi_{B\sigma}(p) = (\Pi_{B0}^1 + \Pi_{B0}^p \hat{p}) + (\Pi_{B\sigma}^1 + \Pi_{B\sigma}^p \hat{p}) \sigma + O(\sigma^2), \] (3.14)

where \( \hat{p} = p^\mu \gamma_\mu \) is the four momentum of the baryon. Here \( \Pi_{B0}^1 \) and \( \Pi_{B0}^p \) represent the invariant functions in the absence of the external field, which can be used to construct the mass sum rules for the relevant baryons, and \( \Pi_{B\sigma}^1 \) and \( \Pi_{B\sigma}^p \) denote the invariant functions in the presence of the external field. Using these invariant functions, one can derive the sum rules at the structures 1 and \( \hat{p} \). In Fig 3.1, we present the diagrams that were used to calculate the Wilson coefficients of the correlation functions \( \Pi_{B\sigma}^1 \) and \( \Pi_{B\sigma}^p \). In Ref. [61], it was found that the sum rule at the structure \( \hat{p} \) for the NN\( \sigma \) coupling constant is more stable than the other sum rule at the structure 1, with respect to variations in the Borel mass. Motivated with this, we here present only the sum rules at the structure \( \hat{p} \) and use these for the determination of the coupling constants.

3.2.1 \( \Lambda \) Sum Rules and \( \Lambda\Lambda\sigma \) Coupling Constant

We shall first present the sum rules calculations for \( \Lambda\Lambda\sigma \) coupling constant in detail and in the next subsection, we shall give the sum rules for \( \Lambda\Lambda\sigma \), \( \Xi\Xi\sigma \) and
Figure 3.1: The diagrams that were used to calculate the Wilson coefficients of the correlation functions $\Pi_{qB\sigma}^q$ and $\Pi_{B\sigma}^1$. The solid, wavy, and the dashed lines represent the quark, gluon, and the external scalar field, respectively.
3.2. Construction of the Sum Rules

$\Sigma\Sigma\sigma$ couplings. At the quark level, we have for $\Lambda$:

$$\left\langle 0 \left| \mathcal{T} \left[ \eta_\Lambda(x) \bar{\eta}_\Lambda(0) \right] \right| 0 \right\rangle_\sigma = \frac{2i}{3} \varepsilon^{abc} \varepsilon^{d'b'c'}$$

$$\times \left( Tr \left\{ S_{sd}'(x) \gamma_5 C \left[ S_{sb}'(x) \right]^T \gamma_\mu \gamma_\rho \right\} \gamma_5 \gamma_\mu S_{sd}'(x) \gamma_\nu \gamma_\rho \right)$$

$$+ Tr \left\{ S_{d'}(x) \gamma_5 C \left[ S_{s}'(x) \right]^T \gamma_\mu \gamma_\rho \right\} \gamma_5 \gamma_\mu S_{d'}(x) \gamma_\nu \gamma_\rho$$

$$- \gamma_5 \gamma_\mu S_{d'}(x) \gamma_5 C \left[ S_{s}'(x) \right]^T \gamma_\mu \gamma_\rho \gamma_\nu \gamma_\rho$$

$$- \gamma_5 \gamma_\mu S_{d'}(x) \gamma_5 C \left[ S_{s}'(x) \right]^T \gamma_\mu \gamma_\rho \gamma_\nu \gamma_\rho \right) .$$

Using the quark propagator in Eq. (3.12), the invariant function at the structure $\hat{\rho}$ in the presence of the external field, $\Pi^p_{\Lambda\sigma}$, is calculated as

$$\Pi^p_{\Lambda\sigma}(p) = g_q^\sigma \frac{1}{(2\pi)^4} \left[ \frac{4}{3} a_q(1 - f) \ln(-p^2) - \frac{4}{9} \frac{m_q}{p^2} \chi a_q^2(1 + 2f) \right]$$

$$- (\chi + \chi_G) \frac{m_0^2}{18p^4} a_q^2(1 + 2f) + \frac{5}{12p^4} m_0^2 a_q - \frac{2}{3} (2m_s - m_q) p^2 \ln(-p^2)$$

$$- \frac{2}{3} \chi a_q (2m_s - m_q) \ln(-p^2) + \frac{1}{9} \left[ (5m_s - 3m_q) + 8(m_s - m_q) f \right] a_q^2 ,$$

where we have defined $f = \frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle} - 1$, $a_q = -(2\pi)^2 \langle \bar{q}q \rangle$ and $\langle g_c \bar{q} \sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$.

In order to construct the hadronic side, we saturate the correlator in Eq. (3.8) with $\Lambda$ states and write

$$\Pi_{\Lambda\sigma}(p) = \frac{\langle 0 | \eta_\Lambda | \Lambda \rangle}{p^2 - m_\Lambda^2} \langle \Lambda | \sigma \Lambda \rangle \frac{\langle \Lambda | \bar{\eta}_\Lambda | 0 \rangle}{p^2 - m_\Lambda^2} ,$$

where $m_\Lambda$ is the mass of the $\Lambda$. The matrix element of the current $\eta_\Lambda$ between the vacuum and the $\Lambda$ state is defined as

$$\langle 0 | \eta_\Lambda | \Lambda \rangle = \lambda_\Lambda \nu ,$$

where $\lambda_\Lambda$ is the overlap amplitude and $\nu$ is the Dirac spinor for the $\Lambda$, which is normalized as $\bar{\nu} \nu = 2m_\Lambda$. Inserting Eq. (3.18) into Eq. (3.17) and making use of
the isoscalar scalar-meson–baryon interaction Lagrangian density,

\[ \mathcal{L} = -g_{\Lambda \Lambda \sigma} \bar{\psi} \psi \sigma, \]  

we obtain the hadronic part as

\[ -|\lambda_{\Lambda}|^2 \frac{\hat{p} + m_{\Lambda}}{p^2 - m_{\Lambda}^2} g_{\Lambda \Lambda \sigma} \frac{\hat{p} + m_{\Lambda}}{p^2 - m_{\Lambda}^2}. \]  

