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Parallel Damping Injection for the Quarter Car Suspension System.

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Abstract—In this paper we study an application of Passivity-Based Control (PBC) to a quarter car suspension system. We use Passivity-Based Control in the Brayton-Moser framework (BM-PBC) that has recently been developed for control of switching and non-switching electrical circuits. Via the usual mass-inductor and spring-capacitor analogy, we translate these results to the mechanical domain. This yields new rules for damping injection, where to place the virtual damping, and how to tune it. The concept of parallel damping from the electrical domain is now used for the quarter car model. The results are compared with the industrial sky-hook damping strategy, and in simulation the parallel BM-PBC outperforms the sky-hook damping strategy.

I. INTRODUCTION

Over the last 20 years, the quarter car model has been successfully used for study and design of active suspension systems of cars. A well-known and industrially implemented control strategy is the sky-hook damper [1], [8], [17], [18] which can be interpreted as a damper between the mass of the chassis and a virtual reference in the sky. This damping strategy appears to work better for the comfort in the car than only standard damping in between the two masses (of the chassis and the tire). In this paper, we compare the sky-hook damping strategy with a damping strategy that stems from a Passivity-Based Control (PBC) scheme for electrical circuits.

Recently, Passivity-Based Control for switching and non-switching electrical circuits based on the Brayton-Moser framework has been developed, [5], [6]. The Brayton-Moser equations stem from the early 60’s, [2], and describe the equations of motion with help of the so-called mixed-potential function. This mixed-potential function has the units of power, and can be used for PBC, whereas the usual Euler-Lagrange and Hamiltonian PBC methods, [13], [14] are based on the energy in the system. This gives rise to power shaping, [12], as an alternative to energy shaping, [14]. The advantage of using the Brayton-Moser framework is that for electrical circuits there is no dissipation obstacle, and tuning rules for damping injection, based on the original results of Brayton and Moser can be given. Furthermore, parallel damping injection naturally occurs as a possibility, and in case of switched-mode power-converters, it is shown in [6] that, in contrast to the usual series-damping injection schemes, it is robust against load variations.

In this paper, we use the usual mass-inductor, spring-capacitor analogy so that we can use the Brayton-Moser framework for control of the quarter car suspension system. The parallel damping injection scheme for electrical circuits can be translated to the mechanical domain, resulting in a force controlled damper interpretation, which is new for the quarter car model (and, up to our knowledge, not used in the mechanical domain before). The resulting simulations and comparison with the sky-hook damping schemes look promising.

In Section II, some background on the Brayton-Moser equations is given. In Section III the quarter car model in Brayton-Moser form is obtained, and in Section IV the different control structures (series damping, parallel damping and sky-hook damping) are given. In Section V a simulation study of a realistic quarter car setup is studied. Finally, in Section VI some conclusions, open issues and on-going work are presented.

II. RLC-CIRCUITS: THE BrayTON-MosoEr EQUATIONS (BM)

For complete electrical RLC circuits with possibly nonlinear elements Brayton and Moser have given the equations of motion in [2]. The standard definitions of respectively inductance and capacitance matrices are given by

\[ L(i_\rho) = \frac{\partial \varphi_\rho(i_\rho)}{\partial i_\rho}, \quad C(u_\sigma) = \frac{\partial q_\sigma(u_\sigma)}{\partial u_\sigma}, \]

where \( i_\rho \in \mathbb{R}^r \) represents the currents flowing through the inductors, \( \varphi_\rho(i_\rho) \in \mathbb{R}^r \) is the related magnetic flux vector, \( u_\sigma \in \mathbb{R}^s \) defines the voltages across the capacitors and the vector \( q_\sigma(u_\sigma) \in \mathbb{R}^s \) represents the charges stored on the capacitors. The Brayton-Moser (BM) equations are given by

\[ \dot{Q}(x) \dot{x} = \frac{\partial P(x)}{\partial x}, \quad (1) \]

where \( x = (i_\rho^T, u_\sigma^T)^T \in \mathbb{R}^{r+s} \), and

\[ Q(x) = \begin{pmatrix} -L(i_\rho) & 0 \\ 0 & C(u_\sigma) \end{pmatrix}. \quad (2) \]
Furthermore, the mixed-potential function $P : \mathbb{R}^{r+s} \to \mathbb{R}$, which contains the interconnection and resistive structure (including the sources) of the circuit, is defined as

$$P(x) = F(i_\rho) - G(u_\sigma) + \lambda^T \Lambda u_\sigma.$$  \hfill (3)

