High-precision (p,t) reactions to determine reaction rates of explosive stellar processes

Matić, Andrija

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Chapter 2

Theoretical model of stellar reaction rates

In this chapter we will discuss the basic theoretical model necessary to calculate stellar reaction rates. The complete formalism and the notation are taken from Ref. [4]. In the following natural units will be used. Here, we will discuss only reaction rates induced by a charged particle.

2.1 Stellar reaction rates

The non-degenerate stars have a “simple” structure. The hottest and most dense region is the core. Going from the interior outwards, the stellar temperature and pressure drop. A normal non-degenerate star consists of a plasma containing nuclei and free electrons. The plasma in the stellar core is fully ionized, and consists of various isotopes. Because this plasma is in thermodynamic equilibrium, the velocity distributions of the nuclei can be described by Maxwell-Boltzmann velocity distributions, see Ref. [4],

\[
\phi(\nu) = 4\pi\nu^2 \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( -\frac{m\nu^2}{2kT} \right)
\]  

(2.1)

This equation can be written in terms of the kinetic energy of the nucleus:

\[
\phi(E) \propto E \exp \left( -\frac{E}{kT} \right)
\]  

(2.2)

where the most probable value of the kinetic energy is equal to \( kT \).

The total reaction rate for a nuclear reaction \( a + B \rightarrow c + D \) can be written as:

\[
R = N_a N_B \langle \sigma \nu \rangle (1 + \delta_{aB})^{-1}
\]  

(2.3)

where \( N_a \) and \( N_B \) are the numbers of particles of type \( a \) and \( B \) per cubic centimeter in a stellar plasma, respectively. The factor \( (1 + \delta_{aB}) \) prevents double counting in case identical particles interact with each other. Here,

\[
\langle \sigma \nu \rangle = \int_0^\infty \phi(\nu)v\sigma(v)dv
\]  

(2.4)
2. Theoretical model of stellar reaction rates

is referred to as the reaction rate per particle pair, where \( \sigma(v) \) is the nuclear cross section, and \( \phi(v)dv \) is the probability that the relative velocity \( v \) between the particles involved in the nuclear reaction is between \( v \) and \( v+dv \). The integration for exothermic reactions extends from \( v=0 \) up to infinity, and for endothermic reactions the integration starts at the threshold velocity for a particular reaction.

As mentioned before, for nuclear reactions taking place in the stellar interior, the distribution of the relative velocity \( v \) of the interacting nuclei \( a \) and \( B \) is described by the Maxwell-Boltzmann velocity distribution, and the reaction rate per particle pair is given as:

\[
\langle \sigma v \rangle = \int_0^\infty \int_0^\infty \phi(v_a)\phi(v_B)v_a v_B \sigma(v)dv_adv_B.
\]

After a proper kinematic transformation, using the reduced mass \( \mu \) and total mass \( M \), and integration over the center-of-mass velocity (the cross section depends only on the relative velocity between the interacting particles) we obtain the following equation:

\[
\langle \sigma v \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E)E \exp\left(-\frac{E}{kT}\right) dE
\]

(2.6)

2.2 Non-resonant reaction rates (direct reaction rates)

A nucleus is a positively charged entity, and therefore, two colliding nuclei repel each other with a force proportional to the product of their respective nuclear charges. This repulsive force leads, in the case of a charged particle undergoing an attractive nuclear interaction, to a potential barrier called the Coulomb barrier. For the fusion of two charged particles they have to penetrate through the Coulomb and centrifugal barriers; see Fig. 2.1. The total repulsive potential is given

\[
V = \frac{Z_B Z_a e^2}{r} + \frac{l(l+1)\hbar^2}{2\mu r^2}
\]

(2.7)

where \( Z_B \) and \( Z_a \) are the nuclear charges of the interacting particles, \( r \) is their mutual distance, \( \mu \) is the reduced mass of the projectile-target system, and \( l \) is the orbital angular momentum.