(3.20)

We have also contributions coming from the excitations to higher \( \Lambda \) states which are written as

\[ -\lambda_{\Lambda} \lambda_{\Lambda} \frac{\hat{p} + m_{\Lambda}}{p^2 - m_{\Lambda}^2} g_{\Lambda \Lambda \sigma} \frac{\hat{p} + m_{\Lambda}}{p^2 - m_{\Lambda}^2}, \]  

(3.21)

and the ones coming from the intermediate states due to \( \sigma - \Lambda \) scattering i.e. the continuum contributions. Note that the term that corresponds to the excitations to higher \( \Lambda \) states also has a pole at the \( \Lambda \) mass, but a single pole instead of a double one like in Eq. (3.20). This single pole term is not damped after the Borel transformation and should be included in the calculations. There is another contribution that comes from the response of the continuum to the external field, which is given as

\[ \int_0^{\infty} \frac{-\Delta s_0 b(s)}{s - p^2} \delta(s - s_0) ds, \]  

(3.22)

where \( s_0 \) is the continuum threshold, \( \Delta s_0 \) is the response of the continuum threshold to the external field and \( b(s) \) is a function that is calculated from OPE. When \( \Delta s_0 \) is large, this term should also be included in the hadronic part [91].

Matching the OPE side with the hadronic side and applying the Borel transformation, the sum rule for \( \Lambda \Lambda \sigma \) coupling at the structure \( \hat{p} \) is obtained as

\[
\begin{align*}
&\left\{ - \frac{4}{3} M^4 a_q (1 - f) E_0^\Lambda + \frac{4M^2}{9} \chi a_q^2 (1 + 2f) L^{4/9} \\
&- \frac{5}{12} M^2 m_0^2 a_q L^{-14/27} - (\chi + \chi_G) \frac{m_0^2}{18} a_q^2 (1 + 2f) L^{-2/27} \right\}
\end{align*}
\]  

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\[+ \frac{2}{3} (2m_s - m_q) M^6 E_1^\Lambda L^{-8/9} + \frac{2}{3} \chi a_q (2m_s - m_q) M^4 E_0^\Lambda L^{-4/9} \]
\[+ \frac{1}{9} \left[ (5m_s - 3m_q) + 8(m_s - m_q) f \right] a_q \left\{ e^{m_s^2/M^2} \right\} e^{m_q^2/M^2} = -\tilde{\lambda}_\Lambda^2 \frac{m_\Lambda}{g_q} g_{\Delta\Lambda\sigma} \]
\[+ \tilde{B}_\Lambda \frac{M^2}{8g_q} + \Delta s_0^\Lambda \left[ (s_0^\Lambda)^2 - 4m_f a_q \right] M^2 L^{-4/9} e^{(m_s^2 - s_0^\Lambda)/M^2}, \]

where we have defined \( \tilde{\lambda}_\Lambda^2 = 32\pi^4 \lambda_\Lambda^2 \) and \( M \) is the Borel mass. The continuum contributions are included by the factors

\[ E_0^\Lambda \equiv 1 - e^{-s_0^\Lambda/M^2}, \]
\[ E_1^\Lambda \equiv 1 - e^{-s_0^\Lambda/M^2} \left( 1 + \frac{s_0^\Lambda}{M^2} \right), \]

(3.24)

where \( s_0^\Lambda \) is the continuum threshold. In the sum rule above, we have included the single pole contribution with the factors \( \tilde{B}_\Lambda \). The third term on the RHS of Eq. (3.23) denotes the contribution that is explained in Eq. (3.22). Note that this term is suppressed by the factor \( e^{-(s_0^\Lambda - m_\Lambda^2)/M^2} \) as compared to the single pole term. We have incorporated the effects of the anomalous dimensions of various operators through the factor \( L = \ln(M^2/\Lambda_{QCD}^2)/\ln(\mu^2/\Lambda_{QCD}^2) \), where \( \mu \) is the renormalization scale and \( \Lambda_{QCD} \) is the QCD scale parameter.