$F : \mathbb{R}^r \to \mathbb{R}$ and $G : \mathbb{R}^s \to \mathbb{R}$ being the current potential (content) related with the current-controlled resistors and sources, and the voltage potential (co-content) related with the voltage-controlled resistors (i.e., conductors) and sources, respectively. More specifically, the content and co-content are defined by the integrals

$$F(i_\rho) = \int_0^{i_{\rho}} \dot{u}_R(u'_\rho) dt'_\rho,$$

and

$$G(u_\sigma) = \int_0^{u_\sigma} \dot{u}_G(u'_\sigma) du'_\sigma,$$

where $\dot{u}_R(i_\rho)$ and $\dot{u}_G(u_\sigma)$ are the characteristic functions of the (current-controlled) resistors, sources and conductors (voltage-controlled resistors and sources), respectively. The $r \times s$ matrix $\Lambda$ is given by the interconnection of the network.

### III. THE QUARTER CAR MODEL IN BRAYTON-MOSER FORM

The quarter car model as presented in Figure 1 is standard, linear, and has four energy storing elements, i.e., the two masses, and the two springs. Furthermore, $u$ is the input through which the suspension can be controlled. Using the usual mass-inductor (velocity-current) and spring-capacitor (force-voltage) analogy, we consider the equations of motion in terms of the velocity, $v_1$, of the sprung mass (the car’s chassis), the velocity, $v_2$, of the unsprung mass (the tire and the wheel), the force, $f_1$, corresponding to first linear spring and the force, $f_2$, corresponding to the second linear spring. Thus, the number of ‘capacitive’ and ‘inductive’ elements is $r = s = 2$, respectively. The standard linear damping coefficient between the chassis and the tire is given by $d_1$. The contact between the tire and the road has been modeled as a spring with a high stiffness ($k_2$) and a small damper ($d_2$). This model can be found in e.g., [3], [4], [7]. The parameters used in this paper stem from an experimental set-up, see [3], and are given in Table I.

For the dynamical equations, we first define the displacement coordinates $x_1$ and $x_2$:

$$x_1 = h_1 - h_{10},$$

$$x_2 = h_2 - h_{20},$$

where $h_1$ and $h_2$ are the height of the chassis and the wheel respectively and $h_{10}$ and $h_{20}$ are the heights at equilibrium. Now, the forces $f_1$ and $f_2$ are defined as

$$f_1 = k_1(x_1 - x_2) + m_1 g,$$

$$f_2 = k_2(x_2 - x_b) + m_2 g.$$ 

The mixed-potential function for this mechanical system is given by

$$P(v, f) = f_1(v_1 - v_2) + f_2(v_2 - v_b)$$

$$+ \frac{1}{2} d_1(v_1 - v_2)^2 + \frac{1}{2} d_2 v_2^2 - d_2 v_b v_2$$

$$- u(v_1 - v_2),$$

where, as in equation (3), we have

$$F(v_1, v_2) = \frac{1}{2} d_1(v_1 - v_2)^2 + \frac{1}{2} d_2 v_2^2 - d_2 v_b v_2$$

$$- u(v_1 - v_2),$$

$$G(f_1, f_2) = - f_2 v_b$$

$$\Lambda = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix},$$

and thus the BM-equations are given by

$$Q \dot{z} = \frac{\partial P(z)}{\partial z},$$

with $z = (v_1, v_2, f_1, f_2)^T$, and

$$Q = \text{diag} \left( -m_1, -m_2, \frac{1}{k_1}, \frac{1}{k_2} \right),$$

resulting in the following equation of motion

$$m_1 \ddot{v}_1 = - f_1 - d_1(v_1 - v_2) + u$$

$$m_2 \ddot{v}_2 = f_1 + d_1(v_1 - v_2) - f_2 - d_2(v_2 - v_b) - u$$

$$\frac{1}{k_1} \dot{f}_1 = v_1 - v_2$$

$$\frac{1}{k_2} \dot{f}_2 = v_2 - v_b,$$

which can also be obtained from Newtons’ second law.
IV. BM-PBC OF THE QUARTER CAR

