The Foamy Morphology of the 2dF Galaxy Distribution

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ABSTRACT — We present a reconstruction of the foamy morphology of the galaxy distribution in the 2dF Galaxy Redshift Survey (2dFGRS). The Delaunay Tessellation Field Estimator (DTFE) was used to reconstruct the projected galaxy surface density field as well as the full three-dimensional galaxy density field. The DTFE is a self-adaptive method for a volume-covering reconstruction of continuous density fields which are sampled by a discrete set of points. It is capable of resolving highly complex point distributions such as the large scale galaxy distribution, which is characterized by anisotropic filamentary and planar features, a hierarchy of spatial scales and densities and a cellular geometry with extended and poorly sampled low density voids. Here we present maps of the projected galaxy surface density for different regions in the 2dFGRS as well as three-dimensional renderings of the complete 2dFGRS region. We discuss our results and some of the various possible applications.
7.1 Introduction

Over the last decades ever deeper redshift surveys have provided us with an increasingly refined view of our cosmic neighborhood (examples include the CfA Redshift Survey (Huchra et al. 1990), the Las Campanas Redshift Survey (Shectman et al. 1996), the 2dF Galaxy Redshift Survey (hereafter 2dFGRS, Colless et al. 2001, 2003) and the Sloan Digital Sky Survey (Stoughton et al. 2002). These surveys have shown that galaxies are not randomly distributed in the universe, but reside in an intriguing network, the cosmic web (Bond et al. 1996, van de Weygaert 1991, 2002). At the nodes of this network reside dense and compact galaxy clusters which form deep potential wells trapping hot X-ray emitting gas and containing up to thousands of galaxies. The Virgo and Coma clusters form the most nearby examples. Clusters are connected by filamentary and wall-like structures of intermediate densities ($\delta \rho / \rho \sim 10$), usually consisting of a large number of galaxy groups, each containing tens of galaxies. The largest of these structures are called superclusters, an example of which is the Local Supercluster, which has the Local Group and thus the Milky Way amongst its members. These extended and elongated structures enclose large and empty regions, voids. The result is a foamy network which is pervading the whole observable universe. The highly complex cellular geometry of the cosmic web is one of the most dominant characteristics of the large scale matter distribution in our universe (see for an extensive review van de Weygaert 2002).

The final data release of the 2dFGRS comprises a set of 221 414 galaxies of which the redshift has been determined with a reasonable confidence. The even larger Sloan Digital Sky Survey (Stoughton et al. 2002), in which in total around a million redshifts are being measured, has been released up to the fourth data release (Adelman-McCarthy et al. 2006). It is the mere size of these new samples which is making it finally possible to perform a detailed analysis of the cosmic web and its elements beyond the second-order power spectrum or correlation function analysis on which conventional studies traditionally have focused (see e.g. Park et al. 1994, Lin et al. 1996, Percival et al. 2001, Dodelson et al. 2002, Tegmark et al. 2004, Cole et al. 2005). This task, however, represents a major challenge. An adequate description of the large scale galaxy distribution is not only complicated by its complex cellular geometry, but also by the very different physical and morphological properties of its constituent elements. For example, densities range from values of some tenths in units of the average density in void-like regions (e.g. Hoyle & Vogeley 2002, 2004, Sheth & van de Weygaert 2004, Colberg et al. 2005, Patiri et al. 2006) up to thousands of times the average density at the cores of rich clusters (e.g. Tyson et al. 1998, Rosati et al. 2002 and references therein). In addition, the hierarchical build-up of structure is responsible for another complication in the analysis of large scale structure. Individual objects often contain a lot of substructure and may themselves be embedded within a hierarchy of larger associations. An example is the Local Group, which consists of two smaller groups centered around the Milky Way and Andromeda. At the same time it is a member of the much larger Local Supercluster. Even voids are known to display substructure, with small voids being embedded within more extended voids (Regős & Geller 1991, Dubinski et al. 1993, van de Weygaert & van Kampen 1993, Szomoru et al. 1996, Sheth & van de Weygaert 2004). To complicate things even more, the structural elements of the large scale galaxy distribution have very different shapes. Whereas both voids and clusters are fairly roundish, walls and filaments appear highly anisotropic, having contracted in one (walls) or two (filaments) spatial directions.
Several attempts have been made to quantify and describe the properties of the cosmic web, amongst which are analyses based on minimal spanning tree techniques and percolation studies, for example by employing Minkowski functionals (e.g. Sahni et al. 1997, Sheth et al. 2003, Shandarin et al. 2004). A major downside of most of these methods is that in one or the other way they often make use of filtering aspects, usually a Gaussian smoothing on a typical scale of $5h^{-1}$ or $10h^{-1}$ Mpc. This is also the case for studies which seek to reconstruct the large scale galaxy density field, of which here we mention those based on Wiener filtering techniques (see e.g. Rybicki & Press 1992, Zaroubi et al. 1995, Erdogdu et al. 2004). These also pre-impose certain constraints on the underlying density field, e.g. a specific power spectrum.