3.2.2 \( N, \Xi \) and \( \Sigma \) Sum Rules and \( NN\sigma, \Xi\Xi\sigma \) and \( \Sigma\Sigma\sigma \) Coupling Constants

One can apply the method explained in the previous subsection for the \( NN\sigma, \Xi\Xi\sigma \) and \( \Sigma\Sigma\sigma \) couplings and derive the corresponding sum rules. Using the interpolating fields in Eq. (3.9), we obtain

\[ \left\langle 0 \left| \mathcal{T} [\eta_N(x) \bar{\eta}_N(0)] \right| 0 \right\rangle_\sigma = 2i \epsilon^{abc} \epsilon^{a'b'c'} \]
\[ \times \text{Tr} \{ S^{bb'}(x) \gamma_\nu C [S^{ad'}(x)]^T C \gamma_\mu \} \gamma_5 \gamma_5^\mu S^{c'}_d(x) \gamma^\nu \gamma_5, \]

(3.25)

\[ \left\langle 0 \left| \mathcal{T} [\eta_\Xi(x) \bar{\eta}_\Xi(0)] \right| 0 \right\rangle_\sigma = 2i \epsilon^{abc} \epsilon^{a'b'c'} \]

(3.26)
×Tr\{S^{\alpha\delta'}_s (x) \gamma^\nu \gamma^\lambda C[S^{b\delta'}_u (x)]^T C \gamma_\mu \} \gamma_5 \gamma^\mu S^{\epsilon\delta'}_u (x) \gamma^\nu \gamma_s ,

\left\langle 0 \right| \mathcal{F} [\eta_\Sigma (x) \bar{\eta}_\Sigma (0)] \right| 0 \right\rangle = 2i \epsilon^{abc} \epsilon^{d'b'c'}

\times \text{Tr}\{S^{\alpha\delta'}_u (x) \gamma^\nu \gamma^\lambda C[S^{b\delta'}_u (x)]^T C \gamma_\mu \} \gamma_5 \gamma^\mu S^{\epsilon\delta'}_u (x) \gamma^\nu \gamma_s ,

(3.27)

at the quark level for \( N, \Xi \) and \( \Sigma \), respectively. Using the quark propagator in Eq. (3.12), the invariant functions at the structure \( \hat{p} \) are calculated as

\begin{align*}
\Pi^p_{N\sigma}(p) &= g^\sigma_q \frac{1}{(2\pi)^4} \left[ a_q \ln(-p^2) - \frac{4}{3p^2} a_q^2 + \frac{m_0^2}{2p^2} a_q - (\chi + \chi G) \frac{m_0^2}{6p^4} a_q \right], \\
(3.28) \\
\Pi^p_{\Xi\sigma}(p) &= g^\sigma_q \frac{1}{(2\pi)^4} \left[ \frac{m_0^2}{6p^2} a_q + a_q \ln(-p^2) \right], \\
(3.29) \\
\Pi^p_{\Sigma\sigma}(p) &= g^\sigma_q \frac{1}{(2\pi)^4} \left[ \frac{m_0^2}{3p^2} a_q - (\chi G + \chi) \frac{m_0^2}{6p^4} a_q^2 - \frac{4\chi}{3p^2} a_q^2 - 2m_q p^2 \ln(-p^2) \\
- 2m_q \chi a_q \ln(-p^2) + \frac{2m_q^2}{3p^2} a_q \right].
(3.30)
\end{align*}

The sum rules, which are obtained by matching the OPE side with the hadronic side and applying the Borel transformation, are as follows:

\[ -M^4 a_q E_0^N + \frac{4}{3} \chi M^2 a_q^2 L^{4/9} - \frac{m_0^2}{2} M^2 a_q L^{-14/27} - (\chi + \chi G) \frac{m_0^2}{6} a_q^2 L^{-2/27} \\
+ 2m_q M^6 E_1 L^{-8/9} + 2\chi m_q a_q M^4 E_0 L^{-4/9} + \frac{2}{3} m_q a_q^2 \right] e^{-m_N^2/M^2} \\
= -\tilde{\lambda}_N^2 m_N g_{N\sigma} + \tilde{B}_N \frac{M^2}{\alpha_q^2} \right] e^{m_N^2/M^2},
(3.31)

\[ -\frac{m_0^2}{6} a_q M^2 L^{-14/27} - a_q M^4 E_0^\Xi \right] e^{m_\Xi^2/M^2},
(3.32)
\[\begin{align*}
&= -\tilde{\lambda}^2 \frac{m_\Sigma}{g_\Sigma} g_{\Xi\Xi}\sigma + \tilde{B}_\Xi \frac{M^2}{g_\Sigma} + \frac{(s_0^\Xi)^2}{2g_\Xi} \Delta s_0 \frac{M^2}{L^{-4/9}} e^{(m_\Sigma^2 - s_0^\Xi)/M^2}, \\
&\left[-\frac{m_0^2}{3} a_q M^2 L^{-14/27} - \chi_G + \chi \right] \frac{m_0^2}{6} a_q^2 L^{-2/27} + \frac{4}{3} \chi a_q^2 M^2 L^{4/9} \\
&+ 2 m_q M^6 E_1^\Sigma L^{-8/9} + 2 \chi m_q a_q M^4 E_0^\Sigma L^{-4/9} + \frac{2}{3} m_q a_q^2 \right] e^{m_\Sigma^2/M^2} \\
&= -\tilde{\lambda}_\Sigma^2 \frac{m_\Sigma}{g_\Sigma} g_{\Sigma\Sigma}\sigma + \tilde{B}_\Sigma \frac{M^2}{g_\Sigma} \\
&+ \frac{\Delta s_\Sigma^0}{2g_\Sigma} \left[(s_0^\Sigma)^2 - 2 m_s (f + 1) a_q \right] M^2 L^{-4/9} e^{(m_\Sigma^2 - s_0^\Sigma)/M^2}, \tag{3.33}
\end{align*}\]

for \(NN\sigma, \Xi\Xi\sigma\) and \(\Sigma\Sigma\sigma\) coupling constants, respectively.