The typical Coulomb barrier for the interaction between two light nuclei is of the order of a few hundred keV or higher. At a temperature of 0.0015 \( T_9 \), which is typical for a star like the Sun, energies of the nuclei are of the order of 1 keV. However, in case of the highest predicted supernovae temperature of 9 \( T_9 \), the corresponding energies for the nuclei are of the order of a few hundred keV. Therefore, the typical particle energies in the stellar environment are smaller than the repulsive Coulomb potential between the nuclei. However, Gamow [34] showed that nuclei can penetrate the barriers with a small but finite probability via the quantum-tunneling effect. The penetrability \( P_l \) through the Coulomb
2.2. Non-resonant reaction rates (direct reaction rates)

Figure 2.1: Schematic view of the combined nuclear, Coulomb, and centrifugal potentials. Where $R_C(E)$ is a classical turning point for the Coulomb and centrifugal barriers. Figure taken from Ref. [4].

and centrifugal barriers is energy dependent and can be expressed as:

$$P_l(E, R_n) = \frac{1}{F_l^2(E, R_n) + G_l^2(E, R_n)}$$  \hspace{1cm} (2.8)

where $F_l$ and $G_l$ are, respectively, the regular and the irregular solutions for the Coulomb wave function for a given relative angular momentum, at the nuclear interaction radius of:

$$R_n = 1.35 \times (A_a^{1/3} + A_B^{1/3}) \text{ fm}$$  \hspace{1cm} (2.9)

where $A_a$ and $A_B$ are the masses of the projectile and target, respectively, given in atomic mass units.

Non-resonant reactions are reactions with a one-step process, where a direct transition into a bound state occurs. Radiative capture, presented in Fig. 2.2, is one example of a non-resonant reaction. Other possible non-resonant reactions are: pickup and stripping reactions, Coulomb excitation, and charge-exchange processes.
In case of non-resonant reactions, the cross section is proportional to a single matrix element. In our example for radiative capture it is given as:

\[ \sigma_{\gamma} \propto | \langle D | H_{\gamma} | B + a \rangle |^2 \]  

(2.10)

where \( H_{\gamma} \) is an electromagnetic operator describing the transition. At a relative kinetic energy much smaller than the Coulomb barrier and for an orbital angular momentum \( l=0 \), the tunneling probability can be approximated as [4]:

\[ P = e^{-2\pi \eta} \]  

(2.11)

The quantity \( \eta \) is called the Sommerfeld parameter and it is equal to

\[ \eta = \frac{Z_B Z_a e^2}{\hbar \nu} \]  

(2.12)

The interaction cross section is dependent on the penetrability and the de Broglie wavelength, which describes the geometrical effects of the cross section \( \sigma \propto \pi \lambda^2 \propto \pi k^{-2} \). Including all these contributions, we can write the cross section as:

\[ \sigma(E) = \frac{1}{E} S(E) e^{-2\pi \eta} \]  

(2.13)

The factor \( e^{-2\pi \eta} \) describes the penetration through the Coulomb barrier of point-like nuclei.
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without orbital angular momentum (s-waves). The function $S(E)$ is called the astrophysical S-factor and contains all the nuclear physics of the reaction.

In non-resonant reactions, the cross section varies continuously as a function of the energy, see Fig. 2.3. For astrophysical applications we are usually interested in a cross section at an incident energy of a few keV where data usually do not exist. The astrophysical S-factor function $S(E)$ varies smoothly as a function of the energy for non-resonant reactions in the region well below the Coulomb barrier, see lower panel in Fig 2.3. Because of this characteristic feature, the astrophysical S-factor is used to extrapolate measured cross sections to energies relevant for the astrophysical environment. By combining Eqs. 2.13 and 2.6 we obtain the equation for the reaction rate for non-resonant stellar nuclear reactions

$$\langle \sigma v \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_{0}^{\infty} S(E) \exp \left( -\frac{E}{kT} - \frac{b}{E^{1/2}} \right) dE \quad (2.14)$$

where the quantity $b$ arises from barrier penetrability and is given as:

$$b = (2\mu)^{1/2} \pi e^2 Z_B Z_a / \hbar. \quad (2.15)$$

Since for non-resonant reactions, the astrophysical S-factor varies slowly, the strongest influence on the reaction rate is caused by the exponential penetrability term $-\frac{b}{E^{1/2}}$ and

![Figure 2.3: An example of the cross section and the astrophysical S-factor for a charged-particle non-resonant nuclear reaction. Figure taken from Ref. [4].](image)
Figure 2.4: The Gamow peak is indicated with the shaded area. Figure taken from Ref. [4].

The exponential Maxwell-Boltzmann term \((-\frac{E}{kT})\). The exponential factor which is related to the penetrability through the Coulomb barrier shifts the effective distribution of the reaction rates to a higher energy \(E_0\). The convolution of these two exponential functions results in a peak of the integrand near the energy \(E_0\), which is usually much larger than \(kT\), known as the Gamow peak; see Fig. 2.4.