In this paper we apply the Delaunay Tessellation Field Estimator (hereafter DTFE) to the 2dFGRS public data release. The DTFE is a method to reconstruct continuous density fields which are sampled by a discrete set of points. It is based on the Delaunay tessellation (Delone 1934) of the given discrete point set. This spatial and volume-covering division of space into mutually disjunct triangular (in two dimensions) or tetrahedral (in three dimensions) cells adapts to the local density and geometry of the point distribution. The DTFE exploits these virtues and adapts fully automatically to changes in the density and geometry of the distribution of sampling points. The DTFE involves an extension of the interpolation procedure described by Bernardeau & van de Weygaert (1996), who used Delaunay tessellations for the specific purpose of estimating the cosmic velocity divergence field. They showed the optimal performance of Delaunay tessellations compared to conventional interpolation methods and proved that the obtained field estimates are volume-weighted in stead of mass-weighted. The DTFE was developed for the purpose of reconstructing fully volume-covering and volume-weighted fields from a discrete set of field values, including the density field as defined by the point sample itself. Instead of involving user-defined filters which are based on artificial smoothing kernels, the main virtue of the DTFE is that it is intrinsically self-adaptive, because it involves filtering kernels which are defined by the local density and geometry of the distribution of sampling points.

The main goal of this chapter is to reconstruct the large scale galaxy density field in the 2dFGRS region. We present both projected two-dimensional surface density maps as well as three-dimensional density maps. Apart from the DTFE surface density field reconstructions at full resolution we also present the same maps after filtering with a Gaussian kernel at a resolution of $5h^{-1}$ Mpc for the purpose of serving as a comparison with conventional density field reconstructions. We also discuss several applications. Note that in this chapter we describe the reconstruction of density maps in redshift space. The transformation between redshift and real space involves a dynamical modelling which falls beyond the scope of this chapter.

This chapter is organized as follows. In section 7.2 we describe the 2dFGRS data set. In section 7.3 we explain the fundamentals of the DTFE, while in section 7.4 its main properties are discussed. In section 7.5 we present a surface map of the DTFE 2dFGRS galaxy density field without making any corrections for observational selection effects. The latter are described in section 7.6. In sections 7.7 and 7.8 we present DTFE reconstructed two-dimensional galaxy surface density and three-dimensional galaxy density maps, which have been corrected for observational selection effects. Finally, in section 7.9 we summarize and discuss our results and explore several of the possible applications of the DTFE reconstruction procedure in the context of studies of the large scale matter distribution.
Figure 7.1 — The 2dFGRS regions in Aitoff projection of RA and Dec with individual fields marked as small circles. Also shown are the lines of Galactic latitude $|b| = 0^\circ$, $30^\circ$ and $45^\circ$. Source: Colless et al. 2001, 2003.

Figure 7.2 — The 2dF galaxy redshift survey. The foamy geometry of the cosmic web is strikingly displayed. Courtesy: the 2dF Galaxy Redshift Survey team.
7.2 The data

The 2dFGRS is one of the major spectroscopic surveys in which the spectra of 245 591 objects have been obtained, with the scope of obtaining a representative picture of the large scale distribution of galaxies in the nearby universe (Colless et al. 2001, 2003). It is a magnitude-limited survey, with galaxies selected down to a limiting magnitude of $b_J \sim 19.45$ from the extended APM Galaxy Survey (Maddox 1990a, b, c). The galaxies were selected in three regions, which together cover an area of approximately 1500 squared degrees. These regions include two declination strips, each consisting of overlapping $2^\circ$ fields, as well as a number of ‘randomly’ distributed $2^\circ$ control fields. In Fig. 7.1 a map of the survey fields on the sky is shown. One strip (the SGP strip) is located in the southern Galactic hemisphere and covers about $80^\circ \times 15^\circ$ close to the South Galactic Pole ($21^h 40^m < \alpha < 03^h 40^m$, $-37.5^\circ < \delta < -22.5^\circ$). The other strip (the NGP strip) is located in the northern Galactic hemisphere and covers about $75^\circ \times 10^\circ$ ($09^h 50^m < \alpha < 14^h 50^m$, $-7.5^\circ < \delta < +2.5^\circ$). The redshifts of these galaxies have been measured using the 2dF multibre spectrograph on the Anglo-Australian Telescope, which is capable of simultaneously observing 400 objects in a $2^\circ$ diameter field. Reliable redshifts were obtained for 221 414 galaxies. These data have been made publically available in the form of the 2dFGRS final data release (available at http://msowww.anu.edu.au/2dFGRS/) and are plotted in Fig. 7.2.

7.3 DTFE: method

The DTFE is a procedure to reconstruct a continuous and volume-covering density field from a set of irregularly distributed points sampling this field. It has been extensively described in Chapter 3. Here we shortly describe the main workings and the most important properties of the DTFE. It consists of three main steps. These are illustrated in Fig. 7.3 for a set of points sampling a Gaussian peak. These three steps consist of:

Step 1: Construction of the Delaunay tessellation

The core of the DTFE is the Delaunay tessellation corresponding to the discrete point set. The first step is to construct this Delaunay tessellation for the point sample at hand (Fig. 7.3, top right-hand frame).