We would like to note that the sum rule for the \(\Xi\Xi\sigma\) coupling constant at the structure \(\hat{p}\) is independent of the susceptibilities \(\chi\) and \(\chi_G\). Another feature of the sum rules above is that, up to the dimension we consider, the terms involving the \(s\)-quark mass do not contribute to the OPE side. The contributions that come from the excited baryon states and the response of the continuum threshold are taken into account by the second and the third terms on the RHS of the sum rules, respectively. Comparing the left-hand side (LHS) of the sum rules in Eq. (3.31), Eq. (3.32) and Eq. (3.33) one can derive a basic relation between the \(\Xi\Xi\sigma, \Sigma\Sigma\sigma\) and \(NN\sigma\) coupling constants in the SU(3)-limit, which is

\[g_{NN\sigma} = g_{\Xi\Xi}\sigma + g_{\Sigma\Sigma}\sigma. \tag{3.34}\]

This relation is quite natural because for the \(\Xi\Xi\sigma\) coupling only the \(u\)-quark propagator outside of the trace and for the \(\Sigma\Sigma\sigma\) coupling the \(u\)-quark propagators inside the trace involve the terms that are proportional to the external field. For the \(NN\sigma\) coupling all the three quark propagators involve such terms and this implies that in the SU(3) and isospin symmetric limit, the relation in Eq. (3.34) holds. It is interesting to note that this relation can also be derived from Eq. (2.50) and Eq. (3.7) assuming the ideal-mixing for the \(q\bar{q}\) picture where the \(NNf_0\) coupling vanishes.
3.3 Analysis of the Sum Rules

In this section we analyze the sum rules derived in the previous section in order to determine the values of the $NN\sigma$, $\Lambda\Lambda\sigma$, $\Xi\Xi\sigma$ and $\Sigma\Sigma\sigma$ coupling constants. To proceed to the numerical analysis, we arrange the RHS of the sum rules in the form

$$f(M^2) = A_B + B_B M^2 + C_B M^2 L^{-4/9} e^{(m_B^2 - s_0^B)/M^2},$$  \hspace{1cm} (3.35)

and fit the LHS to $f(M^2)$. Here we have defined:

$$A_B \equiv -\tilde{\lambda} \frac{m_B}{g_q} g_{BB\sigma}, \quad B_B \equiv \frac{\tilde{B}_B}{g_q},$$  \hspace{1cm} (3.36)

together with

$$C_\Lambda \equiv \frac{\Delta s_\Lambda^0}{2g_q} [(s_\Lambda^0)^2 - 4m_s f a_q],$$  \hspace{1cm} (3.37)
$$C_\Xi \equiv \frac{(s_0^R)^2}{2g_q} \Delta s_0^\Xi,$$
$$C_\Sigma \equiv \frac{\Delta s_\Sigma^0}{2g_q} [(s_0^\Sigma)^2 - 4m_s (f + 1) a_q],$$

for $\Lambda\Lambda\sigma$, $\Xi\Xi\sigma$ and $\Sigma\Sigma\sigma$ sum rules, respectively.

In order to calculate the coupling constants from the sum rules, we need to know the values of the scalar susceptibilities $\chi$ and $\chi_G$. The value of the susceptibility $\chi$ can be calculated by using the two-point function [58]

$$T(q^2) = i \int d^4x e^{iq\cdot x} \left\langle 0 \left| \mathcal{T}[\bar{u}(x)u(x) + \bar{d}(x)d(x), \bar{u}(0)u(0) + \bar{d}(0)d(0)] \right| 0 \right\rangle,$$  \hspace{1cm} (3.38)

via the relation

$$\chi \langle \bar{q}q \rangle = \frac{1}{2} T(0).$$  \hspace{1cm} (3.39)
3.3. Analysis of the Sum Rules

The two-point function in Eq. (3.38) at $q^2 = 0$ has been calculated in chiral perturbation theory [92] with the result

$$\chi = \frac{\langle \bar{q}q \rangle}{16\pi^2 f^4_\pi} \left( \frac{2}{3} \bar{\ell}_1 + \frac{7}{3} \bar{\ell}_2 - \frac{11}{6} \right),$$

where $f_\pi = 93$ MeV is the pion decay constant and $\bar{\ell}_1$ and $\bar{\ell}_2$ are low-energy constants appearing in the effective chiral Lagrangian. The values of these low-energy constants have been estimated previously in various works (see e.g. Ref. [93] for a review). A recent analysis of $\pi-\pi$ scattering gives $\bar{\ell}_1 = -1.9 \pm 0.2$ and $\bar{\ell}_2 = 5.25 \pm 0.04$ [93], which is consistent with earlier determinations, but with smaller uncertainties. Using these values of $\bar{\ell}_1$ and $\bar{\ell}_2$ and taking the quark condensate $a_q = 0.51 \pm 0.03$ GeV$^3$, we find $\chi = -10 \pm 1$ GeV$^{-1}$. The value of the susceptibility $\chi_G$ is less certain. Therefore, we allow $\chi_G$ to vary in a wider range. We also adopt $b = 4.7 \times 10^{-1}$ GeV$^4$, and $m_0^2 = 0.8$ GeV$^2$ [13,94]. We take $m_N = 0.94$ GeV, the renormalization scale $\mu = 0.5$ GeV, and the QCD scale parameter $\Lambda_{QCD} = 0.1$ GeV. It is relevant to point out that the choice of the two-point function in Eq. (3.38) does not imply a theoretical prejudice about the structure of the scalar mesons. What is calculated is just the susceptibility pertaining to the response of the quark condensates $\langle \bar{q}q \rangle$ to the scalar $q\bar{q}$ field, as shown in Eq. (3.10).