Passivity-Based Control in the Brayton-Moser framework, e.g., [5], [6], follows the same principle as Passivity-Based Control for the Euler-Lagrange equations, e.g., [13]. This means that first a copy of the system in terms of the controller states $z_d$ is made and then damping is added in the error terms, i.e., the controller is given by

$$Q z_d = \frac{\partial}{\partial z_d} (P(z_d) - P_a(\ddot{z})),$$

where $\ddot{z} := z - z_d = (\ddot{v}^T, f^T)^T = (v_1 - v_{1d}, v_2 - v_{2d}, f_1 - f_{1d}, f_2 - f_{2d})^T$ is the error state, and the scalar function $P_a(\ddot{z}) := F_a(\ddot{v}) - G_a(\ddot{f})$ is the added damping mixed-potential in which $F_a$ represents the injected velocity-controlled damping (content) and $G_a$ represents the injected force-controlled damping (co-content). This process can be viewed as a ‘shaping’ of the mixed-potential, since our closed loop error mixed-potential is now given by

$$P_a(\ddot{z}) := \ddot{z}_3(\ddot{z}_1 - \ddot{z}_2) + \ddot{z}_4(\ddot{z}_2 - v_b) + \frac{1}{2} d_1(\ddot{z}_1 - \dot{z}_2)^2 + \frac{1}{2} d_2 z_2^2 - d_2 v_b \ddot{z}_2 + P_a(\ddot{z}).$$

A. Series damping injection

In the case of series damping injection, we choose $G_a(\ddot{f}) = 0$ and

$$F_a(\ddot{v}) = \frac{1}{2} d_1(\ddot{v}_1 - \ddot{v}_2)^2,$$

with $d_i$ the damping coefficient. This corresponds to adding a virtual velocity-controlled damper between $m_1$ and $m_2$ in the error coordinates. As a result, the (implicit) controller equations can now be calculated straightforwardly as

$$m_1 \ddot{v}_{1d} = -f_{1d} - d_1(v_{1d} - v_{2d}) + d_1(\ddot{v}_1 - \ddot{v}_2) + u,$$

$$m_2 \ddot{v}_{2d} = f_{1d} + d_1(v_{1d} - v_{2d}) - d_1(\ddot{v}_1 - \ddot{v}_2) - f_{2d} - d_2(v_{2d} - v_b) - u$$

$$\frac{1}{k_1} \dot{f}_{1d} = v_{1d} - v_{2d},$$

$$\frac{1}{k_2} \dot{f}_{2d} = v_{2d} - v_b.$$

As closed-loop objective we have $v_1 = 0$. This means that in the controller equations we set $v_{1d} = 0$ and $\dot{v}_{1d} = 0$, resulting in

$$u = f_{1d} - d_1 v_{2d} - d_i(v_1 - \ddot{v}_2).$$

The tuning of $d_i$ can now be determined based on the criteria given by Brayton and Moser in [2], which has been worked out for control purposes in [5], [6]. This results in $d_i = 6788$ Nsm$^{-1}$.

B. Parallel damping injection

In the case of parallel damping injection we add force controlled damping. In mechanical systems this does not often occur, and therefore, in the setting of the quarter car model, up to our knowledge, this concept is new. Choose $F_a = 0$, and

$$G_a(\ddot{f}) = \frac{1}{2d_{1d}} \ddot{f}_{1d}^2 + \frac{1}{2d_{2d}} \ddot{f}_{2d}^2.$$

This corresponds to adding virtual force controlled dampers in series with spring 1 and 2. In a similar fashion as before, the (implicit) controller equations can then be calculated straightforwardly as

$$m_1 \ddot{v}_{1d} = -f_{1d} - d_1(v_{1d} - v_{2d}) + u,$$

$$m_2 \ddot{v}_{2d} = f_{1d} + d_1(v_{1d} - v_{2d}) - f_{2d} - d_2(v_{2d} - v_b) - u,$$

$$\frac{1}{k_1} \dot{f}_{1d} = v_{1d} - v_{2d} + \frac{1}{d_{1d}} \ddot{f}_{1d},$$

$$\frac{1}{k_2} \dot{f}_{2d} = v_{2d} - v_b + \frac{1}{d_{2d}} \ddot{f}_{2d}.$$

As closed-loop objective we again have $v_1 = 0$. This means that in the controller equations we set $v_{1d} = 0$ and $\dot{v}_{1d} = 0$, resulting in

$$u = f_{1d} - d_1 v_{2d}.$$

The tuning of $d_{1d}$ and $d_{2d}$ can now be determined based on the criteria given by Brayton and Moser in [2], which has been worked out for control purposes in [5], [6]. This results in $d_{1d} = 68.65$ Nsm$^{-1}$.