Step 2: Estimation of the density field at the locations of the sampling points

In the next step the density is estimated at the locations of the sampling points by making use of the observation that the area of the union of all Delaunay triangles surrounding a point, the contiguous Voronoi cell of that point, is inversely proportional to the local density of sampling points. This is illustrated in the bottom right-hand frame of Fig. 7.3. The contiguous Voronoi cells are shaded: one at the lower right-hand corner, in a low density region, with a relatively large area, and one near the center, in a high density region, with a relatively small area. The DTFE defines the density at the location of a sampling point $i$ as the inverse of the volume $V$ of its contiguous Voronoi cell $W_i$ times a normalization constant:

$$\rho(x_i) = \frac{(D + 1)m_i}{V(W_i)}.$$  \hspace{1cm} (7.1)

Here $m_i$ is the mass of sampling point $i$ and $D$ the dimension of space.
DTFE reconstruction procedure:

Figure 7.3 — Overview of the DTFE reconstruction procedure. Given a point distribution (top left), one constructs the corresponding Delaunay tessellation (top right), estimates the density at the position of the sampling points by taking the inverse of the area of their corresponding contiguous Voronoi cells (bottom right) and assumes that the density varies linearly within each Delaunay triangle, resulting in a volume-covering continuous density field (bottom left).

**Step 3: Interpolation of the density field to all other points in space**

In the final step the density field is defined at all other points in space by making use of the fact that the Delaunay triangles form the multi-dimensional equivalent of one-dimensional linear interpolation intervals. Given the \((D + 1)\) density estimates at the vertices of each Delaunay triangle, the value \(\bar{\rho}(x)\) at location \(x\) inside triangle \(m\) is defined through a straightforward linear interpolation:

\[
\bar{\rho}(x) = \bar{\rho}(x_i) + \left. \nabla \rho \right|_m \cdot (x - x_i),
\]

(7.2)

where \(x_i\) is the location of one of the Delaunay vertices of the triangle. It involves an evaluation of the (linear) density gradient \(\left. \nabla \rho \right|_m\) inside triangle \(m\), which follows rather straightforwardly from the set of density estimates \(\rho(x_i)\) at the locations of the vertices of Delaunay triangle \(m\). This step is illustrated in the bottom left-hand frame of Fig. 7.3.

Once a density field has been reconstructed it may subsequently be processed. Processing may involve various operations. These vary from relatively straightforward applications such as image reconstruction and smoothing to more complex and sophisticated applications such as feature detection and various statistical analyses.
7.4 DTFE: properties

The DTFE is a first-order reconstruction procedure, resulting in a linearly varying and continuous density field. The derivative of the reconstructed field is constant inside each Delaunay triangle, but discontinuous at their edges (Fig. 7.3, bottom left-hand frame).

7.4.1 Advantages

The DTFE has a number of properties which makes it ideally suited for the analysis of patterns in the large scale galaxy distribution. Firstly, it automatically adapts to the local density of the galaxy distribution under consideration. This finds its basis in the Delaunay tessellation of the corresponding galaxy distribution, which automatically adapts to the local density. This can be appreciated from Fig. 7.4. In the top frame the 2dFGRS galaxies are shown, while the subsequent frames zoom in on the corresponding Delaunay tessellation. The figure illustrates the adaptive nature of the Delaunay tessellation, which makes the DTFE better suited than conventional reconstruction procedures for resolving point distributions which are characterized by objects of widely varying densities (see Chapter 4).

Fig. 7.4 shows that DTFE does not only adapt to the density, but also to the geometry of the local galaxy distribution. Unlike most conventional reconstruction procedures the DTFE does not involve an intrinsically spherical smoothing kernel. This results in a much more accurate description of the real intrinsic anisotropies of reconstructed objects (see Chapter 4). A telling example of this property is present in the bottom right-hand frame of Fig. 7.4, where just to the right of the center a thin, anisotropic filamentary structure can be recognized, a finger of God.

A third virtue of the DTFE is its capability of an accurate description of low density regions without introducing the shot-noise effects by which conventional methods are beset. In Fig. 7.5 the DTFE reconstruction of a void-like region taken from the 2dFGRS survey is compared to several Triangular-Shaped Clouds (TSC) reconstructions at different resolutions. The TSC procedure is a conventional fixed grid-based reconstruction procedure in which the resolution has to be pre-specified by the user (e.g. Hockney & Eastwood 1981). In the top row of the figure the galaxy distribution and the DTFE reconstruction are shown, while in the bottom row three different TSC reconstructions at increasing resolution are shown. It is clear that at none of the resolutions all characteristics of the void are accurately described by TSC. At the lowest resolution the overall shape of the void is recovered, but the sharp edges visible in the galaxy distribution are smeared out into rather featureless blobs. To a lesser extent the same is true for the reconstruction at average resolution. Here, however, one starts to recognize the inner structure of the void breaking up into distinct density blobs, indicating that the grid resolution in the interior of the void is too high. This is in particular true for the high resolution reconstruction, in which the edges of the void seem to be described much better, but in which the interior of the void is dominated by shot-noise effects. The DTFE recovers the sharp edges of the void as such, while at the same time the interior of the void is reconstructed as a gently varying low density region. It is important to note that this void is just one of the many objects present in the large scale galaxy distribution. Any grid- or filter-based reconstruction method will be inappropriate for describing all but some of the relevant features.
Figure 7.4 — Illustration of the adaptive properties of the DTFE. Top frame: the galaxy distribution of the 2dFGRS SGP sample. The subsequent frames zoom in on the Delaunay triangulation of the region indicated by the solid squares.
7.4.2 Limitations