3.3.1 Scalar-Meson–Baryon Coupling Constants in the SU(3)-Symmetric Limit

We shall first consider the sum rules in the SU(3)-flavor symmetric limit, where we take $m_q = m_s = 0$ and $f = 0$. In this limit we also set the physical parameters of all the baryons equal to the ones of the nucleon; $m_B = m_N = 0.94$ GeV, $\tilde{\lambda}_B^2 = \tilde{\lambda}_N^2 = 2.1$ GeV$^6$ [13], $s_0^B = s_0^N$.

In Figs. 3.2-3.5 we present the Borel mass dependence of the LHS and the RHS of the sum rules for $NN\sigma$, $\Lambda\Lambda\sigma$, $\Sigma\Sigma\sigma$ and $\Xi\Xi\sigma$, respectively, for $s_0^B = 2.3$ GeV$^2$ and $\chi_G \equiv \chi = -10$ GeV$^{-1}$. As stressed above, in the SU(3)-limit we choose the Borel window $0.8$ GeV$^2 \leq M^2 \leq 1.4$ GeV$^2$ which is commonly iden-
Figure 3.2: The Borel mass dependence of LHS and the fitted RHS of the sum rule for $\Lambda\Lambda\sigma$ coupling in Eq. (3.23) for $s_0^\Lambda = 2.3$ GeV$^2$ and $\chi_G \equiv \chi = -10$ GeV$^{-1}$. We also present the terms on the RHS individually. Note that the LHS curve (solid) overlies the RHS curve (dashed).

Figure 3.3: Same as Fig. 3.2 but for the sum rule for the $NN\sigma$ coupling in Eq. (3.31).
3.3. Analysis of the Sum Rules

Figure 3.4: Same as Fig. 3.2 but for the sum rule for the $\Xi\Xi\sigma$ coupling in Eq. (3.32).

Figure 3.5: Same as Fig. 3.2 but for the sum rule for the $\Sigma\Sigma\sigma$ coupling in Eq. (3.33).
tified as the fiducial region for the nucleon mass sum rules [13]. It is seen from these figures that the LHS curves (solid) overlie the RHS curves (dashed). In order to estimate the contributions that come from the excited baryon states and the responses of the continuum threshold, we plot each term on the RHS individually. We observe that the single-pole terms (dotted) give very small contributions to the sum rules except to the one for $\Xi\Xi\sigma$ coupling. The responses of the continuum thresholds (dot-dashed) for all the couplings are quite sizable.

In order to see the sensitivity of the coupling constants on the continuum threshold and the susceptibility $\chi$, we plot in Figs. 3.6-3.8 the dependence of $g_{NN\sigma}/g_q^\sigma$, $g_{\Lambda\Lambda\sigma}/g_q^\sigma$ and $g_{\Sigma\Sigma\sigma}/g_q^\sigma$ on $\chi$ for three different values of the continuum thresholds, $s_0^B = 2.0, 2.3, \text{and } 2.5 \text{ GeV}^2$, and taking $\chi \equiv \chi_G$. One sees that these coupling constants change by approximately 10% in the considered region of the susceptibility $\chi$. The values of the coupling constants are not very sensitive to a change in the continuum threshold, which gives an uncertainty of approximately 7% to the final values. Taking into account the uncertainties in $\chi$, $s_0^B$, and $a_q$, the predicted values for $NN\sigma$, $LL\sigma$, $\Xi\Xi\sigma$ and $\Sigma\Sigma\sigma$ coupling constants in terms of the quark-$\sigma$ coupling constant read:

\begin{align}
g_{NN\sigma}/g_q^\sigma &= 3.9 \pm 1.0, \\
g_{\Lambda\Lambda\sigma}/g_q^\sigma &= 1.9 \pm 0.5, \\
g_{\Xi\Xi\sigma}/g_q^\sigma &= 0.4 \pm 0.1, \\
g_{\Sigma\Sigma\sigma}/g_q^\sigma &= 3.8 \pm 1.0.
\end{align}

To determine the coupling constants, one next has to assume some value for the quark-$\sigma$ coupling constant $g_q^\sigma$. Adopting the value $g_q^\sigma = 3.7$ as estimated from the sigma model [95], we obtain:

\begin{align}
g_{NN\sigma} &= 14.4 \pm 3.7, \\
g_{\Lambda\Lambda\sigma} &= 7.0 \pm 1.9, \\
g_{\Xi\Xi\sigma} &= 1.5 \pm 0.4, \\
g_{\Sigma\Sigma\sigma} &= 14.1 \pm 3.7.
\end{align}
3.3. Analysis of the Sum Rules

Figure 3.6: The dependence of $g_{\Lambda\Lambda}\sigma/g_{q}^{\sigma}$ on the susceptibility $\chi$ for three different values of $s_{q}^{0} = 2.0$, 2.3, and 2.5 GeV$^2$; here we take $\chi \equiv \chi_{G}$.

Figure 3.7: Same as Fig. 3.6 but for the sum rule for the $g_{NN}\sigma/g_{q}^{\sigma}$ coupling constant.