For a given stellar temperature \(T\), nuclear reactions can take place in a relatively narrow region of energies around \(E_0\). Because \(S(E)\) varies slowly as a function of the energy, it can be approximated by a constant value over the Gamow peak:

\[
S(E) = S(E_0) = \text{constant},
\]  

and the reaction rate has the form:

\[
\langle \sigma v \rangle = \left(\frac{8}{\pi \mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} S(E_0) \int_0^\infty \exp \left(-\frac{E}{kT} - \frac{b}{E^{1/2}}\right) dE.
\]  

By taking the first derivative of this formula, the position of the Gamow peak for some
2.3 Resonance reaction rates through narrow resonances

A resonant process is a two-step process, in which an excited state \( E_r \) of the compound nucleus is formed that subsequently decays into lower-lying states. The resonant reaction shows a rapid variation of the cross section over a small energy range. An example of a resonant reaction is presented in Fig. 2.5.

The reaction cross section for resonant reactions is proportional to two matrix elements:

\[
\sigma_\gamma \propto | \langle D \mid H_D \mid E_r \rangle |^2 | \langle E_r \mid H_D \mid B + a \rangle |^2 .
\]

(2.20)

Where the matrix element involving the operator \( H_D \) describes the formation of the compound state \( E_r \), and the second matrix element describes the subsequent \( \gamma \)-decay. This process can happen only if the energy of the entrance channel matches closely with the nuclear reaction can be determined. It is given by the following formula:

\[
E_0 = \left( \frac{b k T}{2} \right)^{2/3} = 0.122 (Z_B^2 Z_a^2 \mu)^{1/3} T_9^{2/3} \text{ MeV} \tag{2.18}
\]

The width of the Gamow peak is given by:

\[
\Delta E = \frac{4}{\sqrt{3}} \sqrt{E_0 k T} = 0.237 (Z_B^2 Z_a^2 \mu)^{1/6} T_9^{5/6} \text{ MeV} \tag{2.19}
\]

In Section 2.5 we will show the relevant parameters for the Gamow windows for the \(^{18}\text{Ne}(\alpha,p)^{21}\text{Na},^{21}\text{Na}(p,\gamma)^{22}\text{Mg},^{25}\text{Al}(p,\gamma)^{26}\text{Si},\text{ and }^{22}\text{Mg}(\alpha,p)^{25}\text{Al} \) reactions discussed in this work.

2.3 Resonance reaction rates through narrow resonances

**Figure 2.5:** Resonant radiative capture as an example of resonant reactions. \( Q = m_a + m_B − m_D \) is the reaction \( Q \)-value. \( E_R \) is the center-of-mass projectile energy needed to populate the centroid of a resonance state.
energy of the resonance involved:

\[ E_R + Q = E_r. \]  

(2.21)

This process can occur for all excited states above the threshold energy \( Q \), when \( E_R \) satisfies the condition given by Eq. 2.21. In addition, a resonant state can be formed via a given reaction channel if selection rules are fulfilled (angular momentum and parity conservation laws).

Resonance phenomena often occur in physical systems. The cross section for a resonant reaction can be written in the form

\[ \sigma(E) \propto \frac{\Gamma_A \Gamma_c}{(E - E_R)^2 + (\Gamma/2)^2} \]  

(2.22)

using the analogy with a damped oscillator driven by an external force. \( \Gamma_A \) and \( \Gamma_c \) are the partial widths of the entrance and exit channels, respectively, and \( \Gamma \) is the total width of the resonant state. The cross section is exactly given by the Breit-Wigner formula:

\[ \sigma_{BW}(E) = \pi \lambda^2 \frac{2J + 1}{(2J_a + 1)(2J_B + 1)} (1 + \delta_{aB}) \frac{\Gamma_A \Gamma_c}{(E - E_R)^2 + (\Gamma/2)^2} \]  

(2.23)

Where \( \lambda \) is the de Broglie wave length and \( J, J_a \) and \( J_B \) are the spins of the resonant state, of the projectile, and of the target, respectively. The term

\[ \omega = \frac{2J + 1}{(2J_a + 1)(2J_B + 1)} (1 + \delta_{aB}) \]  

(2.24)

is known as the spin statistical factor, which can be obtained by summing over all final states and averaging over initial states. The summing over the final states reflects that the probability for a given process increases with an increasing number of available final states. Because in the entrance channel the colliding nuclei can have \((2J_a+1)\) and \((2J_B+1)\) substates, the factor \( \frac{1}{(2J_a + 1)(2J_B + 1)} \) reflects the probability that in the entrance channel the nuclei are in one particular substate.