Apart from these advantages the DTFE also involves a number of limitations and disadvantages. The first is that due to the very high resolution of its effective smoothing kernel, the DTFE is very sensitive to sampling noise in the point distribution. Any variations in the local density due to sampling noise will have repercussions on the reconstructed density field. In Chapter 8 the effects of sampling noise and the significance of reconstructed features are discussed in detail. Note that an advantage of the DTFE is that the effects of sampling noise are local. A second disadvantage of the DTFE is that at the scale of the local smoothing kernel, which is as large as the local contiguous Voronoi cell, the reconstructed density field contains triangular artefacts. These artefacts are the imprints of the linear interpolation procedure within each Delaunay triangle. Obviously, this issue is not specific to the DTFE as any reconstruction scheme contains the imprint of its effective smoothing kernel at small scales. Another limitation of the DTFE is that it is not capable of reconstructing regions of density zero. Any region with a finite size corresponds to Delaunay triangles with a finite size and therefore a non-zero density. The final limitation of the DTFE we mention involves the boundary conditions. In the case of non-periodic boundary conditions the outermost triangles extend outwards to infinity, inducing a density of zero inside their interior. The density reconstruction is therefore not accurate for the outermost triangles.

7.5 Galaxy surface density field reconstructions

We have reconstructed the galaxy surface density field in redshift space corresponding to the 2dFGRS galaxy distribution. The projected galaxy distribution and the resulting DTFE re-
Figure 7.6 — The 2dFGRS galaxy distribution.
Figure 7.7 — The DTFE reconstructed galaxy surface density field.
constructed density field are shown in Figs. 7.6 and 7.7. In the galaxy distribution in Fig. 7.6 five regions are indicated, which are shown in more detail in Fig. 7.8. All density field reconstructions are DTFE reconstructions on the basis of the measured galaxy positions without correcting for any observational selection effects.

Fig. 7.7 shows the ability of the DTFE to reveal the strong contrasts in densities present in the large scale galaxy distribution at an automatically adapted resolution. The resolution of the DTFE is optimal in the sense that the smallest interpolation units it employs are also the smallest units defined by the data. At the same time the DTFE manages to bring out the fine structural detail of the intricate and often tenuous filamentary structures. Notice the sharp rendering of thin edges surrounding void-like regions.

To underline the capacity of the DTFE to dissect the internal structure of the various structural components of the cosmic web, in Fig. 7.8 we zoom in on several interesting regions. Region 1 focuses on the major mass concentration in the NGP region, the Sloan Great Wall (Gott et al. 2005). Various filamentary regions emanate from the high density core. In region 2 a void-like region is depicted. The DTFE renders the void as a low density area surrounded by various filamentary and wall-like features. Two fingers of God are visible in the upper right-hand corner of region 2, which show up as very sharply defined linear features. Many such features can be recognized in high density environments. It is interesting that the void is not reconstructed as a totally empty or featureless region. Substructures are present inside the void, which appears to contain a number of smaller ‘sub’-voids. This in accordance with current theories of the formation of voids (Dubinski et al. 1993, Sheth & van de Weygaert 2004). Region 3 is one of the most conspicuous structures in the 2dF field. This ‘cross’ consists of four tenuous filamentary structures emanating from a high density core located at the center of the region. Region 4 zooms in on some of the larger structures in the SGP region. In the bottom of this region part of the Pisces-Cetus supercluster is visible, while the concentration at the top of this region is the upper part of the Horologium-Reticulum supercluster. Finally, region 5 zooms in on the largest structure in the SGP region, the Sculptor supercluster (SCL9 in Einasto et al. 1997).

Even though the DTFE clearly offers a sharp image of the cosmic web, on the smallest scales (Fig. 7.8) the triangular imprint of the DTFE kernel is clearly visible. It is important to realize that any reconstruction procedure produces such artefacts whose shapes and scale are determined by the smoothing kernel. However, for most methods a smooth and spherically symmetric kernel is used, which tends to stand out less conspicuously.

A considerable amount of noise is visible in the reconstructions. This is a direct consequence of the high resolution of the DTFE reconstruction. Since no smoothing is applied, any noise present in the data will have a clear imprint on the reconstruction. Part of this noise is due to the statistical nature of the galaxy formation process. An extra source of noise is due to the fact that the galaxy positions have been projected on a two-dimensional plane before the density field reconstruction. This leads to additional Poisson noise as the DTFE algorithm connects galaxies which lie closely together in the projection, but may in reality be quite distant from each other. One way to circumvent this problem would be to reconstruct the three-dimensional galaxy density field and then do a two-dimensional projection, or by smoothing the field with a filter with a size equal to the local inter-galaxy separation.
Figure 7.8 — Selected regions in the 2dFGRS galaxy surface density field.
7.6 Selection effects

The 2dFGRS is subject to a number of selection effects. In order to obtain a uniformly defined galaxy density field, one needs to take into account a number of observational selection effects which are present in the 2dFGRS sample:

- Non-uniform sampling;
- Varying redshift completeness;
- Magnitude limit variations;
- Varying radial selection function.

The scope of this subsection is to discuss these issues and the applied correction procedures.

7.6.1 Non-uniform sampling

The first effect one needs to take into account that in the 2dFGRS the observed 2° fields are not uniformly distributed. In Fig. 7.9 these fields are plotted as a function of position on the sky for the SGP and NGP strips (see also Colless et al. 2001, 2003). At certain values of the right ascension more 2° fields at different declinations have been observed than at other locations and the width of the observed area on the sky is larger. This is for example the case in the NGP strip for $9^h 50^m < \alpha < 11^h 45^m$ and in the SGP strip for $21^h 50^m < \alpha < 23^h 25^m$. At other places fields are missing, for example around $(\alpha, \delta) = (24^h 0^m, -28^\circ)$ in the SGP strip. If not taken into account this will have a direct repercussion on the projected galaxy surface density field. Although in principle it is possible to correct for the extra galaxies by dividing by the amount of observed fields at fixed right ascension, this would not yield a fair reconstruction as different parts of the reconstructed surface density map would correspond to larger regions. Therefore we have selected several slices of the same thickness (2°) for the projected galaxy surface density field reconstructions.