Note that, the coupling constants in Eq. (3.42) are defined at $t = 0$, i.e. $g_{BB}\sigma \equiv g_{BB}\sigma(t = 0)$. As stressed above, the value of the susceptibility $\chi_{G}$ is less certain than the value of $\chi$. If we let $\chi_{G}$ change in a wider range, say $6 \text{ GeV}^{-1} \leq -\chi_{G} \leq 14 \text{ GeV}^{-1}$, this brings an additional 15% uncertainty to the values of $NN\sigma, \Lambda\Lambda\sigma$ and $\Sigma\Sigma\sigma$ coupling constants but the $\Xi\Xi\sigma$ coupling constant remains intact because the sum rule is independent of the susceptibilities as stressed above. In
order to stay consistent with the analysis in Ref. [61], we also here have taken $\Lambda_{QCD} = 0.1$ GeV. A change in the value of this parameter, say an increase to $\Lambda_{QCD} = 0.2$ GeV, does not have any considerable effect on the $\Xi\Xi\sigma$ coupling constant, but the $NN\sigma$ and $\Sigma\Sigma\sigma$ coupling constants are increased by approximately 8%, while the increase in the value of the $\Lambda\Lambda\sigma$ coupling constant is 5%.

Our next concern is to investigate the SU(3) relations for the scalar-meson–baryon interactions and see if the coupling constants above as obtained from QCDSR are consistent with these relations. The values of the three coupling constants as determined from QCDSR together with the first equation in Eq. (3.7) are sufficient to determine the three parameters of the flavor SU(3) structure of scalar-meson–baryon couplings namely $g_1$, $g$, and $\alpha_s$. For this purpose, we calculate the coupling constants in Eq. (3.41) with the central values of the parameters $\chi \equiv \chi_G = -10$ GeV$^{-1}$, $a_q = 0.51$ GeV$^3$ and $s_0^R = 2.3$ GeV$^2$ and we obtain:

$$g_{NN\sigma}/g_q^\sigma = 4.0,$$

$$g_{\Lambda\Lambda\sigma}/g_q^\sigma = 1.7,$$

$$g_{\Xi\Xi\sigma}/g_q^\sigma = 0.3,$$

$$g_{\Sigma\Sigma\sigma}/g_q^\sigma = 3.6.$$
Table 3.1: The scalar-meson–baryon coupling constants in the SU(3)-limit where the $q\bar{q}$ picture for the scalar mesons with the ideal-mixing is assumed.

<table>
<thead>
<tr>
<th>M</th>
<th>NN M</th>
<th>ΛΛ M</th>
<th>ΞΞ M</th>
<th>ΣΣ M</th>
<th>ΛΣ M</th>
<th>ΣN M</th>
<th>ΛN M</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>14.6</td>
<td>6.2</td>
<td>1.3</td>
<td>13.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_0$</td>
<td>0</td>
<td>−12.0</td>
<td>−18.9</td>
<td>−1.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_0$</td>
<td>12.0</td>
<td>1.3</td>
<td>13.3</td>
<td>6.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>κ</td>
<td></td>
<td>−1.3</td>
<td>−14.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We first assume the $q\bar{q}$ structure with the ideal-mixing angle $\theta_s \simeq 35.3^\circ$ and use $g_{NN\sigma}$, $g_{ΞΞ\sigma}$ and $g_{ΣΣ\sigma}$ from Eq. (3.43). The $F/(F+D)$ ratio, $\alpha_s$, can directly be calculated via the relation

$$
\frac{(g_{ΞΞ\sigma} - g_{NN\sigma})}{(g_{ΣΣ\sigma} - g_{NN\sigma})} = \frac{-2\alpha_s}{1 - 2\alpha_s}.
$$

(3.44)

With straightforward algebra, the values of the $F/(F+D)$ ratio and the octet and the singlet couplings for the $q\bar{q}$ picture are determined as follows:

$$
\alpha_s \equiv \frac{F}{F+D} = 0.55,
$$

(3.45)

$$
g/g_q^\sigma \equiv g_{NNa_0}/g_q^\sigma = 3.3,
$$

$$
g_1/g_q^\sigma = 3.2.
$$

Inserting $\alpha_s$ and $g$ into the SU(3) relations in Eq. (2.50) and using the mixing scheme for the singlet and the octet couplings as in Eq. (3.7) with the value of $g_1$ in Eq. (3.45), we observe that the coupling constants as determined from QCDSR in Eq. (3.43) are consistent with the SU(3) relations. This also gives $g_{NNf_0} = 0$ with the second equation in Eq. (3.7), which is justified by the non-strange content of the nucleon and by the ideal-mixing scheme. In Table 3.1 we give all the scalar-meson–baryon coupling constants, obtained from these relations, assuming $g_q^\sigma = 3.7$.

In the case of the $q^2\bar{q}^2$ picture, we have an ideal-mixing scheme for the scalar mesons which is quite distinct from that for the $q\bar{q}$ picture. The ideal-mixing
Table 3.2: Same as Table 3.1 but for the $q^2q^2$ picture for the scalar mesons.