The criterion for narrow resonances is that the resonance width is much smaller than the resonance energy: \( \Gamma \ll E_R \). Ref. [4] presents a quantitative criterion:

\[ \frac{\Gamma}{E_R} \ll 10\%. \]  

(2.25)

An example of a narrow resonance is given in Fig. 2.6. Under this circumstance the Maxwell-Boltzmann function changes very little over the resonance region and the term \( E \exp \left( -\frac{E}{kT} \right) \) in Eq. 2.6 can be approximated by \( E_R \exp \left( -\frac{E_R}{kT} \right) \) and taken outside the integral in Eq. 2.6:
2.3. Resonance reaction rates through narrow resonances

\[ \langle \sigma \nu \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} E_R \exp\left(-\frac{E_R}{kT}\right) \int_0^{\infty} \sigma_{BW}(E)dE \quad (2.26) \]

For the integration of the Breit-Wigner cross-section yield for a narrow resonance, we can neglect the energy dependence of \( \lambda, \Gamma, \Gamma_a \) and \( \Gamma_c \):

\[ \int_0^{\infty} \sigma_{BW}(E)dE = \pi^2 \lambda^2 \frac{2J + 1}{(2J_a + 1)(2J_B + 1)} (1 + \delta_{aB}) \frac{\Gamma_a \Gamma_c}{\Gamma} \quad (2.27) \]

where the product

\[ \omega_\gamma = \omega \frac{\Gamma_a \Gamma_c}{\Gamma} = \frac{2J + 1}{(2J_a + 1)(2J_B + 1)} (1 + \delta_{aB}) \frac{\Gamma_a \Gamma_c}{\Gamma} \quad (2.28) \]

is referred to as the strength of the resonance. Low-energy narrow resonances well below the Coulomb barrier will have a larger probability for decay by \( \gamma \)-rays as compared to particle decay. For example, in case of the \((p, \gamma)\) reaction we have \( \Gamma_\gamma \gg \Gamma_p \) and \( \Gamma_\gamma \approx \Gamma \). Consequently the resonance strength depends on the partial decay width by proton emission \( \omega_\gamma = \omega \frac{\Gamma_a \Gamma_c}{\Gamma} \approx \omega \Gamma_p \). However, in case the resonance energy is well above the Coulomb barrier, \( \Gamma_p \) will be much larger than \( \Gamma_\gamma \), and \( \Gamma_\gamma \) will be the dominant factor in the resonance strength.
The particle partial width can be calculated as shown in Ref. [9] as

$$\Gamma_{\text{particle}} = \frac{3\hbar^2}{\mu R_n^2} P_l C^2 S_{\text{particle}}$$

(2.29)

where $\mu$ is the reduced mass of the interacting particle, $S_{\text{particle}}$ is its spectroscopic factor (in our case proton or alpha), $C$ is the isospin Clebsch-Gordan coefficient, and $P_l$ is the penetrability through the Coulomb barrier and centrifugal barrier for orbital-momentum $l$ evaluated at an interaction radius $R_n$; see Eq. 2.9. To calculate the particle penetrability through the Coulomb and centrifugal barriers we used the code PENE [35].

In the case where several narrow resonances contribute to the reaction rates, their contributions are simply summed to obtain the reaction rates for a particular reaction;

$$\langle \sigma \nu \rangle = \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \sum_i (\omega \gamma)_i exp \left( -\frac{E_i}{kT} \right)$$

(2.30)

We obtained this formula by combining Eqs. 2.26, 2.27 and 2.28.

Inserting all the relevant quantities, the reaction rate can be calculated by the formula:

$$N_A \langle \sigma \nu \rangle = 1.54 \times 10^5 (\mu T_9)^{-3/2} \sum_i (\omega \gamma)_i \times exp(-11.605E_i/T_9) \text{ [cm}^3\text{s}^{-1}\text{mol}^{-1}]$$

(2.31)

where the resonance strength $(\omega \gamma)_i$ is in units of eV, and the resonance energy $E_i$ is in units of MeV.