Unfortunately it is not straightforward to correct for missing fields. These regions form a problem for the DTFE reconstruction procedure and should be excluded from the analysis. The reason for this is that in the DTFE procedure no a priori assumptions are made about the density field and an in principle arbitrary amount of galaxies could lie within a missing field. However, by imposing certain properties on the galaxy distribution, for example a particular power spectrum, one may fill in these blank regions. This can be done by using for example constrained field reconstruction techniques (Hoffman & Ribak 1991, Zaroubi et al. 1995, van de Weygaert & Bertschinger 1996). Such an analysis falls outside the scope of this chapter.

7.6.2 Varying redshift completeness

Another selection effect in the 2dFGRS concerns the fact that the fraction of redshifts which have been determined varies as a function of location on the sky. This redshift completeness, the local fraction of galaxies for which the redshift has been determined, is indicated in Fig. 7.9 by the color scaling (see Colless et al. 2001, 2003). The variations in redshift completeness occur for a number of reasons. Firstly, the redshift completeness is in general different for each observed 2° field, because of different observing conditions. Secondly, at many locations different fields overlap and contribute to the completeness. Finally, at some
locations the completeness equals zero. In such regions no galaxies have been observed, e.g. because of the presence of bright stars or due to satellite trails.

In order to obtain a uniformly defined galaxy surface density field it is important to correct for the variation of the redshift completeness and for the presence of holes where no galaxies have been observed. In the DTFE procedure the galaxy distribution is assumed to be a homogeneous Poisson point process of the underlying galaxy density field. If the point process is inhomogeneous Eqn. 7.1 is not applicable. Here it would underestimate the density by the inverse of the redshift completeness and we may therefore correct for the variations in redshift completeness by giving each galaxy a weight which is equal to the inverse of the redshift completeness. The resulting expression for the estimate of the density at the locations of the galaxies is then given by

$$\bar{\rho}(x_i) = \frac{(D + 1)m_i}{\psi(\alpha, \delta) V(W)}.$$  \hspace{1cm} (7.3)

In this expression \(\psi(\alpha, \delta)\) denotes the redshift completeness at location \((\alpha, \delta)\) and we have assumed that the sampling is uniform along redshift (see section 7.6.4). In the two-dimensional case of reconstructing the galaxy surface density field the redshift completeness \(\psi(\alpha, \delta)\) should be replaced by the average redshift completeness \(\psi(\alpha)\) at right ascension \(\alpha\). The latter is de-
fined as
\[ \psi(\alpha) \equiv \int d\delta \frac{\psi(\alpha, \delta)}{\int d\delta}. \] (7.4)

A complication occurs for locations at which the redshift completeness is very low or equal to zero. For these locations the above correction procedure may not be applied. Regions with redshift completeness zero will in fact be automatically excluded from the correction procedure as no galaxies have been observed at these locations. Since no data has been observed these locations should be treated in the same way as missing fields (see section 7.6.1). Because of the large uncertainty of the correction in regions with a very low redshift completeness, we have ignored regions with a redshift completeness smaller than 0.2 and set the density inside such regions equal to zero.

The correction for the varying redshift completeness comes at a price. Reconstructed structures inside regions with a lower redshift completeness have been sampled by a smaller amount of galaxies and therefore have a lower spatial resolution and a lower significance.

### 7.6.3 Magnitude limit variations

Another complication is that although the 2dFGRS was originally selected to have a uniform magnitude limit of $b_J = 19.45$, this limit in fact varies slightly with position on the sky (Colless et al. 2001). The reason for this is that the photometric calibrations on which the parent 2dFGRS catalogue has been based and the extinction corrections towards each galaxy have been improved since the beginning of the survey. In Colless et al. (2003) the magnitude limit is plotted as a function on the sky. In the NGP strip the magnitude limit is $b_J = 18.88$ at lowest, while in the SGP strip the magnitude limit is $b_J = 19.17$ at lowest. In order to get a proper magnitude-limited sample over these two strips, we selected only those galaxies with magnitudes smaller or equal to $b_J = 18.88$.

### 7.6.4 Varying radial selection function

The 2dFGRS is a magnitude-limited survey, which means that with increasing distance less and less galaxies are observed. This effect is clearly visible in both the galaxy distribution and in the DTFE galaxy surface density field reconstruction in Fig. 7.7. Here we describe how one may obtain a uniformly defined galaxy surface density field from a magnitude-limited sample.