<table>
<thead>
<tr>
<th>M</th>
<th>NN</th>
<th>ΛΛ</th>
<th>ΞΞ</th>
<th>ΣΣ</th>
<th>ΛΣ</th>
<th>ΣN</th>
<th>ΛNM</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>14.6</td>
<td>6.2</td>
<td>1.3</td>
<td>13.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_0$</td>
<td>10.3</td>
<td>16.2</td>
<td>19.7</td>
<td>11.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_0$</td>
<td>-8.5</td>
<td>-0.9</td>
<td>-9.4</td>
<td>-4.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>κ</td>
<td>0.9</td>
<td>10.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

angle corresponds to $\theta_s \simeq -54.8^\circ$ in this case. In this picture, we assume that the $u$- or the $d$-quark in the baryon couples to the $q\bar{q}$ component in the scalar meson but not to the $s\bar{s}$ component and the $\langle \bar{q}q \rangle$ condensates are modified in the same way. Accordingly, the s-quark only couples to the $s\bar{s}$ component. Applying the same procedure as in the $q\bar{q}$ case with $\theta_s = -54.8^\circ$, we obtain the values of $F/(F + D)$ ratio, and the octet and the singlet couplings for the $q^2q^2$ picture as

\[
\alpha_s \equiv \frac{F}{F + D} = 0.55, \quad (3.46)
\]

\[
g / g_\sigma = g_{NNa_0} / g_q = -2.3, \quad g_1 / g_\sigma = 4.6.
\]

Inserting these values into the SU(3) relations in Eq. (2.50), we observe that the scalar-meson–baryon coupling constants as found from QCDSR are consistent with the SU(3) relations, as in the $q\bar{q}$ picture.

In Table 3.2, we present the scalar-meson–baryon coupling constants in the $q^2q^2$ picture, assuming $g_q^\sigma = 3.7$. Comparing what we have found for the scalar-meson–baryon couplings in the two pictures, the value of the $F/(F + D)$ ratio remains intact, as apparent from Eq. (3.44), however, the values of $g$ and $g_1$ in the two pictures are quite different from each other. On the other hand, the $BBf_0$ couplings differ very much with regard to the structure of the scalar mesons. In the $q^2q^2$ picture for the scalar mesons, contrary to the $q\bar{q}$ picture, $f_0$ strongly couples to the nucleon due to $\bar{u}u$ and $\bar{d}d$ components it has. The strengths of the $I = 1$ couplings in the two pictures differ by a factor of $\sqrt{2}$. 
3.3 Analysis of the Sum Rules

\[
C_ΛM^2L^{-4/9}e^{(M^2_Λ+s_Λ^0)/M^2}B_ΛM^2 \leq \frac{4}{9}e\left(M^2_Λ+s_Λ^0\right)/M^2
\]

\[M^2 (\text{GeV}^2)\]

Figure 3.9: The Borel mass dependence of LHS and the fitted RHS of the sum rule for the $ΛΛσ$ coupling in Eq. (3.23) with the SU(3)-breaking effects and for $χ_G \equiv χ = -10$ GeV$^{-1}$. We also present the terms on the RHS individually. Note that the LHS curve (solid) overlies the RHS curve (dashed).

3.3.2 Sigma-Meson–Baryon Coupling Constants with SU(3)-Breaking Effects

Next, we turn to the effect of SU(3)-breaking, where we allow $m_s = 0.15$ GeV and $f = -0.2$, keeping $m_q = 0$. We also restore the physical values for the masses, overlap amplitudes and the continuum thresholds of the baryons [56, 96]:

\[
\tilde{λ}_Λ^2 = 3.3 \text{ GeV}^6, \quad \tilde{λ}_Ξ^2 = 4.6 \text{ GeV}^6, \quad \tilde{λ}_Σ^2 = 3.3 \text{ GeV}^6, \quad (3.47)
\]

\[s_0^Λ = 3.1 \pm 0.3 \text{ GeV}^2, \quad s_0^Ξ = 3.6 \pm 0.4 \text{ GeV}^2, \quad s_0^Σ = 3.2 \pm 0.3 \text{ GeV}^2.
\]

The corresponding Borel windows are chosen as

\[
\text{for } Λ, \quad 1.0 \text{ GeV}^2 \leq M^2 \leq 1.4 \text{ GeV}^2, \quad (3.48)
\]

\[
\text{for } Ξ, \quad 1.5 \text{ GeV}^2 \leq M^2 \leq 1.9 \text{ GeV}^2,
\]

\[
\text{for } Σ, \quad 1.2 \text{ GeV}^2 \leq M^2 \leq 1.6 \text{ GeV}^2.
\]

In Figs. 3.9-3.11 we present the Borel mass dependence of the LHS and the RHS of the sum rules for $ΛΛσ$, $ΣΣσ$ and $ΞΞσ$, respectively, with the SU(3)-
Figure 3.10: Same as Fig. 3.9 but for the sum rule for the $\Xi\Xi\sigma$ coupling in Eq. (3.32).

Figure 3.11: Same as Fig. 3.9 but for the $\Sigma\Sigma\sigma$ coupling in Eq. (3.33).
breaking effects. We plot each term on the RHS individually as we did in the SU(3)-limit. We observe that the responses of the continuum thresholds for all the couplings are quite sizable. The contributions of the single pole terms to the $\Xi\Xi\sigma$ and $\Sigma\Sigma\sigma$ sum rules are large and opposite in sign to the third terms on the RHS. Therefore the contributions of these two terms tend to cancel each other which leads to a very stable sum rule for these couplings. The contribution of the single pole term for the $\Lambda\Lambda\sigma$ coupling is very small.