### 2.4 Reactions through broad resonances

Here, we consider resonances which are broader than the relevant energy window for a given stellar temperature. According to the definition given in Ref. [4], a broad resonance is a resonance where

$$\frac{\Gamma}{E_R} \geq 10\%.$$ 

(2.32)

The cross section $\sigma(E)$ extends over a large range of energies, and the dependence of the cross section on energy needs to be taken into account for the calculation of the stellar reaction rate according to Eq. 2.6. The energy dependence of the cross section can be calculated as:

$$\sigma(E) = \sigma_R \frac{E_R}{E} \frac{\Gamma_a(E)}{\Gamma_a(E_R)} \frac{\Gamma_c(E)}{\Gamma_c(E_R)} \frac{(\Gamma_R/2)^2}{(E-E_R)^2 + \Gamma^2(E)/4}$$

(2.33)

where the cross section and the total width are known at the resonance energy: $\sigma_R = \sigma(E_R)$, $\Gamma_R = \Gamma(E_R)$. Obviously, knowledge of the energy dependence of the partial
widths is necessary for the calculation of the cross section. In general, charged particles need to penetrate Coulomb and centrifugal barriers. The particle partial width can be calculated by

\[ \Gamma_l(E) = \frac{2\hbar}{R_n} \left( \frac{2E}{\mu} \right)^{1/2} P_l(E, R_n) \theta_l^2, \]  

(2.34)

where the quantity \( \theta_l^2 \) is called the reduced width of the nuclear state, which represents the probability of finding the excited state in the configuration \( l \). Usually \( \theta_l \) is determined experimentally. The penetrability factor is given in Eq. 2.8. With an increasing orbital angular momentum \( l \), the centrifugal barrier becomes larger than the Coulomb barrier and \( \Gamma_l \) drops rapidly. As an example, the calculated partial proton widths for different values of \( l \) for the \( ^{16}\text{O}+p \rightarrow ^{17}\text{F} \) reaction are presented in Fig. 2.7.

Figure 2.7: The calculated partial proton width \( \Gamma_l \) for the reaction channel \( ^{16}\text{O}+p \rightarrow ^{17}\text{F} \) as function of proton energy for values of the orbital angular momentum \( l = 0 \) to \( 6 \hbar \). Figure taken from Ref. [4].

The energy dependence of \( \Gamma_\gamma \) is given as:

\[ \Gamma_\gamma(E_\gamma) = \alpha_L E_\gamma^{2L+1} \]  

(2.35)

where \( L \) is the multipolarity of the emitted \( \gamma \)-ray and \( \alpha_L \) is constant for each multipolarity
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depending on the nuclear structure of the resonance. Still, the energy dependence of the partial $\gamma$-widths is not as strong as for particle emission, because of the particle penetrability through the Coulomb barrier. Partial $\gamma$-widths are of the order of 1 eV or smaller. In contrast, the particle widths can be very small well below the Coulomb barrier, and they can be in the order of MeV above the Coulomb barrier.

If the resonance is near or above the Coulomb barrier, the partial particle width varies little over the resonance region ($E=E_R \pm \Gamma_R/2$); see Fig. 2.7. In contrast, for a resonance well below the Coulomb barrier, the partial particle width for the entrance channel varies very rapidly. On the other hand, the partial particle width for the outgoing channel, in case of ($\alpha$,p) reactions discussed here, varies more slowly, because the emitted particle has an energy, which is increased by the amount of the $Q$-value of the reaction. The cross section is not symmetric with respect to $E_R$ because it varies much more for energies below the resonance energy $E_R$ as compared to energies above the resonance energy.

2.5 The astrophysically relevant excitation-energy regions for $^{22}$Mg and $^{26}$Si

In the previous sections we discussed direct reactions and the simplest two resonant reactions, which are important for the calculation of the stellar reaction rates. The formalism given in the previous sections will be enough to calculate rates for the $^{18}$Ne($\alpha$,p)$^{21}$Na, $^{21}$Na(p,$\gamma$)$^{22}$Mg, $^{25}$Al(p,$\gamma$)$^{26}$Si, and the $^{22}$Mg($\alpha$,p)$^{25}$Al reactions discussed in this thesis. The Gamow window concept is directly applicable in the case of direct reactions. However, it is useful for resonant reactions to calculate the position of the Gamow window for a particular stellar temperature $T$ and to see in which excitation-energy region a particular resonance will dominate.