One well-known way is to construct a volume-limited sample from the magnitude-limited 2dFGRS sample. Norberg et al. (2001, 2002) describe this procedure for the purpose of studying the dependence of galaxy clustering on luminosity and spectral type. However, as they point out, this procedure has the disadvantage that a large number of galaxies in the flux-limited sample does not satisfy the selection criteria for being included in the volume-limited sample. The reason for this is that the 2dFGRS sample has both a bright and a faint apparent magnitude limit. This means that galaxies are only included in the sample if their redshifts fall within a certain inclusion redshift range, which is dependent on their absolute magnitude. The latter means that a proper volume-limited sample can only be constructed in the redshift range where these inclusion ranges overlap. The major disadvantage of this approach is thus that one loses information and therefore structural resolution. Another disadvantage of this
approach is that the results are dependent on the assumed cosmological model, because the relation between apparent and absolute magnitude depends on cosmology.

Another approach is to model the radial selection function and to subsequently correct for it. Tegmark, Hamilton & Xu (2002) describe this procedure for determining the power spectrum of galaxies in the 2dFGRS 100k sample, as do Erdogdu et al. (2004) in a Wiener filter analysis of the full 2dFGRS. The advantage of this approach is that all observed galaxies can be used for the analysis over the whole observed redshift range. Also, the results are not dependent on the adopted cosmology. However, the disadvantage of this approach is that the results critically depend on the accuracy by which the radial selection function has been modeled. Also, one implicitly assumes that galaxies of different luminosities have the same spatial distribution, which in reality is not the case.

Because one is able to use all the observed data over the whole observed redshift range, the approach of our choice is to model the radial selection function and to subsequently correct the reconstructed field for it.

### 7.6.4.1 Modeling of the radial selection function

The radial selection function denotes the fraction of galaxies which is observable as a function of redshift per volume unit. It is determined by the galaxy luminosity function and the magnitude limit of the survey. The varying radial selection function implies that the observed galaxy sample is not a homogeneous but an inhomogeneous Poisson process of the underlying galaxy density field. Eqn. 7.1 is therefore not applicable. Using this equation would underestimate the density by a factor which is equal to the inverse of the radial selection function and we may therefore correct for it by giving each galaxy a weight which is equal to the inverse of the radial selection function. The resulting expression for the estimate of the density at the locations of the galaxies is therefore given by

\[
\hat{\rho}(x_i) = \frac{(D + 1)m_i}{f(z)V(W_i)} \quad (3D).
\]

Here \(f(z)\) denotes the radial selection function at redshift \(z\). When reconstructing the two-dimensional galaxy surface density field the radial selection function \(f(z)\) should be replaced by \(zf(z)\), where the extra geometrical factor \(z\) accounts for the linear increase of the thickness of the slice with redshift:

\[
\hat{\rho}(x_i) = \frac{(D + 1)m_i}{zf(z)V(W_i)} \quad (2D).
\]

The selection function should be normalized such that it gives the fraction of observed galaxies at a particular redshift. Here we have normalized the selection function such that it specifies the observed amount of galaxies with a luminosity larger than \(L_0\), which corresponds to the lowest magnitude limit \(b_J\) of the 2dFGRS (see section 7.6.3). It is given by

\[
f(z) = \frac{\int_{L(z)}^\infty \Phi(L) dL}{\int_{L_0}^\infty \Phi(L) dL},
\]

where \(\Phi(L)\) is the galaxy luminosity function. We have modelled \(\Phi\) by a Schechter galaxy luminosity function of the form

\[
\Phi(L) dL = \Phi^* \left( \frac{L}{L^*} \right)^\alpha \exp \left( -\frac{L}{L^*} \right) d\left( \frac{L}{L^*} \right),
\]

(7.8)
in which the parameters $\Phi^*$, $L^*$ and $\alpha$ have been measured by Norberg et al. (2002), who found $\alpha = -1.21$, $\Phi^* = 1.61 \cdot 10^{-2} h^3 \text{Mpc}^{-3}$ and $L^*$ following from $M^* - 5 \log_{10} h = -19.67$. These numbers yield a background density of galaxies of $n_b(L > L_0) = 1.39 \cdot 10^{-2} h^3 \text{Mpc}^{-3}$.

In Fig. 7.10 we have plotted the observed redshift distribution of the galaxies in the NGP and SGP strips (the thin solid line). In a redshift slice the amount $dN$ of galaxies in redshift bin $[z, z + dz]$ is related to the radial selection function by

$$dN \propto z^2 f(z) \, dz.$$  \hspace{1cm} (7.9)

The factor $z^2$ is a geometric factor, accounting for the linear increase of both the width and the thickness of the slice with redshift. This analytic approach with the selection function $f(z)$ given by Eqn. 7.7 is also plotted in Fig 7.10 and is indicated by the smooth solid curve. This plot shows that the observed and predicted selection functions are in good agreement.

### 7.6.5 Itinerary

Below we have summarized the reconstruction procedure including the corrections for the different selection effects:

1. Select the galaxies down to a uniform magnitude limit;
2. Construct the corresponding Delaunay tessellation;
3. Estimate the densities at the galaxy locations according to the DTFE procedure;
4. Correct for the redshift completeness;
5. Correct for the radial selection function;
6. Interpolate the density field according to the DTFE procedure.
7.7 2-D galaxy surface density field reconstructions

We have reconstructed the galaxy surface density field in the 2dFGRS NGP and SGP regions. The results are shown in Figs. 7.11 and 7.13. Both regions have been partitioned into several slices with a width in declination of 2°. In this way one may obtain an impression of the three-dimensional structure of the 2dFGRS regions. In Figs. 7.12 and 7.14 the same slices are shown after smoothing with a Gaussian filter with an FWHM of $5h^{-1}$ Mpc, which may serve as a comparison with conventional density field reconstructions. The magnitude of the galaxy surface density is indicated in the color bar, in which the corresponding galaxy surface density is denoted in units of the background galaxy surface density $\Sigma_b$.