Taking into account the uncertainties in $\chi$, $s_0^B$, and $a_q$, the predicted values for $\Lambda\Lambda\sigma$, $\Xi\Xi\sigma$ and $\Sigma\Sigma\sigma$ coupling constants in terms of the quark-$\sigma$ coupling constant with the SU(3)-breaking effects read:

$$g_{\Lambda\Lambda\sigma}/g_q^\sigma = 2.0 \pm 0.5,$$

$$g_{\Xi\Xi\sigma}/g_q^\sigma = 0.5 \pm 0.1,$$

$$g_{\Sigma\Sigma\sigma}/g_q^\sigma = 5.7 \pm 1.4.$$

Adopting again the value $g_q^\sigma = 3.7$ as estimated from the sigma model [95], we obtain:

$$g_{\Lambda\Lambda\sigma} = 7.4 \pm 1.9,$$

$$g_{\Xi\Xi\sigma} = 1.9 \pm 0.4,$$

$$g_{\Sigma\Sigma\sigma} = 21.1 \pm 5.2.$$

A few remarks are in order now. Comparing the values obtained from the sum rules in the SU(3)-symmetric limit and the ones beyond the SU(3)-limit, we observe that the introduction of the SU(3)-breaking effects does not change the $\Lambda\Lambda\sigma$ coupling constant, while the $\Xi\Xi\sigma$ and $\Sigma\Sigma\sigma$ couplings are modified by approximately 30\% and 50\%, respectively. We also note that the obtained value of the $\Lambda\Lambda\sigma$ coupling constant is small as compared to the $NN\pi$ and $NN\sigma$ coupling constants. Since the $\sigma$ exchange gives the dominant contribution in the $\Lambda\Lambda$ system, this suggests that the $\Lambda\Lambda$ interaction is weak, in accordance with the recent experimental result.
3.4 Discussion and Conclusions

We have calculated the $NN\sigma$, $\Lambda\Lambda\sigma$, $\Xi\Xi\sigma$ and $\Sigma\Sigma\sigma$ coupling constants which play significant roles in OBE models of $YN$ and $YY$ interactions, employing the external field QCDSR method. The coupling constants can be determined in terms of the quark-$\sigma$ coupling constant in this method. In order to compare our results with the others in the literature and stay as model-independent as possible, we find it useful to give the ratios of the coupling constants in the SU(3)-limit for the central values of the vacuum parameters:

\[
\frac{g_{\Lambda\Lambda\sigma}}{g_{NN\sigma}} = 0.43, \quad \frac{g_{\Xi\Xi\sigma}}{g_{NN\sigma}} = 0.08, \quad \frac{g_{\Sigma\Sigma\sigma}}{g_{NN\sigma}} = 0.91. \tag{3.51}
\]

We observe that the $\Xi\Xi\sigma$ coupling constant is more than one order of magnitude smaller than the $NN\sigma$ coupling constant. The $\Sigma\Sigma\sigma$ coupling constant is of the same order as the $NN\sigma$ coupling constant and twice as large as the $\Lambda\Lambda\sigma$ coupling constant. We have shown that these coupling constants as determined from QCDSR satisfy the SU(3) relations which lead to a determination of the $F/(F+D)$ ratio for the scalar octet. Although the scalar-meson–baryon coupling constants depend on the picture assumed for the structure of the scalar mesons ($q\bar{q}$ or $q^2\bar{q}^2$), the $F/(F+D)$ ratio remains intact in the two pictures. We also would like to note that the third terms on the RHS’s of the sum rules which represent the responses of the continuum thresholds, affect only the values of the individual coupling constants, which are multiplied by the same factor. Therefore the ratios of the coupling constants and the value of $\alpha_s$ remain unchanged if this term is omitted.

All the Nijmegen soft-core OBE potential models have $\theta_s > 30^\circ$, which points to almost ideal-mixing angles for the scalar $q\bar{q}$ states. Since the ideal-mixing angle for the scalar mesons has been assumed in our QCDSR calculations as well, it is convenient to compare our results with NSC potential models. Our result for the ratio $\frac{g_{\Lambda\Lambda\sigma}}{g_{NN\sigma}}$ is half of the value found in Ref. [86], however, it qualitatively agrees with the one from NSC89 [4], which is $\frac{g_{\Lambda\Lambda\sigma}}{g_{NN\sigma}} = 0.58$. The obtained value of the $NN\sigma$ coupling constant is in agreement with the large
value found in OBE models. The value we have obtained for the $\Lambda\Lambda \sigma$ coupling constant is small as compared to the $NN \sigma$ coupling constant and this implies that the $\Lambda\Lambda$ interaction is weak, since the $\sigma$ exchange gives the dominant contribution to this interaction in terms of OBE models. The value of the $F/(F + D)$ ratio, which is 0.55 as obtained from QCDSR, is about half of the values given in NSCa-f [5], which is $F/(F + D) \approx 1.1$.

In order to estimate the effect of SU(3)-breaking in the coupling constants, we have restored the physical values of the parameters like the strange quark mass and the physical baryon masses. We observe that the SU(3)-breaking effects do not change the $\Lambda\Lambda \sigma$ coupling, while the $\Xi\Xi \sigma$ and $\Sigma\Sigma \sigma$ couplings are modified largely. It is also desirable to derive the sum rules for the $BBf_0$ and $BBa_0$ couplings in order to estimate the SU(3)-breaking in these coupling constants.