In Section 1.2 we already mentioned that at a stellar temperature above 0.8 $T_9$ the $^{18}$Ne($\alpha$,p)$^{21}$Na reaction becomes possible. This reaction is one of the possibilities to link reactions between the hot CNO cycle and the NeNa cycle. This breakout from the hot-CNO cycle gives the energy trigger for the X-ray bursts. For the more accurate X-ray bursts models it is therefore necessary to obtain more precise data for $^{22}$Mg resonances for the entire span of stellar temperatures up to 2.5 $T_9$. On the right side of Fig. 2.8 we present the positions of the Gamow windows (peak position and width) for the $^{18}$Ne($\alpha$,p)$^{21}$Na reaction at 0.8 $T_9$ and 2.5 $T_9$.

In Section 1.4.1 we discussed the astrophysical importance of the $^{21}$Na(p,$\gamma$)$^{22}$Mg reaction as a tool to check present novae models. Since the novae peak temperatures reach values of 0.4 $T_9$, we are interested in $^{22}$Mg levels up to 5.95 MeV. For the X-ray bursts we are interested in temperatures up to 2.5 $T_9$. On the left side of Fig. 2.8 we show the Gamow window for this reaction for the temperature of 2.5 $T_9$. From Fig. 2.8 it can be seen that we are interested in the $^{22}$Mg nuclear structure from the proton-emission threshold up to 11.5
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MeV.

In Section 1.4.2 we already mentioned that $^{25}\text{Al}(\beta^+ \nu)^{25}\text{Mg}(p,\gamma)^{26}\text{Al}_{g.s.}$ can occur at a stellar temperature above 0.035 $T_9$ and $^{25}\text{Al}(p,\gamma)^{26}\text{Si}(\beta^+ \nu)^{26}\text{Al}^*$ at higher stellar temperatures. Because supernovae explosions can be possible sources for $^{26}\text{Al}$ production we are interested in the $^{25}\text{Al}(p,\gamma)^{26}\text{Si}$ reaction rates for temperatures up to the supernovae peak temperature of $4 T_9$. On the left side in Fig. 2.9 we indicate Gamow windows at $2.5 T_9$ and $4 T_9$ for the $^{25}\text{Al}(p,\gamma)^{26}\text{Si}$ reaction at the X-ray bursts and supernovae peak temperatures, respectively. From the same figure it can be seen that for this reaction we are interested in $^{26}\text{Si}$ levels up to 8 MeV.

The $^{22}\text{Mg}(\alpha, p)^{25}\text{Al}$ reaction was not previously studied. On the left side of Fig. 2.9 Gamow windows for this reaction are shown for temperatures of $1 T_9$, for the X-ray bursts peak temperature $2.5 T_9$ and supernovae peak temperature $4 T_9$. From the same figure it can be seen that for this reaction we are interested in $^{26}\text{Si}$ levels up to 13 MeV.
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Figure 2.8: Relevant astrophysical windows for the $^{18}\text{Ne}(\alpha,p)^{21}\text{Na}$ (right) and $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ (left) reactions at temperatures of 0.8 $T_9$ and 2.5 $T_9$ for the $^{18}\text{Ne}(\alpha,p)^{21}\text{Na}$ reaction and at a temperature of 2.5 $T_9$ for the $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ reaction. The full horizontal lines indicate the thresholds for proton and $\alpha$ emission, respectively. The dashed lines indicate for illustrative purposes the positions of two known levels in $^{22}\text{Mg}$. 

$^{18}\text{Ne} + \alpha$\hspace{2cm} $^{21}\text{Na} + p$\hspace{2cm} $^{22}\text{Mg}$

8.141 MeV

10.272 MeV

12.665 MeV

2.5 $T_9$

0.8 $T_9$
2.5. The astrophysically relevant excitation-energy regions for $^{22}\text{Mg}$ and $^{26}\text{Si}$

Figure 2.9: Relevant astrophysical windows for the $^{22}\text{Mg}(\alpha,p)^{25}\text{Al}$ (right) and $^{25}\text{Al}(p,\gamma)^{26}\text{Si}$ (left) reactions at temperatures of 1 $T_9$, 2.5 $T_9$ and 4 $T_9$ for the $^{22}\text{Mg}(\alpha,p)^{25}\text{Mg}$ reaction and at temperatures of 2.5 $T_9$ and 4 $T_9$ for the $^{25}\text{Al}(p,\gamma)^{26}\text{Si}$ reaction. The full horizontal lines indicate the thresholds for proton and $\alpha$ emission, respectively. The line at 13 MeV excitation energy is marked for illustrative purposes.