A visual inspection of the slices shows that the slices appear quite different. This illustrates the dangers involved in analyzing two-dimensional projections such as shown in Fig. 7.7. To a lesser extent the same holds for the slices shown in this section. Nevertheless we can make several observations. Structures at high redshifts appear more extended and blurry than structures at low redshifts. This is due to three effects. The first is that the effective resolution is varying across the map. The DTFE employs an effective smoothing kernel whose resolution is set by the local density (and geometry) of the galaxy distribution. This ensures that the resolution is higher in high density regions in which most information is present. Note that because the sampling is not uniform this also implies that similar structures at different locations are not resolved at the same resolution. Secondly, at higher redshifts the slice is thicker, effectively involving a smoothing over a larger declination range. Thirdly, the dilution of the sampling at higher redshifts means that structures are inherently less resolved. The latter two effects are not unique to the DTFE as may be appreciated in the Gaussian smoothed images in Figs. 7.11 and 7.13.

It is interesting to see how coherent features are over the different slices. The first thing which may be noted is that structures appear most coherent at low redshifts ($z < 0.05$), which is where the slices are relatively thin. At high redshift where the slices are thicker the structures in each slice appear uncorrelated. Most voids do seem to be present in more than one or in all slices. This is in particularly true for the largest voids.

In the NGP strip the most prominent structure is the large concentration at a redshift of about 0.08 and RA ranging from $12^h 30^m$ to $13^h 30^m$. This structure corresponds to the Sloan Great Wall (Gott et al. 2005) and is most prominent in the slice with declination ranging from $\delta = -2^\circ$ to $\delta = 0^\circ$. In the slice with $\delta$ ranging from $0^\circ$ to $2^\circ$ it can also be clearly recognized. In this slice it visible that the Sloan Great Wall is indeed a huge structure, extending out to an RA of $10^h 30^m$. In the SGP strip the most prominent structure is located at a redshift of around 0.12 and an RA ranging from about $0^h$ to $1^h 30^m$, with the precise extent depending on the slice. This is the Sculptor Supercluster (SCL9 in Einasto et al. 1997).

7.8 3-D galaxy density field reconstructions

We have also reconstructed the full three-dimensional galaxy density field in the NGP and SGP regions of the 2dFGRS. The results are shown Figs. 7.15 and 7.16. These figures show a three-dimensional rendering of the NGP and SGP slices out to a redshift $z = 0.1$. To obtain a better impression of the three-dimensional structure inside these slices they are depicted at different viewing angles in Fig. 7.17. The iso-density contour in these figures corresponds to twice the average background galaxy density $n_b$. 
Figure 7.11 — The NGP 2dFGRS galaxy surface density field in several selected slices with a declination width of 2°. The color bar indicates the galaxy number density in units of the average background galaxy surface density (see text for a description).
Figure 7.12 — Same as Fig. 7.11, smoothed with a Gaussian filter with an FWHM of $5h^{-1}$ Mpc.
Figure 7.13 — The SGP 2dFGRS galaxy surface density field in several selected slices with a declination width of 2°. The color bar indicates the galaxy number density in units of the average background galaxy surface density (see text for a description).
Figure 7.14 — Same as Fig. 7.13, smoothed with a Gaussian filter with an FWHM of $5h^{-1}$ Mpc.
Figure 7.15 — The DTFE reconstructed NGP 2dFGRS galaxy density field. The iso-density contour shown corresponds to twice the background density. Several well-known structures are indicated.
Figure 7.16 — The DTFE reconstructed SGP 2dFGRS galaxy density field. The iso-density contour shown corresponds to twice the background density. Several well-known structures are indicated.
Figure 7.17 — The DTFE reconstructed NGP and SGP 2dFGRS galaxy density field at different viewing angles. The iso-density contour shown corresponds to twice the background density. The second row has to the same viewing angle as Figs. 7.15 and 7.16.
The rendering of the large scale galaxy distribution in Figs. 7.15 and 7.16 show that the DTFE is able of recovering the three-dimensional structure of the cosmic web as well as its individual elements. The NGP region is dominated by the large supercluster at a redshift of about 0.8, the Sloan Great Wall (Gott et al. 2005). The structure near the upper edge in Fig. 7.15 at a redshift of 0.05 to 0.06 is part of the upper edge of the Shapley supercluster. In the SGP region several known superclusters can be recognized as well. The supercluster in the center of this region is part of the Pisces-Cetus supercluster. The huge concentration at the top in Fig. 7.16 at a redshift of about 0.07 is the upper part of the enormous Horologium-Reticulum supercluster.

Although less obvious than for the two-dimensional reconstructions, the effective resolution of the three-dimensional reconstructions is also varying across the map. Here the interpretation of the reconstructed maps is further complicated by the fact that the DTFE is not able of handling completely empty regions.

7.9 Summary and discussion

In this chapter we have described the DTFE reconstruction of both the two-dimensional galaxy surface density field and the three-dimensional galaxy density field corresponding to the 2dF galaxy redshift survey. We have argued that the DTFE is well suited for the reconstruction of the large scale galaxy distribution. It is capable of accurately describing the key characteristics of the large scale galaxy distribution. The detailed renderings of the large scale galaxy distribution shown in Fig. 7.7 and in Figs. 7.15 and 7.16 show that the DTFE manages to recover the overall three-dimensional foam-like structure of the cosmic web as well as its individual elements and their intricate interdependencies. Telling examples are the filamentary structures in frame 3 of Fig. 7.8.