C. Sky-hook damping

For comparison with the often applied sky-hook damper, we add the equations for the corresponding controller equations here. The sky-hook damper is nothing else than

\begin{table}[h]
\centering
\caption{Parameters of simulation}
\begin{tabular}{|l|l|}
\hline
Parameters & Values \\
\hline
The mass of the chassis $m_1$ & 243 kg \\
The mass of the wheel $m_2$ & 40.8 kg \\
The coefficient of damping between the chassis and the wheel $d_1$ & 370 Nm$^{-1}$ \\
The coefficient of damping created by the tire $d_2$ & 414 Nm$^{-1}$ \\
The stiffness of the spring between the chassis and the wheel $k_1$ & 14671 Nm$^{-1}$ \\
The stiffness of the spring created by the tire $k_2$ & 124660 Nm$^{-1}$ \\
\hline
\end{tabular}
\end{table}
a damper between the mass \( m_1 \) and some virtual reference in the sky, i.e.,

\[
u = -d_{\text{sky}} v_1\]

Sometimes a so-called rattle velocity is added to this control, i.e. [9], which essentially means that another damper is added to the spring, or in other words, it means that \( d_1 \) is updated, i.e.,

\[
u = -d_{\text{sky}} v_1 - d_{\text{rattle}} (v_1 - v_2).
\]

In our simulations we have not included the rattle velocity, since it made the response worse. We have chosen \( d_{\text{sky}} = 6788 \text{ Nsm}^{-1} \).

V. SIMULATIONS

We have simulated the above three control schemes when a bump in the road occurs using Matlab/Simulink. The bump is specified in Fig. 2. The displacement and acceleration of the chassis are displayed in Fig. 3 and Fig. 4.

From the figures we see that the series damping BM-PBC has large displacement and acceleration right after the bump occurs. In fact, it is even higher than in the open loop case, though in the open loop case it takes longer to converge back to the equilibrium. The sky-hook damper and the parallel damping BM-PBC clearly perform much better. The maximum sky-hook closed-loop displacement is about twice as high as the parallel damping BM-PBC closed-loop displacement, and is about 23 % and 11 %, respectively, of the bump height. On the other hand, the sky-hook damper converges just a little faster back to displacement position 0. In the acceleration figure, it can be seen that the parallel damping BM-PBC outperforms the sky-hook damper as well, i.e., the sky-hook dampers’ acceleration is higher and also oscillating more.

Performing simulations with uncertainty in the mass \( m_1 \) (due to e.g. a changing number of passengers) yields similar results, i.e., the parallel damping BM-PBC outperforms the sky-hook damper.

VI. CONCLUSIONS AND OUTLOOK

In this paper, we have studied a new control scheme for the suspension system of the linear quarter car model. The scheme is based on extension to mechanical systems of a control method developed for electrical circuits. It relies on force feedback. In simulation it is shown that this scheme outperforms the traditional sky hook damping control scheme from industry. Even though most quarter car models studied in the literature and used for control in industry are linear, in practice nonlinearities occur. The Brayton-Moser framework is in principle suitable for nonlinear systems as well, and hence, the framework and the control scheme may be extended to nonlinear quarter car models.

The proposed control scheme uses a parallel damping principle that relies on force feedback, i.e., it requires knowledge of spring forces. This is not very common yet in the car industry. However, new developments and collaborations with the car industry show that force sensors have recently become feasible.

Recently, we have gained access to a realistic industrial simulation model of a full car that simulates among others the roll dynamics of a car. Currently a Brayton-Moser model is developed for that full car model. Controller design via force feedback for anti-roll stabilization based on these models is under way.

REFERENCES

Fig. 3. Chassis displacement; open-loop versus parallel, series, and sky-hook damping injection.

Fig. 4. Chassis acceleration; open-loop versus parallel, series, and sky-hook damping injection.