The galaxy density maps presented in this chapter show that the high spatial resolution of the DTFE makes it rather sensitive to sampling noise. At the smallest scales this sampling noise is visible as the triangular imprint of the linear DTFE interpolation procedure. The sensitivity of the DTFE to sampling noise makes it of crucial importance to be able to determine the statistical significance of reconstructed structures. This is described in Chapter 8. A possible way of reducing the amount of sampling noise is by processing the DTFE reconstructed density fields, e.g. by applying a filter. In principle extensions of the DTFE procedure to higher-order interpolation are possible as well. A particularly promising example is natural neighbor interpolation (Sibson 1981) which produces smooth fields. Such higher-order interpolation schemes have already been successfully implemented for two-dimensional problems in the fields of geophysics (Sambridge et al. 1995, Braun & Sambridge 1995) and solid state physics (Sukumar 1998). However, for various reasons the implementation of such an algorithm in the context of reconstructing density fields is not trivial.

An obvious next step would be to reconstruct the galaxy density field in real space in stead of in redshift space. The latter is distorted by the peculiar velocity field (Kaiser 1987, Hamilton 1998). On small scales the high gravitationally induced peculiar velocities of galaxies in clusters cause a smearing along the line of sight (fingers of God). These cluster motions are highly non-linear and the redshifts of cluster galaxies therefore cannot be straightforwardly mapped to real space locations. Instead one has to assume a dynamical model for the galaxy clusters and assign galaxy locations accordingly. On larger scales infall motions at large
distances from clusters or around filamentary and wall-like structures induce a compression along the line of sight. Conversely, the expansive motions inside voids have the opposite effect and cause a stretching along the line of sight. To be able to transform from redshift to real space the mass distribution responsible for these motions has to be modelled. However, a severe limitation of redshift surveys is that these flows may as well be produced by structures lying outside the survey volume and one needs to incorporate data from other, more extended all-sky surveys. Such an analysis falls beyond the scope of this chapter.

Once the transformation to real space has been done, the high resolution DTFE reconstruction of the large scale galaxy density field may be used in a variety of subsequent analyses. As yet, a physical description of the properties of the cosmic foam has mostly been in qualitative terms. The quantitative studies that do exist have usually been expressed in terms of statistical and ensemble averaged quantities. For a proper understanding of the large scale structure of our universe it is crucial to study both the large scale topology as well as the local structure of the cosmic web and its characteristic constituent elements. Such studies will not only provide insight into the properties and formation of the cosmic web but may also provide important clues to where and how galaxies have formed.

The DTFE is ideally suited for a systematic analysis of the properties of individual elements of the cosmic web. As it is not beset by artificial smoothing and automatically adapts to the local density and geometry of the galaxy distribution, it provides an optimal description of local structure, be they high density clusters, anisotropic filaments and walls, or low density voids. Figs. 7.5 and 7.8 form a telling illustration of this point. Indeed, Aragón-Calvo & van de Weygaert (2006) have devised an advanced algorithm, the Multiscale Morphology Filter, with the purpose of identifying clusters, filaments and walls in galaxy redshift surveys and cosmological $N$-body simulations. Another technique, the Watershed algorithm (Platen & van de Weygaert 2006), can identify void regions. Both techniques are based on the DTFE density field reconstruction.

A particularly prominent characteristic of the large scale structure matter distribution is its interconnectedness. One of the ways in which the topological properties of the large scale galaxy distribution can be studied is by using the percolation properties of the galaxy density field (see e.g. Zel’dovich 1982, Shandarin & Zel’dovich 1989, Sahni et al. 1997, Sheth et al. 2003, Shandarin et al. 2004, Pandey & Bharadwaj 2005). These percolation properties will depend on non-linear effects such as the large scale coherence or the local sub-clumping of structure into smaller entities. The percolation properties will also depend on the structure of the characteristic building blocks of the cosmic web and on how these are connected with the cosmic web. Studies of the percolation properties of the large scale galaxy distribution will therefore provide important information on the properties of the cosmic web.

Previous studies of the percolation properties of the large scale galaxy distribution have been hampered by the use of inappropriate reconstruction techniques with very poor numerical resolution. For example, in Shandarin et al. (2004) a fixed grid-based reconstruction technique has been used. This results in a density field in which most characteristic structures have been smeared out. The anisotropic filaments and walls have been reconstructed as low level, extended and roundish structures. Such inappropriate reconstruction techniques will lead to significant distortions of the percolation properties of the resulting reconstructed density field. Since the DTFE reconstruction technique does not make use of artificial filtering, its use may yield a significant improvement over conventional studies.
The results of this chapter show that the DTFE reconstruction procedure is capable of reconstructing the foam-like morphology of the large scale galaxy distribution. With the advent of ever larger and deeper redshift surveys, such as the Sloan Digital Sky Survey (Stoughton et al. 2002) and the 6dF Galaxy Survey (Jones et al. 2004), the DTFE promises to become an important tool for the analysis of the properties of the large scale galaxy distribution. Ultimately the use of such powerful tools will lead to a better understanding of the properties, formation and evolution of the cosmic web.

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